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Quantum-corrected Hybrid Bohm and Classical Diffusion in a Laser-driven Plasma

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Synopsis

Within the framework of the hydrodynamic guiding-center approximation, we have investigated such quantum effects as the diffraction correction and the symmetry effect on the classical version of the particle diffusion coefficient $D_{\perp}$ across a dc magnetic field through the temperature-dependent pseudo-potentials. Analytic results are explicitly given with recourse to the order-of-magnitude estimate of a set of parameters pertaining to a laser-driven plasma.

1. Introduction

With the advent of laser-driven plasmas, attention has been paid to their transport properties, such as the thermal and the diffusion coefficients. Recently, it was proposed [1] and experimentally measured that, in the underdense region of a pellet, noncolinear density and temperature gradients may develop Mega-gauss sized $\vec{\nabla}n\times\vec{\nabla}T$ magnetic fields $\vec{B}$ during a time interval of several tens of picoseconds.

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In a laser-spot configuration currently discussed, $\vec{\nabla}T$ is directed radially inward to the axis of a laser beam, while $\vec{\nabla}n$ points axially into the target face. An induced magnetic field thus turns out to be troidal and varies over a characteristic scale size of the spot, because the source is confined nearly within the width of a density jump or an ionization layer. For instance, a composition jump may produce Mega-gauss sized magnetic field localized in a region of a few μm in diameter.

From the practical theoretic point of view, the most important transport coefficient, in relation with the above mentionned huge B values, is the thermal conductivity, across the magnetic lines of force, which is expected to control a deleterious outward heat flow. The thermal conductivity is known to be a many-body problem which is far more involved than the particle diffusion coefficient $D_\perp$ itself. We may, however, conjecture that in a large-field limit $\rho_L<\lambda_D$, or equivalently, $\Omega_i>\omega_{pi}$, the two lie on an almost equivalent footing. Moreover, in case where $\Omega_i<\omega_{pi}$, $D_\perp$ appears as an upper limit to the heat conductivity. In fact, a few available pieces of numerical and experimental data lend to support a heat flow intermediate between a Bohm-like ($\sim 1/B$) and a classical ($\sim 1/B^2$) one. We may thereby expect to be capable of drawing many useful informations from this more accessible quantity $D_\perp$. Within the framework of the three-dimensional (3D) guiding-center (GC) plasma model, Montgomery *et al* [2,3] and Vahala [4] have worked out the hydrodynamic transverse diffusion across $\vec{B}$ as well as the velocity-space diffusion along $\vec{B}$.

In this work, we address ourselves to an evaluation of such quantum effects as the diffraction correction arising from the delocalization of charges due to the uncertainty principle through the inequality $\lambda\!\!\!^{-}_{ee}> e^2/k_BT$ ($\lambda\!\!\!^{-}_{ee}$ is the thermal de Broglie wavelength of electrons) and symmetry effects originating in the Fermi statistics operating within the electron component. The former diffraction effect will be non-negligible only for small distances comparable to the Bohr radius [5]. Therefore, the hydrodynamic modes (convective cells, for instance) which convey most of the particle transport across $\vec{B}$ are not affected by these quantum effects. Albeit small in the underdense region of a laser-driven plasma, they are expected to increase near the critical layer and beyond it. For CO_2 laser with wavelength of 10.6 μm, the critical density is roughly of the order of $10^{19}/cm^3$. The latter domain could become important in relation with recent speculations about the possibility that self-generated intense magnetic field may penetrate, frozen, into the denser region because of a high conductivity prevailing in that region[6].

Sec.2 is essentially a review of the hydrodynamic formalism of the diffusion coefficient $D_\perp$ for the 3D guiding center plasma discussed in detail by Montgomery *et al* [2,3] and by Vahala[4].. Upon evaluating the autocorrelation function between the transverse components of fluctuating electric fields, the electron-electron, electron-ion and ion-ion structure factors intervene automatically, accounting for the above mentionned two quantum effects. They are derived in Sec.3 and are used subsequently in Sec.4 in order to evaluate the quantum-corrected diffusion coefficient. The last section is devoted to concluding remarks.

2. Survey of the Hydrodynamic Formalism

By virtue of the Green-Kubo formalism for the linear response theory, the particle diffusion coefficient $D_\perp$ across $\vec{B}$ reads as

$$D_\perp = \frac{c^2}{B^2}\int_0^\infty d\tau \, \langle \vec{E}(\tau)\cdot\vec{E}(0)\rangle \tag{1}$$

in terms of the equilibrium canonical average of the 2-point autocorrelation function for fluctuating electric fields. Since discussions in the case of the 3D [2,3,4] as well as the extended ν-dimensional [5] GC plasmas have already been detailed in References quoted, we content ourselves, for the commodity of presentation, to outline the derivation. Taking for granted the existence of the discrete sum

$$\vec{E}(\tau) = \sum_{\vec{k}} \vec{E}_{\vec{k}}(\tau)\cdot e^{i\vec{k}\cdot\vec{x}(\tau)}$$

where $\vec{x}(\tau)$ is the orbit of the test ion at time τ and ignoring the correlation between the position of the test ion and those of the background plasma particles, we arrive at the statistically factorized result

$$D_\perp = \frac{c^2}{B^2} \sum_{\vec{k}} \int_0^\infty d\tau \langle \vec{E}_{\perp,\vec{k}}(\tau) \cdot \vec{E}_{\perp,-\vec{k}}(0) \rangle \langle e^{i\vec{k}\cdot\vec{x}(\tau)} \rangle . \tag{2}$$

