Memoirs of the School of Engineering, Okayama University, Vol. 17, No. 2, June 1983

Computer Aided Design of Thyristor Phase-Control Circuits

Sen-ichiro NAKANISHI*, Yoshiaki KATSUYAMA** and Toyoji HIMEI*

(Received February 15, 1983)

Synopsis

The paper presents the computer aided design (CAD) method, the program, the design and the experimental results of inverse parallel thyristor phase-control circuits. The calculated values agree well with the measured. The CAD program contains the next two methods which are inquired carefully by authors, such as (i) the optimization by SUMT (Sequential Unconstrained Minimization Technique) method, and (ii) the combined use of the gradient and the cramp calculation methods.

1. Introduction

Recently, the ac phase control circuit by phase retard adjustment of inverse parallel thyristors has been used in various power control systems or in a variety of electric circuit configurations such as series or parallel resonant circuit. Thyristor control circuit with series *RLC* elements [1], and with series and parallel load [2] have been already studied. It is pointed out that their circuit behaviors are apt to become complicated, since the thyristor switching action is sometimes influenced by *LC* resonance [3].

Therefore, the selection of circuit configuration and the decision

* Department of Electrical Engineering,
** Now, The Kansai Electric Power Co. Inc.

of circuit constant values must be carried out based on the precise analysis, because the control characteristics of voltage and current etc., are affected extremely by a little deviation of circuit constant values. In such design problems, an electronic computer with excellent abilities of high-speed calculation and processing large data is very useful for the correct and suitable circuit design.

The authors have tried to apply the Computer Aided Design (CAD) method to the thyristor phase control circuit design. And the two points on the application of the CAD method are studied:

(i) The design of the thyristor phase control circuits is regarded to be an optimization problem with constraints, and the optimization is practiced by SUMT (Sequential Unconstrained Minimization Technique) method which uses Newton method in calculation.

(ii) As the Newton method can't be applied to a convexity problem, the defect is concealed by the combined use of the gradient and the cramp methods.

In this paper, the CAD software, the design calculation, and the experimental results of the thyristor phase control circuit are described. The calculated values agree the experiments, and the usefulness of the CAD program is confirmed.

2. Solution of optimization problem [5)]

In this chapter, the minimized (or optimized) method of mth variables and the objective function $f(x)$ are described.

2.1 SUMT method

SUMT (Sequential Unconstrained Minimization Technique) method is one of the solution method of optimization under given conditions. This SUMT method is based on the fact that the minimum value of the new objective function $P_k(x, rk)$ $(rk \to 0)$, gained from the $f(x)$, converges to the limit value of $f(x)$ under the constraints of

$$g_j(x) \geq 0 \qquad (j=1,\text{--------},l) \tag{1}$$

$$h_j(x) = 0 \qquad (j=1,\text{--------},p). \tag{2}$$

Where $P_k(\boldsymbol{x}, r_k)$ is

$$P_k(\boldsymbol{x}, r_k) = f(\boldsymbol{x}) + r_k \sum_{j=1}^{l} [gj(\boldsymbol{x})]^{-1} + r_k^{-1/2} \sum_{j=1}^{p} [hj(\boldsymbol{x})]^2 \qquad (3)$$

$$(k=1,\ 2,\ ---,\ r_1 > r_2 > --- \rightarrow 0)$$

and r_k is a positive number called "perturbation parameter".

This optimization problem can be solved by calculating the minimum value of $P_k(\boldsymbol{x}, r_k)$ with Newton's method to k=1,2,---, in general.

2.2 Newton's method

The Newton's method is a minimizing method of the objective function $P(\boldsymbol{x})$, where $P(\boldsymbol{x})$ is supposed as a convexity function. We put $\boldsymbol{x}$ as $\boldsymbol{x}=(x_1, x_2, ---, x_m)^t$, and expand $P(\boldsymbol{x})$ into higher power terms Taylor's series near at $\boldsymbol{x}_0$, Ignoring the cubic term and the expanded $P(x)$ is written as follows;

