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An Investigation on Low Frequency Combustion Oscillation

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Synopsis

This paper deals with a combustion stability of diffusion flame. A simplified linear differential equation of second order, which involves the parameters estimated from the states of steady combustion, has been suggested to discuss theoretically the nature of oscillatory combusion about a two-dimensional combustion chamber. Its validity has been testified by comparing calculated results with experimental ones. Results obtained have indicated that factors markedly affecting the low-frequency oscillatory combustion are primary volumetric air-fuel ratio, duct length of combustion side, shape of burner and inlet throttles. But outlet throttle gives little effect on the stability of combustion.

§ 1. Introduction

Installing the burner, which gives stable combustion in an open flow, there can be seen the low frequency self-excited oscillation which leads to troubles such as noise, flame blow-off, low combustion efficiency and destruction of the chamber. The more intensive the combustion is, the severer those troubles become. Hitherto many investigators have carried out the studies on the combustion instability and arrived at the conclusion that the low-frequency combustion oscillation is generated by the coupled fluctuation of heat release and pressure in the chamber. However, there are few reports [1),2),3),4)] in which the combustion instability is related together with the structure of the chamber and the states of flow and supply systems.

Studying the characteristic values of oscillatory combustion, such as the frequency and the blow-off limit in regard to the overall combustion system, seems to be useful to widen the stable combustion extent and also to improve the combustion performance.

In this work, the authors suggested a simple differential equation of second order to investigate the pressure oscillation in a two-dimensional combustion chamber. The theoretical characteristic values of oscillatory combustion were compared with the experimental values to testify the validity of the equation. Although the model of combustion system is simple, it may not be difficult to extend this analytical method to the more complicated practical systems.

§ 2. Test Procedures

A schematic diagram of apparatus is shown in Fig. 1. Fig. 2 shows the details of combustor which is composed of primary and secondary plenum chambers, a burner, a duct and a outlet throttle.

Propane gas whose flow rate is adjusted by a control valve is supplied into the primary plenum chamber through a rotameter and, after being mixed with the primary air, it is

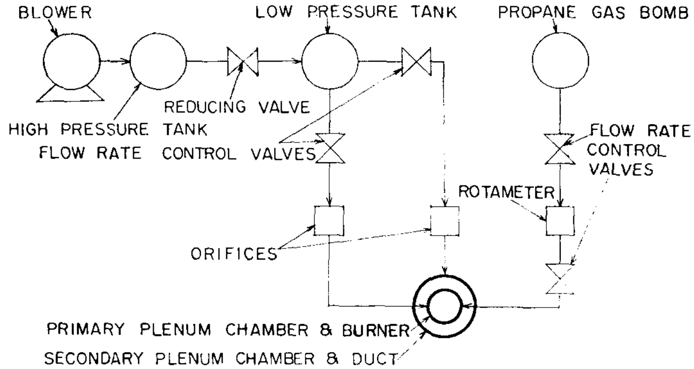

Fig. 1. *Schematic diagram of apparatus.*

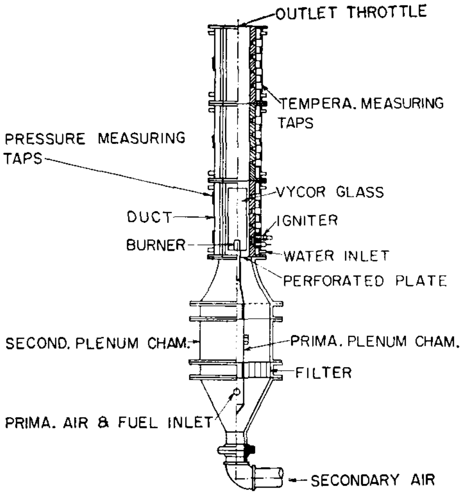

Fig. 2. Combustor.

fed through the burner into the duct, that is, the combustion chamber. The flow rate of primary air is adjusted by a control valve and is measured with the orifice. The air, which is supplied into the secondary plenum chamber through the control valve and the oriffice, flows into the duct through the perforated plates. Five kinds of the plate with 180 holes have been prepared and their hole diameters are 1. 5, 2.0, 2.5, 3.0 and 3.5 mm, respectively. At the inlet of burner a perforated plate has been also mounted on. The inner cross sections of the secondary plenum chamber, the duct and the burner are 238×108 mm^2, 90×45 mm^2 and 8×25mm^2, respectively. An adjustable throttle plate or a rotating butterfly valve has been used as the outlet throttle of the duct.

Pressure fluctuations have been measured from synchroscopic traces by using pressure transducers of semiconductor gauge and piezoelectric types.

§ 3. Theoretical Analysis

3—1 Treatments

Observed characteristics of the low frequency oscillation were as follows :

(1) Pressure and velocity fluctuations became gradually larger with an increase in the secondary flow rate, and in the vicinity of the blow-off limit the flame became quite unstable. Fig. 3 shows both the changes of the static pressure and the amplitudes of pressure oscillation at the different points from the duct inlet (static pressure at 0.038 m, pressure osillation at 0.146 m and 2.182 m).

