

## *Application of Finite Element Analysis to Groundwater Flow in Field Problems*

Iichiro KONO\* and Makoto NISHIGAKI\*

(Received October 31, 1979) —

### Synopsis

The purposes of this paper are primarily to research on behavior of groundwater flow in saturated and unsaturated zone and to develop the most effective methods for solving groundwater flow problems related to civil engineering practice. The mathematical model provides a finite element solution to two- or three-dimensional problems involving transient flow in the saturated and unsaturated domains of nonhomogeneous, anisotropic porous media. Before progressing into the various levels of applications the input data and boundary conditions are discussed and evaluated. To demonstrate the flexibility of the finite element approach and its capability in treating complex situations which are often encountered in the field, the groundwater flow through sand bank at flood water levels and the flow through aquifer due to an excavation were analyzed. As the results there were good qualitative agreements between the numerical results and the informations received.

---

\*Department of Civil Engineering

## 1. Introduction

The validity and the accuracy of the saturated-unsaturated finite element method has been investigated by comparing with laboratory experimental results [1,2]. It can be concluded, with a sense of confidence, that the numerical methods can provide reliable basis for design analysis. In this paper, some application of models to field situation will be demonstrated. To simulate a practical flow problem in the field following data and conditions must be required.

- (1) Hydraulic properties of soils that constitute a flow domain.
- (2) Initial conditions in a flow domain.
- (3) Boundary conditions of a flow domain.

In simulating the model of laboratory experiment, these data and conditions are relatively easily obtained. In many practical situations, however, one may encounter these data and conditions that are impossible to be defined.

The purpose of this paper is twofold: before progressing into the various levels of applications, (1) to discuss and evaluate the above data and conditions; and (2) to describe two example applications for both two- and three-dimensional field problems.

## 2. Hydraulic Characteristics of Soil in Field

The decision as to whether it is necessary to include consideration of the unsaturated zone in the analysis of seepage through porous media involves a tradeoff between the possible additional accuracy and the definite additional complexity. The method requires an increased amount of input data in the form of the characteristic  $\psi$ - $\theta$ - $K$  curves which are strongly dependent on soil texture. The concept of soil characteristics that vary with moisture content is not common in soils engineering but is well established in the solution of irrigation and drainage problems in agricultural engineering. These data are not commonly collected, nor are they familiar to most civil engineers. The curves can be determined in the laboratory by the techniques that are well established [3]. Data for compacted soils, on the other hand, are almost nonexistent and undoubtedly the relationships are more complex. It is clear from compaction theory that saturated permeability is heavily influenced by soil density, compactive effort, and moisture content at compaction [4]. This conclusion undoubtedly holds for the unsaturated curves as well and results suffer from the usual suspicions as to their applicability to actual field sites.

It is encouraging to note that research is proceeding in the soil physics field in developing direct field measurement techniques. Because of the paucity of data on the unsaturated properties of compacted soils, it is difficult to vouch for the suitability of the soil properties, especially in cases of similitude extrapolation. Rather, the emphasis has been on examining the possible implications of the complete analysis.

A number of methods are now available for measuring the unsaturated hydraulic conductivity function of soil profiles in situ. The purpose of this section is to survey the various methods available for the measurement of hydraulic conductivity and water retention characteristics and to identify the principles advantages and disadvantages of each. There are mainly two kinds of method to estimate the hydraulic characteristics of unsaturated soil in situ:

- (1) Direct measurement of the hydraulic conductivity function.
- (2) Calculation of conductivity from water retention data.

In the former, many laboratory methods have been applied in the field [5]. It is generally much more difficult to set steady flow regimes in the field than in the laboratory. Infiltration techniques have been proposed based on a steady application rate by sprinkling or based on ponding infiltration through an impeding crust [6-8]. In unsteady-state methods, the "instantaneous profile" techniques seem to offer the best possibility for hydraulic characterization of field soils. The theory does not assume uniformity of the hydraulic properties of the flow system, and the boundary conditions do not need to be constant, or known in detail. Because in this method a diffusivity in an internally draining profile is measured. Several variations in the method of experimental procedure have been employed. The water content distributions were measured by using neutron or gamma ray or gravimetric sampling [9-15]. The pressure head distributions were measured by using tensiometers of mercury-water or pressure transducers [9-16]. In these techniques reported in the literature, Hillel's method was made to give the most detailed description of a simplified procedure for determining the intrinsic hydraulic properties of a complete soil profile in situ [11]. These methods have proven the feasibility of determining the unsaturated hydraulic conductivity function of soils in the field. However, these methods have complexity in treating the experimental apparatus.

In the latter, there has been considerable interest in the possibility of calculating the conductivity from other properties of the medium that may be easier to measure.

There are many publications that deal with the relationship of conductivity to various aspects of pore space geometry or water retention data. These methods were also summarized and evaluated in the literature and further works were recently proposed [5,17-20]. Though these methods give the merit to calculate hydraulic conductivity from water retention data, water retention data must be obtained on soil cores taken to the laboratory. It should be understood that no field sampling technique yet available provides truly undisturbed samples. Therefore this method suffers from above limitation.

By the way, all of the methods that are reviewed above have been applied for only the hydraulic characteristics of surface layered soils above the depth of three or four meters. This reason is that these methods have been established on the sense of agricultural engineering to solve the problems involving irrigation, drainage, water conservation, nutrient transport and runoff pollution, as well as infiltration. These methods can be also applied to determine the hydraulic properties of center core of rock-fill dam or to check the quality of a constitutive soil of a bank. To solve problems of groundwater recharge and discharge due to pumping or excavation, it is necessary to determine the unsaturated hydraulic properties of the soil texture which is layered above and below the water table as shown in Fig.1 with hatching. Unfortunately, there is no technique to obtain the properties so far as authors know. It may be believed that the case of accomplishing determination of hydraulic properties of soils in situ will improve with additional field experience and improvements in equipment and instrumentation in the near future.

The values of hydraulic conductivity ( $K_g$ ) and storage coefficient ( $S$ ) in the saturated region can be determined by using the techniques of drawdown test analyses. The value of specific yield ( $S_y$ ) can be considered the equivalent value of effective porosity ( $n_e$ ).

### 3. Flow through Sand Bank at Flood Water Level

#### 3.1 Introduction

In July 1972 and September 1976 the basin of the River Ohta in Hiroshima Prefecture suffered damage from leakage or piping water when the water level of the river was raised due to localized torrential downpour. Acting on information received the damaged district in landside was mainly shown in Fig.2 with the shaded region and the

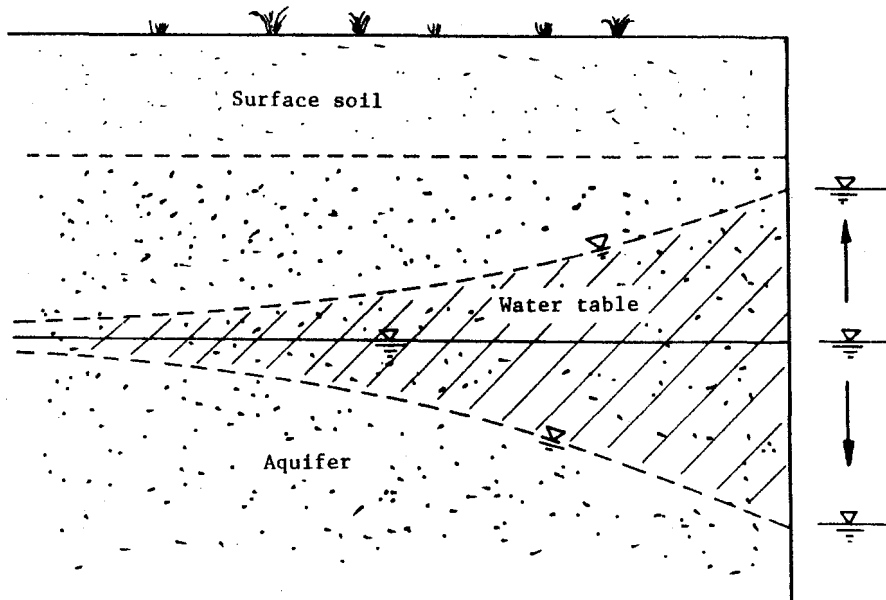


Fig.1 Schematic figure of underground

leakage and piping were happened after two or three hours from the peak of river water level. These phenomena can be considered symptoms of a disaster of embankment failure due to flood, then it is necessary to work out a countermeasure for leak prevention. The purposes of this section are to simulate the flow pattern in the embankment when the river water level is raised to the height of July 1972, September 1976 and high water level (HWL), and to evaluate the effectiveness of bank protection. To ascertain geologic and groundwater conditions six test drillings were driven until 20m depth. Four test holes of this six holes also were used as observation wells for measuring water levels and for conducting drawdown tests. The position of test holes are shown in Fig.2. Fig.3 shows the geologic condition estimated from well logs constructed from drilling samples.

Saturated hydraulic conductivities of the various soil layers were measured in the field by the auger hole method (USBR Method E-18). The mean hydraulic conductivities of each layer are also shown in Fig.3. As mentioned earlier the complex multilayer systems that one can encounter in the field can not be certainly handled analytically. On the other hand, the numerical procedures embodied in the finite