Eq. (2) implies a salient feature of the present GC approximation, in that the above two averages can be expressed in terms of the 2-point electric field correlations, so that we can write

$$\langle \vec{E}_{\perp,\vec{k}}(\tau) \cdot \vec{E}_{\perp,-\vec{k}}(0) \rangle = \sum_{i,j} \frac{(4\pi)^2 e_i e_j}{V^2} \frac{k_\perp^2}{k^4} \langle e^{-i\vec{k}\cdot[\vec{x}_i(\tau) - \vec{x}_j(0)]} \rangle \tag{3}$$

and

$$\langle e^{i\vec{k}\cdot\vec{x}(\tau)} \rangle_{k_\parallel = 0} = \exp\left[-\frac{c^2 k^2}{4B^2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \langle \vec{E}_{\perp,\vec{k}}(\tau') \cdot \vec{E}_{\perp,-\vec{k}}(\tau'') \rangle \right] , \tag{4}$$

where $\sum_{i,j}$ indicates the sum over all particles of the species i and j. In the three-dimensional case, the present approach amounts to modeling the spatial diffusion of the test ion across $\vec{B}$ through small increments $\vec{x}(t)$ and the velocity-space diffusion along $\vec{B}$ via the equations of motion

$$\frac{d\vec{x}_\perp}{dt} = c \frac{\vec{E}_\perp(t) \times \vec{B}}{B^2} \quad \text{and} \quad \frac{d\vec{v}_\parallel}{dt} = \frac{e_i}{m_i} \vec{E}_\parallel(t) . \tag{5}$$

Notice that the initial position $\vec{x}(0)$ may be set equal to zero, while the initial velocity of the test ion is assumed to obey the Maxwell-Boltzmann distribution. Let us also remark that, upon evaluating $\langle exp\{i\vec{k}\cdot\vec{x}(\tau)\}\rangle$ with the aid of the cumulant expansion[4], we need an approximation which renders tractable an expression with $\vec{k}_\parallel \neq 0$. Concentrating ourselves on the low-frequency long-wavelength parts of the electric field spectrum[5], we may therefore expect that the free streaming of the particles along the magnetic lines of force would be dominant in destroying the electric field correlation between two different time points. Thus, if we ignore $E_\parallel$, we obtain

$$\langle \exp\{i\vec{k}\cdot\vec{x}(\tau)\} \rangle = \exp(-k_\parallel^2 V_i^2 \tau^2/2), \quad k_\parallel \neq 0 . \tag{6}$$

Performing the summation over i and j in eq. (3), we obtain

$$\langle \vec{E}_\perp(t) \cdot \vec{E}_\perp(0) \rangle = \frac{(4\pi)^2 n e^2}{V} \{ \sum_{k_\parallel \neq 0} \frac{k_\perp^2}{k^4} H_1(k) e^{-k_\parallel^2 V_i^2 t^2/2}$$

$$+\sum_{k_\parallel=0} \frac{1}{k_\perp^2} H_2(k) \exp\left[-\frac{c^2 k_\perp^2}{4B^2} \int_0^t d\tau' \int_0^t d\tau'' \langle \vec{E}_\perp(\tau')\cdot\vec{E}_\perp(\tau'')\rangle\right] \Big\}, \tag{7}$$

where $V_i\,(=\sqrt{k_B T/m_i})$ is the ion thermal velocity. Two functions $H_1(k)$ and $H_2(k)$ denote appropriate combinations of the electron-electron, electron-ion and ion-ion structure factors given, respectively, as [7]

$$\begin{aligned} H_1(k) = \frac{Z^2}{(1+Z)^2} \Big[& e^{-k_\parallel^2 V_e^2 \tau^2/2} \left(\frac{1+Z}{Z} - n S_{ee}(k) + n S_{ei}(k)\right) \\ & + e^{-k_\parallel^2 V_i^2 \tau^2/2} \left(1+Z - n S_{ii}(k) + n S_{ei}(k)\right)\Big] \end{aligned} \tag{8a}$$

and

$$H_2(k) = Z + n \frac{Z^2}{(1+Z)^2} (-S_{ee} - S_{ii} + 2S_{ei}), \tag{8b}$$

where the structure factor $S_{ij}(k)$ has been introduced through the relation [5]

$$\langle \exp\{ i\vec{k}\cdot[\vec{x}_i(\tau) - \vec{x}_j(0)]\}\rangle = \delta_{\vec{k},0} - \frac{1}{V}\int d\vec{r}\, g_2^{ij}(r)\, e^{i\vec{k}\cdot\vec{r}} \equiv \delta_{\vec{k},0} - \frac{1}{V} S_{ij}(k),\ i \neq j \tag{9}$$

in terms of the pair correlation function $g_2^{ij}(k)$. In the next section, we shall evaluate explicitly these structure factors, in which terms accounting for quantum effects appear. The diffusion coefficient $D_\perp$ is calculated through the mean dispersion of the test ion [2,3,4]

$$R_\perp(t) = \frac{c^2}{4B^2} \int_0^t d\tau' \int_0^t d\tau'' \langle \vec{E}_\perp(\tau''-\tau')\cdot \vec{E}_\perp(0)\rangle \tag{10}$$

with $\dot{R}_\perp(\infty) = D_\perp/2$. $R_\perp(t)$ obeys the second-order ordinary differential equation

$$\begin{aligned} \ddot{R}(t) = {} & \varepsilon_b \sum_{\vec{k}_\perp} \frac{k_D^2}{2k_\perp^2} H_2(k)\, e^{-2k_\perp^2 R_\perp(t)} \\ & + \varepsilon_b \sum_{\vec{k},\, k_\parallel \neq 0} \frac{k_D^2 k_\perp^2}{2k^4} H_1(k)\, e^{-k_\parallel^2 V_i^2 t^2/2 - k_\parallel^2 D_\parallel t^3} \end{aligned} \tag{11}$$

where $\varepsilon_b = (4\pi)^2 e^2 c^2 n / B^2 V k_D^2$. The velocity-space diffusion coefficient, $D_\parallel$, is given by

$$D_\parallel = \frac{e^2}{m_i^2} \int_0^\infty d\tau \langle E_\parallel(\tau) E_\parallel(0)\rangle \cong \frac{1}{8\pi^{3/2}}\, \omega_{pi}^3 \lambda_D^2 \Lambda \ln\frac{1}{\Lambda} \tag{12}$$

with $\Lambda = 1/(n_e\lambda_D^3)$ the plasma parameter. In the limit of a large volume, the discrete sums can be transformed to integrals through the relations

$$\sum_{\vec{k}_\perp} \longrightarrow \frac{V^{2/3}}{(2\pi)^2}\int d\vec{k}_\perp \quad \text{and} \quad \sum_{\vec{k},\, k_\parallel \neq 0} \longrightarrow \frac{V}{(2\pi)^3}\int d\vec{k}\,. \tag{13}$$

