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Freezing of Quiescent Water in a Horizontal Cylinder

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SYNOPSIS

Heat transfer measurements were conducted during freezing of quiescent water in a horizontal cylinder. A horizontal cylinder with inner diameter of 61.1 mm is cooled by air in a constant low temperature room and time variations of the radial distribution of fluid temperature were observed. Experimental results for the velocity of the phase change interface, the time taken for complete freezing and apparent freezing heat transfer coefficient were compared with the simple theoretical model based on the quasi-steady assumption.

INTRODUCTION

Freezing of quiescent water in an enclosure is an important problem in relation to the cold heat storage(1) and food engineering, etc. A comprehensive review of the freezing model is given in Hattori(2). The phase change process is essentially unsteady. A model based on the quasi-steady assumption, however, usually gives a good prediction if the latent heat of freezing is larger than the sensible one. The present study was undertaken to confirm the model based on the quasi-steady assumption during freezing of quiescent water in a horizontal cylinder.

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ANALYSIS BASED ON QUASI-STEADY ASSUMPTION

Consider the growth rate of the annular ice layer in a horizontal cylinder with inner radius of R_i and the outer one of R_o. The heat liberated at the phase change interface flows outward through the ice layer and tube wall to the ambience. If the fluid is kept at its freezing temperature t_f during the phase change and also the wall resistance can be neglected in comparison with the other ones, an energy balance can be written as

$$\frac{dy}{d\tau} 2\pi r \rho_i L = \frac{t_f - t_\infty}{\ell n(R_i/r)/(2\pi\lambda_i) + 1/(2\pi R_o \alpha_o)}$$

where y is the radial distance measured from the inner tube wall to the phase change interface, $r = R_i - y$ is the radius of the phase change interface, τ is the time, t_∞ is the ambient temperature, α_o is the heat transfer coefficient at the outer surface of the cylinder. Physical properties L, ρ_i and λ_i are the latent heat of freezing, ice density and thermal conductivity of ice, respectively. The above equation can be reduced to

$$r\frac{dy}{d\tau} = \frac{t_f - t_\infty}{\rho_i L\{\ell n(R_i/r)/\lambda_i + 1/(R_o\alpha_o)\}} \tag{1}$$

If the relation $\ell n(R_i/r)/\lambda_i \ll 1/(R_o\alpha_o)$ holds (e.g., the cylinder is located in a low temperature room without air flow effects), equation (1) gives the following equivalent results

$$r\frac{dy}{d\tau} = \frac{t_f - t_\infty}{\rho_i L} R_o \alpha_o \tag{2}$$

or

$$(R_i - y)\frac{dy}{d\tau} = \frac{t_f - t_\infty}{\rho_i L} R_o \alpha_o \tag{3}$$

Solution to equation (3) subject to the initial condition, $y = 0$ at $\tau = 0$, gives

$$y = R_i - \sqrt{R_i^2 - 2(t_f - t_\infty)R_o\alpha_o\tau/\rho_i L} \tag{4}$$

Time taken for complete freezing τ can be obtained by substituting $y = R_i$ into equation (4) as

$$\tau = \frac{R_i{}^2\rho_i L}{2(t_f - t_\infty)R_o\alpha_o} \tag{5}$$

Apparent freezing heat transfer coefficient α_i is defined on the basis of the phase change interface basis as

$$\alpha_i = \frac{\lambda_i}{r\ln(R_i/r)} \tag{6}$$

Equation (6) indicates that the α_i value depends only on r value and takes a minimum at r_{min} given by

$$r_{min} = R_i/e \tag{7}$$

EXPERIMENT

Experimental Apparatus and Procedure Schematic view of the test cylinder is shown in Figure 1. The cylinder, filled with water, is placed horizontally in a constant low temperature room and is cooled by air. The cylinder is made of copper with dimensions as follows: outer diameter, 63.1 mm; wall thickness, 1.0 mm; tube length, 500 mm. Both ends of the cylinder are covered with rubber sheets with 1 mm in thickness to ensure that abrupt pressure rise due to freezing is avoided.