(2) Stable combustion region was limited by a continuous blow-off limit curve and there was no so-called "detached territory".

(3) In the vicinity of the blow-off limit, the oscillation was the self-excited one and its wave form was similar to the sine wave.

On the basis of the above mentioned characteristics, it is possible to express the equation for pressure oscillation as the second order type, with which the stability can be easily investigated. If the equation is valid, the following conditions should be satisfied.

(1) The characteristic values obtained theoretically, such as frequency, blow-off limit and others, agree with the experimental ones or their tendency is similar to that obtained experimentally.

(2) If the forced vibration is imposed on the combustion system, the smooth combustion

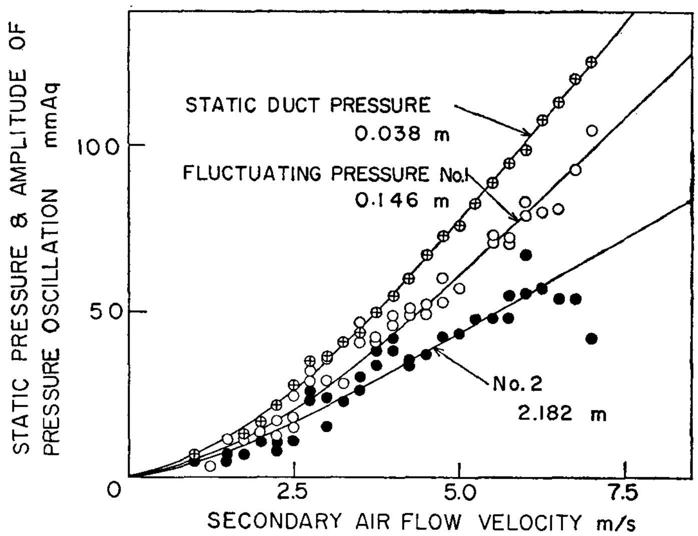

Fig. 3. Static duct pressure and amplitude of pressure oscillation as a function of the secondary air flow velocity. $\varphi=2.5$, $L_u=0.356$ m, $L_b=1.956$ m, $\psi=2/9$, $d=2$mm, $b=4.5$mm. Measuring point distances from the duct inlet are written in the figure.

region becomes narrower than the self-excited case and the amplitude of pressure or velocity oscillation becomes maximum at the forced frequency near the natural one of the system.

3—2 Assumptions

In order to simplify the differential equation for pressure oscillation, the following assumptions are introduced.

(1) Equations for unsteady behavior of the process used in the analysis are able to be derived from equations for steady behavior.

(2) A second order and higher order terms are able to be neglected in contrast with a time-mean and a first order term because of their smallness.

(3) As the wave length of the pressure oscillation is larger than the dimensions of construction elements of the chamber, oscillation occurs over the system in the same pressure phase.

(4) The time period of the ignition delay after reactants were injected is a half cycle, that is, the phase of heat release rate is in the antiphase against the pressure oscillation[2),5)].

(5) The flow in the combustion chamber is one dimensional.

(6) There is no pressure oscillation in the plenum chambers. From the assumption (1), it becomes possible to discuss the state of oscillation by using the known data of the steady combustion state.

Experiments show that the pressure oscillations in the plenum chambers are not so small to be able to be neglected. But if we consider the pressure oscillation contained the phase lags both in the fuel and the secondary air sides, the differential equation becomes a third or a higher order one and hard to be dealed with.

3—3 Analytical Equation

Let it be the combustion model as follows. Secondary air is flowed into the duct from the secondary plenum chamber through the secondary perforated plate. After being mixed in the primary plenum chamber, fuel and primary air are flowed into the duct from the burner, and then they burn together with the secondary air. The combusion products are exhausted from the outlet throttle.

On the assumption that a second and higher order components are negligible, an arbitrary fluctuating quantity W is written as

$$W=\overline{W}+\widetilde{W}, \tag{1}$$

where $\overline{W}$ is the mean value and $\widetilde{W}$ is the first order fluctuating component.

Neglecting the effects of the pressure oscillation in the plenum chambers, the mass flow rates of secondary air through the perforated plate of duct inlet, primary air-fuel mixtures through the burner, and combustion products through the duct outlet are expressed in the following equations, respectively.

$$m_4=\frac{\overline{P}_1-\overline{P}_3}{\overline{R}_4}-\frac{\widetilde{P}_3}{\widetilde{R}_4}, \tag{2}$$

$$m_5=\frac{\overline{P}_2-\overline{P}_3}{\overline{R}_5}-\frac{\widetilde{P}_3}{\widetilde{R}_5}, \tag{3}$$

and

$$m_6=\frac{\overline{P}_3-\overline{P}_0}{\overline{R}_6}+\frac{\widetilde{P}_3}{\widetilde{R}_6}. \tag{4}$$

As the relations between the mass flow rate and the pressure difference are parabolic, the resistance for the fluctuating mass flow rate should be twice as large as the registance for the steady mass flow rate.