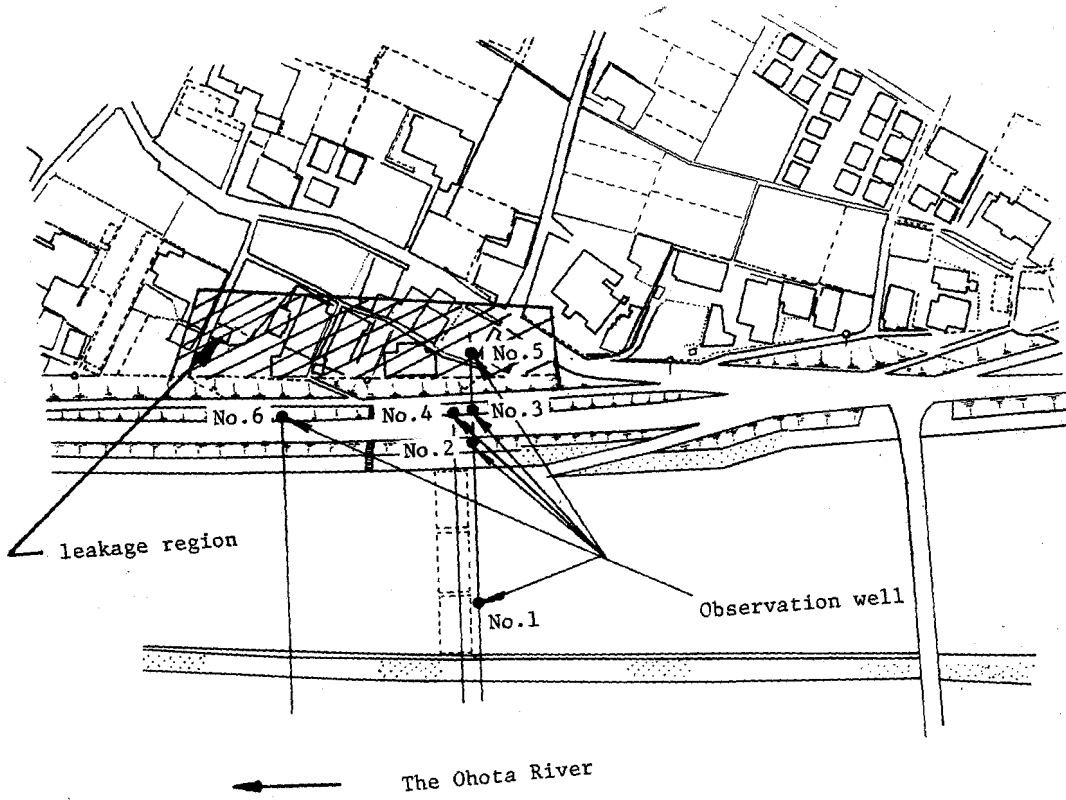


Fig.2 Plane view of damaged district

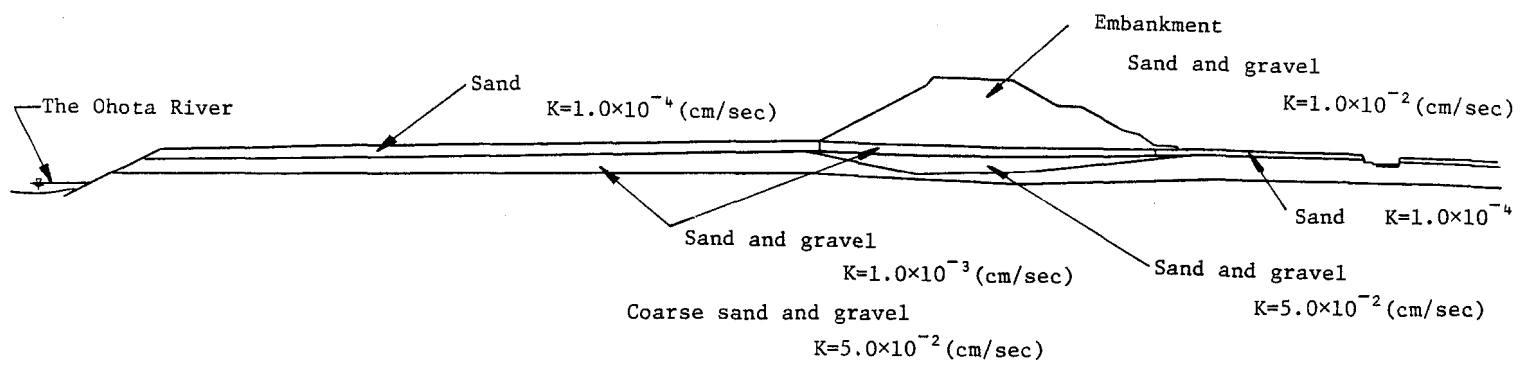


Fig.3 Hydrogeological cross-section

element method provide a practical means of analyzing complex systems. Therefore the change of flow pattern in the embankment due to flood water level and the other environmental effects were investigated by the saturated-unsaturated finite element method.

### 3.2 Selection of Boundary Condition

In many practical situations, one may encounter geometries and boundary conditions that cannot be defined. For instance, in the case of flow in or out of riverbanks, tidal beaches, and extensive aquifers, one has to deal with infinite extents of the media. It is then necessary to include only significant finite zones in an analysis, and one has to make proper assumptions concerning potential and flow conditions on the discretized boundaries. Proper choice of these conditions will depend upon the geological properties and conditions of groundwater flow and will require engineering judgement. Some criteria were proposed to determine extents of discretized zones for free surface flow through earth banks [21]. It was found that if the end boundary is placed beyond a distance of about  $8H$  to  $12H$ , measured from the final point of drawdown (Fig.4), the behavior of the free surface near the sloping face of the bank will not be influenced significantly. The assumption of an "impervious" base in an infinite medium will be approximately valid if the bottom boundary is placed beyond a distance of about  $3H$  to  $6H$  (Fig.4).

Three possible boundary conditions were assumed to occur at the discretized end boundary, namely impervious, constant level, and equipotential as in Fig.5. Both the impervious and constant-level conditions yielded about the same results, which compared well with observations, whereas the equipotential condition gave results that differed from the other two and from the observations [21]. Hence, it was concluded that for long homogeneous banks, the boundary condition at large distances can be assumed to be impervious or constant level. In this work, the boundary as shown in Fig.6 was applied to simulate this boundary problem. Fig.6 also shows the finite element mesh. The aquifer has been divided into 211 elements with a total of 240 nodes, the grid being denser in embankment than at the outskirts.



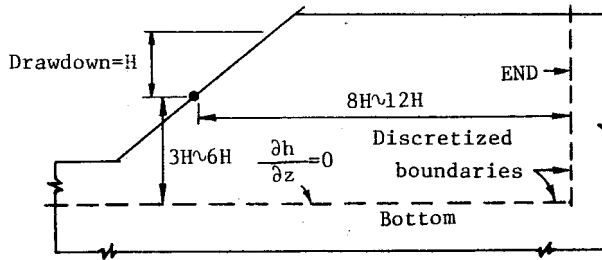


Fig.4 Discretized end and bottom boundaries (after Desai)[21]

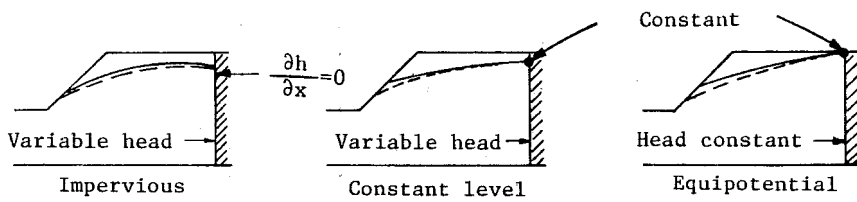


Fig.5 Different boundary conditions (after Desai) [21]

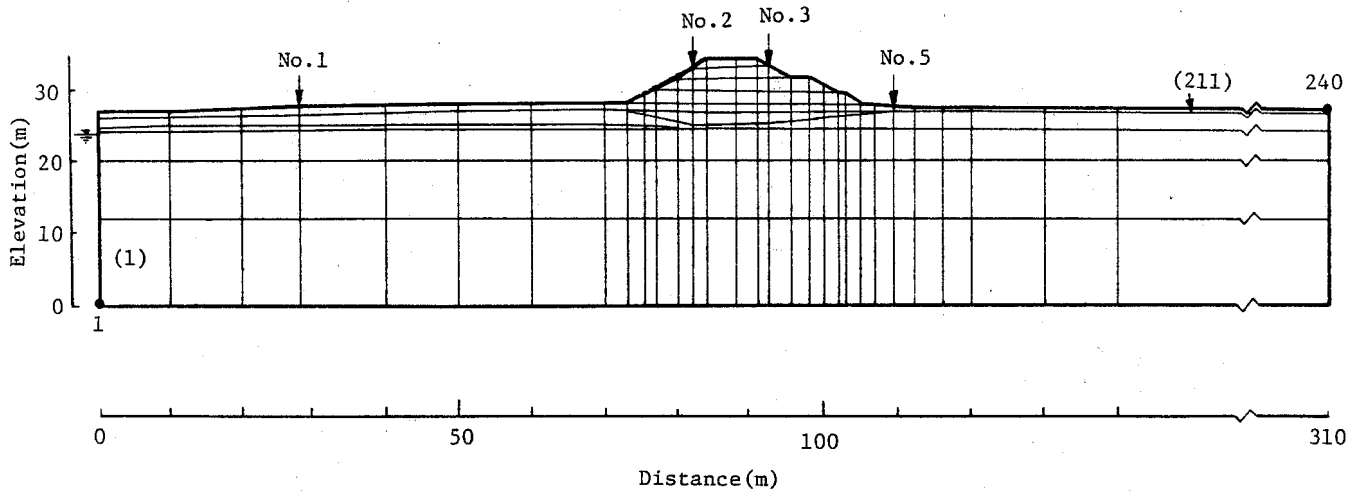


Fig.6 Finite element network of field problem

### 3.3 Determination of Hydraulic Properties in Unsaturated Region

The water level in the coarse sand and gravel layer is about G.L. -3.5m and the aquifer of coarse sand and gravel layer is revealed an unconfined aquifer. There is a need to determine unsaturated hydraulic properties of these region to simulate the flow through these region. Unfortunately these properties were not reported, so some other method must be taken to obtain these properties. A numerical method was used to investigate in detail certain physical aspects [22]. Using this numerical method, it can be shown that if the saturated conductivity were accurately determined, slight changes in the shape of the rest of the conductivity-water content relationship will cause small changes in the calculated discharge-time curves. Thus a computer method of using an accurately determined value of several shapes for the rest of conductivity-water content relationships to calculate several discharge-time is proposed. These calculated curves could then be compared with the experimentally observed discharge-time curve to select the appropriate conductivity-water content relationship. In this problem, discharge-time curve could not be obtained, so the groundwater table - time curve which was the results of measuring the groundwater fluctuations at the observation well No.1, No.2, No.3 and No.5 (as shown in Fig.6) were used to compare with the calculated curves. Fig.7 shows the observation results of water level charges of river and water table changes for each observation well with time for the period from December 9 to December 13, 1977. In Fig.7 the change of the river level in a day might be due to the variation of flood from the dam for water power plant.

The hydraulic property of unsaturated flow domain which was determined with trial and error method by the numerical approach is shown in Fig.8 and Fig.9 compares the computed water table variation with the measured data for each observation well. A period, 12 o'clock to 21 o'clock on December 10, was selected as the comparison period. In computation the river level change during this period was used as boundary condition at the river. There is a reasonably good agreement between the computed and the measured data during the first 6 hours. During the last 3 hours the agreement is somewhat less satisfactory. Here the measured data are lower than those indicated by the computed results. At least part of this discrepancy may be due to the adoption of a single soil moisture retention curve and a single hydraulic conductivity curve for the entire soil profile and the effect of hysteresis.