Eleven enamel-coated type-K thermocouples are used for measuring the radial distributions of the fluid temperature. A thin acrylic rod with these thermocouples is fixed at the mid cross-section of the cylinder. Thermocouple installation is shown in Table 1. Tube wall temperature is observed by the same type one soldered to the outer surface of the cylinder. Ambient air temperature variation around the

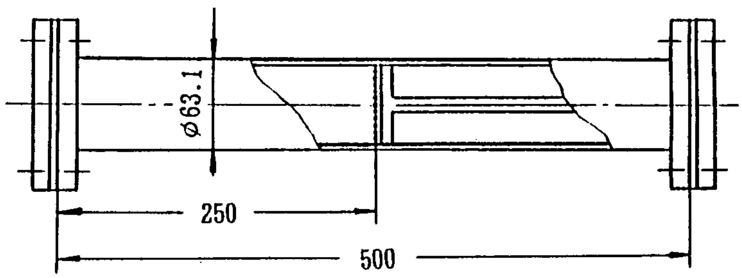

Figure 1 Side view of test cylinder

Table 1 Installation of thermocouple for measuring radial temperature distribution

No of thermocouple	1	2	3	4	5	6	7	8	9	10	11
Distance measured from inner tube wall (mm)	3.05	5.55	8.05	10.55	13.05	15.55	18.05	20.55	23.05	25.55	30.55

cylinder are also measured. The outer surface of the cylinder is thermally insulated by fiberglass insulating material.

Preliminary experiments, conducted using transparent end-sheets for the rubber sheets, revealed that one-dimensional radial heat flow is considered to be predominant since little axial variation of the phase change interface was observed during the freezing process. It is also confirmed that the measurements with high accuracy were carried out since an agreement of within 12.3% was found between the observed heat transfer rate and the calculated total amount of the latent heat of freezing (481.4 kJ).

Results and Discussion Tables 2 and 3 summarize the results obtained at $t_\infty = -18^\circ C$ and $-28^\circ C$, respectively. In Tables 2 and 3, values of q_ℓ, α_i and ε are obtained respectively from

$$q_\ell = 2\pi R_o \alpha_o (t_w - t_\infty) \tag{8}$$

$$\alpha_i = q_\ell / \{2\pi r (t_f - t_\infty)\} \tag{9}$$

and

$$\varepsilon = 1 - (r/R_i)^2 \tag{10}$$

where q_ℓ, α_i and ε are the heat transfer rate at the tube surface per tube length, the apparent freezing heat transfer coefficient and the ratio of the cross-sectional area of the ice layer to the tube cross-sectional value, respectively. The results are compared with the prediction method described in the previous chapter.

Table 2 Measured results obtained at ambient temperature of $t_\infty = -18^\circ C$

τ	hr	2.5	4.38	-	6.84	8.25	9.05	10.12	10.55	10.75	11.39	11.75	12.19
y	mm	3.05	5.55	8.05	10.55	13.05	15.55	18.05	19.31	20.55	23.05	25.55	30.55
t_w	°C	-0.4	-0.8	-	-1.3	-1.6	-1.8	-2.0	-2.5	-2.5	-2.9	-3.8	-4.1
α_o	W/(m²K)	5.94	5.94	-	5.83	5.81	5.79	5.77	5.72	5.72	5.69	5.62	5.57
q_ℓ	W/m	20.69	20.22	-	19.27	18.86	18.57	18.27	17.55	17.55	17.01	16.02	15.32
α_i	W/(m²K)	299.4	160.9	-	118.0	107.2	109.5	116.3	99.4	111.7	124.5	141.6	-
ε	%	19.0	33.0	-	57.1	67.2	75.9	83.3	86.5	89.3	94.0	97.3	100.0

Table 3 Measured results obtained at ambient temperature of t_∞ = -28°C

τ	hr	1.6	2.82	3.36	3.98	4.66	5.52	6.26	6.5	6.68	6.98	7.20	7.58
y	mm	3.05	5.55	8.05	10.55	13.05	15.55	18.05	19.31	20.55	23.05	25.55	30.55
t_w	°C	-0.5	-1.0	-1.1	-1.2	-1.7	-2.2	-2.8	-3.3	-3.5	-3.7	-3.7	-4.7
α_o	$W/(m^2K)$	6.65	6.62	6.62	6.61	6.58	6.55	6.51	6.51	6.46	6.45	6.45	6.38
q_ℓ	W/m	36.20	35.38	35.25	35.06	34.25	33.45	32.47	31.58	31.33	31.02	31.02	29.42
α_i	$W/(m^2K)$	419.0	225.2	226.7	232.5	183.2	161.3	147.7	135.5	142.5	177.9	266.9	-
ε	%	19.0	33.0	45.8	57.1	67.2	75.9	83.3	86.5	89.3	94.0	97.3	100.0

Figure 2 shows the effects of the ambient temperature t_∞ on the variation of the phase change interface with time. In Fig.2, lines 1 and 2 show the comparison at t_∞ = -28°C and lines 3 and 4 at t_∞ = -18°C. It is seen from Fig. 2 that the observed velocity of the phase change interface (lines 2 and 4), $dy/d\tau$, increases monotonically with increasing τ, and the $dy/d\tau$ value at t_∞ = -28°C is larger than that at t_∞ = -18°C. In both cases, the observed $dy/d\tau$ value is somewhat smaller than the prediction of equation (4) so that the observed time taken for complete freezing (τ value at y = 30.55 mm) is somewhat longer than the prediction of equation (5). A possible source of the discrepancies between the prediction and the measurement includes that the thermal conductance of the ice layer is neglected in the present analysis (average ratio of the conductance of the ice layer to the wall-to-ambient value is estimated to be less than 5%).