Therefore,

for steady state terms, $\overline{R}=D\overline{U}$, (5)

and for fluctuating terms, $\widetilde{R}=2D\overline{U}$. (6)

The burned side mean temperature is given by

$$\overline{T}_b=\overline{T}_0+\frac{\overline{c_{pu}(T_u-T_0)}\overline{m}_u+\gamma\overline{H}}{\overline{c}_{pb}\cdot\overline{m}_b}. \tag{7}$$

For the difficulty to estimate the over-all combustion efficiency, it was assumed in this report that the efficiency is unit and equal to a over-all excess air ratio for the cases of the excess air ratio larger and less than unit, respectively.

The following relations, including oscillatory terms, between the mass of fluid and the pressure are given for the fluids on the unburned and the burned sides of duct, respectively.

$$M_u=\frac{\overline{M}_u}{\overline{P}_3}\cdot P_3, \tag{8}$$

$$M_b=\frac{\overline{M}_b\cdot\overline{T}_b}{\overline{P}_3}\cdot\frac{P_3}{T_b}. \tag{9}$$

The equation of mass conservation between the inlet and the outlet of duct is

$$\frac{dM_t}{dt}=m_u-m_b, \tag{10}$$

and the equation of energy conservation between them is

$$\bar{c}_{pu}(T_u-T_0)m_u+\gamma H=\bar{c}_{pb}(T_b-T_0)m_b+\frac{d}{dt}\int_{L_t}\bar{c}_v\rho(T-T_0)dL. \tag{11}$$

Using the assumption (4), and assuming the amplitude of fluctuation of heat release rate is proportional to that of the fuel feeding rate, the fluctuating heat release rate becomes,

$$\widetilde{H}=-K\frac{A_1}{A_3}h_0\,\widetilde{m}_{1f}. \tag{12}$$

The coefficient of fluctuating heat release rate K is the ratio of the fuel flow rate, which is converted into the fluctuating heat release rate, to the fluctuating flow rate of fuel, and is considered to be proportional to the fuel concentration in the axis of the duct. Since the theoretical estimation of the coefficient is difficult, the coefficient has been represented by the following empirical equation obtained from the blow-off limit and is the function of fuel flow rate.

$$K=C_1+C_2\bar{m}_{1f}+G_3\bar{m}_{1f}^2+C_4\bar{m}_{1f}^3. \tag{13}$$

Rearranging the above fundamental equations by the linearlyzation method, we get the next equation for the duct pressure.

$$X\frac{d^2\widetilde{P}_3}{dt^2}+Y\frac{d\widetilde{P}_3}{dt}+Z\widetilde{P}_3=0, \tag{14}$$

where,

$$X=\bar{c}_{vu}(\bar{T}_u-T_0)\frac{\bar{M}_u}{\bar{P}_3}+\tau\bar{c}_{vb}\,\bar{m}_b\frac{\bar{T}_b}{\bar{P}_3}\frac{\bar{M}_t}{\bar{M}_b}-\bar{c}_{vb}(\bar{T}_b-T_0)\frac{\bar{M}_u}{\bar{P}_3}, \tag{15}$$

$$Y=\bar{c}_{vb}(\bar{T}_b-T_0)\frac{1}{\widetilde{R}_6}+\bar{c}_{pu}(\bar{T}_u-T_0)\left(\frac{A_1}{A_3}\frac{1}{\widetilde{R}_4}+\frac{A_2}{A_3}\frac{1}{\widetilde{R}_5}\right)+\tau\bar{c}_{vb}\bar{m}_b\frac{\bar{T}_b}{\bar{M}_b}\left(\frac{A_1}{A_3}\frac{1}{\widetilde{R}_4}+\frac{A_2}{A_3}\frac{1}{\widetilde{R}_5}+\frac{1}{\widetilde{R}_6}\right)+\frac{\bar{M}_t}{\bar{M}_b}\frac{\bar{T}_b}{\bar{P}_3}\bar{U}_u(\bar{c}_{pb}\bar{\rho}_u-\bar{c}_{pu}\bar{\rho}_b)-K\frac{\rho_f}{\rho_f+\rho_a\varphi}\frac{h_0}{}\frac{A_1}{A_3}\frac{1}{\widetilde{R}_4}-\bar{c}_{vb}(\bar{T}_b-T_0)\left(\frac{A_1}{A_3}\frac{1}{\widetilde{R}_4}+\frac{A_2}{A_3}\frac{1}{\widetilde{R}_5}+\frac{1}{\widetilde{R}_6}\right), \tag{16}$$

$$Z=\frac{\bar{T}_b}{\bar{M}_b}\bar{U}_u(\bar{c}_{pb}\bar{\rho}_u-\bar{c}_{pu}\bar{\rho}_b)\left(\frac{A_1}{A_3}\frac{1}{\widetilde{R}_4}+\frac{A_2}{A_3}\frac{1}{\widetilde{R}_5}+\frac{1}{\widetilde{R}_6}\right). \tag{17}$$