$$P(\boldsymbol{x}) \simeq P(\boldsymbol{x}_0) + (\boldsymbol{x}-\boldsymbol{x}_0)^t \cdot \text{grad } P(\boldsymbol{x}_0) + \frac{1}{2}(\boldsymbol{x}-\boldsymbol{x}_0)^t U(\boldsymbol{x}-\boldsymbol{x}_0) \qquad (4)$$

where

$$\text{grad } P(\boldsymbol{x}) = \begin{bmatrix} \frac{\partial P}{\partial x_1}(\boldsymbol{x}) \\ \\ \frac{\partial P}{\partial x_m}(\boldsymbol{x}) \end{bmatrix},$$

$$U = (uij) = \left[\frac{\partial^2 P}{\partial x_i \partial x_j}(\boldsymbol{x}) \right] = \begin{bmatrix} \frac{\partial^2 P}{\partial x_1^2} & --- & \frac{\partial^2 P}{\partial x_1 \partial x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 P}{\partial x_1 \partial_m} & --- & \frac{\partial^2 P}{\partial x_m^2} \end{bmatrix}. \qquad (5)$$

If $P(\boldsymbol{x})$ has minimum value in $\boldsymbol{x}=\boldsymbol{x}^{(1)}$, it is written as follows;

$$\text{grad } P(\boldsymbol{x}^{(1)}) = 0. \qquad (6)$$

Differentiating Eq.(4), and setting $x=x^{(1)}$, the following equation is obtained.

$$x^{(1)} = x_0 - U^{-1}\cdot\text{grad } P(x_0) \ . \tag{7}$$

Eq.(7) isn't always satisfied with Eq.(6) gained from $P(x)$ approximated as Eq.(4). Therefore, in such a case, the minimization problem can be solved till it becomes as follows;

$$\text{grad } P(x^{(n)}) = 0 \tag{8}$$

repeating next procedure;

$$x^{(n+1)} = x^{(n)} - U^{-1}\cdot\text{grad } P(x^{(n)}) \ . \tag{9}$$

But, if $P(x)$ isn't a convexity function in an region, it has not the minimum value in the region. Where a convexity function must be content as follows;

$$P(x+\Delta x) - P(x) > (\Delta x)^t\cdot\text{grad } P(x) \ . \tag{10}$$

If Δx is very small and Eq.(4) is valid the condition of the convexity function is written from Eq.(4) and (10) as follows;

$$(\Delta x)^t U(\Delta x) > 0 \ . \tag{11}$$

As this equation means that $U=(uij)$ is positive, it is necessary that D_i is decided whether plus or not.

$$D_i = \det \begin{bmatrix} u_{11} & --- & u_{1i} \\ & & \\ u_{i1} & --- & u_{il} \end{bmatrix} \ . \tag{12}$$
$$(i=1,---, m)$$

When Newton's method can't be applied, ($D_i \leqq 0$), other methods, need to be used. Actually, it is found that the character of $P_k(x, r_k)$ is improved rather in comparison with only $f(x)$ by the effect of the terms except $f(x)$, that is, the penalty terms ($r_k\Sigma[g_j(x)]^{-1}+ r_k^{-1/2}\Sigma[h_j(x)]^2$) in the objective function shown by Eq.(3). The action of the penalty terms is the next two points.

(i) The constraints are added to the objective function.

(ii) The form of the objective function is improved to a convexity form.

Newton's method (the second-order derivative method) has generally the merit that it is possible to obtain the solution with a fewer repeat times in comparison to the gradient method (the first-order derivative method), but has the defect that it can not be minimized except the convexity function. The defect is onvercomed by the action of the penalty terms and the treatment described in 3.2, so the SUMT method of Newton's method is used in this CAD program.

3. Design program

Here, the outline of the general analytical program and the design program made according to chap.2 are described, and the procedure to gain $x(n+1)$ from $x(n)$ is explained.

3.1 Analytical method

This method uses the state variable method. The analytical circuit is the inverse parallel thyristor phase control circuits with *RLC* element in the series and parallel shown in Fig.1 and modified circuit removed any elements from the circuit shown in Fig.1 . And, therefore the 511 kinds of modified circuits removed any element from the above circuit can be analyzed. It is the generality which the circuits of 511 kinds can be analyzed. The program is composed of the next two parts.

