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Generalized Analytical Program of Thyristor Phase Control Circuit with Series and Parallel Resonance Load

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Synopsis

The systematic analytical method is required for the ac phase control circuit by means of an inverse parallel thyristor pair which has a series and parallel L-C resonant load, because the phase control action causes abnormal and interesting phenomena, such as an extreme increase of voltage and current, an unique increase and decrease of contained higher harmonics, and a wide variation of power factor, etc.

In this paper, the program for the analysis of the thyristor phase control circuit with a series and parallel connected load of series R-L-C circuit units, is been developed. By means of the program, the transient and steady state characteristics of the circuit can be calculated and then comparative study of various versions of circuits can be carried out systematically. The usefulness of the program is demonstrated by some numerical calculated examples.

1. Introduction

The ac phase control circuit with an inverse parallel thyristor

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pair has various load configurations such as a series R-L-C circuit, a parallel circuit of R, L, C elements and the combination circuit, etc., as the applications of the phase control circuit have been increased. In the L-C resonance circuit, the behavior form of reactive power becomes complicated. Then, the ac phase control circuit represents at times abnormal and/or interesting phenomena, such as an extreme increase of voltage and current, an unique increase and decrease of higher harmonics, and a wide variation of power factor, etc. The circuit analysis have been reported on the series R-L-C load[1)2)] and the combination load[3)4)], to clarify above-mensioned phenomena. But the each paper intends to analyze only a special circuit, and there is not any paper of systematic comparative study.

For the optimal design of circuit, it is necessary to determine not only circuit's constants but also a circuit's configuration. In this case, the use of a digital computer may be most practical. Already, the simulation programs of a thyristor circuit have been reported and an electronic circuit analysis program may be also used[5)6)]. However, these programs require relatively large memory capacity and long calculating times, because they are made for a general purpose and not exclusive for the analysis and design of an inverse parallel thyristor circuit.

Thus, the authors have developed an exclusive, generalized analytical program of the thyristor phase control circuit with series and parallel resonance load[7)8)].

The advantages of this program are as follows:

(1) The above-mentioned R-L-C series and/or parallel connected circuit is preliminaly setted in the program, which is contrived that the circuit is reduced to the configuration, wanted to analyze by the input data. Therefore, the input data are relatively fewer and calculating times shorten.

(2) It is contrived that the phase difference between load voltage and current of various configurations can be determined automatically. Then, the limits of the thyristor phase control angle can be obtained and the whole characteristics on control can be easily calculated.

(3) A three dimensional vector, representing the classification of loads, circuit elements of state variables and operation modes of circuit is introduced into the analytical program. The three dimensional vector makes the correspondence of state variables from a mode to the next mode easily.

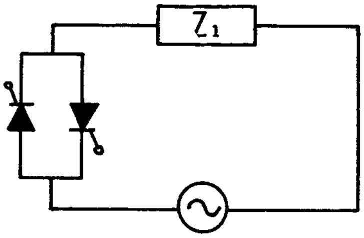
The numerical calculations are carried out using the state transition matrix obtained from a matrix form of differential equations, which is induced from the circuit by the graph theory.

2. Analytical method

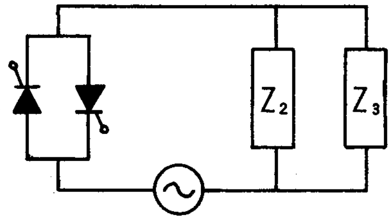
2.1 Circuits

There are many load forms controlled by an inverse parallel thyristor pair in a single phase ac power source systems such as series, parallel and series-parallel impedance shown in Fig.1. In this paper, we deal with the load of Fig.1(c). Fig.1(a) and Fig.1(b) may be derivered from the Fig.1(c).

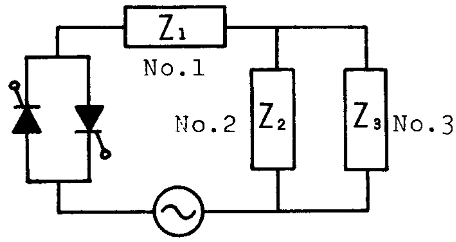
For a generalized example of the circuit of Fig.1(c), we have adopted the circuit of Fig.2 with series R-L-C elements as a series and parallel impedance. This circuit has 511 kinds of load form whether existence of



(a) Circuit with aseries load element.



(b) Circuit with parallel load elements.



(c) Circuit with series-parallel load elements.

Fig.1. Circuit.

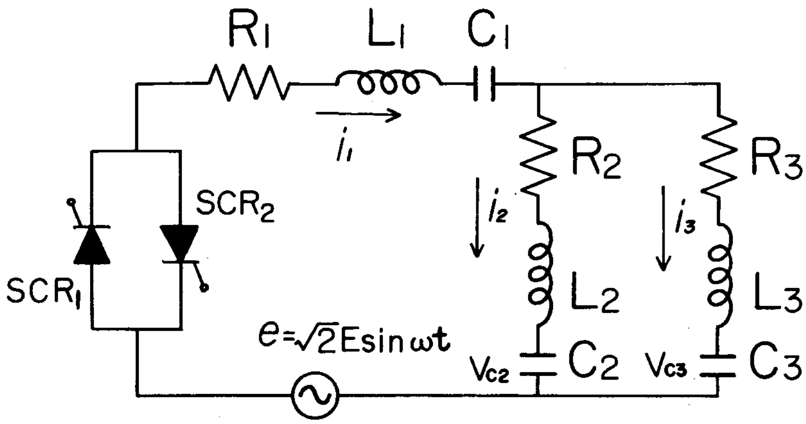


Fig.2. Thyristor phase control circuit with series-parallel R-L-C elements.

elements or not. For a practical circuit, there are 115 kinds considering next two conditions (see Appendix I)[9]:

(i) The reactance L has resistance R in usual.