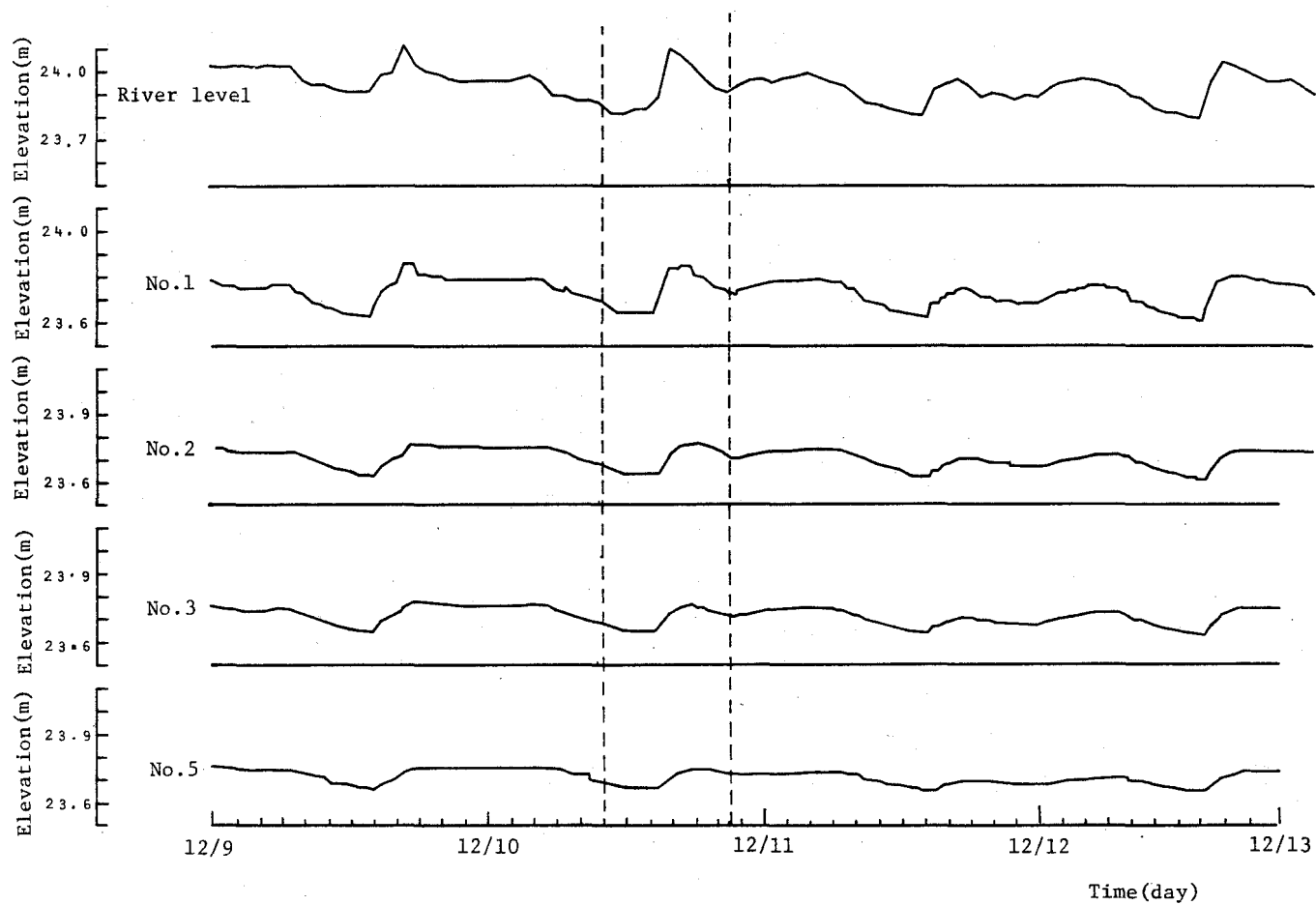


Fig.7 Observation results of water level of river and water table for each observation well with time

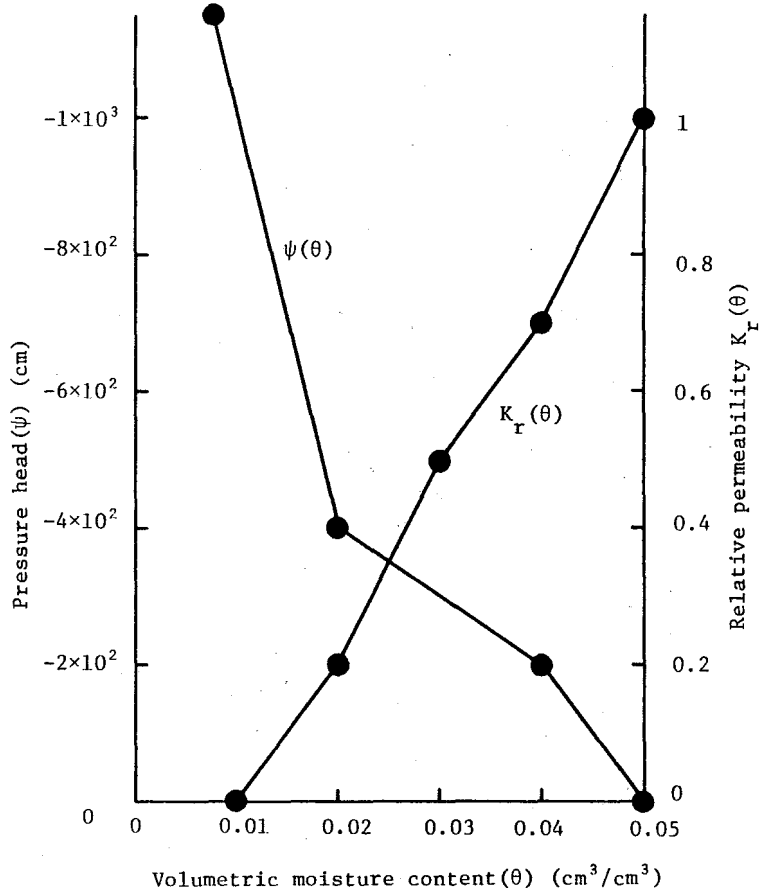


Fig.8 Unsaturated property of soil

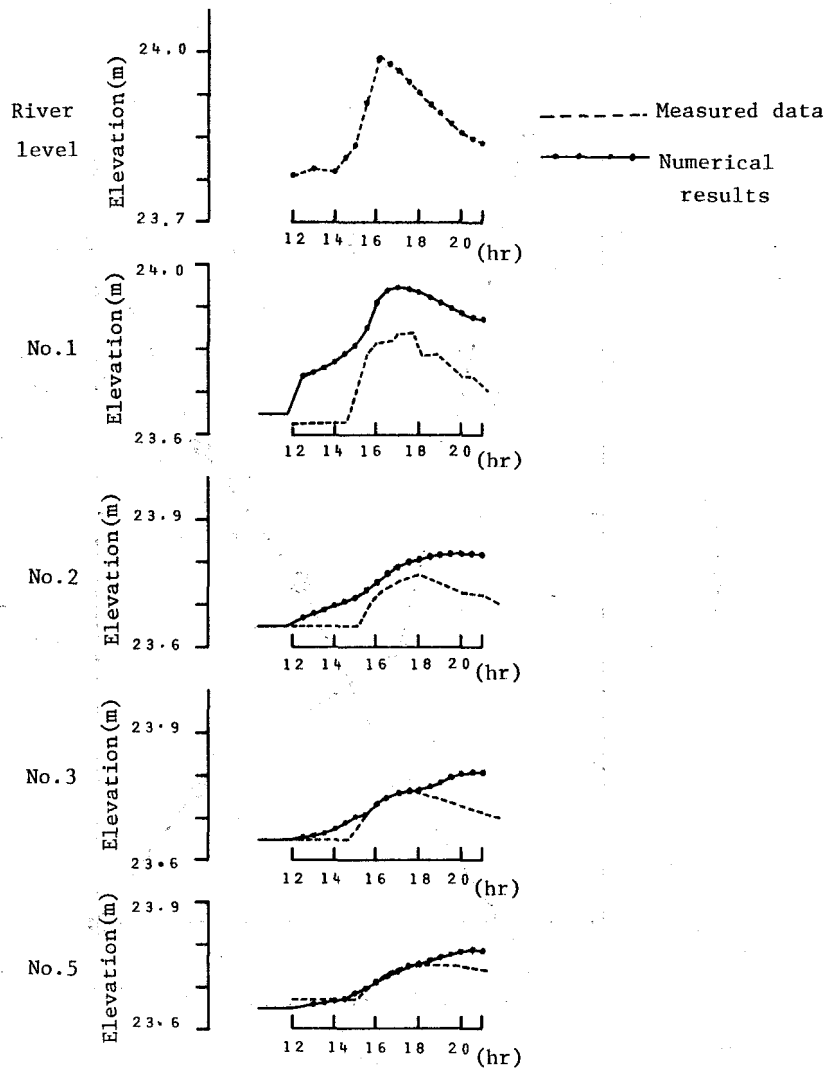


Fig.9 Comparison of measured and computed water table

This general agreement is certainly good enough to simulate the river level raises suddenly to the flood water level and water begins to flow through the embankment by using the relationships as shown in Fig.8.

#### 3.4 Simulation of Earth Embankment Subject to Sudden Raise in a River Level

As simulated, the following four cases were calculated with the saturated-unsaturated finite element analysis. The solution advances in time by means of a fully implicit finite difference scheme. Hysteresis was not taken into account.

- Case 1. At time  $t$  equal to zero the water level of the river is suddenly raised to the height of 29.630m from the bottom of of flow domain as shown in Fig.6. This value of the river level is adopted the highest value of the flood it occurred in July 1972 as shown in Fig.10.
- Case 2. The river level is 29.167m from the bottom of flow domain (Simulation of September 1976 flood).
- Case 3. Simulation of the flow through the embankment when the river level reaches the high water level of 33.451m.
- Case 4. To evaluate the efficiency of bank protection the hydraulic conductivity of this protection is chosen  $K=1.0 \times 10^{-6}$  cm/sec. The river level is in H.W.L.