(i) The graphs of the circuits is drawn, and the normal differential equation; $\dot{X}=AX+Bu$ by means of the state variable, is induced.

(ii) The equation is solved with Liou's method, and the RMS values of voltage, current and power, the power factor, the efficiency and the distortion factor are given.

In the analysis, Mode I is thyristor conducting period and Mode II is unconducting.

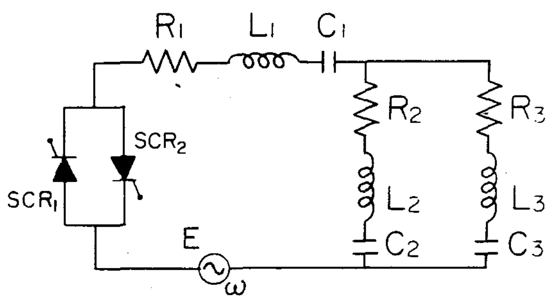

Fig.1 Circuit's form.

The flow-chart is shown in Fig.2. Where, this program is one of the subroutine in the design program.

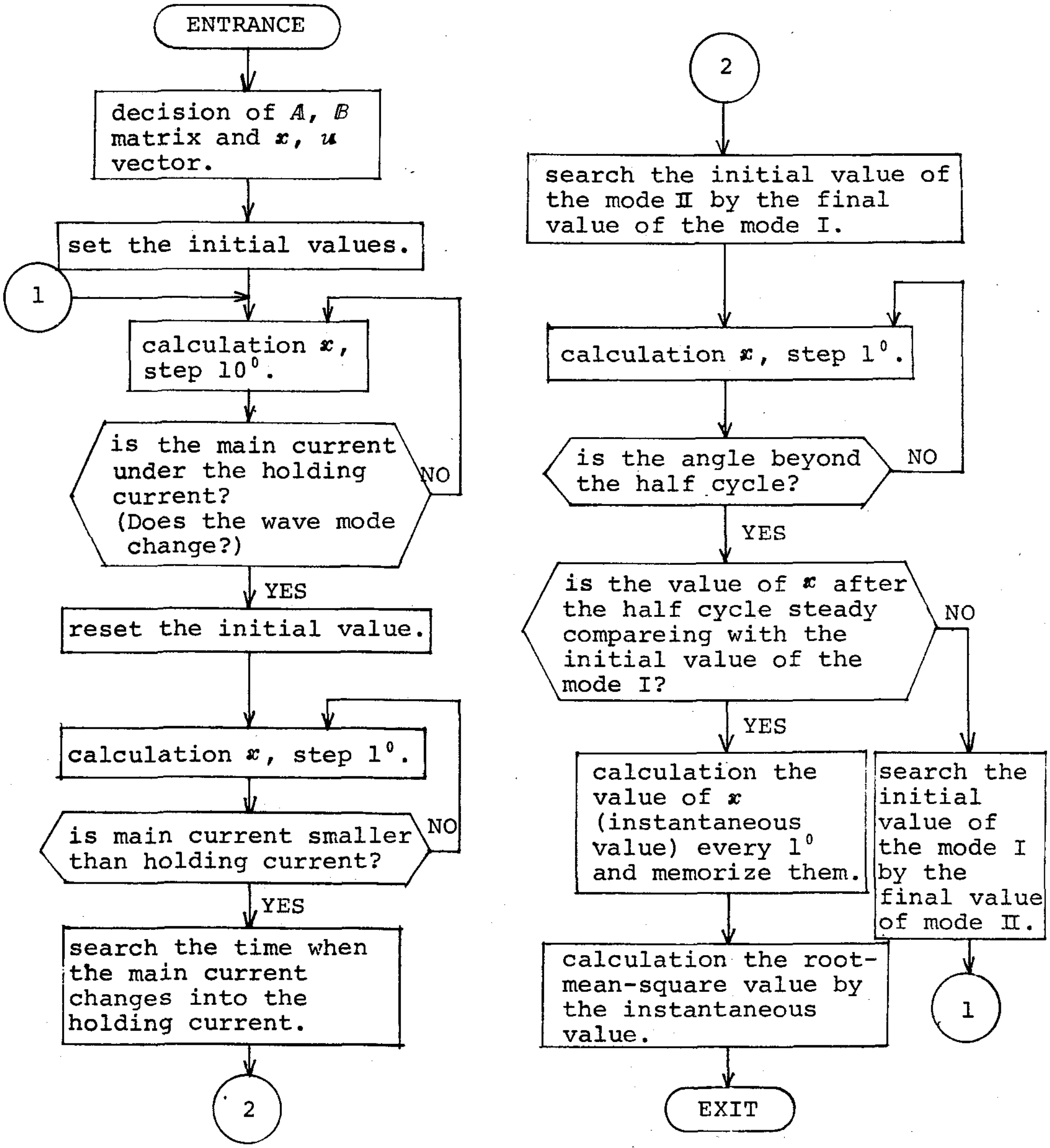

Fig.2 General flow-chart for analytical program.

3.2 Improvement of convergent calculation

As mentioned in Sect. 2.2, if $P_k(\boldsymbol{x}, r_k)$ is not convex, Newton's method can be not used. Accordingly, the convergent calcualtion method is improved with the following treatment.

(i) The maximum and minimum of the paremeter x_i are decided with the use of the gradient of P_k to x_i, $\partial P_k/\partial x_i$. This method is based on the fact that the maximum ($xhgi$) and the minimum ($xlwi$) are obtained from that the sign of $\partial