(ii) The circuits containing only a capacitor or capacitors connected to the source are eliminated as inrush currents appear.

To analyze the circuit, we set up the following assumptions on Fig.2:

(a) The power supply produces a sinusoidal voltage and its internal impedance is zero. There is no fluctuation of its frequency and magnitude.

(b) The leakage current flows of thyristors are neither the forward nor the backward direction and no voltage drop appears in the forward direction.

(c) The turn-on and the turn-off times are negligibly small.

(d) Two thyristors are triggered alternately by applying the nallow triggering pulses of positive and negative half-waves.

The resistor and the capacitor have the linear voltage-current characteristics.

The circuit has two modes following to flow or not to flow of current through the thyristors. We state the conducting period as "mode I", and the non-conducting period as "mode II". The transitional conditions and initial values at each mode are given by inspection as follows.

[A] Transition from mode I to mode II.

Condition : $i_1(t_c)=0$, where t_c is the periods of mode I.

Initial values : The initial values of v_{c2}, v_{c3}, i_2 and i_3 at mode II are equal to the last values of the mode I respectively.

[B] Transition from mode II to mode I.

Condition : The time t_e of mode II is equal to $(1/120-t_c)$.

Initial values : The value of i_1 is equal to zero, and the initial values of v_{c2}, v_{c3}, i_2 and i_3 are equal to the last values of the mode II respectively. Where, we initiate the original time at starting point of each mode.

2.2 Construction of a graph from a circuit

We will make a graph from a circuit numbering the branch and the node of the circuit. The numbers of branches are set from voltage sources, capacitors, resistors, thyristors and inductors in order. The numbers of nodes are set at will. A graph of the circuit of Fig.2 is shown in Fig.3. It's constructed above-mentioned procedure. The graph

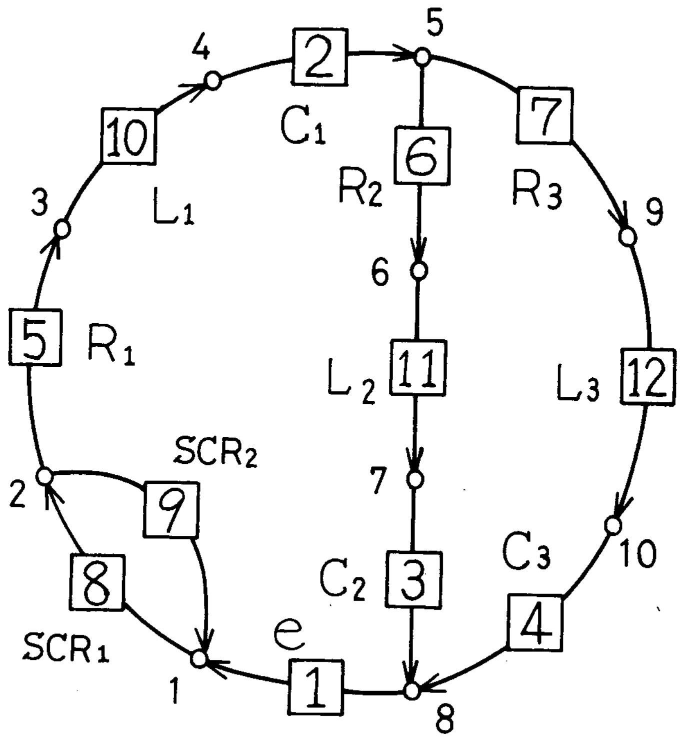


Fig.3. Graph of the circuit.

may be reduced to a circuit which is needed to analysis by a method shown later. Based on the reduced graph, the circuit's connective matrix $\mathbb{A}_m$, modified connective matrix $\mathbb{A}_{am}$, and a normal tree are determined. In the circuit of Fig.3, the branch 1~10 is "normal tree", and branch 11 12 is "link". Where, "Normal Tree" is a tree, proposed by P.R. Bryant[10], containing all voltage sources, no current source, and as many capacitors as possible.

2.3 Derivation of a standard differential equation with a matrix form and the solution

The matrixes $\mathbb{A}$ and $\mathbb{B}$ of a standard differential equation

$$\dot{x}(t) = \mathbb{A}x(t) + \mathbb{B}u(t) \qquad (1)$$

are derived from the fundamental cut-set matrix Q_f[11][12]

$$Q_f = A_{f1}^{-1} \cdot A_f \tag{2}$$

Where, A_f is a matrix which is eliminated a low of fundamental node (node of the largest number) from a connected matrix A_{am} of modified circuit. The partial matrix A_{f1} is gained, by dividing A_f into a partial matrixes A_{f1} and A_{f2} corresponding to a normal tree and a link, respectively.

