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Analysis of the A-C Voltage Control Circuit with Parallel Connection of SCR and Reactor*

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The a-c voltage control circuit, composed of parallel connection of a SCR and a reactor, gives the similar performance as the control circuit of a back to back SCR pair. However, the control characteristics of this circuit is affected by magnetizing characteristics of reactor. In this paper, the circuit with the resistive load has been analyzed by using the approximated models of hysteresis loop of core materials, where the SCR's forward voltage drop is taken into account.

The results of the analysis have revealed the facts as follows,

1) The control characteristics of the a-c voltage in average value is independent on the magnetizing characteristics of reactors except in the vicinity of minimum output point. But, in effective value, it is not valid.

2) A SCR's forward voltage drop results in the reduction of not only SCR's current but also reactor's current.

3) The a-c output current does not include the d-c component, if the winding resistance of the reactor is negligible.

§ 1. Introduction

The silicon controlled rectifier (*SCR*) is a sort of synchronous switch and the circuit of a back to back *SCR* pair in parallel is an a-c voltage control circuit. On the other hand, the circuit with saturable reactors *i.e.*, a magnetic amplifier, is too a similar a-c voltage control circuit. The circuit introduced in this paper is a new type of the a-c voltage control circuit that, though composed of parallel connection of a *SCR* and a reactor, has the similar control characteristics.

Here, in order to investigate the dependence of the control characteristics upon the magnetizing characteristics of reactor, the circuit with the resistive load has been analyzed by using the four models of hysteresis loop, where the *SCR*'s voltage drop in conducting is taken into account.

§ 2. Performance of the Circuit

The circuit is shown in Fig. 1. To illustrate the performance of the circuit, it is assumed that the reactor has the ideal magnetizing characteristics, as shown in Fig. 3 (a). First, in the half-cycle in which the a-c source voltage applies across the *SCR* in forward direction, until the *SCR* is fired by the gate input, the full voltage appears across the reactor and the flux level of reactor will be moved toward the positive saturation level. In this period, the load current is negligible. Once, the *SCR* is turned on, the load current is flowed through the *SCR* and the source voltage e_s appears across the load, and the flux level of reactor stays still since the reactor voltage is negligible.

Second, in the succeeding half cycle, the source voltage e_s in turn, appears across the *SCR* in the backward direction and the load

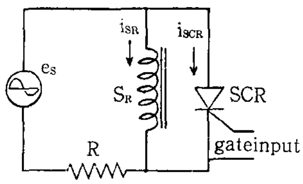

Fig. 1, Circuit diagram.

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current is blocked. In turn, the source voltage across the reactor cause the flux level to move toward the negative saturation. When the flux level of the reactor reaches to the negative saturation level, the load current begins to flow through the reactor in the reverse direction.

Since, in steady state, the flux change of the reactor during one half cycle, must be equal that during the succeeding one half cycle, the following equation will be valid.

$$\frac{1}{N\omega}\int_0^{\alpha_1} e_s\, d(\omega t) + \frac{1}{N\omega}\int_\pi^{\alpha_2} e_s\, d(\omega t) = 0 \qquad (1)$$

where N: number of turns of reactor.
ω: angular velocity of source voltage.
α_1: firing angle of SCR
α_2: firing angle of reactor.
e_s: source voltage in instantaneous value.

From Eq. 1, the following relation between α_1 and α_2 may be obtained.

$$\alpha_1 + \pi = \alpha_2 \qquad (2)$$

From the last equation, it is clear that α_2 depend on α_1 which is controlled by the gate input to the SCR.

§ 3. Analysis of the Circuit

3.1 The assumptions. To simplify the circuit analysis, the following assumptions are introduced.

1) The leakage current of the SCR is negligible and the SCR has the constant voltage drop E_{SCR} when conducting.

2) The source voltage e_s is expressed by Eq. (3).

$$e_s = E_m \sin \omega t \qquad (3)$$

3) The resistance of windings of reactor is negligible.