In the above equation τ is the stay time of fluid in the duct and expressed as,

$$\tau=\frac{\bar{c}_{vu}\bar{\rho}_b L_u\bar{U}_b+\bar{c}_{vb}\bar{\rho}_u L_b\bar{U}_u}{\bar{c}_{vb}\bar{\rho}_u\bar{U}_u\bar{U}_b}. \tag{18}$$

Equation for the resistance of the rotating butterfly valve at the outlet of the duct is given by,

$$R_6=\bar{R}_6+\hat{R}_6\cdot\cos\omega t, \tag{19}$$

where, $\bar{R}_6$ is the time mean resistance, $\hat{R}_6$ is the amplitude of the fluctuating component and ω is the forced angular frequency. Therefore, in the case that the rotating butterfly valve is installed at the outlet, the equation for the pressure fluctuation becomes as follows:

$$X\frac{d^2\widetilde{P}_3}{dt^2}+Y\frac{d\widetilde{P}_3}{dt}+Z\widetilde{P}_3=\hat{R}_6\sqrt{(F\omega)^2+G^2}\cdot\cos(\omega t+\alpha), \tag{20}$$

where,

$$F=\left[\tau\bar{c}_{vb}\,\bar{m}_b\frac{\bar{T}_b}{\bar{M}_b}+(\bar{c}_{pb}-\bar{c}_{vb})(\bar{T}_b-T_0)\right]\cdot\frac{\bar{P}_3-P_0}{\widetilde{R}_6^2}, \tag{21}$$

$$G=\bar{c}_{pb}\bar{m}_b\frac{\bar{T}_b}{\bar{M}_b}\frac{\bar{P}_3-\bar{P}_0}{\widetilde{R}_6^2}, \tag{22}$$

$$\alpha=\tan^{-1}\frac{F\omega}{G}. \tag{23}$$

Solving the equation (20), the amplitude of pressure oscillation $\hat{P}_3$ is obtained as,

$$\hat{P}_3 = \hat{R}_6 \frac{\sqrt{(F\omega)^2 + G^2}}{\sqrt{(Z-\omega^2 X)^2 + (\omega Y)^2}} . \quad (24)$$

The coefficients X and Z do not become negative except extreme cases. Equations (14) and (20) for pressure oscillation are aplicable for both the states of laminar and turbulent flows. Therefore, the stability of combustion can be investigated by the sign of the coefficient Y or a coefficient ratio of viscous damping γ. Y or γ is negative in blow-off region, and positive in stable combustion region. It becomes zero at the blow-off limit.

The steps of calculating procedure are as follows. The mean values of combustion temperature, specific heat, throttle registances and others are estimated from the setting flow rates of fuel, primary air and secondary air. Then, introducing them into the equations for pressure oscillation, the characteristic values such as frequency, coefficient ratio of viscous damping and amplitude of pressure oscillation are calculated.

Empirical equations were used in calculating the non-dimensional throttle registances. Flow resistances through the duct were included in the outlet throttle resistance. Mean specific heat of fluid in the burned state was calculated by using the equations of molecular heat capacity [6] on the assumption that the combustion products consist of CO_2, H_2O, N_2, O_2 and C_3H_8.

In this report, the empirical coefficients C_1, C_2, C_3 and C_4 were obtained from the measured values of blow-off limit at the conditons of the primary air-fuel ratio $\varphi=2.5$, the duct length of burned side $L_b=1.378$ m, the duct length of unburned side $L_u=0.356$ m, the throttle ratio of duct outlet $\psi=2/9$ and the hole diameter of secondary perforated plate $d=2$ mm.

Those coefficients were used for calculating the theoretical values of blow-off limit at the other conditions in the primary air-fuel ratio $\varphi=2.5$. Numerical values used in the calculation are tabulated in Table 1.

Table 1. Numerical values used in calculations

$A_1=2\times10^{-4}\text{m}^2$, $A_2=3.71\times10^{-3}\text{m}^2$, $A_3=4.05\times10^{-3}\text{m}^2$,
$(c_{pu})_f=0.404$ kcal/kg°c, $(c_{pu})_a=0.24038$ kcal/kg°c,
$(c_{vu})_f=0.359$ kcal/kg°c, $(c_{vu})_a=0.17145$ kcal/kg°c,
$h_0=11079$ kcal/kg, $L_u=0.356$m, $P_0=10332$ kg/m²
$\rho_f=0.2058$ kg s²/m⁴, $\rho_a=0.1221$ kg s²/m⁴,
$D_1=6.389$,
$D_2=10^{3.1462-715d}$, where d is in meter unit,
$D_3=10^{1.47712-3.3235\psi}+2L_t$, where L_t is in meter unit,
$C_1=0.013135$, $C_2=0.29749$, $C_3=-0.41943$,
$C_4=0.50110$ for $\varphi=2.5$, respectively.