Usually, all the voltages across the capacitors C and all the currents through the reactors L are chosen as state variables of vector x. Here, however, we will chose the state variables to capacitor's voltages contained only normal tree and reactor's current contained a link. This may be possible to reduce the rank of state variables. As the input voltage source of a thyristor phase control circuit is sinusoidal wave, we set the input vector U as

$$U = U_0 E_m \sin(\omega t+\phi) \tag{3}$$

where, U_0 is a vector with the value of ±1, 0 obtained from tie-set. The time domain solution of a differential equation of standard type is as follows,

$$x(t) = e^{At}[x_0+(A^2+\omega^2 I)^{-1}(A\sin\phi+\omega I\cos\phi)B_u] - (A^2+\omega^2 I)^{-1}[A\sin(\omega t+\phi)+\omega I\cos(\omega t+\phi)]B_u \tag{4}$$

$$x_0 = x(0) \tag{5}$$

where, I is a unity matrix and vector x_0 is an initial value at $t=0$[13].

The calculating times of Eqs.(4) (5) may be very numerous if the value of x is calculated of the step size H one after another and the mode change must be checked at every step. In order to save the calculating times, we choose H for $1<|A|\cdot H<10$ preliminary and calculate[14],

$$e^{AH} \doteqdot I + AH + \frac{A^2H^2}{2!} + \frac{A^3H^3}{3!} + --- + \frac{A^kH^k}{k!} \equiv \Phi \tag{6}$$

$$(A^2+\omega^2 I)^{-1} \equiv D \tag{7}$$

If we put

$$A\sin(\omega nH+\phi) + \omega I\cos(\omega nH+\phi) \equiv Y_n \tag{8}$$

then, the last term S_n in the right side of Eq.(4) becomes

$$S_n = D(Y_n(B_u)) \tag{9}$$

where, $t=nH$. On the other hand, the first term r_n is

$$r_n = \Phi^n(x_0+S_0) = \Phi\cdot r_{n-1} \tag{10}$$

The Eq.(10) can be calculated cyclically, if $r_1=\Phi(x_0+S_0)$ is gained preliminary. Therefore, Eq.(4) is represented as

$$x_n = r_n - S_n \tag{11}$$

and the calculating times are saved.

2.4 Digital computer program for circuits

Using the above mentioned analytical method, we composed of a program for a digital computer. In this section, are described the flow-chart, the items to be attention and a generalized method for different configurations of load circuit.

2.4.1 Reduction of the input data of circuit

Input data into a digital computer are branch number(BN), starting node number of the branch(NF), arriving node number(NT), constants to classify the element(MT), and constants to classify the load(MF). Usually, these data must be punched on a card. In this method, however, so many cards are necessary when many circuits are analyzed at same time. Therefore, we developed a new procedure to reduce a circuit with series and parallel R-L-C elements of Fig.3, into a circuit which wanted to analyze, using preliminary input constant ME. The ME represents connective state of branches. On this way, the data cards decrease only two to represent state ME and constant values VC.

The state of branch is represented by the value of ME as follows:

ME=0; open branch,

ME=1; branch of normal state,

ME=2; branch to be reduced.

The reduction procedure of circuit using the ME is shown subsequently

(i) To gain a matrix which is constructed from a circuit to reduce branches for ME=2.

(ii) To gain a branch M whose node number from start to arrival is minimum in the all reduced branches.

(iii) To rearrange the node number of the branch M from larger to smaller sequentially from the start to the arrival node.

(iv) To repeat the procedure (ii) and (iii) for all the branches.

(v) To remove the input data NB, NF, NT, MT, MF, ME corresponding to the branch which must be reduced.

(vi) To renumber the discontinuous node number continuously.

2.4.2 The angle of displacement between current and voltage at a load

The extent of the control angle in a thyristor phase control circuit is gained from the displacement angle between current and voltage waveform when sinusoidal voltage is applied to the load. The phase angle is induced with a impedance Z_t of the load. The Z_t, however, has various forms by the configuration of load in Fig.1. In order to treat the equation of the displacement angle unification, we introduce a contrivance as follow:

Substituting the constants C_i, R_i, L_i to the vector $\mathbb{C}(i)$, $\mathbb{R}(i)$, $\mathbb{L}(i)$ as elements, where i=1, 2, 3. The impedance $\mathbb{Z}(i)$ of the ith load is given as

$$\mathbb{Z}(i) = \mathbb{R}(i) + j(\omega\mathbb{L}(i) - \frac{1}{\omega\mathbb{C}(i)}) \qquad (12)$$

The impedance of load can be generally represented as Eq.(13) by using the $Q(i)$=0 when $Z(i)$ is a zero vector and $Q(i)$=1 when $Z(i)$ is not a zero vector.

$$Z_t = Q(1)\cdot Z(1)+(Q(2)+Q(3)-Q(2)\cdot Q(3)) \Big/ \left(\frac{Q(2)}{Z(2)} + \frac{Q(3)}{Z(3)} \right) \qquad (13)$$

If Z(2)=0, Z(3)=0, or Z(2)=Z(3)=0, then the denominator of the Eq.(13) became zero. In such cases there are no problems if the equation is dealed as 0/0=0. The angle of displacement between current and voltage in the load is given as

$$\phi = \tan^{-1}(\text{the imaginary part of } Z_t/\text{the real part of } Z_t). \quad (14)$$

2.4.3 Correspondence of state variables at mode transition

Branches of the state variables, or numbers of state variable become different as the load forms are changed. Then, we considered the correspondence of state variables based on the definition of a matrix NFUKA (3×2×2) as follows:

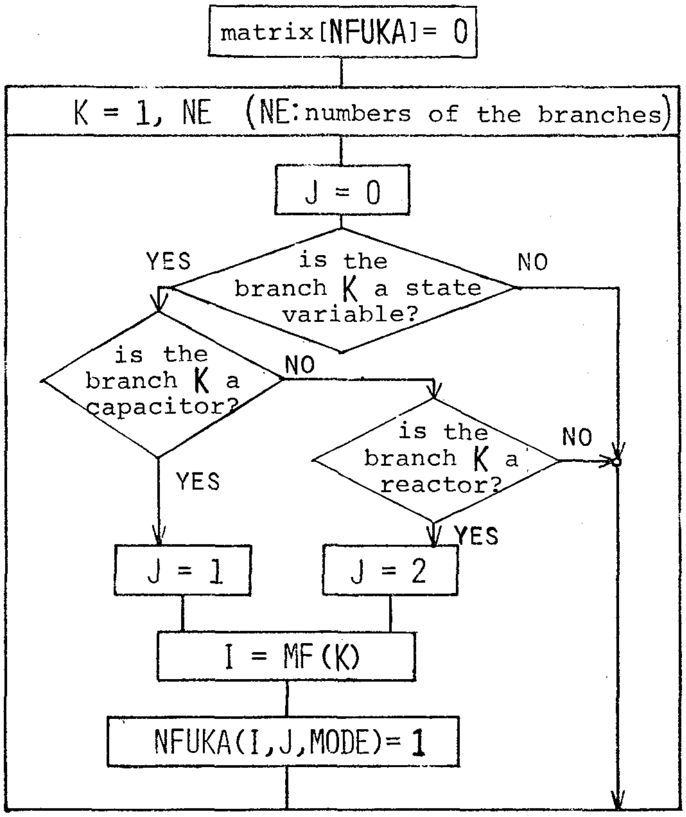


Fig.4. Flow-chart of matrix "NFUKA".

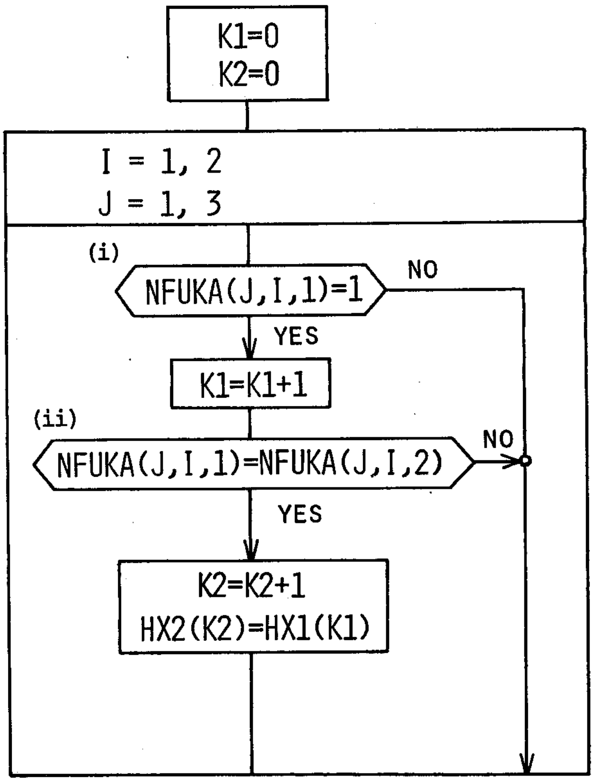
The row of the matrix is corresponded to the load classification, that is, the first row represents the first load, the second row represents the second load, the third row represents the third load. The rank of the matrix is corresponding to the classification of elements which become state variables, that is, the first rank is a capacitor of a normal tree, the second is a reactor of a link. The height of the matrix is corresponding to a mode, that is,

the first height is a mode I,

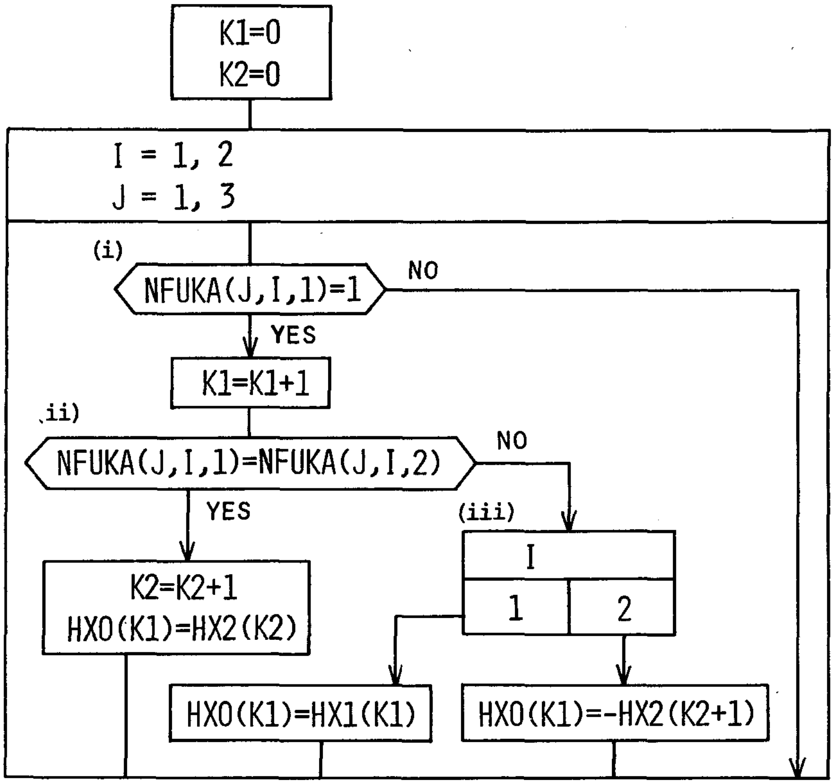
the second is a mode II .