P_k/\partial x_i(\boldsymbol{x}^{(n)}) \times \partial P_k/\partial x_i(\boldsymbol{x}^{(n+1)})$ is negative when the minimum of P_k is hold between $x_i^{(n)}$ and $x_i^{(n+1)}$. The flow-chart is shown in Fig.3.

(ii) When D_i shown in Eq.(12) is not positive, the next operation is carried out in order to apply Newton's method.

(a) When the maximum and the minimum described in (i) have been already decided, $x_i^{(n+1)}$ is searched with the cramp method.

(b) When the maximum and the minimum is not decided, $x_i^{(n+1)}$ is obtained with the gradient $\partial P_k/\partial x_i$ and $\partial^2 P_k/\partial x_i^2$. This is based that supposing of $x_i>0$, the solution is smaller than $x_i^{(n)}$ in $\partial P_k/\partial x_i>0$ and bigger than $x_i^{(n)}$ in $\partial P_k/\partial x_i<0$.

When $\partial^2 P_k/\partial x_i^2 \leqq 0$, the varying ratio of x_i is programed to enlarge so that P_k is not convexity about x_i. The flow-chart is shown in Fig.4.

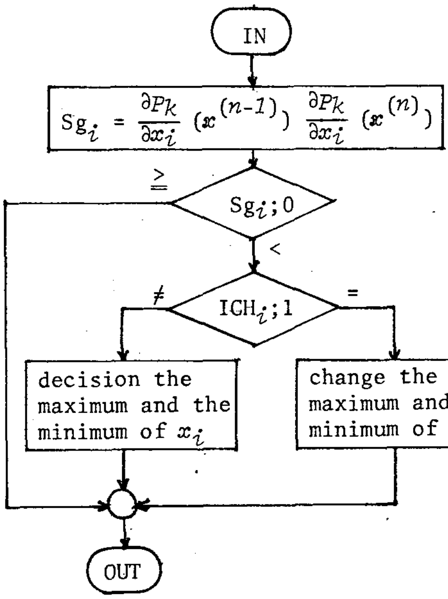

Fig.3 Decision of maximum and minimum.

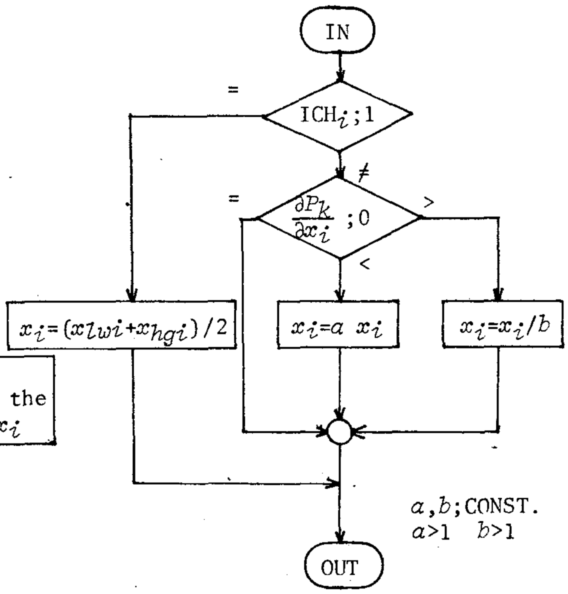

Fig.4 Operation when Newton's method can not be applicated.