§ 4. Results and Discussions

The relation between the pressure oscillation and the forced frequency, when the butterfly valve at the outlet of duct was rotated, has been measured. The resonance curves obtained are shown in Fig. 4. With an increase in the forced frequency, the amplitude of oscillation increases, and after becoming maximum, it decreases. At the frequencies above twenty cycles per second, the amplitude is held nearly constant. The forced frequency whch gives a maximum pressure amplitude increases as the secondary air flow velocity becomes larger. Flame blew off at the secondary flow velocity of 6.5 m/s. However, when there was no external forced disturbance, that is, the thro-

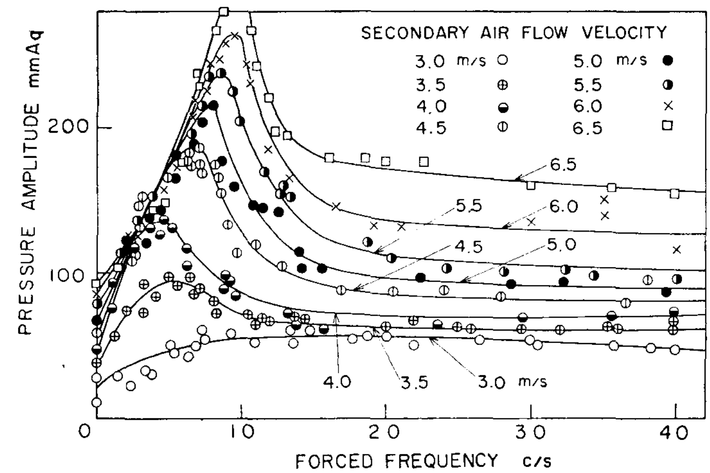

Fig. 4. Pressure amplitude obtained by rotating the butterfly valve at the outlet of duct. $\varphi=2.5$, $L_u=0.356$m, $L_b=1.378$ m, mean $\psi=2/9$, $d=2$mm, $b=4.5$mm, Measuring point distance from the duct inlet is 0.146m.

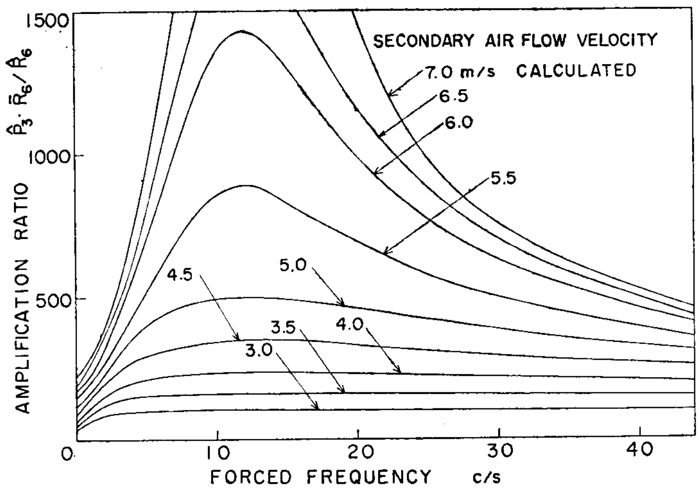

Fig. 5. Calculated resonance curves expressed by the relation between amplification ratio $\widehat{P}_3 \cdot \bar{R}_6/\widehat{R}_6$ and forced frequency. $\varphi=2.5$, $L_u=0.356$m, $L_b=1.378$m, $d=2$mm, $b=4.5$mm.

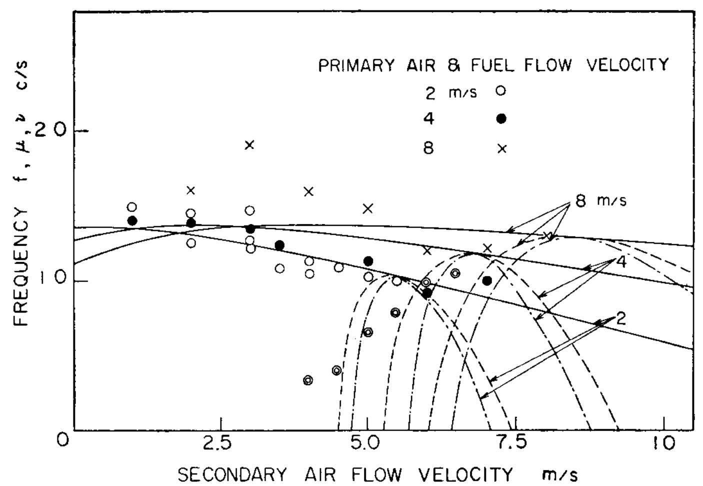

Fig. 6. Frequency as a function of the secondary air flow velocity. $\varphi=2.5$, $L_u=0.356$m, $L_b=1.378$m, $\psi=2/9$, $d=2$mm, $b=4.5$mm. Solid, broken and dot-dash lines indicate the theoretical curves of natural frequency f, damped natural frequency ν, and resonance frequency μ, respectively.

ttle plate was installed at the duct outlet, flame blew off at 7 m/s in the same conditions indicated in the under part of the figure. Fig. 5 shows a theoretical resonance curves computed from the equation (24). They agree qualitatively well with the experimental curves in Fig. 4. These results indicate that the forced oscillation can be generated by imposing forced disturbances upon the combustion system.