Figure 3 shows the variation of the apparent freezing heat transfer coefficient α_i with radius of phase change interface r, where line 1 shows the prediction of equation (6), and lines 2 and 3 the observed value at t_∞ = -28 and -18°C,

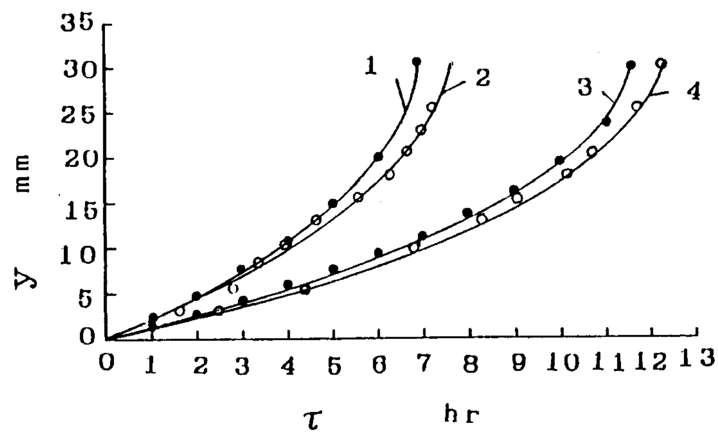

Figure 2 Variation of phase change interface with time

respectively. The α_i value first show a sharp decrease with decreasing of r, and then it takes a minimum and shows a rapid upturn with further decreasing of r. The two data sets show a good agreement with equation (6) in trend, but show a dependence on t_∞ not predicted by the equation.

Neglecting the effect of the wall resistance, α_i can be related to α_o as

$$\alpha_i 2\pi r(t_f - t_w) = \alpha_o 2\pi R_o(t_w - t_\infty) \quad (11)$$

Figure 4 compares the observed and predicted tube wall temperatures t_w, where the latter was obtained by substituting a conventional correlation equation of natural convection heat transfer coefficient into equation (11). In Fig.4, lines 1 and 2 show the comparison at t_∞ = -28°C and lines 3 and 4 at t_∞ = -18 °C. The observed t_w value at t_∞ = -28°C (line 1) decreases more rapidly than that for t_∞ = -18°C (line 3).

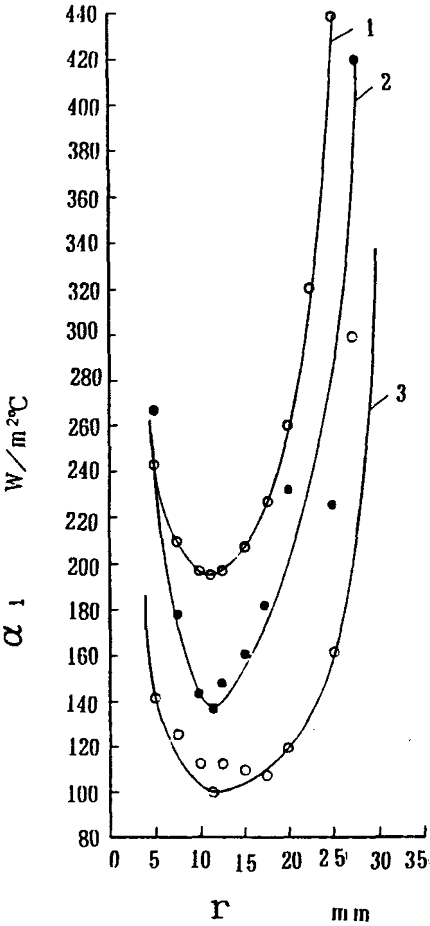

Figure 3 Relationship between apparent freezing heat transfer coefficient and radius of phase change interface