For example, the element "(3,2,1)" represents a reactance element of the link of the third load at a mode I, and the value of the array is one when the element is in existence, and zero when the element is not in existence. The flow charts to gain the matrix [NFUKA] and to represent the correspondence of a state vector at a mode change are showed in Figs.4 and 5, respectively.

In the flow chart of Fig.5, the judgement of "yes" at (i)



(a) From mode I to mode II .



(b) From mode II to mode I.

Fig.5. Flow chart of transition of state variable vectors.

represents being a state variable, and "yes" at (ii) represents that the correspondence of a state variable vector has been completed. I=1 at (iii) of Fig.5 means that the voltage value of a normal tree of mode I remains to mode II if the first load has a capacitor of a normal tree in a series and parallel load's form. And I=2 means that the current through the second load at a mode I is gained from the third load at a mode II if the each second and third load has a reactor of a link. The HX0, HX1 and HX2 in Fig.5 represent the values of a vector x at initial, mode I, and mode II, respectively. A general flow chart is shown in Fig.6 for the generalized anaytical program to a thyristor phase control circuit using the above mentioned method. In the practical program, the data of to be or not to be is

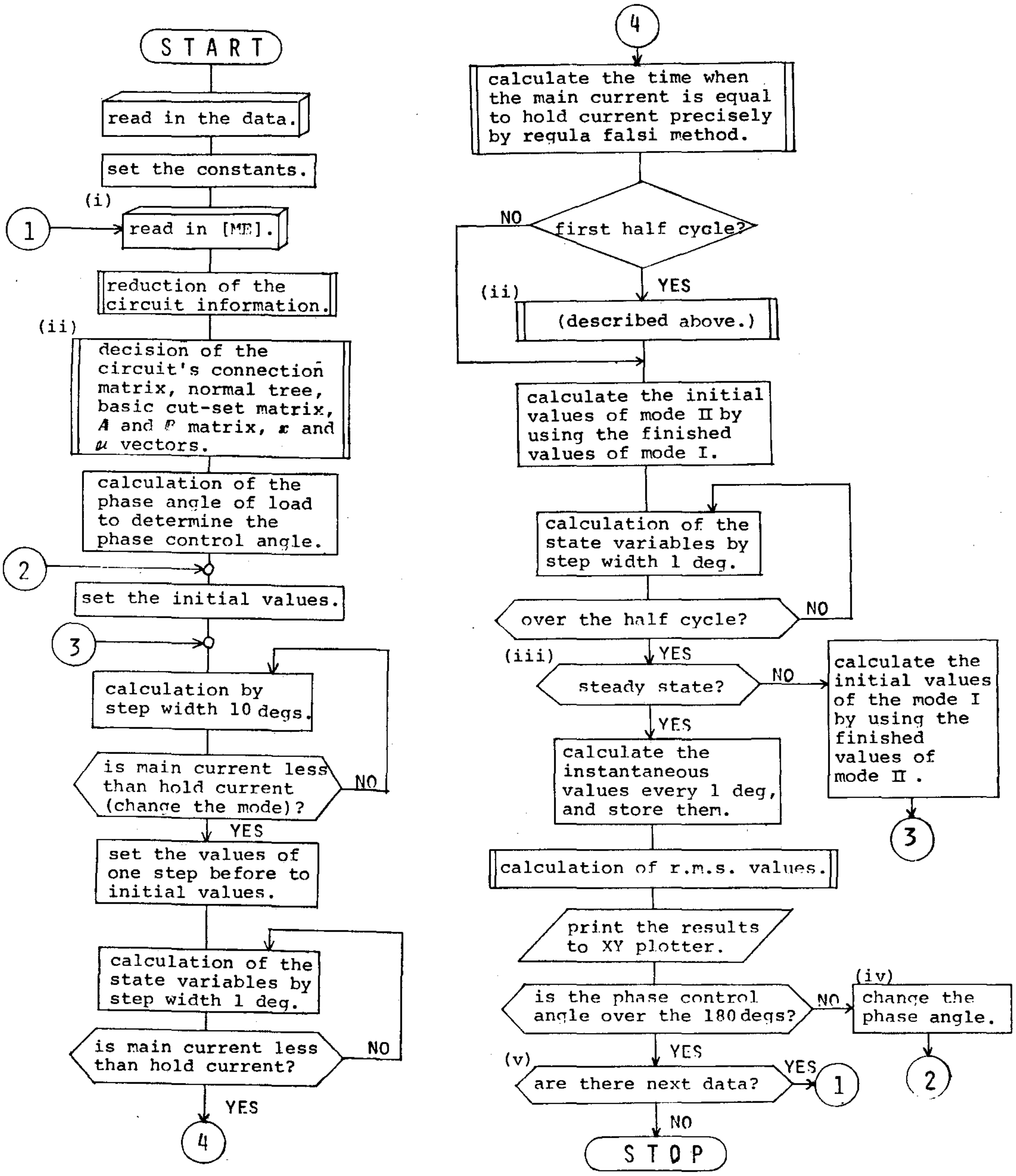


Fig.6. Flow chart of analytical program.

judged from the input data of (i), and the matrixes $\mathbb{A}$, $\mathbb{B}$ at (ii) are gained before the calcualtion of the mode II. The calculation is carried out step by step, and a steady state is regard if the difference between the calculated values of n and n+1 cycles becomes less than 1%. The angle of a phase control at (iv) in Fig.6 is changed by 10 degs.

