The Friction Factors of Unsteady Pipe Flows*

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In this study, the friction factors of unsteady pipe flows have been experimentally investigated. The normal temperature water was used as working fluid and pulsating flow superposed on the steady turblent flow, having the variation of velocity approximately represented by sine curve, was used.

In the result, the time mean friction factors of the unsteady flow agree with that of steady flow, and the momentary friction factors in the accelerating state are smaller and in the decelerating state larger than that in steady flow for each Reynolds numbers.

§ 1. Introduction

The general topic of periodic flow includes such as the exhaust of reciprocating pumps and engines, acoustical instruments, the surging phenomena in power plants and so forth.

When we concider the flow of fluid is classified two types, i. e. the laminar flow and the turblent flow. The type of actual flows in many cases is almost turblent flow.

In regard to the friction factors of the unsteady pipe flows, theoretical analysis has been establshed for the laminar flow[1)2)]. But for the turblent flow, it is difficult to develop the theoretical analysis, and there are few experimental reports[3)].

In this study, the influence of the sectional mean acceleration of fluid on the friction factors has been experimentally studied.

§ 2. Analytical Studies

The unsteady pipe flow of incompressible fluid through the straight circular pipe is considered.

The equation of motion is expressed by

$$\frac{\partial w}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial s}-\lambda\frac{w^2}{2d} \qquad (1)$$

where w is the sectional mean velocity, p is the pressure, ρ is the density of fluid, d is the inside diameter of the pipe, λ is the friction factor, s is the distance along the pipe and t is the time.

Integrating the equation (1) from $s=0$ to $s=l$, it becomes as follows.

$$\begin{aligned}\Delta p&=\rho l\frac{\partial w}{\partial t}+\lambda\frac{l}{d}\frac{\rho}{2}w^2\\&=\rho l\alpha+\Delta p_{ul}\end{aligned} \qquad (2)$$

where l is the distance between two pressure taps, Δp is the pressure difference, Δp_{ul} is the pressure loss caused by the friction on the inside wall and α is the sectional mean acceleration of the fluid.

Then the equation (2) is integrated from 0 to T with respect to t and divided by T.

$$\frac{1}{T}\int_0^T\Delta p dt-\frac{1}{T}\int_0^T\rho l\alpha\, dt=\frac{1}{T}\int_0^T\lambda\frac{l}{d}\frac{\rho}{2}w^2dt \qquad (3)$$

In these experiments, the variation of velocity is approximately represented by sine curve, and so

$$\frac{1}{T}\int_0^T\rho l\alpha\, dt=0$$

It is able to express the right side term of equation (3) as follows,

$$\frac{1}{T}\int_0^T\lambda\frac{l}{d}\frac{\rho}{2}w^2\, dt=\bar{\lambda}\frac{l}{d}\frac{\rho}{2}(\overline{w})^2$$

and ultimately, the equation (3) is reduced as follows,

$$\overline{\Delta p}=\bar{\lambda}\frac{l}{d}\frac{\rho}{2}(\overline{w})^2 \qquad (4)$$

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where $\bar{\lambda}$ is named the time mean friction factor, and $\bar{w}$ and $\overline{\Delta p}$ represent the time mean velocity and the time mean pressure difference or loss respectively.

If the value of the friction factor λ is considered as follows,

$$\lambda = \lambda_s + \lambda_t \tag{5}$$

equation (2) becomes as follows,

$$\Delta p - \rho l \alpha = \lambda_s \frac{l}{d} \frac{\rho}{2} w^2 + \lambda_t \frac{l}{d} \frac{\rho}{2} w^2 \tag{6}$$

The first term of right side of equation (6) means pressure loss by friction when the flow is regarded as steady flow, and the second term of the right side of equation (6) means the additional pressure loss accuring from the condition when the flow state is unsteady.

§ 3. Experimental Studies

The outline sketch of experimental apparatus is shown in Fig. 1. The water is pumped up to the head tank which locates 6.60 m high, and drawn through the test pipe maked of hard vinyle chloride, and returned to the reservoir tank. The test section is the horizontal straight pipe of circular section of internal diameter d=25 mm, and the inner wall is enough smooth.

The distance between two pressure taps is L=200 cm. Two by-pass pipes are connected between the test pipe and the cylinder of the apparatus for pulsating flow.