4) The load is pure resistance.

5) The approximated magnetizing characteristics of reactor are indicated in Fig. 3(a), (b), (c) and (d).

3.2 Equivalent circuits. The modes of operation in a cycle may be classified into (A), (B), (C) and (D), as shown in Table 1. The equivalent circuits corresponding to each modes are shown in Fig. 2.

3.3 Boundary conditions and behaviors of modes of operation. The four approximated models of B-H loops of the reactor utilized in this analysis are shown in Fig. 3 and the wave

Table 1 Classification of modes of operation.

reactor \ SCR	conducting	nonconducting	
		forward blocking	backward blocking
unsaturated	B	A	C
saturated		D	

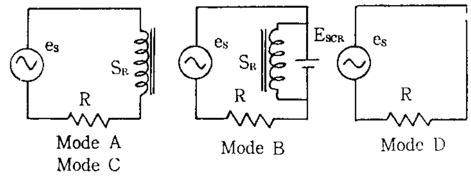

Fig. 2 Classification of circuit modes.

forms of output voltage e_R and reactor voltage e_{SR} are schematically illustrated too.

Here, the following analysis will be carried out only with regard to the B-H loop in Fig. 3(d), because the analysis of the others in Fig. 3, (a), (b) and (c), is contained in this example. Mode D_1 of operation; When the phase angle of the source voltage is zero, i. e., $\omega t=0$, the operating point of reactor is supposed to be on the point (0), the intersection of the negative saturation level $-\Phi_s$ and the vertical axis. The period from point (0) to point (1) is mode D.

The phase angle of source voltage $\omega t=\beta_1$ when the operating point comes to the point (1) is given by

$$\beta_1 = \sin^{-1} I_{01} R/E_m \qquad (4)$$

Mode A of operation; At $\omega t=\beta_1$, the mode A of operation starts, because the reactor becomes unsaturated and the SCR is blocked in forward direction. The load current i_R is expressed as follows.

$$i_R = \frac{E_m}{\sqrt{R^2+\omega^2L^2}}\Big\{\sin(\omega t-\theta) - \varepsilon^{-\frac{R}{L}\left(t-\frac{\beta_1}{\omega}\right)}\sin(\beta_1-\theta)\Big\} + I_{01}\,\varepsilon^{-\frac{R}{L}\left(t-\frac{\beta_1}{\omega}\right)} \qquad (5)$$

And also the reactor voltage e_{SR} is given by

$$e_{SR} = L\frac{di_R}{dt} = L\Big[\frac{E_m}{\sqrt{R^2+\omega^2L^2}}\Big\{\omega\cos(\omega t-\theta) + \frac{R}{L}\varepsilon^{-\frac{R}{L}\left(t-\frac{\beta_1}{\omega}\right)}\sin(\beta_1-\theta)\Big\} - \frac{R}{L}I_{01}\,\varepsilon^{-\frac{R}{L}\left(t-\frac{\beta_1}{\omega}\right)}\Big] \qquad (6)$$

where L: inductance of reactor.
θ: arc tan $\omega L/R$.