In Figs.11 through 14, a series of numerical solutions for unsteady state seepage are presented for various river water levels, accounting the effect of bank protection. It is worth noting in the simulation of 1972 flood that after three hours, seepage face appears at the toe of embankment. This result well agrees with the information received. In the simulation of 1976 flood level seepage face also appears after about 4 hours. These simulation can be considered the most dangerous situations for the practical flow problems. The comparison of Fig.13 and Fig.14 is the most interesting result of the numerical analysis. Due to the effect of bank protection the water table profile in Fig.14 is extremely different from that in Fig.13. The computed results is shown for the water flow out of the embankment (Fig.15). It is evident with comparing case 4 and case 3 that if bank protection is worked out as a countermeasure for leak prevention outflow rate from seepage face is reduced to about one-third of that of case 3, and the time lag of seepage face appearing is three times

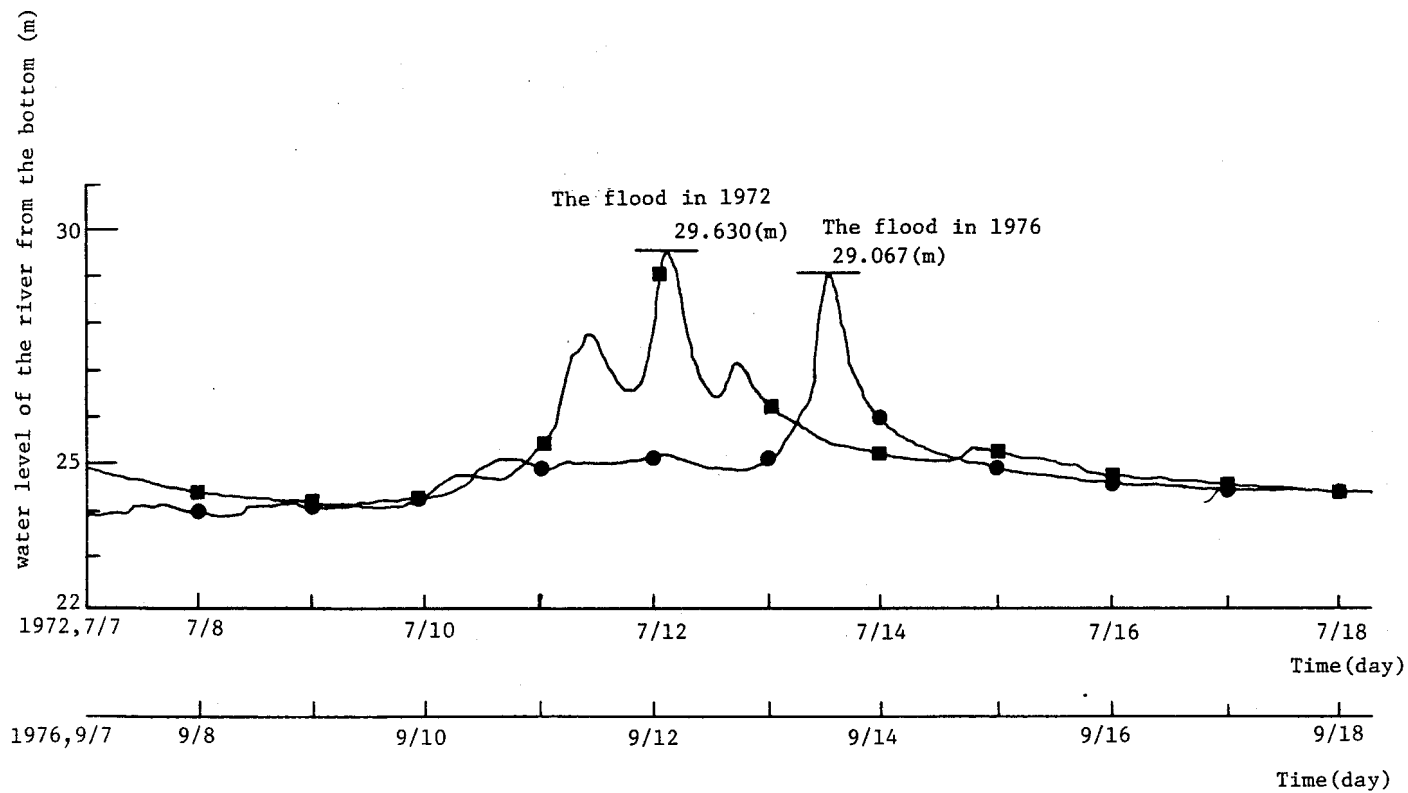


Fig.10 The change of the river level during the flood in July 1972 and September 1976.



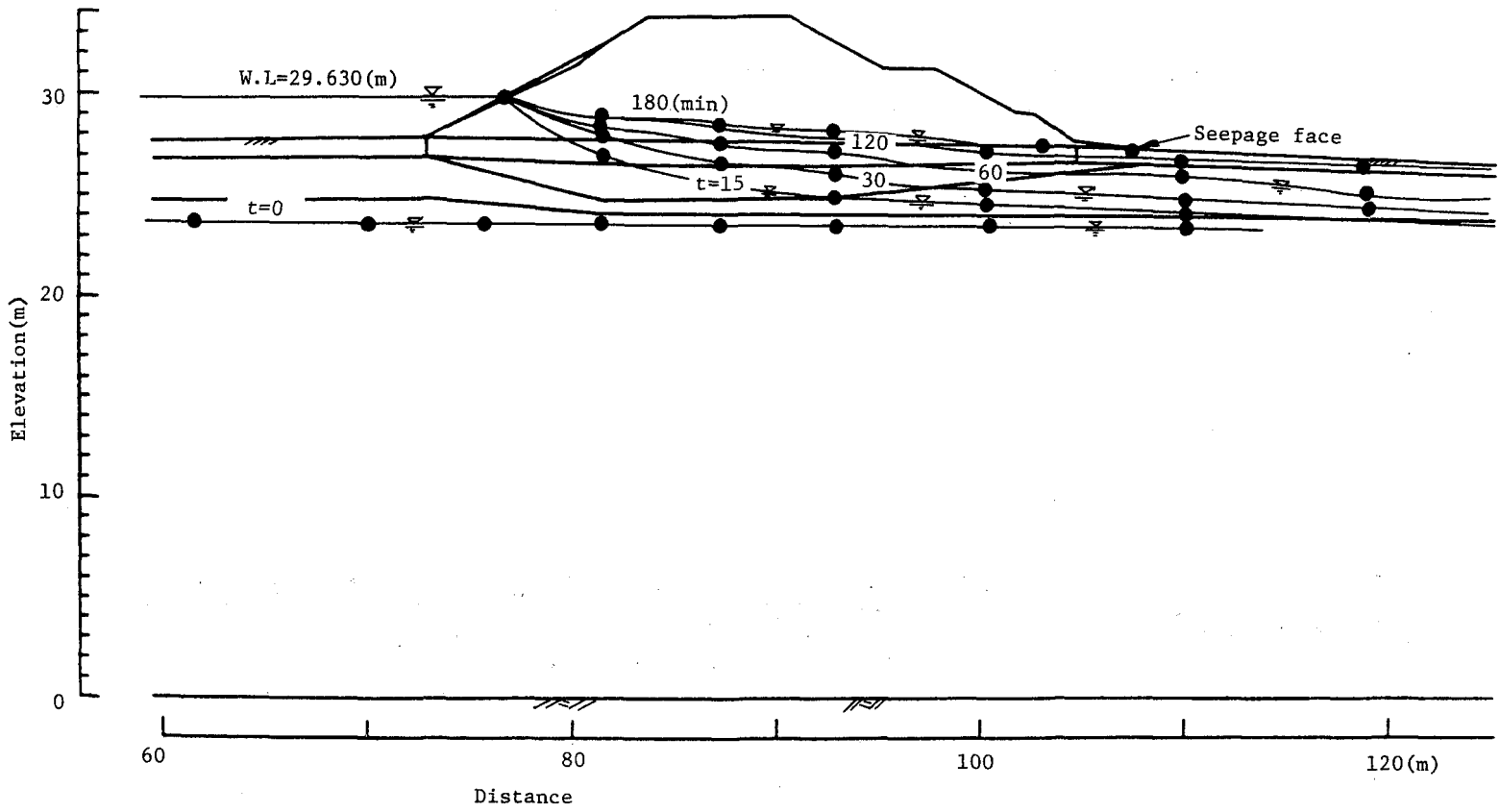


Fig.11 The numerical results of flow problem at the July 1972 flood level water

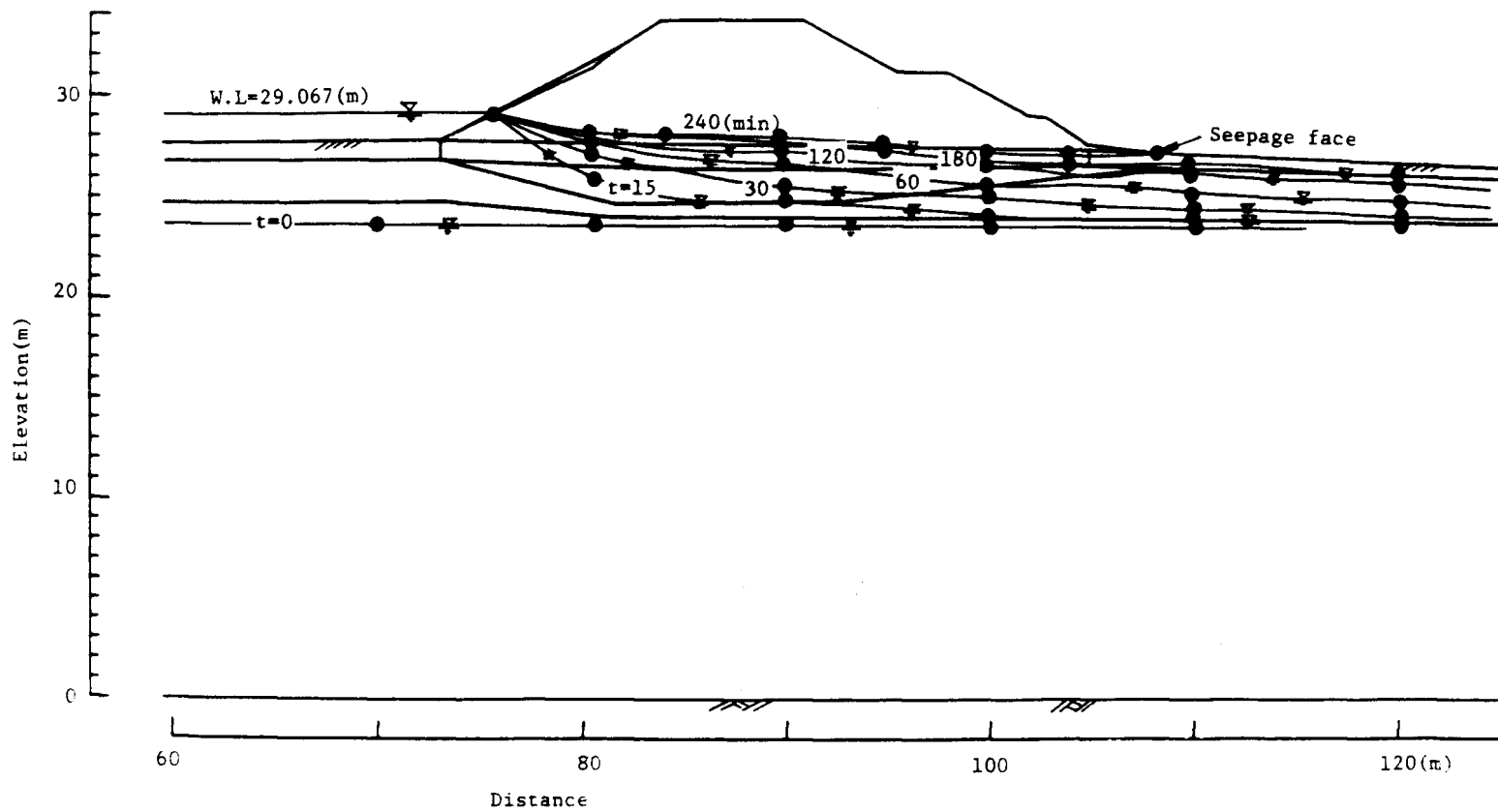


Fig.12 The numerical results of flow problem at the September 1976 flood water level