The apparatus for pulsating flow is constructed with two couples of the cylinder and piston, and each piston reciprocates symmetrically by the variable speed motor. The stroke of the piston can be changed stepwise so that it is possible to change the amplitude of the velocity variation.

The electromagnetic flow-meter (HOKUSHIN DENKI EL 281) is installed at the downstream of the test section so that the volume rate of flow is measured. The pressure transducers are set up at the pressure taps. The phosphor bronze diagram which is 0.5 mm in thickness and 40 mm in diameter is set in the transducers, and the four sheets of the strain guages are stuck on the diagram. The pressure is converted into electrical signal by the stsain meter (SANEI SOKKI 6L-1701).

The variation of rate of flow and the pressure difference are finally converted into electric signals and they are recorded on the chart of servo recorder (WATANABE SOKKI SR-501) with the marker signals.

§ 4. Results and Discussion

The experinental values of the friction factors λ_s of the steady state versus Reynolds numbers Re are shown in Fig. 2 and agree well with the equation of Blasius. Caluculating the equations (5) or (6), equation of Blassius, namely $\lambda_s = 0.3164\, Re^{-0.25}$, is taken as the values of λ_s.

Fig. 3 shows an example of the measurement

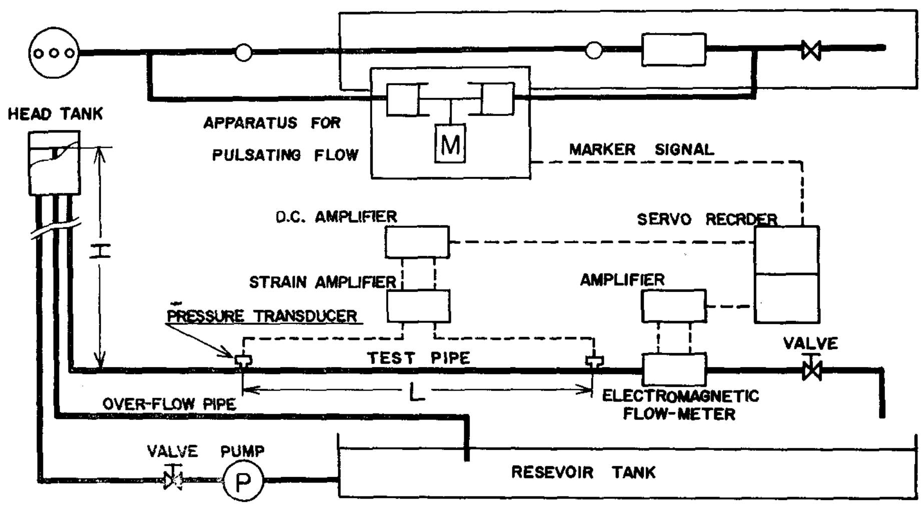

Fig. 1. Outline sketch of experimental apparatus

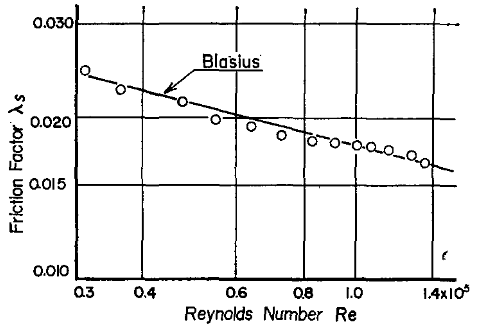

Fig. 2. Friction factors λ_s of the steady state versus Reynolds number *Re*

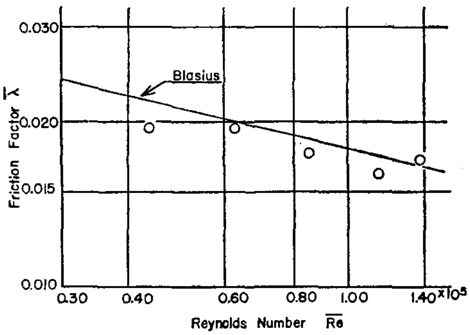

Fig. 6. Relation between $\bar{\lambda}$ and $\overline{Re}$ ($\omega=9.42$ rad/sec)