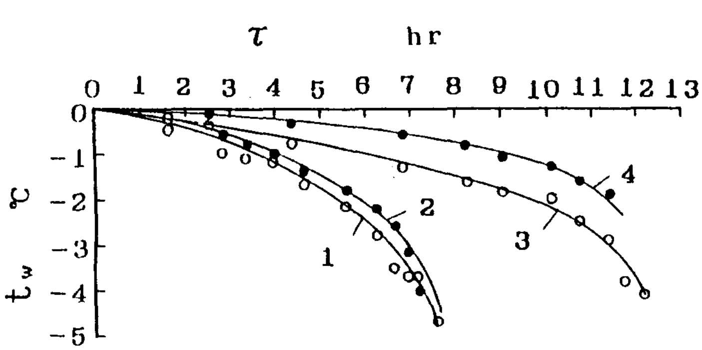

Figure 4 Variation of tube wall temperature with time

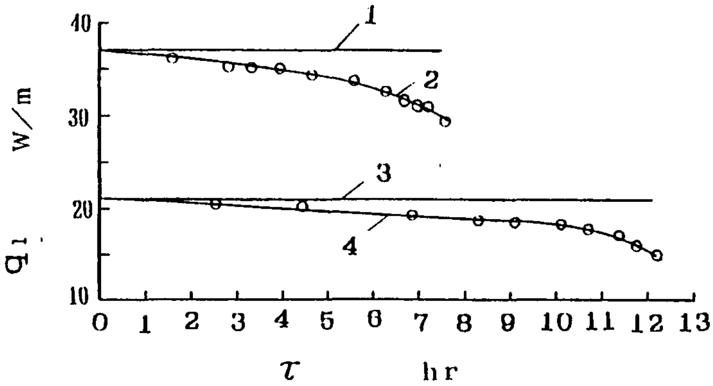

Figure 5 Variation of heat transfer rate per tube length with time

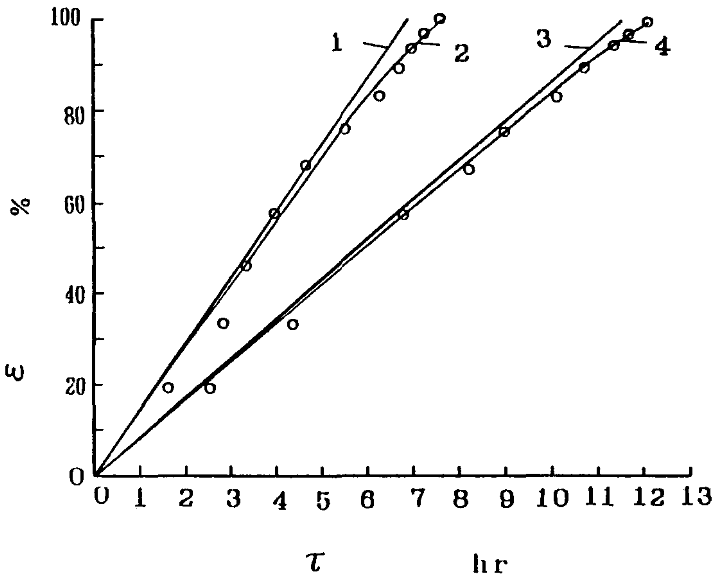

Figure 6 Variation of cold heat stored with time

Figure 5 shows the variation of q_ℓ with τ, where lines 1 and 3 are the predictions obtained from equation (8) and lines 2 and 4 are the observed results. The observed value decreases moderately with increasing τ, and consequently then decreases rapidly near the end of the phase change.

Figure 6 shows the ratio of the cross-sectional area of the ice layer to the tube value plotted on the coordinates of ε and τ. Lines 1 and 3 show the prediction of equation (10) for t_∞ = -28 and -18°C, respectively. The observed results take lower ε value than the prediction, with the difference increasing with τ.

CONCLUSIONS

Heat transfer measurements have been conducted during freezing of quiescent water in a horizontal cylinder cooled by natural convection of air. The results have been compared with a simple analytical model for predicting the freezing process. Conclusions obtained in the present study can be summarized as:

1. The velocity of the phase change interface increases as the freezing proceeds. The radial distance y measured from the tube wall to the phase change interface can be calculated using equation (4). Also the time taken for complete freezing τ can be obtained from equation (5).

2. The apparent freezing heat transfer coefficient α_i depends only on the y value and takes a minimum value at the position given by equation (7). The tube wall temperature can be calculated using equations (6) and (11).

3. The ratio of the cross-sectional area of the ice layer to the tube value, ε, increases lineally with time. The ε value can be obtained from equations (4) and (10).

4. The quasi-steady model can be used to make an approximate estimate for the freezing process as that studied in the present paper.

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