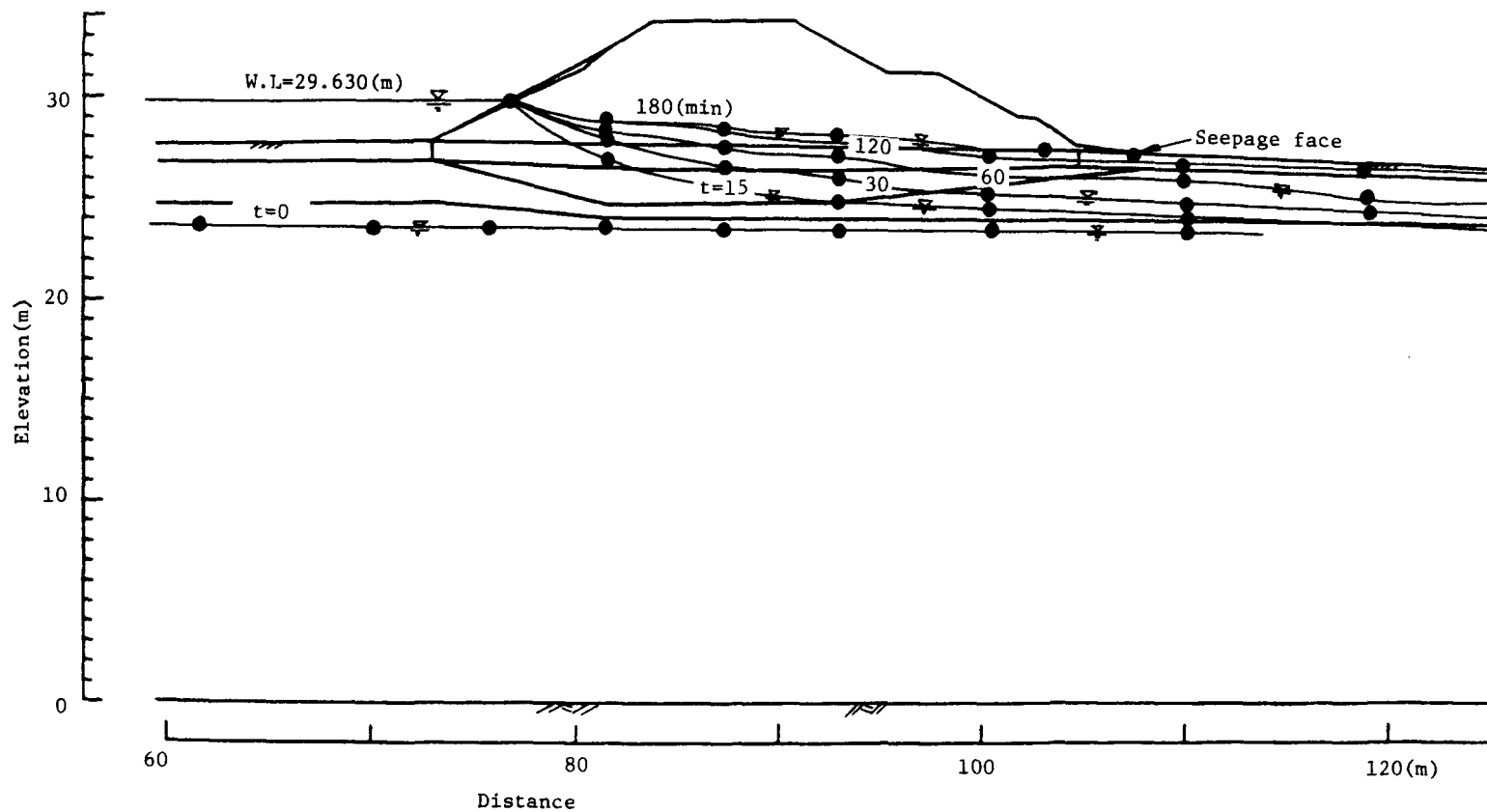


Fig.11 The numerical results of flow problem at the July 1972 flood level water

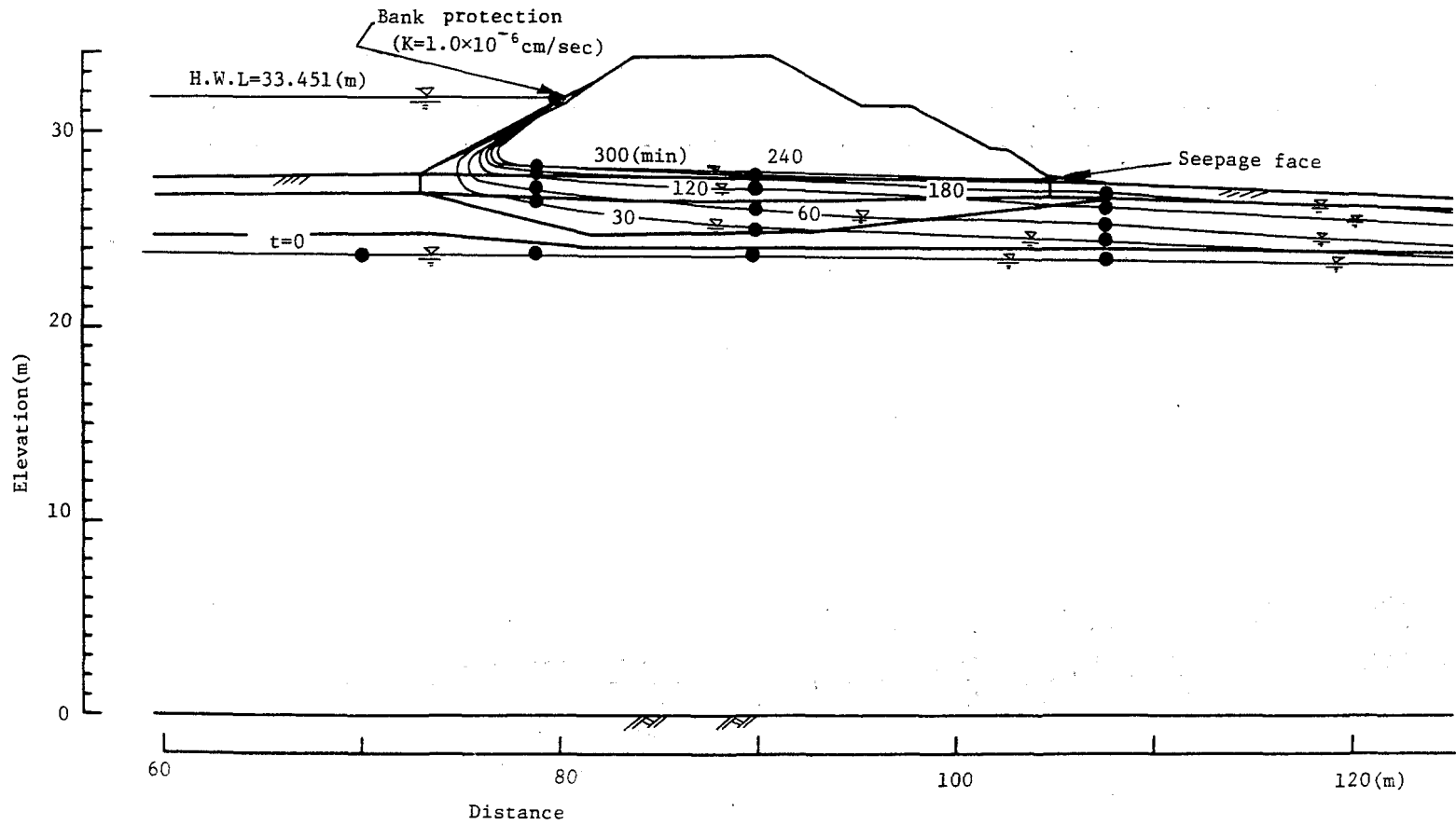


Fig.14 The numerical results of flow problem at H.W.L. taking account of the effect of bank protection.

longer than that in the case without bank protection.

To summarize the results of this section, it would appear that this finite element analysis can be adapted to solve complicated practical problems involving soil stratification and variations in soil hydraulic conductivity and this method is very effective to detect the realistic change of the flow pattern.

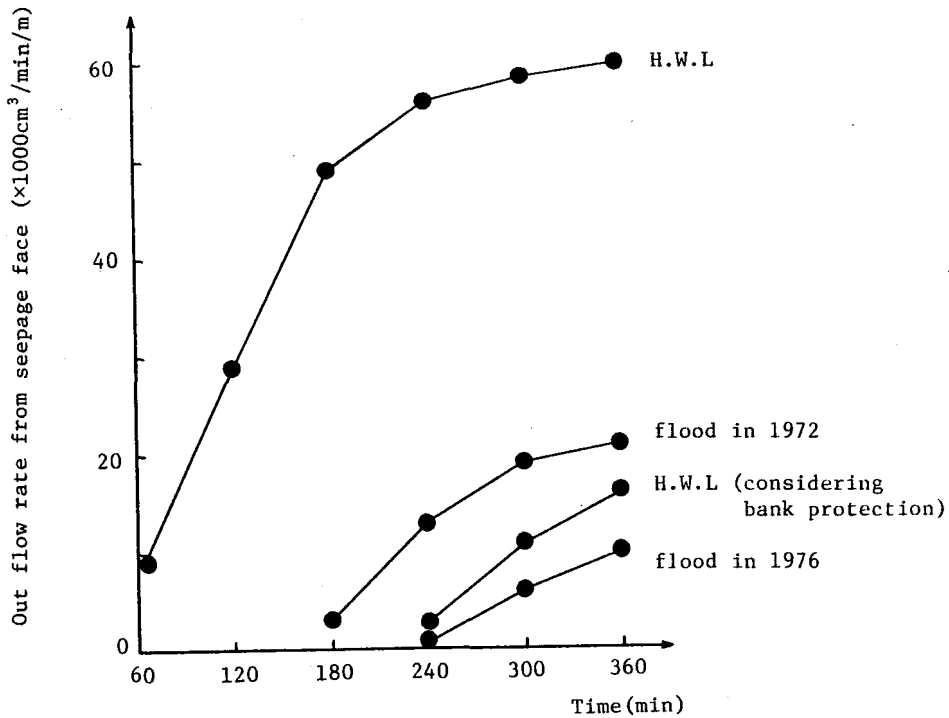


Fig.15 Out flow rate from seepage face with time

#### 4. Open Cut Excavation Model

##### 4.1 Introduction

The problem of groundwater controlling for foundation excavations will be considered as practical problem which is some interest to the foundation engineer. Many types of engineering construction require the excavation of soil and rock below the natural groundwater table. If the formations are well cemented, water control may be simply a matter of allowing the water to seep down the excavation slopes into shallow ditches or sumps from which it is removed by pumping. On the other hand, if the water bearing materials have low strengths, extensive dewatering systems may be required.