For a program language, FORTRAN IV is used and the statement cards are about 1500 sheets.

3. Application of the analytical program

3.1 Comparison of the calculated with the measured

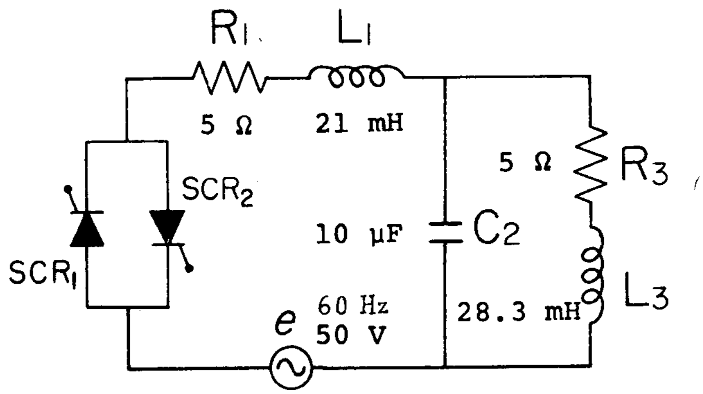


Fig.7. Example circuit with series parallel load elements.

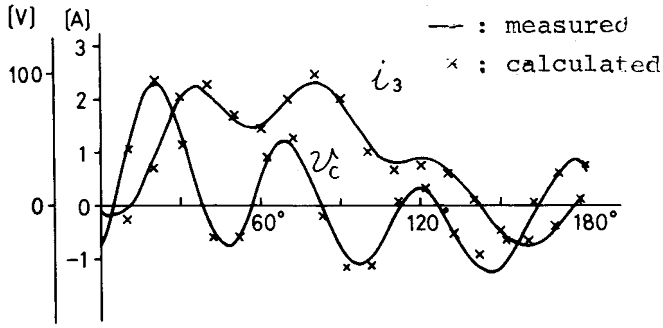
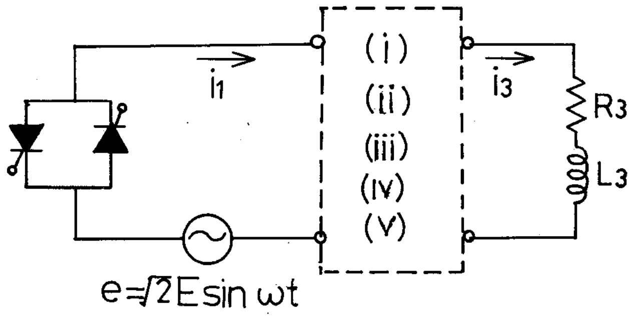


Fig.8. Comparison with analytical results and experimental ones.

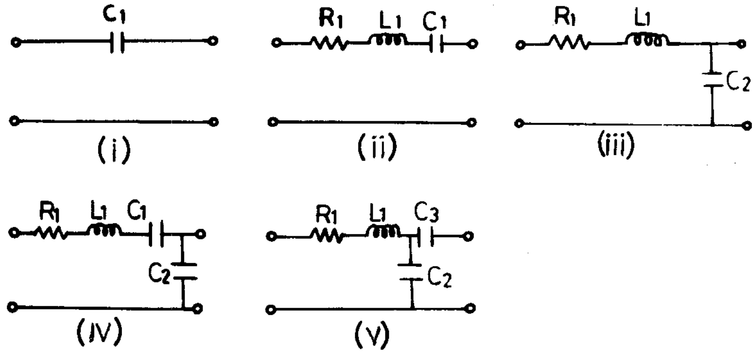
Fig.7 shows a calculation example of a circuit with a series and parallel R-L-C load. The measured i_3, v_c and the calculated ones are compared in Fig.8, when a phase control angle $\alpha=100^{0}$. Both the calculated and the measured agrees comparatively good, then the propriety of the analytical program may be accepted. The numerical calculations have been carried out by using the computer NEAC 2200/500 of Okayama University Computer Center. The used memories are about 130KC, and the CPU times are about five minutes contained compile and linkload times.

3.2 Application of the program to filter circuits

The calculation is carried out to the circuit of Fig.9(a) which



(a) Basic circuit.



(b) Filters for insertion.

Fig.9. Circuit for analysis.

are to inserted the filter circuit of Fig.9(b) into enclosed with a dotted line. The values of the constants are shown in Table 1.

Table 1. Circuit constants.

E(V)	$R_1(\Omega)$	L_1(mH)	$C_1(\mu F)$	$C_2(\mu F)$	$R_3(\Omega)$	L_3(mH)	$C_3(\mu F)$
100.0	2.83	25.0	25.0	10.0	20.0	50.6	28.0

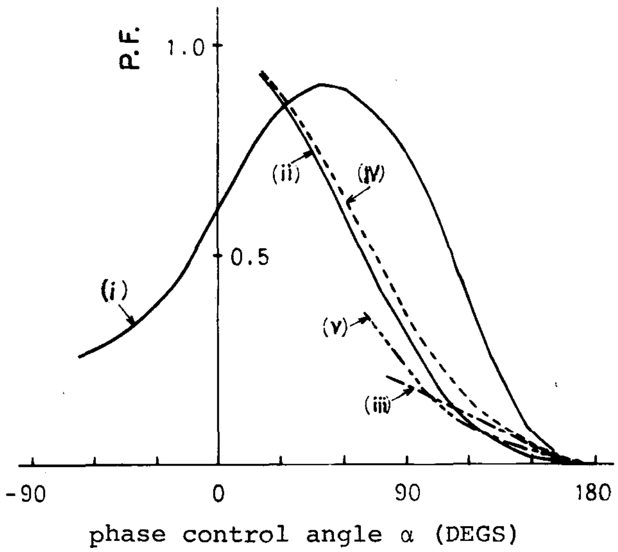


Fig.10. Power factor versus phase control angle α.