For numerical integrations, it is more convenient to introduce dimensionless quantities by the transformations

$$z = 8\pi^2\left(\frac{\Omega_i}{\omega_{pi}}\right)^2 \frac{P^{1/2}}{\Lambda}\frac{R_\perp}{\lambda_D^2} \quad \text{and} \quad \tau = P^{1/2}\omega_{pi}t \tag{14}$$

where $P = (2\pi\lambda_D/L)^2$. $L(=V^{1/3})$ denotes an average size of the toroidal magnetized region of a plasma spot and ω_{pi} and Ω_i are the ion plasma and the ion cyclotron frequencies, respectively. Substitution of eqs. (13) and (14) into eq. (11) yields the non-dimensional equation

$$\frac{d^2 z}{d\tau^2} = \frac{Z+1}{2Z}\left[\int_P^{1 \text{ or } Q} \frac{dx}{x} H_2(x)\, e^{-axz} + \frac{1}{2P^{1/2}}\int_P^{Q} \frac{dy}{\sqrt{y}}\int_P^{Q} dx \frac{x}{(x+y)^2} H_1(x,y)\, e^{-(1+b\tau)\tau^2 y/P}\right], \tag{15}$$

where

$$Q = \left(\frac{8\pi}{\Lambda}\right)^2, \quad a = \frac{1}{4\pi^2}\left(\frac{\omega_{pi}}{\Omega_i}\right)^2 \frac{\Lambda}{P^{1/2}}, \quad b = \frac{\Lambda \ln(1/\Lambda)}{8\pi^{3/2} P^{1/2}}$$
$$x = (k_\perp \lambda_D)^2 \quad \text{and} \quad y = (k_\parallel \lambda_D)^2. \tag{16}$$

As for the above equation, several remarks should be given. (1) The transformations (14) ensure a rapid decay in τ of the second term of the right-hand side (r.h.s.), since the coefficient of τ^2 in the exponent is of the order of or larger than unity. They are thus more advantageous than those used by Vahala, which give very small coefficients of τ^2. Our procedure serves thus to shorten considerably computation time upon numerical solution of eq. (15). (2) Since Vahala's instructive reasoning [4] of splitting into two parts of the y-integration in the second term of the r.h.s. of eq. (15), according as whether $y > (\Lambda \ln 1/\Lambda)^2$ or not, only results in an overestimation of the y-integrals, we do not follow his procedure. (3) The upper limit 1 or Q in the first term of the r.h.s. indicates that the original $k_\perp$-integration is cut either at $k_{\perp,max} = k_D$ (the so-called fluid limit) or at $k_{\perp,max} = k_BT/e^2$ (the kinetic limit). As will be shown later, the distinction into two cases of the first term on the r.h.s. does not affect a result seriously, so far as $a>>1$, condition typical of laser-driven plasmas.

Once $H_2(x)$ and $H_1(x,y)$ are known, eq.(15) can be solved numerically with the initial conditions $z(0) = \dot{z}(0) = 0$. $D_\perp$ is finally determined by $\dot{z}(\infty)$ through the relation

$$D_\perp = \frac{1}{4\pi^2} \frac{\omega_{pi}^3}{\Omega_i^2} \Lambda \lambda_D^2 \dot{z}(\infty) . \tag{17}$$

3. Quantum-corrected Structure Factors

In order to properly take into account such quantum effects as diffraction corrections and symmetry effects through the quantum-corrected structure factors $S_{ee}(k)$, $S_{ei}(k)$ and $S_{ii}(k)$, we start from the pseudo-potentials [8] (see Appendix A)

$$u_{ee}(r) = \frac{e^2}{r}(1 - e^{-r/\lambda\!\!\!^-}) + k_B T \ln 2 \, \exp[-\frac{1}{\pi \ln 2}(\frac{r}{\lambda\!\!\!^-})^2] , \tag{18a}$$

$$u_{ei}(r) = -\frac{Ze^2}{r}(1 - e^{-\sqrt{2}r/\lambda\!\!\!^-}) , \tag{18b}$$

and $$u_{ii}(r) = \frac{Z^2 e^2}{r} , \tag{18b}$$

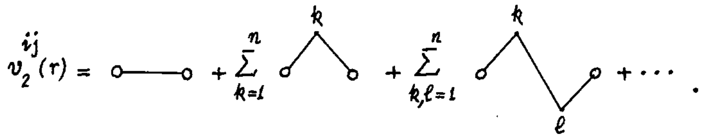
where $\lambda\!\!\!^- = \lambda\!\!\!^-_{ee}/\sqrt{2\pi}$. For instance, the first term in $u_{ee}(r)$ represents the diffraction effect, while the second term stands for the symmetry one. Two other expressions are self-explanatory. Our next task is to evaluate explicitly $S_{ee}(k)$, $S_{ei}(k)$ and $S_{ii}(k)$. To this end, we first recall the definition of $S_{ij}(k)$ given by eq.(9). In the limit of small Λ values, the structure factor $S_{ij}(k)$ can be explained through the linearized pair correlation function

$$g_2^{ij}(r) = \exp[w_2^{ij}(r)] \simeq 1 + w_2^{ij}(r) , \tag{19}$$

where $w_2^{ij}(r)$ is the normalized potential of average force which, in the present application, is no other than a resummed pseudo-potential $v_2^{ij}(r)$. Thus we have

$$\frac{1}{V}\int d\vec{r}\, g_2^{ij}(r)\, e^{-i\vec{k}\cdot\vec{r}} \simeq \delta_{\vec{k},0} + \frac{1}{V}\int d\vec{r}\, v_2^{ij}(r)\, e^{-i\vec{k}\cdot\vec{r}} = \delta_{\vec{k},0} + \frac{1}{V} v_2^{ij}(k) \,. \tag{20}$$

In other words, $S_{ij}(k)=v_2^{ij}(k)$. The resummation of the pseudo-potentials (18) has now to be generalized to the case of a multi-component plasma[9]. Given n species of charged particles, the resummed pseudo-potential $v_2^{ij}(r)$ is written as