In Fig.3 and 4, *ICHi*=1 shows the decision of the maximum and the minimum and *ICHi*$\neq$1 shows not.

3.3 Numerical differentiation

The differential coefficient $\partial P_k/\partial x_i$, $\partial^2 P_k/\partial x_i \partial x_j$ is required in optimizing. In CAD $P_k(\boldsymbol{x}, r_k)$ is not always a integral function and whether the derivative is gained or not is not made clear. So, the difference equation is used. About $\partial P_k/\partial x_i$, $\partial^2 P_k/\partial x_i^2$ the next approximate equation well known is used.

$$\frac{\partial P_k}{\partial x_i}(\boldsymbol{x}^{(n)}) = \frac{1}{2\Delta x_i}[P_k(x_1^{(n)}, ---, x_i^{(n)}+\Delta x_i, ---, x_m^{(n)}) - P_k(x_1^{(n)}, ---, x_i^{(n)}-\Delta x_i, ---, x_m^{(n)})] \tag{13}$$

$$\frac{\partial^2 P_k}{\partial x_i^2}(\boldsymbol{x}^{(n)}) = \frac{1}{(\Delta x_i)^2}[P_k(x_1^{(n)}, ---, x_i^{(n)}+\Delta x_i, ---, x_m^{(n)}) - 2P_k(x_1^{(n)}, ---, x_i^{(n)}, ---, x_m^{(n)}) + P_k(x_1^{(n)}, ---, x_i^{(n)}-\Delta x_i, ---, x_m^{(n)})] \ . \tag{14}$$

In the both equations, the error is the order of $(\Delta x_i)^2$, $\partial^2 P_k/\partial x_i \partial x_j$ $(i \neq j)$ is gained as follows. Setting Cpd as Eq.(15),

$$\begin{aligned} \mathrm{Cpd} = {} & P_k(x_1^{(n)}, ---, x_i^{(n)}+\Delta x_i, ---, x_j^{(n)}+\Delta x_j, ---, x_m^{(n)}) \\ & + P_k(x_1^{(n)}, ---, x_i^{(n)}-\Delta x_i, ---, x_j^{(n)}-\Delta x_j, ---, x_m^{(n)}) \\ & - P_k(x_1^{(n)}, ---, x_j^{(n)}+\Delta x_i, ---, x_j^{(n)}-\Delta x_j, ---, x_m^{(n)}) \\ & - P_k(x_1^{(n)}, ---, x_i^{(n)}-\Delta x_i, ---, x_j^{(n)}+\Delta x_j, ---, x_m^{(n)}) \end{aligned} \tag{15}$$

Cpd is expanded up to the third order terms, as $\Delta x = \max\{|\Delta x_i|, |\Delta x_j|\}$.

$$\mathrm{Cpd} = 4\frac{\partial^2 P_k}{\partial x_i \partial x_j}\Delta x_i \Delta x_j + O(\Delta x^4) \tag{16}$$