Either of the two fundamental methods of controlling seepage can be used for the control of groundwater during construction: (1) Those that keep the water out or (2) those that depend on its control by drainage processes. Chemical grout, cement grout, sheet pile walls, and caissons are means that serve to keep out most of the water. Usually when these methods are used, pumps are required to maintain dry conditions in excavations. Most excavations in water bearing formations such as gravels, sands, silts, and stratified clays are stabilized by wellpoints, deep pumped wells, or other groundwater control systems.

Groundwater control for foundation excavations may be accomplished in a number of different ways. The most appropriate method for a given job should be determined by adequate soil surveys and test borings to delineate important soil strata and locate sources of water. On important soil strata and permeability of the formations should be determined by field drawdown tests or other adequate methods. For any dewatering project in which failures could lead to extensive structural damage or serious flooding the design and installation of water control systems should be carried out with deep considerations.

Most dewatering systems are flexible with respect to discharge capacity and can be enlarged in capacity to take care of unexpectedly large rates of flow. Nevertheless, the approximate rate of discharge should be known in advance so that approximate power requirements will be known. The design of dewatering systems involves two important steps

1. Evaluation of the magnitude of the dewatering project, including an estimate of the probable rate of inflow and power consumption.

2. Design of a system capable of providing the required ground water lowering for the length of time needed for the construction that is to be carried out below the natural groundwater level. To estimate the probable inflow rates to dewatered excavations and to provide the ground water lowering seepage systems must be analyzed. All fluid systems must necessarily extend in three dimensions, but in former methods seepage systems analyzed are predominately two-dimensional flow with assumption of the infinite length of excavation as shown in Fig.16.

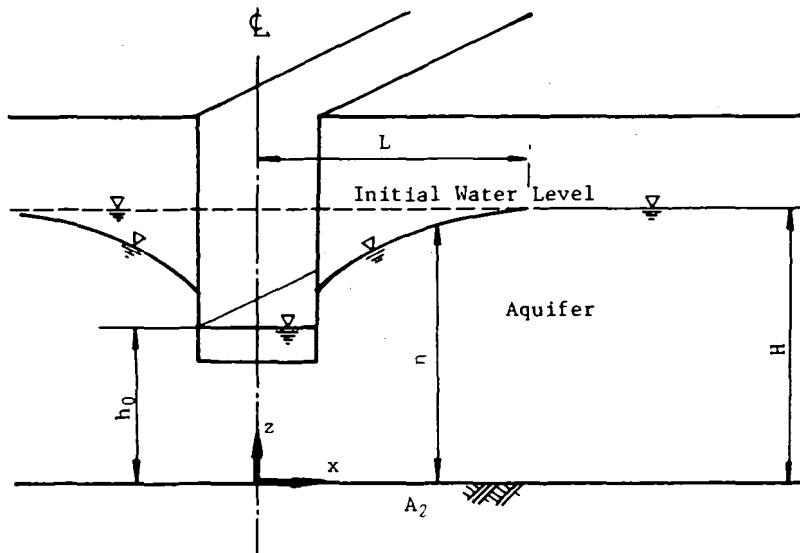


Fig.16 Two-dimensional ditch drainage

In practical excavations the length of excavation is finite as shown in Fig.17. This system has the effect of sheet pile walls so that there is no known nonsteady analytical solution. In this section the nonsteady state flow analysis will be performed in this hypothetical case of an open cut excavation.

#### 4.2 Simulation of Seepage through Three-dimensional Aquifer

The problem is as follows: A 20m sandy aquifer is under hydrostatic equilibrium with fluid potential  $h$  everywhere equal to 15m. In this

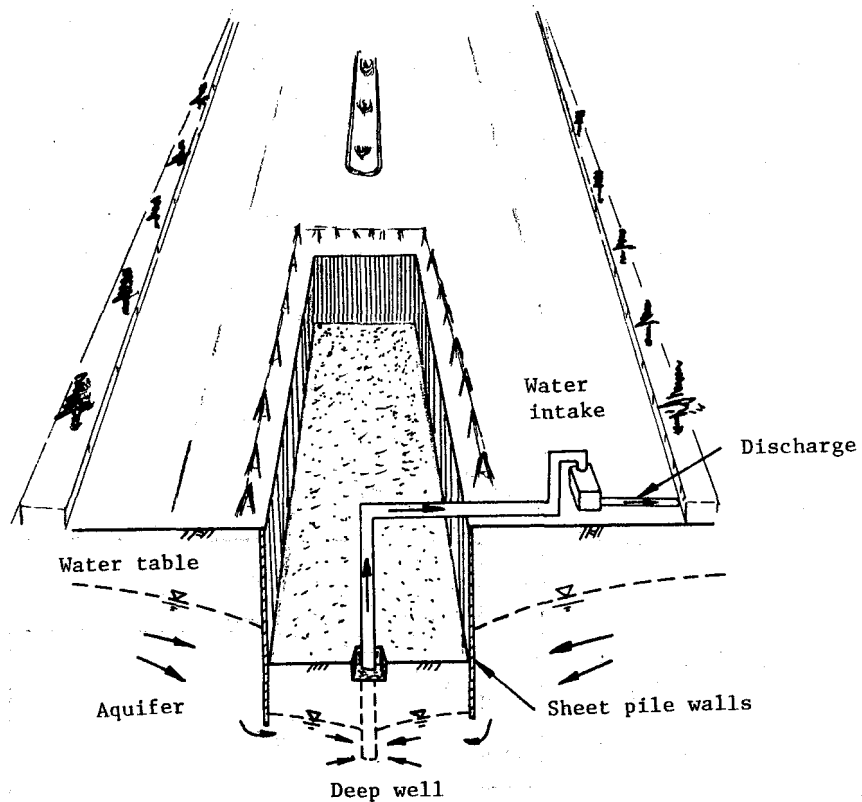


Fig.17 Excavation stabilized with deep well system (Artist's view of a deep well system including cutaway sketch of an deep well.)

aquifer an excavation (10m wide, 10m deep and 60m long) is made. The problem is to study the drainage pattern imposed within the sandy aquifer due to the excavation, that is, due to a rapid 5m drawdown of groundwater table in the excavation. A quarter of the flow domain was indentified by a system of finite elements as shown in Fig.18. The model is composed of 288 nodal points, and 168 eight node elements.

The boundary conditions are illustrated in Fig.19. The floor of the excavation is assumed to be constantly covered with a thin film of water so as to form a fixed potential boundary, while the wall of the excavation is an impermeable boundary (due to sheet pile walls). The bed rock is also an impermeable boundary. All the other boundaries of the flow region are assumed constant head.



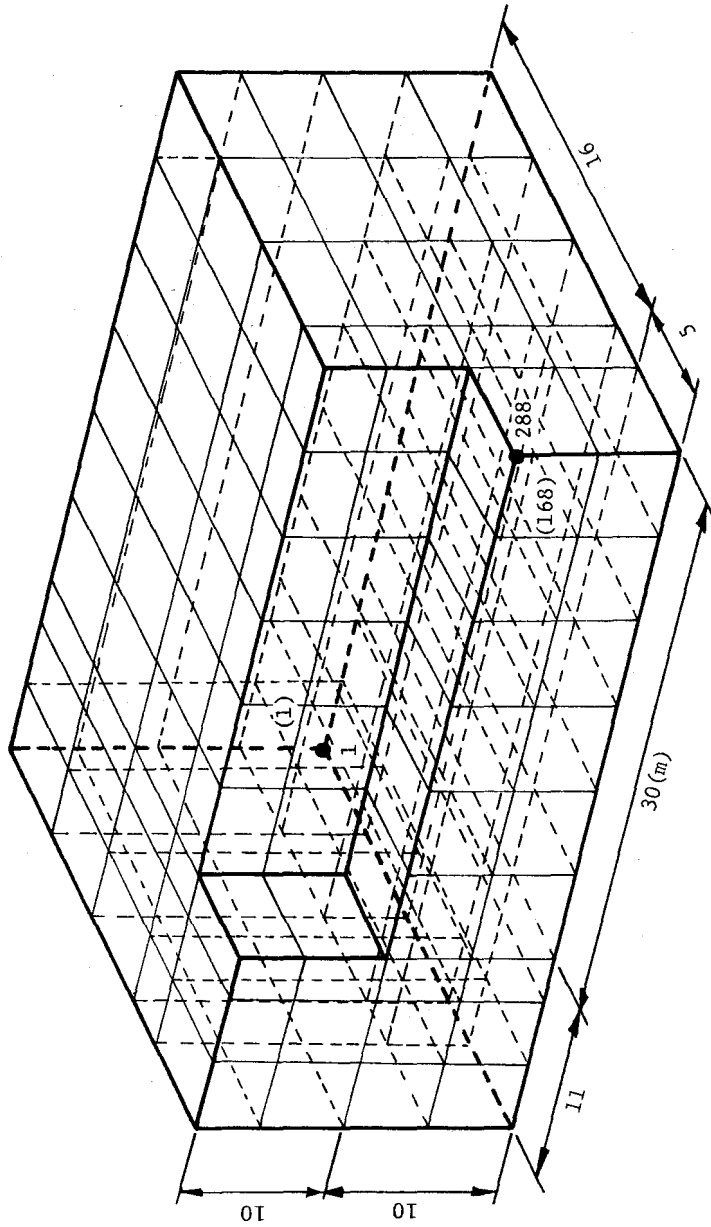


Fig.18 Three-dimensional finite element grid for the sandy aquifer

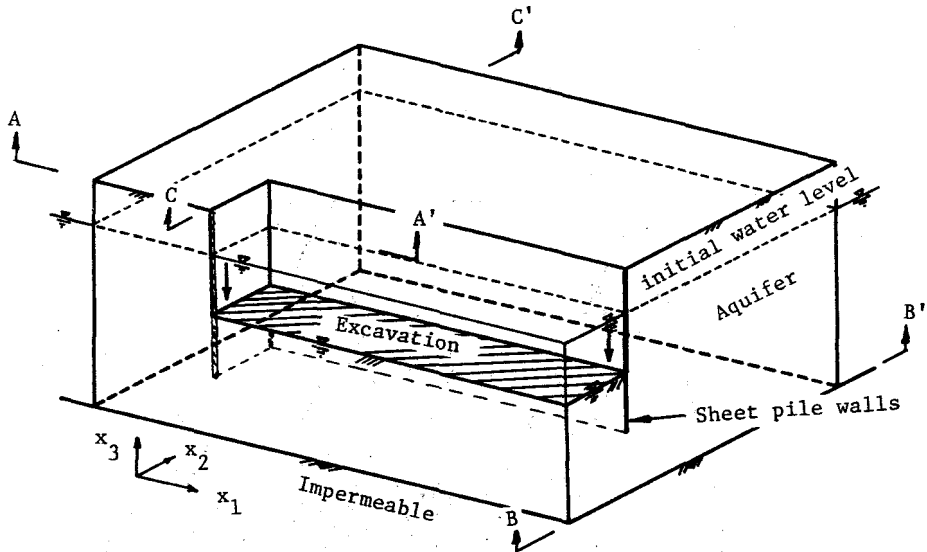


Fig.19 Transient flow toward an excavation face following rapid drawdown

For this hypothetical problem Fig.20 was used as a set of curves for unsaturated properties of the soil in the sandy aquifer. When  $\psi$  is equal to zero,  $K_s = 1.0 \times 10^{-2}$  cm/sec and  $\theta_o = 0.46$ . The solution advances in time by means of a fully implicit finite difference schema. Hysteresis was not taken into account.