Mode A of operation continues until the

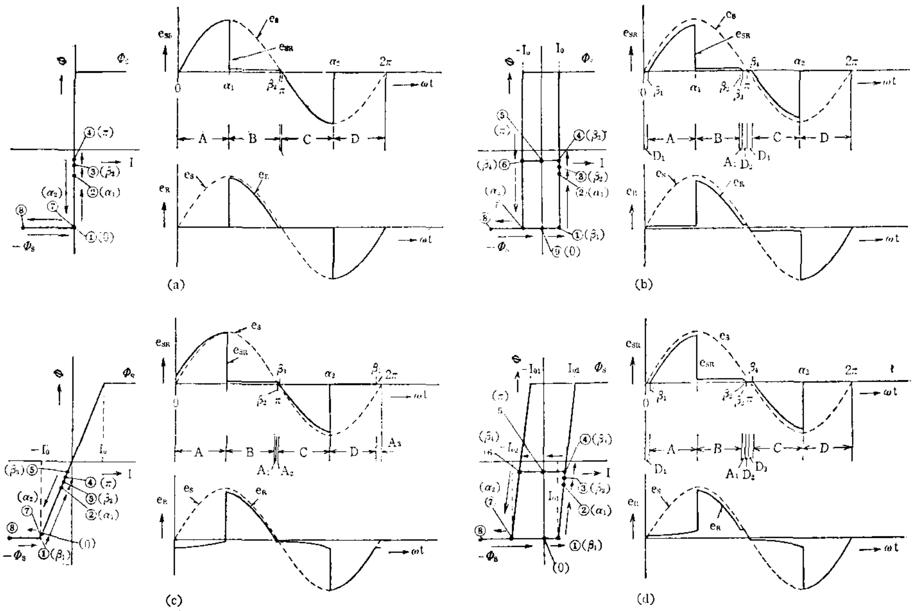

Fig. 3 Models of approximated B-H loops, reactor and load voltage waveforms.

point (2), α_1, when the *SCR* turns on by the gate input. The flux change Φ_1 (the flux resetting value) from the point (1) to (2) is

$$\Phi_r = \frac{1}{N\omega}\int_{\beta_1}^{\alpha_1} e_s d(\omega t) \tag{7}$$

The correlation of the current and flux gives the flux resetting value as follows.

$$\Phi_r = \frac{2\Phi_s}{I_{02}-I_{01}}(i_{SR1}-I_{01}) \tag{8}$$

where i_{SR1}: Reactor current at α_1.

Mode *B* of operation; After the *SCR* turns on, the circuit condition is mode *B*. The load voltage e_R, the reactor voltage e_{SR} and the reactor current i_{SR} are given respectively.

$$e_R = e_s - E_{SCR} \tag{9}$$

$$e_{SR} = E_{SCR} \tag{10}$$

$$i_{SR} = \frac{E_{SCR}}{L}\left(t - \frac{\alpha_1}{\omega}\right) + i_{SRI} \tag{11}$$

The phase angle $\omega t = \beta_2$, when the *SCR* turns off, is obtained from the relation of Ep. (12)

$$\left.\begin{aligned} &i_{SCR} = i_R - i_{SR1} = 0 \\ &\frac{E_m}{R}\sin\omega t = \frac{E_{SCR}}{L}\left(t - \frac{\alpha_1}{\omega}\right) + \frac{E_{SCR}}{R} + i_{SR1} \end{aligned}\right\} \tag{12}$$

The flux resetting value Φ'_r in the duration of mode *B* of operation is

$$\Phi'_r = \frac{E_{SCR}}{N\omega}(\beta_2 - \alpha_1) \tag{13}$$

Mode A_1 of operation; After β_2, the circuit condition is mode A_1 of operation similar to mode *A* of operation. The load current i_R and the reactor voltage e_{SR} are given in Eq. s (5) and (6) by substituting β_2 for β_1 and also i_{SR2} at β_2 for I_{01}. The boundary point β_3 between this mode and the succeeding, is determined by $e_{SR} = 0$. The operating point of reactor is then at (4) in Fig. 3(d).

Mode D_2 and D_3 of operation; After β_3, the operating point moves into the inner of the B-H loop. At $\omega t = \pi$, the operating point is on the vertical axis and at $\omega t = \beta_4$, reaches to the left side of the B-H loop. The duration from β_3 to π corresponds to mode D_2, and the duration from π to β_4 does to mode D_3. β_4 is as follows.

$$\beta_4 = \sin^{-1}\frac{R}{E_m}(i_{SR3} - I_{01} - I_{02}) \tag{14}$$

where i_{SR3}: reactor current at β_3.

Mode *C* of operation; After β_4, the circuit condition is mode *C* of operation. The load current i_R and the reactor voltage are expressed in Eq. (5) and (6) by substituting β_4 for β_1 and i_{SR4} for I_{01}.