The relation between the secondary air flow velocity and the frequency was investigated in the case of installing the throttle plate at the duct outlet. Taking the primary flow velocity as a parameter, the results obtained are shown in Fig. 6. It seems that the measured values of frequency agree well with those of the theoretical natural frequency f which is expressed by the equation in the later section of normenclature and is indicated by the solid lines in the figure. Broken lines and dot-dash lines indicate damped natural frequency ν and resonance frequency μ, respectively. The data plotted by double circles in the figure indicate the forced frequency at the primary air and fuel flow velocity of 4m/s, at which frequency the rotating butterfly valve gives a maximum pressure amplitude. Although the theoretical resonance frequency has the similar tendency as the double circles, there is a quantitative difference between them. The discrepancy seemed to be owing to the errors of the coefficient ratio of viscous damping, which are caused by estimating the coefficient of heat release rate K only from the blow-off limit. In the vicinity of the blow-off limit, the wave form of pressure oscillation was similar to the sine wave. With aparting from the limit, it became the combined form of two or more waves, and it was observed that only the wave of higher frequency remained at low flow velocity. Therefore the higher frequency was used as the frequency of the oscillatory combustion.

The experimental blow-off limits are shown in Fig. 7, taking a primary volumetric air-fuel ratio as a parameter. It is convinced that the combustible region is extended as a primary air-fuel ratio becomes larger and the primary

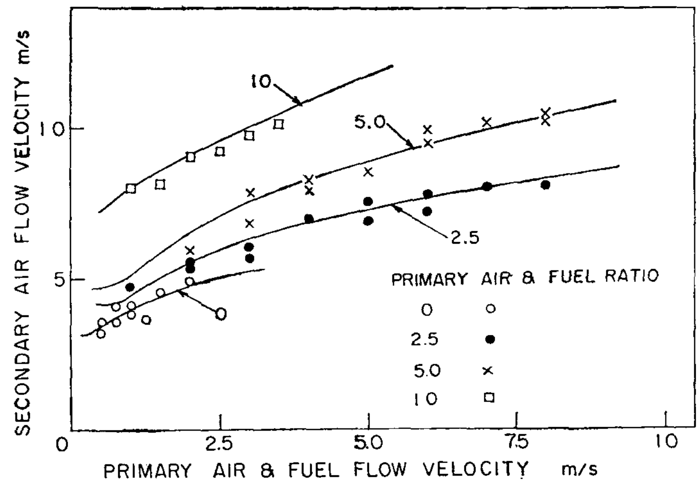

Fig. 7. Experimental blow-off limit curves taking a primary volumetric air-fuel ratio as a parameter. $L_u=0.356$ m. $L_b=1.378$, $\psi=2/9$, $d=2$mm, $b=4.5$mm.

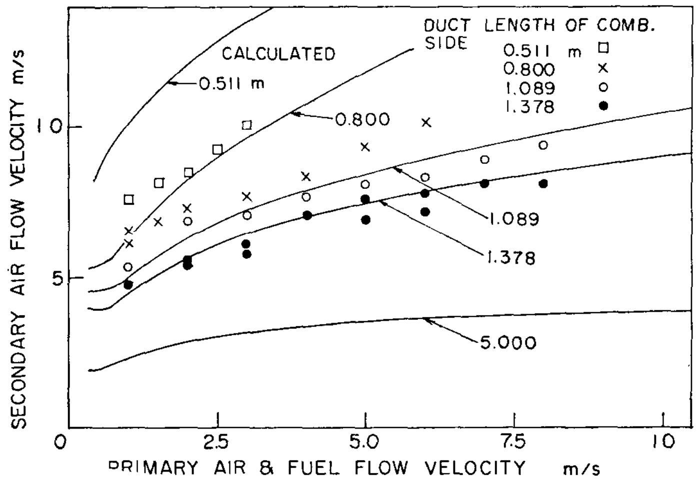

Fig. 8. Blow-off limit curves when the duct length of combustion is varied. $\varphi=2.5$, $L_u=0.356$m, $\psi=2/9$, $d=2$mm, $b=4.5$mm.

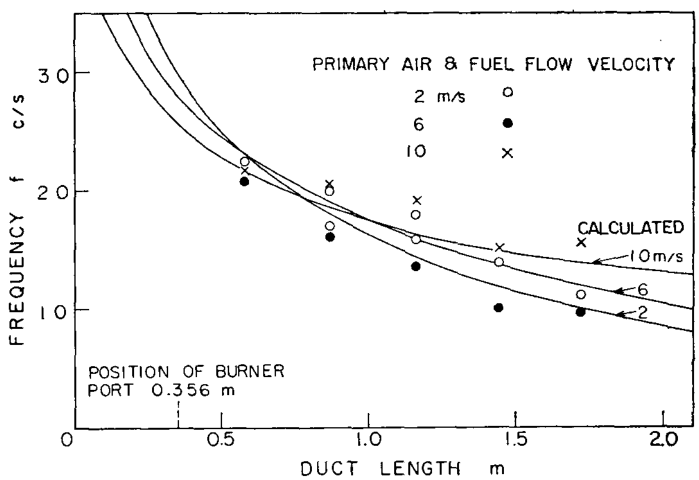

Fig. 9. Natural frequency as a function of the duct length. $\varphi=2.5$, $L_u=0.356$m, $\psi=2/9$, $d=2$mm, $b=4.5$mm.

flow velocity becomes higher. This means that the ratio of the oscillatory part of heat release rate to the steady part decreases by making the primary air-fuel ratio larger and consequently the temperature fluctuation on the combustion side decreases. Therefore, it can be said that oscillatory combustion becomes more difficult to occur according as the primary air-fuel ratio was increased. Since the heat release rate is reduced by increasing the primary air-fuel ratio, the operation region of an actual combustor must be determined by taking both the combustion stability and a combustion intensity into consideration.