(21)

In terms of the Fourier transforms in matricial representation, eq. (21) can be put into the form

$$\overleftrightarrow{V}(k) = \overleftrightarrow{U}(k)\cdot[\overleftrightarrow{I} + \overleftrightarrow{U}(k)\cdot\overleftrightarrow{C}]^{-1} , \tag{22}$$

where $\overleftrightarrow{U}(k)$ and $\overleftrightarrow{V}(k)$ are, respectively, the matrix composed of the Fourier transformed bare and resummed pseudo-potentials. $\overleftrightarrow{C} = n\beta C_i \delta_{ij}$, with $C_i\,(=N_i/\Sigma N_i)$ the relative concentration of the i-th species and $\beta = 1/k_B T$ as usual. When a two-component plasma (i, j = electron, ion) is concerned, elementary but heavy algebra yields the structure factors as [8]

$$n\, S_{ee}(\kappa) = \frac{1}{Z}\left[\frac{1}{1+\kappa^2+\eta^2\kappa^4} + \zeta\left\{1 - \frac{2}{(1+Z)(1+\kappa^2)} + \frac{1}{(1+Z)^2(1+\kappa^2)^2}\right\} e^{-\alpha'\kappa^2}\right] , \tag{23a}$$

$$n\, S_{ei}(\kappa) = -2\left[\frac{1+\eta^2\kappa^2}{(1+\kappa^2+\eta^2\kappa^4)(2+\eta^2\kappa^2)} - \frac{\zeta}{8(1+Z)^2(1+\kappa^2)^2} e^{-\alpha'\kappa^2}\right] , \tag{23b}$$

and $$n\, S_{ii}(\kappa) = Z\left[\frac{1}{1+\kappa^2} + \frac{\zeta}{1+Z}\left\{\frac{1}{1+\kappa^2} - \frac{1}{(1+Z)(1+\kappa^2)^2}\right\} e^{-\alpha'\kappa^2}\right] , \tag{23c}$$

where $\kappa = k\lambda_D$, $\eta = \sqrt{2\pi}\,\bar{\lambda}/\lambda_D$, $\zeta = Z\pi^3(\ln 2)^{5/2} n_e \bar{\lambda}^3$ and $\alpha' = (\pi/4)\eta^2 \ln 2$. The parameter η^2 reflects the diffraction correction, while the symmetry effect is characterized by the parameter $\eta'\,(=4\pi n_e \bar{\lambda}^3)$.

4. Evaluation of $\dot{z}(\infty)$ and Results

We now proceed on to evaluate $H_2(k)$ and $H_1(k)$ which appeared in the differential equation (15). For this purpose, we first devide the structure factors in the classical and the quantum parts and retain terms up to the order of η^2 in the latter. Substitution of the resulting expressions into eqs.(8a) and (8b) then yields

$$H_1(\kappa)_{cl} = \frac{Z}{1+Z}\,\frac{\kappa^2}{1+\kappa^2}\left[\, e^{-(m_i/m_e)\tau^2 y/P} + Z\, e^{-\tau^2 y/P}\right], \tag{24a}$$

$$H_1(\kappa)_{qu} = e^{-(m_i/m_e)\tau^2 y/P}\,\frac{Z}{(1+Z)^2}\left[\eta^2\,\frac{\kappa^2\{(2+Z)\kappa^2 - Z\}}{2(1+\kappa^2)^2} + \zeta\left\{1 - \frac{2}{(1+Z)(1+\kappa^2)} + \frac{4+Z}{4(1+Z)^2(1+\kappa^2)^2}\right\} e^{-\alpha'\kappa^2}\right]$$
$$+ \frac{Z^2}{2(1+Z)^2}\, e^{-\tau^2 y/P}\left[-\eta^2\,\frac{\kappa^2(1-\kappa^2)}{(1+\kappa^2)^2} - \frac{\zeta}{(1+Z)(1+\kappa^2)}\left\{2Z - \frac{1+4Z}{2(1+Z)(1+\kappa^2)}\right\} e^{-\alpha'\kappa^2}\right], \tag{24b}$$

$$H_2(\kappa)_{cl} = Z\,\frac{\kappa^2}{1+\kappa^2}, \tag{25a}$$

and

$$H_2(\kappa)_{qu} = \frac{Z}{(1+Z)^2}\left[\eta^2\,\frac{\kappa^2\{(1+Z)\kappa^2 - Z\}}{(1+\kappa^2)^2} - \zeta\left\{1 - \frac{2-Z^2}{(1+Z)(1+\kappa^2)} + \frac{2-Z-2Z^2}{2(1+Z)^2(1+\kappa^2)^2}\right\} e^{-\alpha'\kappa^2}\right]. \tag{25b}$$

In order to render eq.(15) more tractable, we integrate it once over τ from 0 to ∞, after having multiplied both sides by $2\dot{z}d\tau = 2dz$. Integrating the resulting expression over x and deleting terms proportional to $\exp\{-(m_i/m_e)\tau^2 y/P\}$ because of the large mass ratio $m_i/m_e >> 1$, procedure of neglecting the free streaming of electrons along $\vec{B}$, we obtain

$$[\dot{z}(\infty)]^2 \cong \beta + \frac{1}{2P^{1/2}}\int_P^Q dy\, f(y) \int_0^\infty d\tau\, \dot{z}(\tau)\, e^{-(2+b\tau)\tau^2 y/P}, \tag{26a}$$

where

$$f(y) = \frac{1}{\sqrt{y}}\left[\, Z\left\{ \ln\frac{Q+y}{P+y} - (1+y)\ln\frac{(Q+y)(1+P+y)}{(1+Q+y)(P+y)}\right\}\right.$$

$$
-\frac{Z\eta^2}{2(1+Z)}\Big[-\ln\frac{Q+y}{P+y}+(1-y)\ln\frac{(Q+y)(1+P+y)}{(1+Q+y)(P+y)}-2(1+y)\Big(\frac{1}{1+Q+y}-\frac{1}{1+P+y}\Big)\Big]
$$