At the phase angle α_2 when the reactor is saturated, the relation $i_R = -I_{02}$ is valid. Further more, since the total flux changes over one cycle in steady state is zero, α_2 is given by

$$\Phi_r + \Phi'_r + \Phi''_r + \Phi_g = 0 \tag{15}$$

where Φ_r'' : Flux resetting value in mode A_1 of operation.

Φ_g : Flux gating value in mode C of operation, *i.e.*, "flux gating value" means the flux change in the direction from $+\Phi_s$ to $-\Phi_s$.

Mode D of operation; After α_2, the circuit condition is mode D of operation. This mode continues until $\omega t=2\pi$. The load current i_R is e_s/R.

3.4 Control characteristics. As above mentioned, since boundary conditions and behaviors of modes of operation over one cycle are determined for any firing angle α_1, the wave forms of load voltage and therefore the control characteristics are determined for α_1.

The above analysis of the circuit operation will be valid to other B-H loops in Fig. 3 by using the following expressions, that is, $I_{01}=I_{02}=0$ in Fig. 3(a), $I_{01}=I_{02}=I_0$ in Fig. 3(b), and $I_{01}=-I_{02}=-I'_0$ in Fig. 3(c).

3.5 Calculated. According to the analysis, an example has been calculated numerically. Circuit constants and boundary conditions, wave forms and control characteristics are shown in Table 2, Fig. 4 and Fig. 5 respectively.

Table 2 Circuit constants and boundary conditions.

I_{01}	-80 mA
I_{02}	120 mA
L	5 H
R	100
E_m	141.4 V
f	60
E_{SCR}	1 V
β_1	-3
α_1	$90°$
β_2	$180° - 0.56°$
β_3	$180° - 0.22°$
β_4	$190° + 1.47°$
α_2	$180° + 95°$

The calculated in the case of idealized B-H loop, is also shown with dotted line in Fig. 5 for comparison.

§ 4. Experiments results

In order to investigate experimentally the dependence of the control characteristics upon the magnetizing characteristics of reactors, the measurement are carried out used with four reactors, that is, Fe 50%—Ni 50% oriented

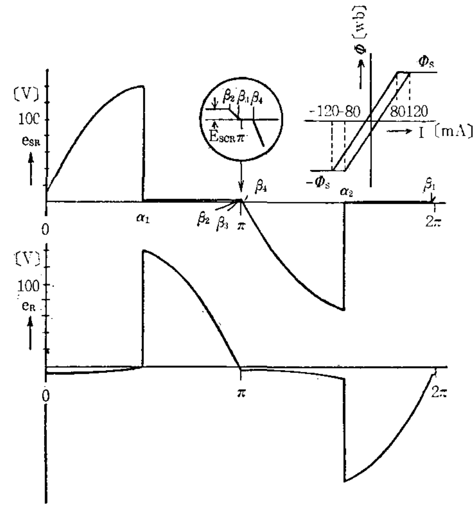

Fig. 4 Calculated waveforms of reactor voltage and load current.