Fig. 8 shows the blow-off limit curves when the combustion side duct length from the burner port to the duct outlet is varied. The shorter the combustion side length is, the wider the stable region becomes, and the less the theoretical limits coinside with the experimental ones. The disagreement between the theoretical and the experimental blow-off limits is owing to the errors in estimating the combustion efficiency which is lowered according as making the combustion length shorter. The measured frequency and the calculated natural frequency are presented in Fig. 9 against the duct length. The theoretical values agree with the experimental ones. The same qualitative trend on the frequency have been pointed out in the reports [7),8)] dealt with the low frequency combustion oscillation from the view point of acoustic oscillation.

The blow-off limit curves, in which the hole diameter of secondary perforated plate were changed, are shown in Fig. 10. The combustible region is extended by increasing the throttle registance, that is, by decreasing the hole diameter. Frequency curves are shown in Fig. 11 under the same conditions. It is evident that the frequency is lowered by making the hole diameter smaller.

Fig. 12 shows the blow-off limit when the duct outlet is throttled with the throtting plate. The theoretical trend of the throttling effect is indicated to be opposite to the experimental trend, but it seems that the blow-off limit is little affected by throttling the duct outlet. If we discuss how to stabilize the combustion on the stand poind of the structure of the combustor, the following matter should be noticed: When the length of combustion chamber can not be made short from the point of the combustion efficiency it is necessary to throttle the chamber inlet for rasing the flame stability.

When the wall thickness of burner port was altered, the blow-off limit was changed remarkably as shown in Fig. 13. It is considered that the recirculation on the burner port stabilizes the flame and suppresses the fluctuation of heat release rate. Therefore, the combustible region is extended by increasing the wall thickness, because of becom-

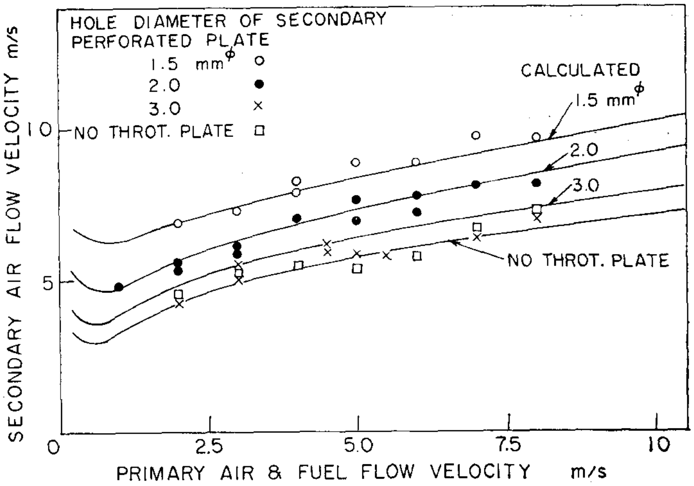

Fig. 10. Blow-off limit curves when the hole diameter of secondary perforated plate is changed. $\varphi=2.5$, $L_u=0.356$m, $L_b=1.378$m, $\psi=2/9$, $b=4.5$mm.

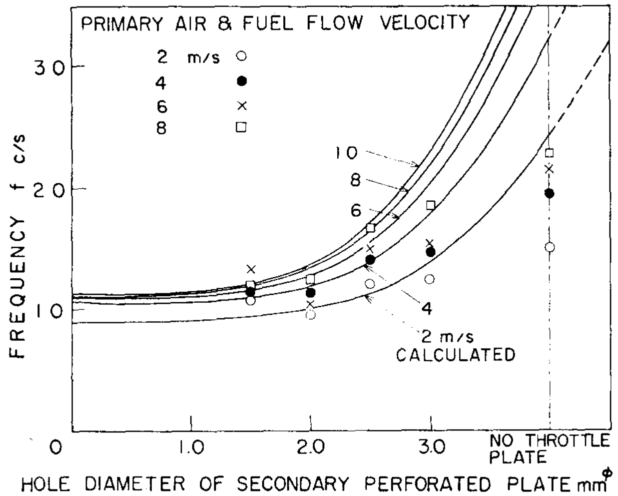

Fig. 11. Natural frequency as a function of the hole diameter of secondary perforated plate. $\varphi=2.5$, $L_u=0.356$m, $L_b=1.378$m, $\psi=2/9$, $b=4.5$mm.