$$
+\frac{Z\zeta}{2(1+Z)}e^{-\alpha' y}\Big[-\frac{2Z}{1+Z}\Big\{y\Big(\frac{e^{-\alpha' Q}}{Q+y}-\frac{e^{-\alpha' P}}{P+y}\Big)+(1+y+\alpha' y)e^{\alpha' y}\big(E_1[\alpha'(P+y)]-E_1[\alpha'(Q+y)]\big)
$$

$$
-(1+y)e^{\alpha'(1+y)}\big(E_1[\alpha'(1+P+y)]-E_1[\alpha'(1+Q+y)]\big)\Big\}
$$

$$
+\frac{1+4Z}{2(1+Z)^2}\Big\{y\Big(\frac{e^{-\alpha' Q}}{Q+y}-\frac{e^{-\alpha' P}}{P+y}\Big)+(1+y)\Big(\frac{e^{-\alpha' Q}}{1+Q+y}-\frac{e^{-\alpha' P}}{1+P+y}\Big)
$$

$$
+(1+2y+\alpha' y)\,e^{\alpha' y}\big(E_1[\alpha'(P+y)]-E_1[\alpha'(Q+y)]\big)
$$

$$
-\{1+2y-\alpha'(1+y)\}\,e^{\alpha'(1+y)}\big(E_1[\alpha'(1+P+y)]-E_1[\alpha'(1+Q+y)]\big)\Big\}\Big]\Big] \tag{26b}
$$

and

$$
\beta=\frac{1}{a}\Big[(1+Z)\ln\frac{Q(1+P)}{(1+Q)P}+\frac{\eta^2}{1+Z}\Big\{-Z\ln\frac{Q(1+P)}{(1+Q)P}+(1+2Z)\Big(\frac{1}{1+P}-\frac{1}{1+Q}\Big)\Big\}
$$

$$
-\frac{\zeta}{1+Z}\Big[\Big\{1+\frac{Z^2-2}{1+Z}-\frac{2Z^2+Z-2}{2(1+Z)^2}\Big\}\Big(\frac{e^{-\alpha' P}}{P}-\frac{e^{-\alpha' Q}}{Q}\Big)-\frac{2Z^2+Z-2}{2(1+Z)^2}\Big(\frac{e^{-\alpha' P}}{1+P}-\frac{e^{-\alpha' Q}}{1+Q}\Big)
$$

$$
+\Big\{-\alpha'-\frac{Z^2-2}{1+Z}(1+\alpha')+\frac{2Z^2+Z-2}{2(1+Z)^2}(2+\alpha')\Big\}\{E_1(\alpha' P)-E_1(\alpha' Q)\}
$$

$$
-\Big\{\frac{2-Z^2}{1+Z}+\frac{2Z^2+Z-2}{2(1+Z)^2}(2-\alpha')\Big\}e^{\alpha'}\{E_1[\alpha'(1+P)]-E_1[\alpha'(1+Q)]\}\Big]\Big] \tag{27}
$$

with $E_1(x)$ the exponential integral[10]. Since an analytic form of $\dot{z}(\tau)$ in eq.(26) is unknown, we shall rewrite eq.(26) symbolically as

$$
[\dot{z}(\infty)]^2 \cong \beta+(\alpha-\gamma)\,\dot{z}(\infty) \tag{28a}
$$

with

$$
\alpha \doteq \frac{1}{2P^{1/2}}\int_P^Q dy\, f(y)\int_0^\infty d\tau\, e^{-(2+b\tau)\tau^2 y/P}
$$

and (28b)

$$
\gamma=\frac{1}{2P^{1/2}}\int_P^Q dy\, f(y)\int_0^\infty d\tau\,\Big\{1-\frac{\dot{z}(\tau)}{\dot{z}(\infty)}\Big\}\, e^{-(2+b\tau)\tau^2 y/P}.
$$

Then, $\dot{z}(\infty)$ is formally given by

$$\dot{z}(\infty) = \frac{1}{2}\left(\alpha - \gamma + \sqrt{(\alpha-\gamma)^2 + 4\beta}\right). \tag{29}$$

Here we remark that Vahala's approximation is tantamount to neglecting the last term $\gamma\dot{z}(\infty)$ on the r.h.s. of (28a) with respect to two other terms. This implies that the term $\dot{z}(\infty) - \dot{z}(\tau)$ is expected to be negligibly small except for very small τ values. In view of the fact that $f(y)$ is a well sharped function for small y values, there is no guaranty that his subtle approximation be satisfactory. We rather claim, that, in certain circumstances such as a laser-driven plasma, the neglected last term $\gamma\dot{z}(\infty)$ could be of the same order of magnitude as $\alpha\dot{z}(\infty)$. With this proviso in mind, we now evaluate α analytically. Since $y/P>1$, we can readily carry out the τ-integration to give

$$\int_0^\infty d\tau\, e^{-(2+b\tau)\tau^2 y/P} = \frac{1}{2}\left(\frac{P}{2y}\right)^{1/2}\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{b}{2\sqrt{2}}\right)^n\left(\frac{P}{y}\right)^{n/2}\Gamma\left(\frac{3n+1}{2}\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{P}{y}\right)^{1/2}\left[1 - \frac{b}{2(2\pi)^{1/2}}\left(\frac{P}{y}\right)^{1/2} + \frac{15}{128}b^2\frac{P}{y} - \cdots\right]. \tag{30}$$

As for the y-integrations, some are elementary and others complicated. As an illustrative example, we perform in Appendix B the integral of the type $\int_P^Q dy E_1[\alpha'(P+y)]/y$, which appears on the third line of eq.(26b).

Since a general expression for α is lengthy and is likely to obscure a physical content involved, we shall first give an order-of-magnitude estimation of all parameters characterizing both the classical and the quantum contributions. In connexion with a laser-produced plasma, we choose a set of parameters [1] pertaining to a glass microballoon target currently used in Nd laser experiments. They are: $n_e = 10^{21}$ cm^{-3}, $T = 500$ eV, $L = 10$ μm and $B = 10^6$ Gauss. Correspondingly, we have $P = 5.4542 \times 10^{-6}$, $Q = 1.6657 \times 10^6$, $\Lambda = 1.9473 \times 10^{-2}$, $a = 3.9898 \times 10^6$, $\eta = 4.6969 \times 10^{-3}$, $\zeta = 4.1904Z \times 10^{-6}$ and $\alpha' = 1.2010 \times 10^{-5}$. These values suggest that we could make use of the inequalities $Q>>1$ and $P<<1$, valid at least in the example of our present interest, in order to obtain compact expressions for α and β.