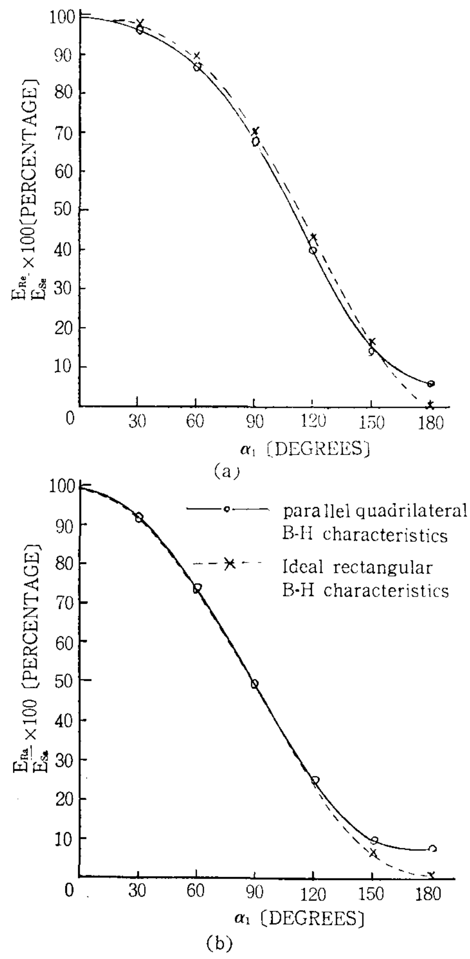

Fig. 5 Calculated control characteristics in effective value (a) and in average value (b).

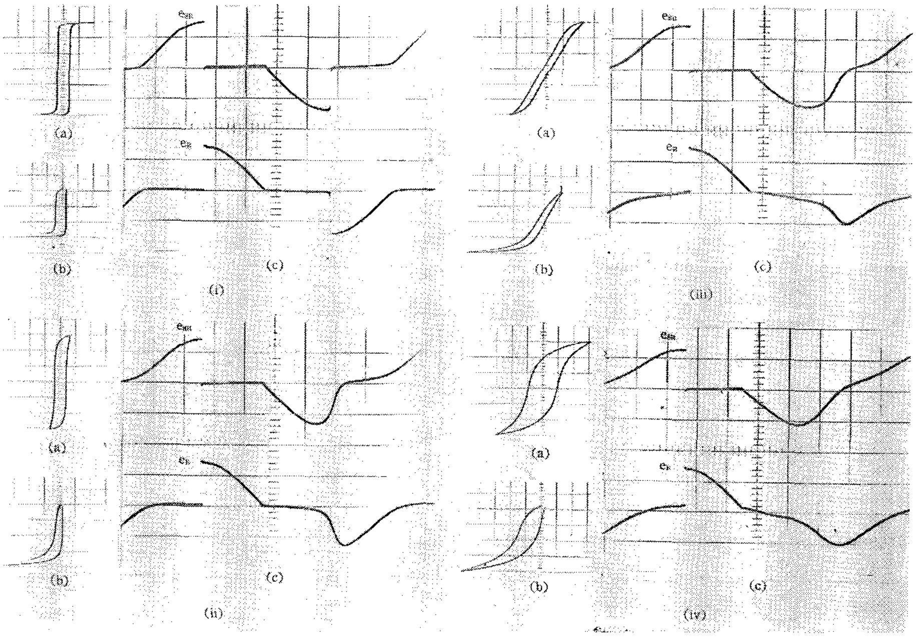

Fig. 6 Observed B-H loops (a), miner loops (b), reactor voltage wave forms (c, upper side) and load current wave forms (c, lower side).
(i) Fe 50%—Ni 50% oriented core. (ii) oriented silicon steel core.
(iii) oriented silicon steel cut core. (iv) silicon steel stack core.

core, oriented silicon steel core, oriented silicon steel cut-core with air gaps and silicon steel stack core for power transformer.

The B-H loops, the minor loops at $\alpha_1=\pi/2$ and the wave forms of the reactor and voltages at $\alpha_1=\pi/2$ are shown in Fig. 6(a), (b) and (c) respectively. The measured of voltage control characteristics with resistive load are shown in Fig. 7, where (a) is in effective value and (b) is in average.

§ 5. Discussion

Though it seems to be the good identification between the calculated and the measured, the discussion on the foregoing analysis and measurements more over reveals some features of the control characteristics as follows.