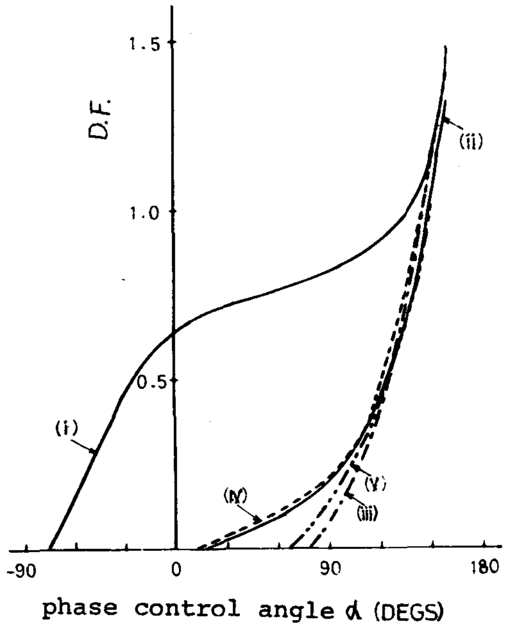


Fig.11. Distortion factor versus phase control angle α.

Fig.10 shows the power factor of the load. From this, the power factor is good at the (B) (C) (E) of Fig.9(b). Fig.11 shows distortion factors of current I_1. The distortion factor of only the circuit (B) represents a different tendency of the others. Fig.12 shows a power P_{R3} at the resistance R_3, and could be understood that the power dissipation is larger in circuits of (B), (C) and (E). Fig.13 shows a peak voltage across the thyristor. The maximum value appears at

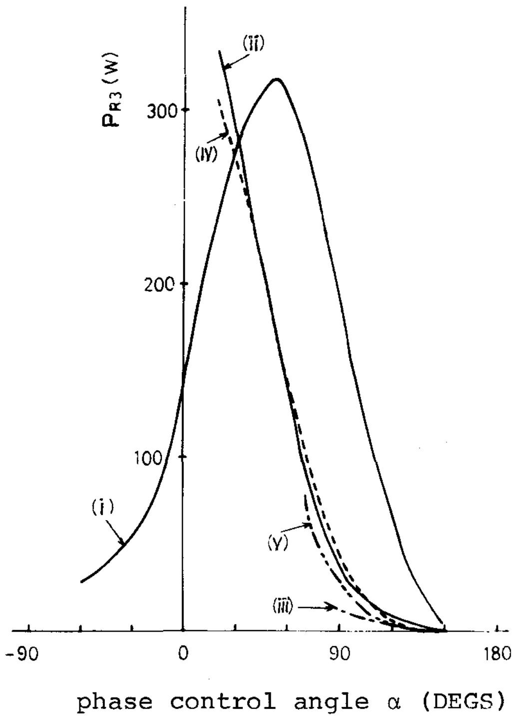


Fig.12. Power P_{R3} versus phase control angle α.

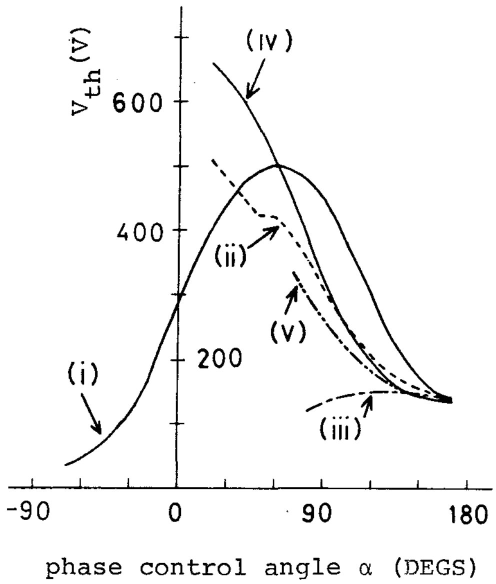


Fig.13. Thyristor voltage versus phase control angle α.

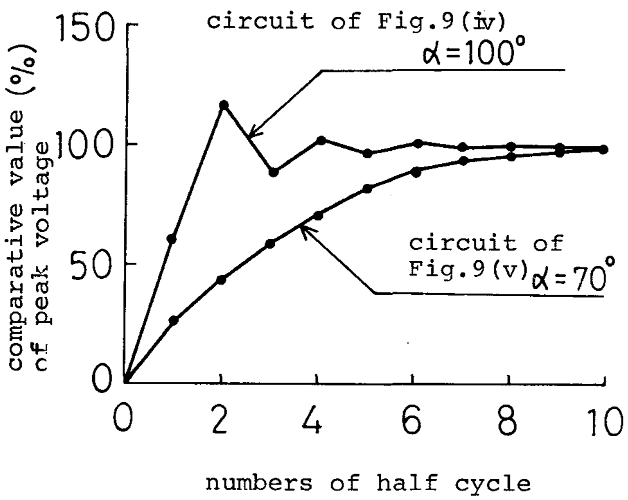


Fig.14. Transient phenomenon.

the circuit of (C).

Fig.14 represents numerical examples of transient phenomenon of the circuits. It shows that the reponse curves for time are exponential or oscillatory.