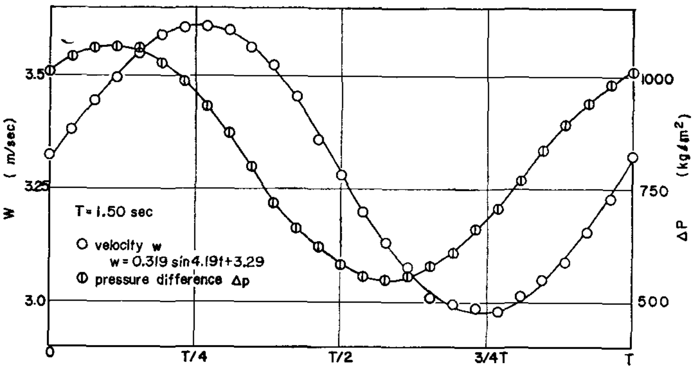

Fig. 3. Variation of velocity and pressure difference

on the velocity and pressure difference. The velocity versus the time are resembled to the form $w=A\sin\omega t+B$, and so the acceleration is as follows,

$$\alpha=\frac{\partial w}{\partial t}=A\omega\cos\omega t$$

Figs. 4, 5 and 6 show the time mean friction fractors $\bar{\lambda}$ versus the time mean Reynolds numbers $\overline{Re}$, which are defined as $\overline{Re}=\bar{w}d/\nu$.

These experiments are performed within the limits that is from $\overline{Re}=3\times10^4$ to 1.3×10^5 and the ratio of the variation amplitude of the velocity to the time mean average velocity, i.e. A/B, is from 0.035 to 0.75, and the values of $\bar{\lambda}$ agree approximately with λ_s for above range.

Fig. 7 shows the friction factor λ_t versus non-dimensional number $2d\alpha/w^2$.

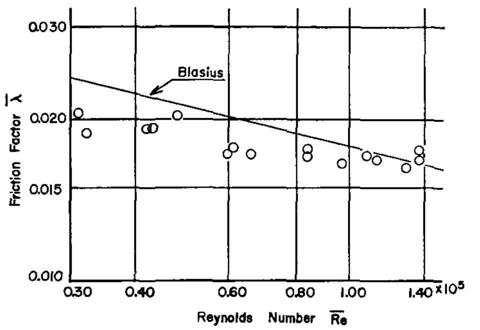

Fig. 4. Relation between $\bar{\lambda}$ and $\overline{Re}$ ($\omega=3.14$ rad/sec)

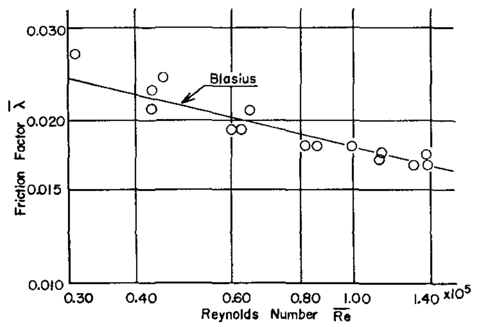

Fig. 5. Relation between $\bar{\lambda}$ and $\overline{Re}$ ($\omega=6.28$ rad/sec)

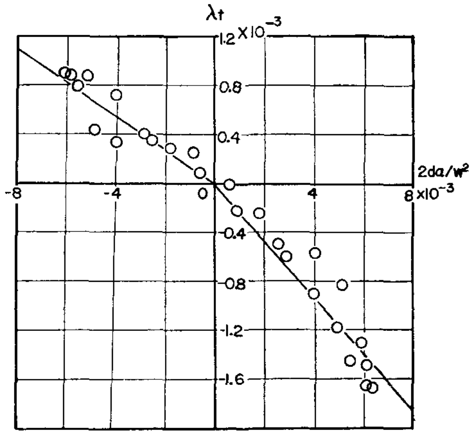

Fig. 7. Relation between λ_t and $2d\alpha/w^2$

The values of $\lambda_t/2d\alpha/w^2$ obtained from the results using a least squares method is -0.137 for $\alpha<0$ and is -0.230 for $\alpha>0$. From the facts described above, in the decelerating zone, the friction factor λ is larger than the friction facor λ_s, and in the accelersting zone that is smaller than λ_s.

References

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