The drainage computations were carried out for a period of 4 days from the start of the drainage process and the results of the computations are summarized in Figs.21 through 24. The time-dependent changes in the evolution of the surface  $\psi=0$  (water table) are presented in Figs.21, 22 and 23 along three cross sections, A-A', B-B' and C-C'. The topographic contours for approximately steady state (after 4 days) are also shown in Fig.24. The comparisons of three dimensional and two dimensional analytical results are shown for steady state water table in Figs.25 a. through 25 c.. Along sections A-A' and B-B' there are good agreements with two-dimensional results. Along section C-C', however, three-dimensional result differs significantly from the two-dimensional result. This discrepancy may be due to the effect of the

three-dimensional flow.

The rates of seepage into dewatered excavations was  $Q_{3-D}=1.72\text{m}^3/\text{min}$  by using three-dimensional analysis, while the rates of seepage was  $Q_{2-D}=1.56\text{m}^3/\text{min}$  which was estimated by using the two-dimensional analysis and the next equation.

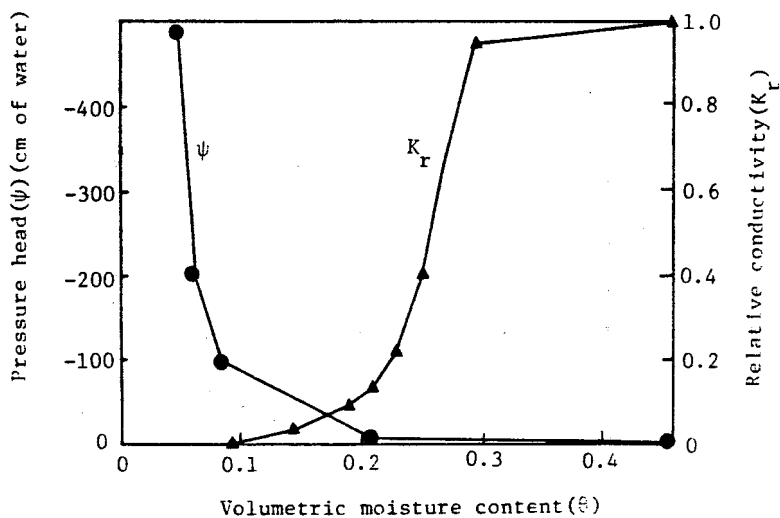


Fig.20 Unsaturated property of soil

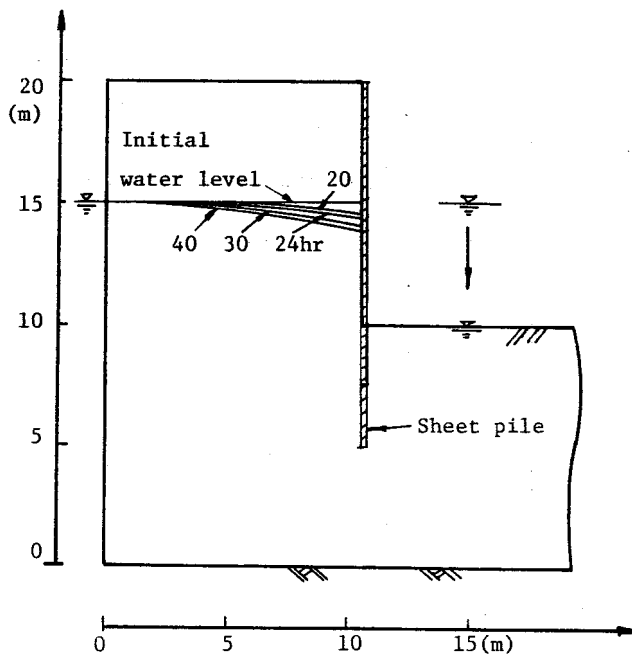


Fig.21 Numerical results (cross section A-A')

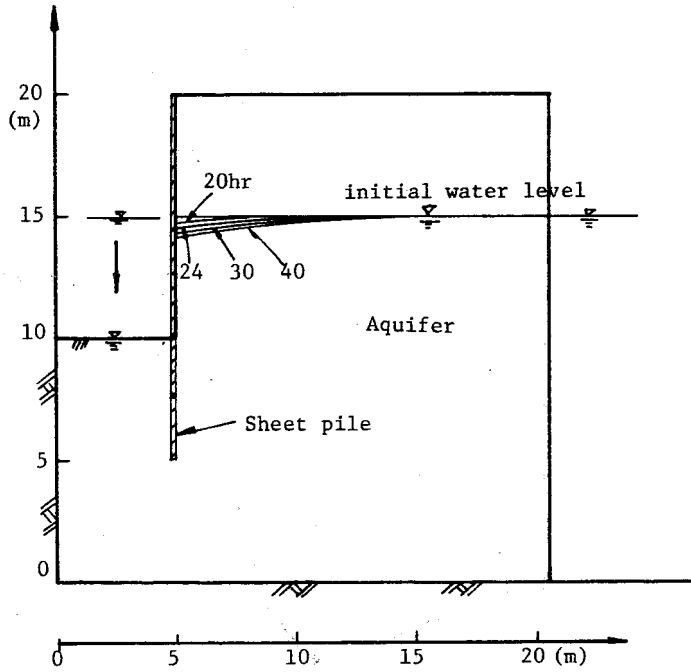


Fig.22 Numerical results (cross section B-B')

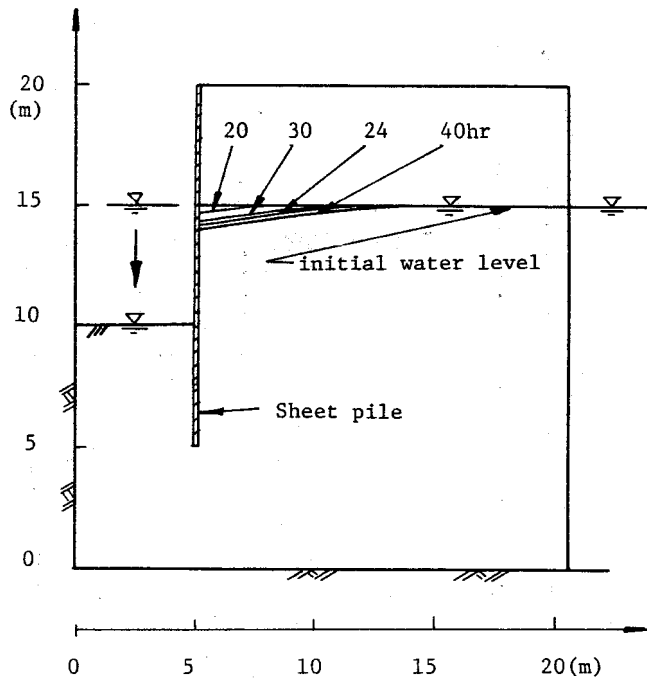


Fig.23 Numerical results (cross section C-C')

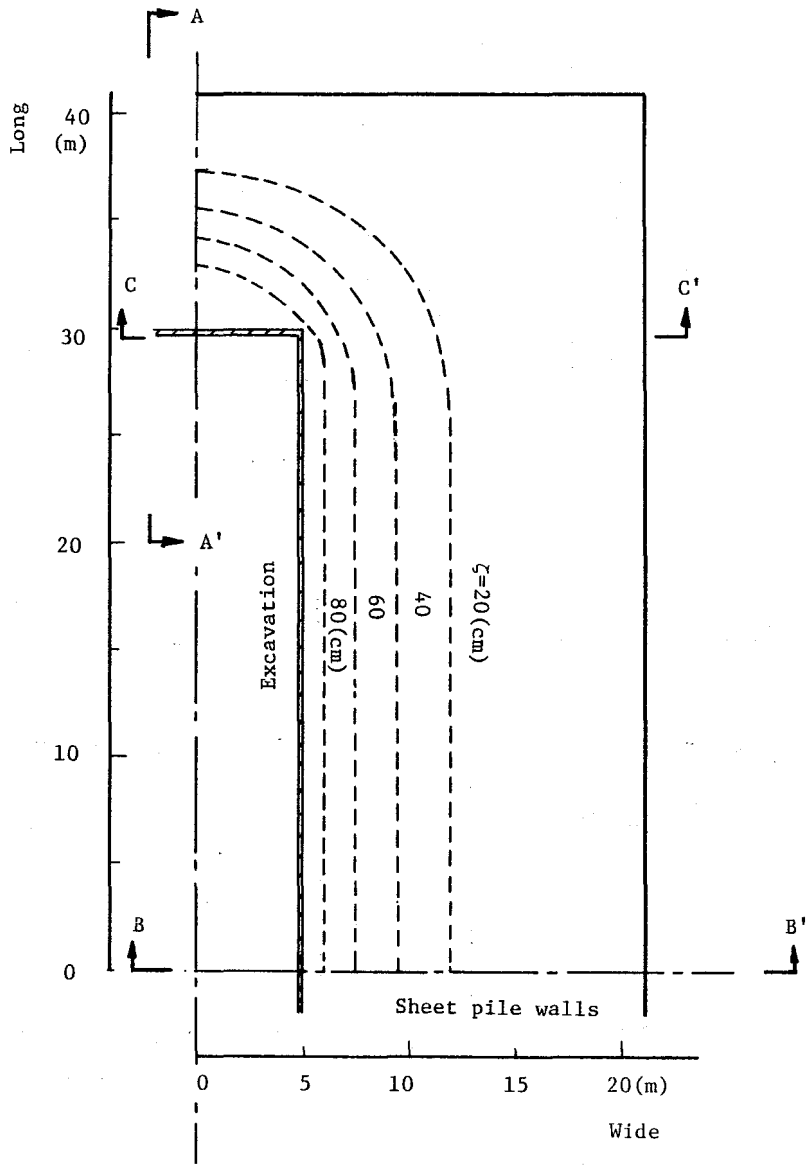


Fig.24 Topographic contours (in cm below original water level( $\zeta$ ))  
(after 4 days)