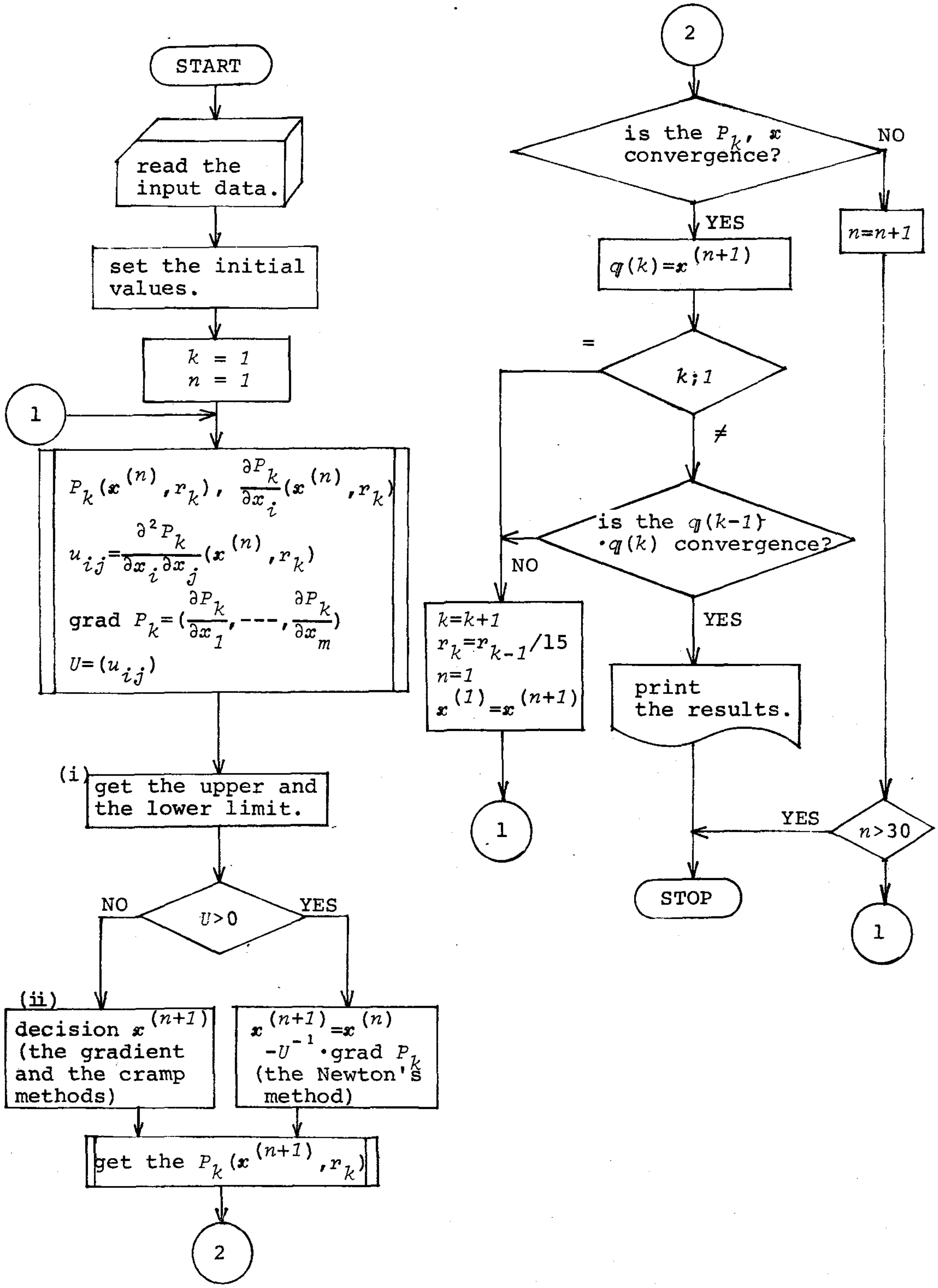

Fig.5 General flow-chart for design program.

Where $O(\Delta x^4)$ is the error of Δx^4 order. And Δx_i is as follows ;

$$\Delta x_i = |x_i|/1000 \tag{17}$$

Though x_i is the value of the parameter, each x_i can be changed to the same order with a suitable transformation. Then, $\Delta x_i \cong \Delta x_j$, and from Eq.(16)

$$\frac{\partial^2 P_k}{\partial x_i \partial x_j} = \frac{\mathrm{Cpd}}{4\Delta x_i \Delta x_j} \tag{18}$$

is gained. The error is about the order of Δx^2. The flow-chart of the design program by means of the above-mentioned methods is shown in Fig.5. This design program can be applied to the circuit forms which any element are omitted from the circuit shown in Fig.1. The input data are as follows;

(i) The information which is needed in drawing the graphs of the circuit.

(ii) The initial values of the parameters.

4. Design values and experiment results of trial circuit

The formulation of the objective function when the above design program is used, is described in this chapter. And the calculated values are compared with the experimental values and the circuits obtained from the design is discussed.