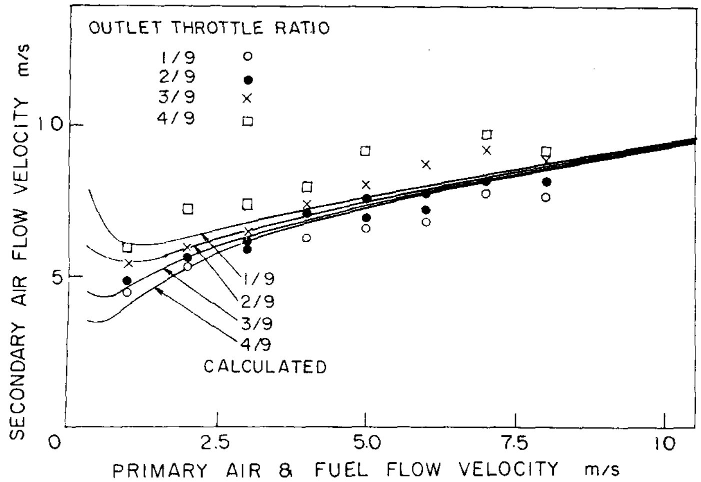

Fig. 12. Blow-off limit curves when the throttle ratio of the duct outlet is changed. $\varphi=2.5$, $L_u=0.356$m, $L_b=1.378$m, $d=2$mm, $b=4.5$mm.

ing the recirculation more intensive. In this calculation, the empirical equation for the coefficient of fluctuating heat release rate K does not include the factors concerning with the wall thickness of burner port and the primary air-fuel ratio. However, it is not difficult to make the empirical equation, in which all of the conditions of combustion such as the kind of fuel, the cross sectional area, the wall thickness of burner port and the primary air-fuel ratio are taken into consideration, if more detailed experiments are carried out.

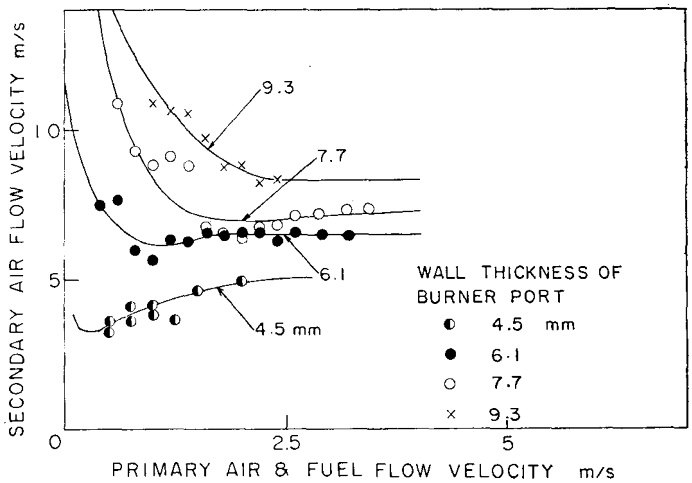

Fig. 13. Experimental blow-off limit curves when the wall thickness of burner port is altered. $\varphi=0$, $L_u=0.356$m, $L_b=1.378$m, $\psi=2/9$, $d=2$mm.

§ 5. Conclusions

The results obtained through the experimental and theoretical investigations are summarized as follows ;

(1) Applying the linearization method, the pressure oscillation in a simple combustion system could be expressed by a differential equation of second order type. The results calculated agreed qualitatively well with the experimental ones. Moreover, the blow-off limit could be estimated from the setting conditions of steady state by using the differential equation.

(2) It was convinced that the forced oscillation could be raised by imposing the forced external disturbance on the combustion system.

(3) Stability of the combustion was much affected by the primary volumetric air-fuel ratio, the duct length of combustion side, the shape of burner port and the inlet throttle, but little by the outlet throttle.

Nomenclature

A cross-sectional area
b wall thickness of burner port
c_p specific heat at constant pressure
c_v specific heat at constant volume
D non-dimensional throttle resistance
d hole diameter of the secondary perforated plate
f natural frequency $=\frac{1}{2\pi}\sqrt{\frac{Z}{X}}$
H heat release rate
h_0 lower calorific value of propane gas
K coefficient of fluctuating heat release rate
L duct length
m mass flow rate per unit area
M mass of fluid per unit area
P pressure
R throttle registance
T absolute temperature
t time
U flow velocity
γ coefficient ratio of viscous damping $=\frac{Y}{2\sqrt{XZ}}$
η over-all combustion efficiency
ρ density of fluid
μ resonance frequency $=f\sqrt{1-2\gamma^2}$
ν damped natural frequency $=f\sqrt{1-\gamma^2}$
φ primary volumetric air-fuel ratio
ψ throttle ratio of duct outlet
ω forced angular frequency

Subscripts

1 primary plenum chamber
2 secondary plenum chamber
3 duct
4 at the primary throttle
5 at the secondary throttle
6 at the outlet of duct
0 at the reference state
a air
b mean value on the burned side
f fuel
t for the total duct length
u mean value on the unburned side

Superscripts

$-$ mean value
$\sim$ first order fluctuating component
$\wedge$ amplitude

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