Neglecting terms of the order of $1/Q$, P, etc, we finally obtain

$$\alpha \cong \frac{Z}{4}\left(\frac{\pi}{2}\right)^{1/2}\Big[\frac{1}{2}(\ln Q)^2 + \ln Q \ln\frac{1}{P} - \ln Q - \left(1 - \ln 2 + \frac{\pi^2}{12}\right)$$

$$+ \frac{\eta^2}{2(1+Z)}\left[\frac{1}{2}(\ln Q)^2 + \ln Q \ln\frac{1}{P} + \left(\frac{\pi}{2}\right)^{3/2}(\ln 2)^{5/2}\frac{Z(4Z^2-1)}{2(1+Z)^2}\frac{\eta}{\Lambda}\left\{\frac{1}{2}(\ln Q)^2 + \ln\frac{1}{\alpha'}\ln\frac{1}{P}\right\}\right]\Big] \quad (31)$$

and

$$\beta \cong \frac{1}{a}\Big[(1+Z)\ln\frac{1}{P}$$

$$+ \eta^2\left\{1 + \frac{Z}{1+Z}\left(1 - \ln\frac{1}{P}\right) - \left(\frac{\pi}{2}\right)^{3/2}(\ln 2)^{5/2}\frac{\eta}{\Lambda}\frac{Z^2(2Z^2+2Z-1)}{2P(1+Z)^3}\right\}\Big], \quad (32)$$

where use has been made of the identity $\zeta = Z\pi^3(\ln 2)^{5/2} n_e \lambda\!\!\!^{-3} = Z(\pi/2)^{3/2} \times (\ln 2)^{5/2}\eta^3/\Lambda$, which indicates that the symmetry effect is, in the present application, $Z\eta/\Lambda$ times the diffraction correction. An interesting remark on the order of magnitude of two quantities α and β illustrates well the B-dependence of $\dot{z}(\infty)$. While α is independent of B, β depends on it only through the parameter a. Thus, whenever $a \gg 1$, β provides a minor correction to $\alpha-\gamma$, yielding a very weak Bohm dependence by virtue of eq.(29). In this case, $D_{\perp}$ becomes substantially of the classical type through eq.(17) and quantum effects can be practically neglected. An interesting situation could come about with increasing β (or with decreasing a), in which case the hybrid classical and Bohm diffusion may be observable. A condition under which two quantum effects become important will be discussed in the next section.

5. Concluding Remarks

So far, we have discussed the diffraction and the symmetry effects on the hybrid Bohm and classical diffusion coefficient, within the framework of the hydrodynamic guiding center approximation.

In close connexion with a laser-driven plasma, we have chosen a set of parameters typical of a glass microballoon target currently encountered in Nd laser experiments. With the use of nondimensional parameters obtained therefrom, we have evaluated explicitly two quantities α and β which give an indication to the order of magnitude of a required $D_\perp$. Our main results are enumerated thus.

(1) The symmetry effect characterized by the parameter $Z\eta^3/\Lambda$ is, in the present application, $Z\eta/\Lambda$ times the diffraction parameter η^2. The importance of the former thus crucially depends on a magnitude of the ratio η/Λ. If this is small, the diffraction correction predominates the symmetry effect. Now, it is instructive to show in which domain of the parameter space (see Fig.1) the diffraction effect becomes important. For our set of parameters, we obtain $\eta^2 = 2.2061 \times 10^{-5}$ and $\eta/\Lambda = 0.24120$, indicating that two quantum effects are very weak. Since the dependence on n_e and T of η^2 is explicitly given by

$$\eta^2 = 2\pi\left(\frac{\lambda\!\!\!^-}{\lambda_D}\right)^2 = 0.55153\,\frac{[n_e]}{T^2} \tag{33}$$

with $[n_e]$ in units of $10^{20}/\mathrm{cm}^3$ and T in eV, the condition $\eta^2 > 1$ requires $T \leqslant 0.74265[n_e]^{1/2}$. Thus, when $[n_e] = 10$, $T \leqslant 2.9433$ eV, which shows that the quantum effects will become important when a temperature is sufficiently low. We notice with reserve, however, that a favorable situation can be realized in a strongly magnetized semiconductor even at a room temperature, for which the quantum-corrected part of $D_\perp$ can really compete with the classical part.

(2) As was discussed in Sec.4, $D_\perp$ is of the classical type so far as $a >> 1$, since β is negligibly small compared with $\alpha - \gamma$. When β increases, however, the hybrid classical and Bohm diffusion is expected to occur.

To conclude, we note that the parameter β calculated in the text can be modified to include finite gyro-radius effects [11] by multiplying them with the factor $\{1 + (\omega_{pi}/\Omega_i)^2\}^{-1}$, so that $\beta \sim 1/B$ when $\Omega_i > \omega_{pi}$ (Bohm behaviour) or $\beta \sim constant$ when $\Omega_i < \omega_{pi}$ (plateau regime). From the numerical point of view, we only need to inject n_e, T, B and L values, in order to implement the present evaluation of $D_\perp$. We hope that such an argument may arouse some interest of code makers.