5.1 No d-c component in load current. In every modes of operation, the following equation is always true.

$$e_s = e_{SR} + e_R \tag{16}$$

But, in steady state, since the total flux changes over one cycle is zero, the following equation may be obtained.

$$\int_0^{2\pi} e_{SR}\, d(\omega t) = 0 \tag{17}$$

From Eq. s (16) and (17),

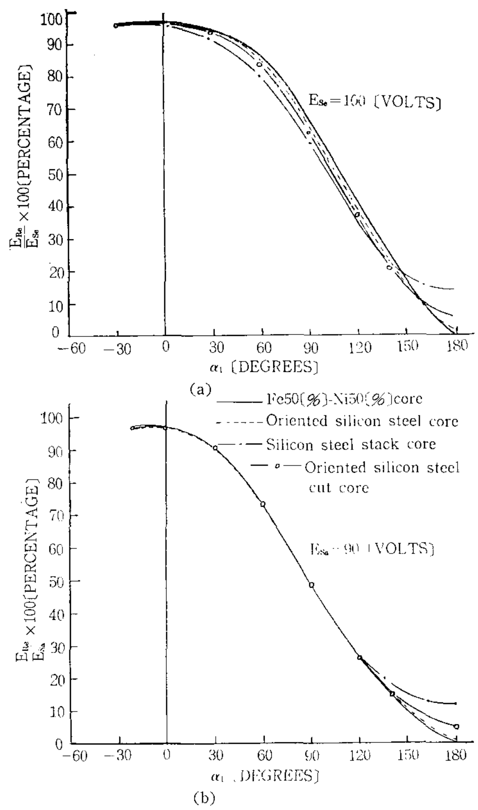

Fig. 7 Measured control characteristics in effective value (a) and in average value (b).

$$\int_0^{2\pi} e_R d(\omega t) = 0 \tag{18}$$

Eq. (18) tells that the d-c component is not included in the load current. This fact is good for the case in which the resistance of reactor windings is negligible.

Otherwise, it is not and the following equation have to be taken into account in mode B of operation.

$$e_{SR} = E_{SCR} - R_{SR}\, i_{SR} \tag{19}$$

Where, if i_{SR} is negative, the flux resetting value Φ_r' is increased. Φ_r' affects to increase α_2 in mode D of operation.

And also, in mode D of operation, the load voltage e_R is

$$e_R = \frac{R}{R+R_{SR}} e_s \tag{20}$$

The increment of α_2 and the decrement of e_R in mode D of operation result in the existence of d-c component in the load current.

5.2 Dependence of control characteristics upon the magnetizing characteristics of reactors. As α_1 approaches to β_2, i_R decreases and minimized at $\alpha_1=\beta_2$. Then, in the case of the magnetizing characteristics in Fig. 3(a), the exciting current of reactor does not flow through the load, but, it does in the cases of Fig. 3 (b), (c) and (d).

As α_1 decreases and leaves away from β_2, the load current flowing through the *SCR* increases, but, in the forward load current, that is, the load current in the direction of the *SCR* current, the component of the exciting current decreases, and then, the proportion of the exciting current to the load one decreases extremely. Therefore, the forward current in effective value and also in average value, is independent on the magnetizing characteristics of reactor except in the vicinity of $\alpha_1=\beta_2$. And more, the bacdkward load current in average value is true, since the load current has not d-c component. However, the backward load current in effective value is not, because the backward current waveform is not the same to the forward one.

5.3 Effect of *SCR* forward voltage drop E_{SCR}. As the load voltage e_R when the *SCR* is conducting is indicated in Fq. (9), and decreases by E_{SCR}. Further more, E_{SCR} resets the flux by Φ_r' in Eq. (13), which results in the increased α_2. In other word, E_{SCR} results in the reduced not only *SCR*'s but also reactor's current.

§ 6. Conclusion

The a-c voltage control circuit with a resistive load has been analyzed. The results of analysis and measurement have revealed the fact as follows.

1) The control characteristics of the a-c output voltage in average value is independent on the magnetizing characteristics of reactors except in the vicinity of minimum output point. But, in effective value, it is not valid.

2) A *SCR*'s forward voltage drop results in the reduction of not only *SCR*'s current but also reactor's current.

3) The a-c output current does not include the d-c component, if the winding resistance of the reactor is negligible.