From the above examples, it clears that a comparative study on the circuit behavior of different configurations is possible easily by using this program.

4. Conclusion

The generalized analytical program, developed by authors, of the thyristor phase control circuit with a series and parallel load and its numerical examples has been described. The features of this program are summarized as follows:

(1) As a series and parallel connected circuit of R-L-C series elements is included in the program, the partial reconstruction and the modification of the load are easy by changing the input data.

(2) Consequently, comparative studies of the circuits can be carried out easily as one pleases.

(3) The required input data to computer are minimized, such as branches of the circuit, node numbers and sorts of elements, etc.

From these input data, the circuit's configuration and a standard type of differential equation with a state variable vector are automatically determined based on the graph theory, and the characteristics about input and output power, current and voltage in each elements are calculated.

(4) The program is written by FORTRAN IV and the statement cards are about 1500 sheets. The CPU times of the computer are about a few minutes.

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References

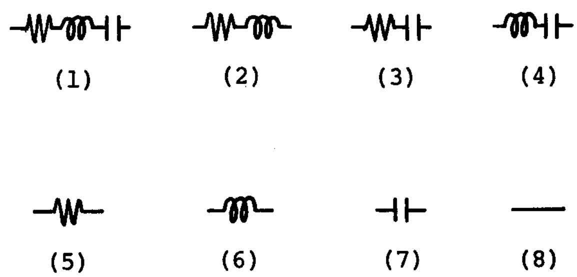
1. T. Himei, et al.:"Steady state characteristics of the thyristor control circuit with a series RLC load," J.I.E.E., Vol.90, No.1, pp.135-144, Jan. 1970 (in Japanese).
2. T. Himei, et al.:"A Step-up Phenomenon of the Thyristor Control Circuit with Series RLC Element," IEEE Trans. Ind. Appl., Vol.IA-11, No.5, p.531, 1975.
3. J. Takeuchi:"Analysis of Single-Phase Back-to-Back SCR Circuit of Inductive-Load Having Condenser," J.I.E.E., Vol.86, No.10, pp.1691-1696, Oct. 1966 (in Japanese).
4. EL-Sayed EL-Bedweihi, et al.:"SCR Control of Power for Induction Melting," IEEE Trans. Ind. Appl., Vol.IA-10, No.4, p.501, 1974.
5. F. F. Kuo & G. Magnuson, Jr. ed:"Computer Oriented Circuit Design," Prentice Hall, 1969.
6. Y. Murakami, et al.:"Simulation Program for Thyristor Circuits and Its Application," J.I.E.E., Vol.91, No.7, p.1301, July 1971 (in Japanese).
7. T. Himei, et el.:"On Analytical Program of the Thyristor Phase Control Circuit with a Series and Parallel Load," 1975-Chugoku Branch Joint Conv. Four Insts. Elec. Engrs. of Jap., No.20501.
8. T. Himei, et al.:"Analytical Program of the Thyristor Phase Control Circuit with a Series and Parallel Load and it's Application," 1976-Chugoku Branch Joint Conv. Four Insts. Elec. Engrs. of Jap., No.50517.
9. T. Himei, et al.:"Classification of the Thyristor Phase Control Circuit with a Series and Parallel Load," 1975-Chugoku Branch Joint Conv. Four Insts. Elec. Engrs. of Jap., No.20505.
10. P. R. Bryant:"The Explicit Form of Bashkow's A Matrix," IRE Trans., Vol.CT-9, p.303, 1962.
11. S. Seshm, et al.:"Linear Graph and Electrical Networks," Addison-Wesley, 1961.
12. E. S. Kuh, et al.:"The State-Variable Approach to Nerwork Analysis, Proc. IEEE, Vol.53, p.672, 1965.
13. M. L. Liou:"Closed-Form Solutions of Linear Stationary State Equations," Proc. IEEE, Vol.55, p.1244, 1967.
14. M. L. Liou:"Evaluation of transition matrix," Proc. IEEE, Vol.55, No.2, p.228, 1967.
15. T. Himei, et al.:"Generalized Analytical Program of Thyristor Phase Control Circuit with Series and Parallel Load," Papers of

Technical Meeting on Information Processing, IEEJ No.IP-77-29, 1977.

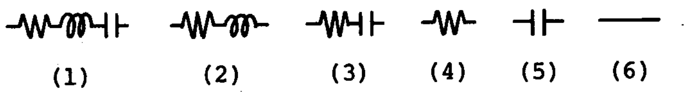
Appendix 1. Classification of load circuit forms

The configurations of circuits, gained by reduction from a series R-L-C unit, are 8 kinds as shown in app.Fig.1. The combinations of these circuits are 8×8×8-1=511 kinds. Where, the circuit is eliminated when all elements are short circuit with (8) in app.Fig.1. Considering the condition §2.1 (i), the series loads reduced from a series R-L-C unit are 6 circuits shown in app.Fig.2 and the combinations of the circuits are 6×6=36 kinds. When the load (5) and (6) in app.Fig.2 are constructed one series load, the circuit forms are 27 kinds by considering the condition §2.1 (ii) . The numbers of circuit configurations are 36×4+27+26=197 kinds in all.

On the other hand, the parallel loads have 21×4+16+5=115 kinds of configurations in the same way as the series load provided that the circuit configuration is identical if the parallel loads Z_2 and Z_3 in Fig.1 are interchanged the positions each other.



app. Fig.1. Combination of elements.



app. Fig.2. Combination of elements for practical load.