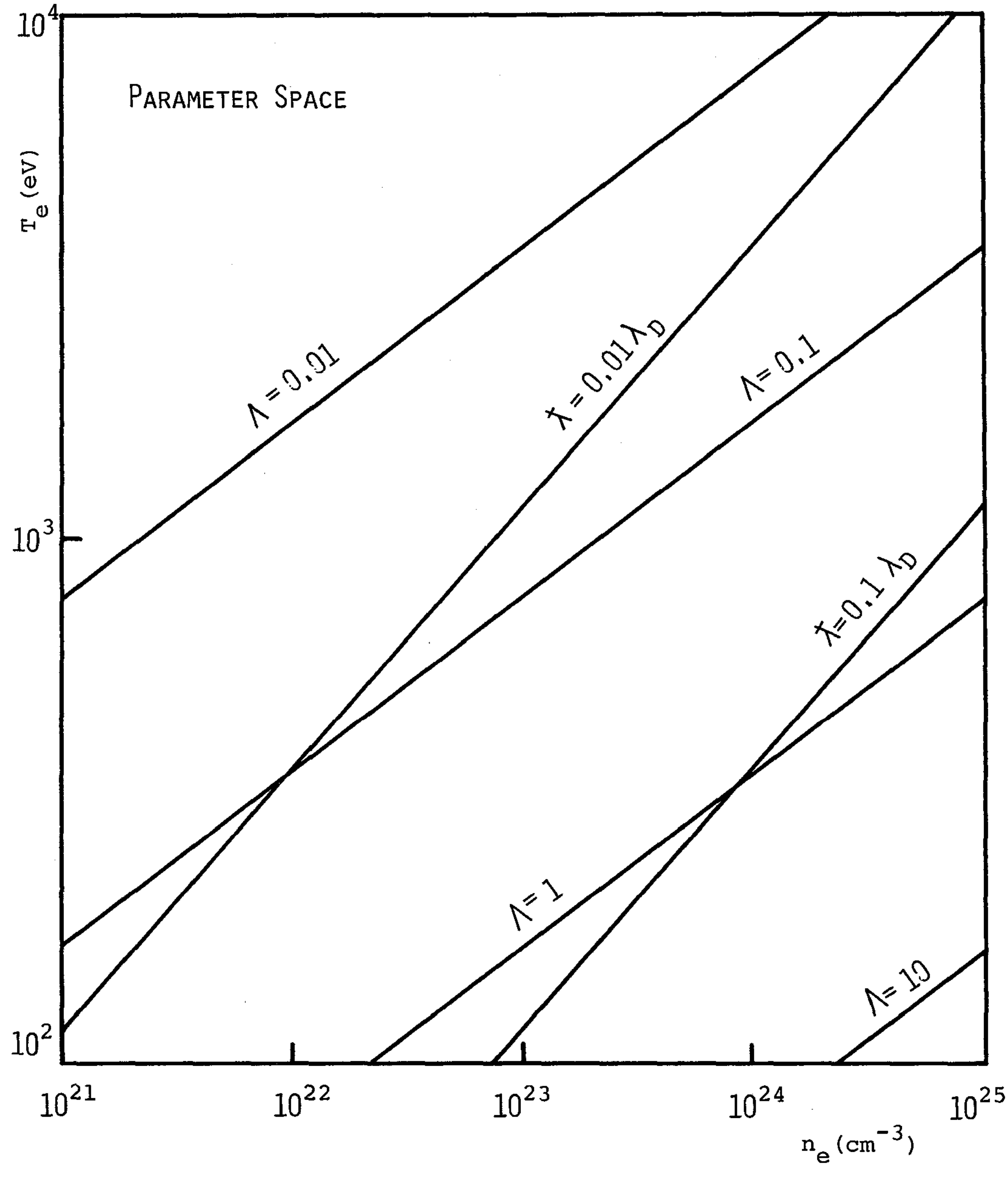


Fig. 1

Parameter space pertaining to a laser-driven plasma. $\ln T$ is plotted versus $\ln n_e$, with T in eV. Λ, $\bar{\lambda}$ and λ_D are the plasma parameter, the thermal de Broglie wavelength and the Debye length, respectively.

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Appendix A

Survey of the Two-body Correlation Function

The effective pair potential $u_{ij}(r)$ is defined, as a function of the two-particle radial distribution function (r.d.f.), $g_{ij}(r)$, by $[\beta = (k_B T)^{-1}]$

$$\beta u_{ij}(r) = -\ln g_{ij}(r). \tag{A1}$$

$g_{ij}(r)$ denotes the usual two-body correlation function, in a N-body system, for two specified particles i and j. Expressed in terms of the one- and two-particle density matrices, it is given by

$$g_{ij}(r) = [\rho_2(\vec{x}_i, \vec{y}_j, \vec{x}_i, \vec{y}_j, \beta) - \delta_{ij}\rho_2(\vec{x}_i, \vec{y}_j, \vec{y}_j, \vec{x}_i, \beta)] / \rho_1(\vec{x}_i, \vec{x}_i, \beta)\,\rho_1(\vec{y}_j, \vec{y}_j, \beta) \tag{A2}$$

where

$$\rho_1(\vec{x},\vec{y},\beta) = \langle \vec{x}|\, exp(-\beta H_1)|\vec{y}\rangle$$

and $\rho_2(\vec{x},\vec{y},\vec{u},\vec{v},\beta) = \langle \vec{x},\vec{y}|\, exp(-\beta H_2)|\,\vec{u},\vec{v}\rangle$.

H_1 and H_2 denote, respectively, the one- and two-particle Hamiltonians, $\vec{x},\vec{y},\ldots$ are the particles positions, $r = |\vec{x} - \vec{y}|$, and

$$\begin{aligned} &\delta_{ij} = 0 \text{ when } i \neq j \text{ (distinguishable particles)}, \\ &\delta_{ij} = 1 \text{ when } i = j \text{ (indistinguishable particles)}. \end{aligned} \tag{A3}$$

For a system of identical particles, two cases are to be envisaged:

a. Particles with parallel spin ($i\uparrow\ i\uparrow$).

b. Particles with antiparallel spin ($i\uparrow\ i\downarrow$).

In the first case, the particles are completely indistinguishable, and $\delta_{i\uparrow i\uparrow} = 1$. In the second case, the particles are distinguishable by their spin orientation, and $\delta_{i\uparrow i\downarrow} = 0$.