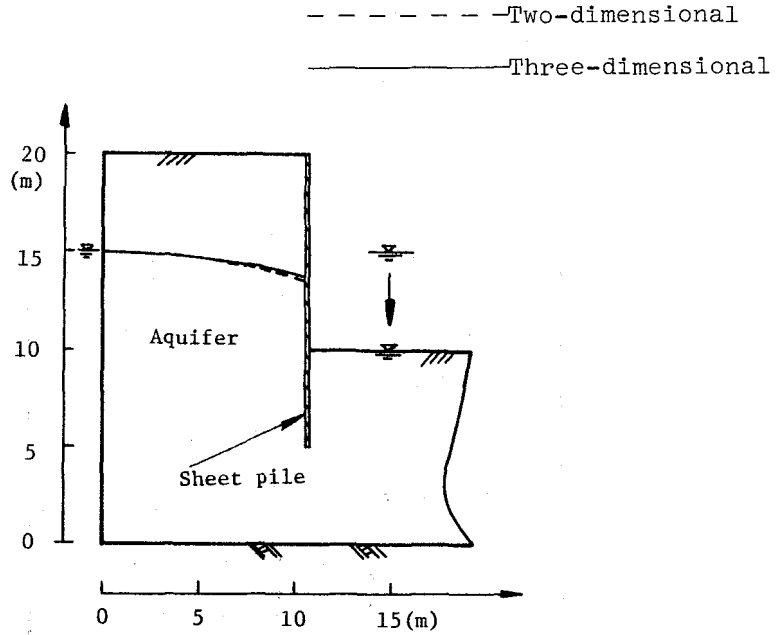


Fig.25 a (cross section A-A')

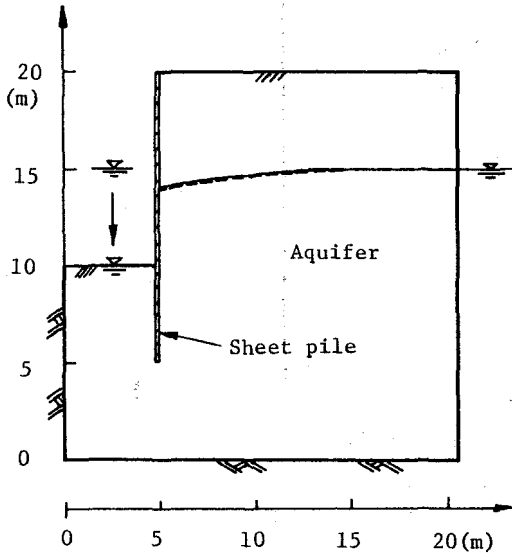


Fig.25 b (cross section B-B')

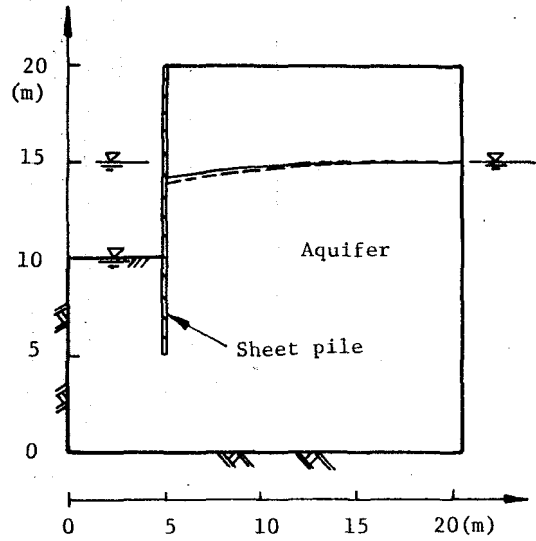


Fig.25 c (cross section C-C')

Fig.25 Comparisons of three-dimensional and two-dimensional numerical analysis results

$$Q_{2-D} = 4 \times ( a \cdot q_{A-A'} + b \cdot q_{B-B'} ) \quad (1)$$

where  $a$  and  $b$  are the length shown in Fig.24, and  $q_{A-A'}$  and  $q_{B-B'}$ , are the rate of inflow along cross section A-A' and B-B', respectively. From this result the estimated value of  $Q_{2-D}$  is smaller than that of  $Q_{3-D}$ . This discrepancy may be due to the difference in the profile of water table along cross section C-C'.

As it was said earlier, three-dimensional analyses are used only in those cases for which two-dimensional models are grossly inappropriate. It is therefore of interest to know the corresponding additional cost. The two-dimensional equivalent of this model along cross section A-A' has 28 nodes. The simulation in 34 time steps of 4 days of drainage required 10.2 sec of computer time on the ACOS-700 and 10K of core storage. As a comparison, the similar three-dimensional analyses performed with this model required 60.5min of computer time and 45K of core storage. To summarize the results of this section, it would appear that the use of three-dimensional finite element analysis is a step forward from the two-dimensional analysis used so frequently for analyzing excavation seepage problem because it allows the variation of flow in the third dimension. It is evident that the three-dimensional analysis is accurate enough to evaluate the behavior of the groundwater in the complex soil media qualitatively and quantitatively. However, the computer program, limited to the core storage of the available computer facility, is intended only to solve the simple illustrative problems. With increased capacity, it would be possible to handle problems of complex geometry and arbitrary boundary conditions. It is concluded that three-dimensional analysis is an expensive but valid alternative.

## 5. Conclusions

In this paper some attempts have been made to apply the two- and three-dimensional finite element analysis of seepage to the field problem. These examples clearly demonstrate the flexibility of this finite element approach and its capability in treating complex situations which are often encountered in the field. Consideration of anisotropy is clearly warranted in seepage analysis. Since the effects of saturated anisotropy have been widely studied in the seepage field

[23,24], any anisotropic examples have not been included in this paper. Throughout of this paper, the following conclusions are obtained.

- (1) The results of simulating the groundwater flow pattern in an inhomogeneous embankment when a river water level is raised to the flood heights and high water level have been shown.
- (2) There are good qualitative agreements between the numerical results and the informations recieved.
- (3) For three-dimensional flow example model, the seepage into dewatered excavations has been shown.
- (4) The three-dimensional analysis is accurate enough to evaluate the behavior of groundwater and it is a step forward from the two-dimensional analysis. However, by using the three-dimensional analysis it accentuates computer limitations by reducing the maximum size of problem that can be simulated on any given computer installation and increasing the computer time required to solve it.
- (5) The two- and three-dimensional saturated-unsaturated finite element method was found to be very effective to detect the realistic change of the flow pattern.
- (6) Finally main problems to use the saturated-unsaturated finite element procedure are to obtain the material properties of soils, especially the water retention curve in the unsaturated zone. And so there is a need to determine on a systematic basis the spectrum of problems for which consideration of the unsaturated flow domain retains engineering importance.

## References

- [1] Akai, K., Y. Ohonishi and M. Nishigaki: Finite element analysis of saturated-unsaturated seepage in soil, Pro. JSCE, 1977, Vol. 264, pp. 87-96, (in Japanese)
- [2] Akai, K., Y. Ohonishi and M. Nishigaki: Finite element analysis of three-dimensional flows in saturated-unsaturated soils, Third International Conference on Numerical Methods in Geomechanics, 1979, pp. 227-239.
- [3] Kono, I., M. Nishigaki and K. Yoriki: Measurement of water content changes in soil by using low-energy gamma ray attenuation, 14th Annual Meeting of JSSMFE, 1979, pp. 1137-1140, (in Japanese).
- [4] Mitchell, J. K., D. R. Hooper and R. G. Campanella: Permeability of compacted clays, ASCE, 91(SM4), 1965, pp. 41-66.



- [5] Klute, A.: The determination of the hydraulic conductivity and diffusivity of unsaturated soils, *Soil Sci.* Vol.113(4), 1972, pp.264-276.
- [6] Youngs, E.G.: An infiltration method of measuring the hydraulic conductivity of unsaturated porous materials, *Soil Sci.* 107, 1964, pp.307-311.
- [7] Hillel, D. and W.R.Gardner: Measurement of unsaturated conductivity and diffusivity by infiltration through an impeding layer, *Soil Sci.* 109, 1970, pp.149-153.
- [8] Bouma, J., D.I.Hillel, F.D.Hole, and C.R.Amerman: Field measurement of unsaturated hydraulic conductivity by infiltration through artificial crusts, *Soil Sci. Soc. Amer. Proc.*, 32, 1971, pp.362-364.
- [9] van Bavel, C.H.M., Stirk, G.B., and Brust, K.J.: Hydraulic properties of a clay loam soil and the field measurement of water uptake by roots. I. Interpretation of water content and pressure profiles. *Soil Sci. Soc. Amer. Proc.* 32, 1968, pp.310-317.
- [10] Davidson, J.M., L.R.Stone, D.R.Nielsen and M.E.LaRue: Field measurement and use of soil-water properties, *Water Res. Res.* 5, 1969, pp.1312-1321.
- [11] Hillel, D., V.D.Krentons and Y.Stylianou: Procedure and test of an internal drainage method for measuring soil hydraulic characteristics in situ. *Soil Sci.* Vol.114, No.5, 1972, pp.395-400.
- [12] Robins, J.S., L.L.Kelly, and W.R.Hamon, Reynolds Creek in south west Idaho: An outdoor hydrologic laboratory, *Water Res. Res.* 1(3), 1965, pp.407-413.
- [13] Jeppson, R.W., W.J.Rawls, W.R.Hamon and D.L.Schreider: Use of axisymmetric infiltration model and field data to determine hydraulic infiltration model and field data to determine hydraulic properties of soils. *Water Res. Res.* 11(1), 1975, pp.127-138.
- [14] Richards, L.A., Gardner, W.R., and Ohata, G.: Physical processes determining water loss from soil, *Soil Sci. Soc. Amer. Proc.* 20, 1956, pp.310-314.
- [15] Ogata, G. and Richards, L.A.: Water content changes following irrigation of bare field soil that is protected from evaporation, *Soil Sci. Soc. Amer. Proc.* 21, 1956, pp.355-356.
- [16] Gardner, W.R.: Field measurement of soil water diffusivity. *Soil Sci. Soc. Amer. Proc.* 34, 1970, pp.832-833.
- [17] Jackson, R.D.: On the calculation of hydraulic conductivity, *Soil Sci. Soc. Amer. Proc.* 36, 1972, pp.380-382.
- [18] Gardner, W.R.: The permeability problem, *Soil Sci.* 117, 1974, pp.243-249.

- [19] Cassel, D.K. and A. Baver: Spatial variability in soils below depth of tillage: Bulk density and fifteen atmosphere percentage, Soil Sci. Soc. Amer. Proc. 39, 1975, pp.247-250.
- [20] Carvalho, H.O., C.K. Cassel, J. Hammon, and A. Baver: Spatial variability of in situ unsaturated hydraulic conductivity of Maddock sandy loam, Soil Sci., 121, 1976, pp.1-8.
- [21] Desai, C.S.: Seepage analysis of earth banks under drawdown, ASCE, Vol. 98, (SM11), 1972, pp.1143-1162.
- [22] Whisler, F.C. and K.K. Watson: One-dimensional gravity drainage of uniform columns of porous materials, J. Hydrology, 6, 1968, pp. 277-296.
- [23] Cedergren, H.R.: Seepage, Drainage and Flow Nets, John Wiley, New York, 1967.
- [24] Selim, H.M.: Water flow through a multilayer stratified hill side, Water Res. Res. 11(6), 1975, pp.949-957.