In several specific problems such as these convected with the thermodynamic functions of the dense multi-component plasmas, it is required to know the mean effective pair potential of identical particles with different spin orientations. This can be obtained from $g_{ij}(r)$ through the definition:

$$\begin{aligned} g_{ij}(r) &\equiv \frac{1}{2}\,[\,g_{i\uparrow i\uparrow}(r) + g_{i\uparrow i\downarrow}(r)] \\ &= [\,\rho_2(\vec{x_i},\vec{y_i},\vec{x_i},\vec{y_i},\beta) - \frac{1}{2}\rho_2(\vec{x_i},\vec{y_i},\vec{y_i},\vec{x_i},\beta)]/\rho_1(\vec{x_i},\vec{x_i},\beta)\,\rho_1(\vec{y_i},\vec{y_i},\beta) \ . \end{aligned} \tag{A4}$$

When particles i and j have charges of opposite sign, their r.d.f. can be considered as a sum of two terms:

$$g_{ij}(r) = g_b(r) + g_s(r) \ , \tag{A5}$$

where $g_b(r)$ and $g_s(r)$ stand, respectively, for the contribution to the r.d.f. from the bound and the scattered states. In other cases, there are no bound states and $g_{ij}(r)$ is simply equal to $g_s(r)$.

Appendix B

Evaluation of the Integral : $\int_P^Q dy\, E_1[\alpha'(P+y)]/y$

We wish to evaluate the integral

$$I \equiv \int_P^Q \frac{dy}{y} E_1[\alpha'(P+y)] , \tag{B1}$$

where $$E_1[\alpha'(P+y)] = \int_{\alpha'(P+y)}^{\infty} dt\, \frac{e^{-t}}{t} \tag{B2}$$

is the exponential integral. Substituting (B2) into (B1) and permuting the order of integration, we first obtain

$$I = \int_{2\alpha'P}^{\alpha'(Q+P)} dt\, \frac{e^{-t}}{t} \ln(t-\alpha'P) - \ln(\alpha'P)\{E_1(2\alpha'P) - E_1[\alpha'(Q+P)]\} + \ln\frac{Q}{P} E_1[\alpha'(Q+P)] . \tag{B3}$$

Evaluation of the first term on the r.h.s. requires some artifice. If we write it as

$$\int_{2\alpha'P}^{\alpha'(Q+P)} dt\, \frac{e^{-t}}{t} \ln(t-\alpha'P) = \int_{\alpha'P}^{\alpha'Q} du\, \frac{e^{-(u+\alpha'P)}}{u+\alpha'P} \ln u \cong e^{-\alpha'P} \int_{\alpha'P}^{\alpha'Q} du\, \frac{e^{-u}}{u} \ln u ,$$

the above approximation is equivalent to retaining the first term in the expansion

$$\frac{1}{u+\alpha'P} = \frac{1}{u}\left\{1 - \frac{\alpha'P}{u} + \frac{(\alpha'P)^2}{u^2} - \cdots\right\} ,$$

because of $\alpha'P \ll 1$. Now, the integration by parts yields

$$\int_{\alpha'P}^{\alpha'Q} du\, \frac{e^{-u}}{u} \ln u = e^{-\alpha'P}\left[\frac{1}{2}\{[\ln(\alpha'Q)]^2 e^{-\alpha'Q} - [\ln(\alpha'P)]^2 e^{-\alpha'P}\} + \frac{1}{2}\int_{\alpha'P}^{\alpha'Q} du\, e^{-u}(\ln u)^2\right] . \tag{B4}$$

By the change of variable $v = u/\alpha'P$, the last integral is expressed as

$$\int_{\alpha'P}^{\alpha'Q} du\, e^{-u}(\ln u)^2 = e^{-\alpha'P}[\ln(\alpha'P)]^2 + 2E_1(\alpha'P)\ln(\alpha'P) + \alpha'P\int_1^{\infty} dv\, e^{-\alpha'Pv}(\ln v)^2 , \tag{B5}$$

because of $Q/P \gg 1$. In view of evaluating the integral of the type $\int_1^{\infty} dv e^{-\mu v}(\ln v)^n$ with $n \geqslant 2$, we start from the identity

$$\int_1^\infty du\, u^\alpha e^{-\mu u} = \frac{1}{\mu^{\alpha+1}}\{\Gamma(\alpha+1)-\gamma(\alpha+1,\mu)\}, \qquad \Re e\ \mu>0 \tag{B6}$$

where $\gamma(\alpha,x)$ is the incomplete gamma function. Differentiating twice both sides with respect to α and setting $\alpha=1$ in the resulting expression, we obtain

$$\int_1^\infty du\, e^{-\mu u}(\ln u)^2 = \frac{1}{\mu}[(\ln\mu)^2\{\Gamma(1)-\gamma(1,\mu)\}-2\ln\mu\{\Gamma'(1)-\gamma'(1,\mu)\}+\Gamma''(1)-\gamma''(1,\mu)]$$

$$= \frac{1}{\mu}(\ln\mu)^2 + 2\gamma\frac{\ln\mu}{\mu} + \frac{1}{\mu}\{\zeta(2)+\gamma^2\} - 2\sum_{n=1}^{\infty}\frac{(-\mu)^{n-1}}{n!\,n}, \tag{B7}$$

where use has been made of the identities [10]

$$\psi'(x) = \frac{1}{\Gamma^2(x)}\{\Gamma''(x)\Gamma(x)-[\Gamma'(x)]^2\}$$

and $\psi^{(n)}(1) = (-)^{n+1}n!\,\zeta(n+1)$ with $n\geq 1$. γ is Euler's constant. Setting $\mu=\alpha' P$ in (B7) and collecting the results obtained, we then obtain

$$\int_{2\alpha' P}^{\alpha'(Q+P)} dt\, \frac{e^{-t}}{t}\ln(t-\alpha' P) = \frac{1}{2}\{[\ln(\alpha' Q)]^2 e^{-\alpha' Q} - [\ln(\alpha' P)]^2 e^{-\alpha' P} + \zeta(2)+\gamma^2+1\}. \tag{B8}$$

By virtue of the power expansion of $E_1(2\alpha' P)$ in (B3), the final result reads as

$$I \simeq \ln\frac{Q}{P}\, E_1(\alpha' Q) + (\gamma+\ln 2)\ln(\alpha' P) + \frac{1}{2}\{\zeta(2)+\gamma^2+1\}. \tag{B9}$$