4.1 Formulation of objective function and design values[6)]

As an example of the design, the CAD of the circuit in Fig.6 is executed satisfying the specification shown in Table 1. This circuit is the figure that C_1, R_2, L_2 and C_3 are omitted in Fig.1 and, R_3 and L_3 are loads, L_1 and $C2$ are filters. Where the phase control angle α, L_1 and C_2 are parameters of optimization, and R_1 is constant 3Ω. The constraints are given as follows;

Load power R_L:

$$h_1 = R_L - 265 = 0 \tag{19}$$

$$g_1 = 300 - P_L \geqq 0 \tag{20}$$

$$g_2 = P_L - 200 \geqq 0 \tag{21}$$

RMS value E_L of load volt:

$$h_2 = E_L - 100 = 0 \tag{22}$$

$$g_3 = 120 - E_L \geqq 0 \tag{23}$$

$$g_4 = E_L - 70 \geqq 0 \tag{24}$$

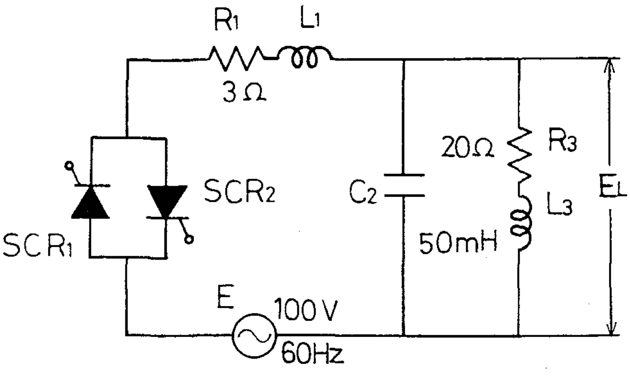

Fig.6 Design circuit.

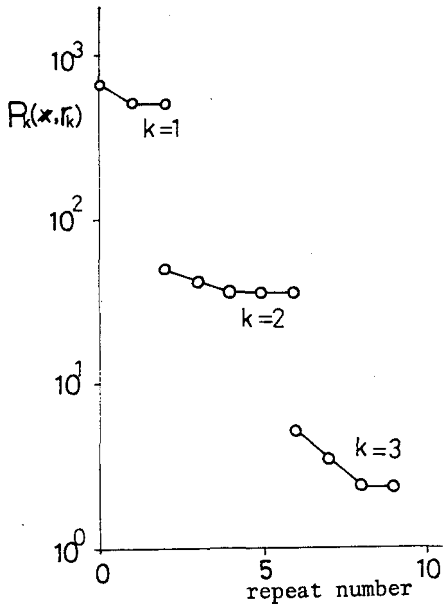

Fig.7 Convergence trace of $P_k(x, r_k)$.

Table 2 Values of K_{hj} and K_{gj}.

K_{h1}	0.005
K_{h2}	0.03
K_{g1}	400
K_{g2}	300
K_{g3}	60
K_{g4}	50
K_{g5}	100
K_{g6}	100
K_{g7}	4
K_{g8}	250
K_{g9}	750
K_{g10}	500
K_{g11}	500
K_{g12}	400
K_{g13}	400

Table 1 Specifications.

output power	P_L (W)	265
output voltage (r.m.s)	E_L (V)	100
power factor	$P.F.$ (%)	over 85
efficiency	η (%)	over 80
distortion factor	$D.F.$	less 1.0

Load power factor

$$g_5 = \mathrm{P.F.} - 85 \geqq 0 \tag{25}$$

Efficiency :

$$g_6 = \eta - 80 \geqq 0 \tag{26}$$

Distortion factor D.F.:

$$g_7 = 1.0 - \mathrm{D.F.} \geqq 0 \tag{27}$$

Elements L_1, C_2:

$$g_8 = L_1 \geqq 0 \tag{28}$$

$$g_9 = 200 - L_1 \geqq 0 \tag{29}$$

$$g_{10} = C_2 \geqq 0 \tag{30}$$

$$g_{11} = 200 - C_2 \geqq 0 \tag{31}$$

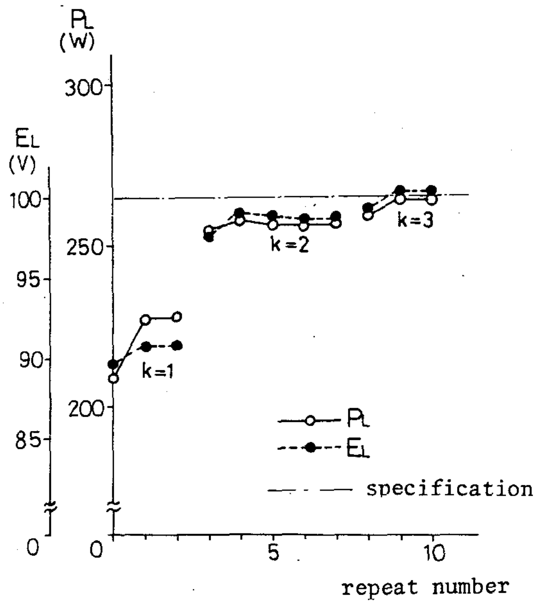

Fig.8 Convergence trace of P_L and E_L.

Where the unit of L_1, C_2 is mH and μF respectively.

Phase control angle α(deg):

$$g_{12} = 180 - \alpha \geqq 0 \tag{32}$$

$$g_{13} = \alpha - \phi \geqq 0 \tag{33}$$

Where α(deg) is the displacement angle of the load.

If these constraints is straightly substituted to Eq.(3), the effects of each equation to the objective function are different. So introduction of the weight constants K_{hi} and K_{gi} is need. The objective function as Eq.(34) is induced with the modification of Eq.(3). The values of K_{hi} and K_{gi} is shown in Table 2.

$$P_k(\boldsymbol{x},\ r_k) = r_k^{-1/2} \sum_{j=1}^{2} K_{hj}\{hj(\boldsymbol{x})\}^2 + r_k \sum_{j=1}^{13} K_{gj}\{gj(\boldsymbol{x})\}^{-1} \tag{34}$$

Where there is no term corresponding to $f(x)$ of Eq.(3), because α=48(deg) as the initial value, L_1=19(mH), C_2=69(μF) and r_1=5 are introduced. The convergence trace to the repeat times of $P_k(\boldsymbol{x},\ r_k)$,

P_L and E_L are shown in Fig.7 and Fig.8. The employed digital computer was ACOS700 of the Okayama University Computer Center, and the used CPU time was about 6 minites.

4.2 Experiment results of trial circuit

The experimental values are shown in Table 3, compared with the design values and characteristics calculated. It is clear that the design values agree approximately with the experimental values. Though the experimental value of the distortion factor is not measured, the distortion factor seems also agree, because the fact the instantanous values of voltage and current agree, is confirmed by the general analytical program [4] used here.

Table 3 Comparison of design values with experimental values.

		design values	experimental values
R_1	(Ω)	3.0	3.0
R_3	(Ω)	20.0	20.0
L_1	(mH)	16.6	16.2
L_3	(mH)	50.0	50.0
C_2	(μF)	103.7	103.8
α	(deg)	17.0	17.0
P_L	(W)	264.8	260.4
E_L	(V)	100.3	99.7
P.F.	(%)	93.4	93.8
η	(%)	89.9	90.1
D.F.		0.318	——

5. Conclusions

The CAD of inverse parallel thyristor phase control circuits and the experimental results are described. The good agreement between the design values and the experimental values is confirmed. This result suggests that the CAD method is useful to the thyristor phase control circuit with resonance elements.

References

1) W. Shepherd;"Steady-State Analysis of the Series Resistance-Inductance Circuit Controlled by Sillicon Controlled Recifier", IEEE, Vol.IGA-1, No.4, P259 (1965)
2) B. W. Lingard, R. W. Johnson and W. Shepherd;"Steady-State

Performance and Analysis of Series Resistance-Capacitance Circuit with Control by Adjustable Thyristor Triggering", IEEE, Vol.IGA-4, No.6, P644 (1968).
3) T. Himei, J. Inouye, S. Nakanishi and I. Ukita;"A Step-up Phenomenon of the Thyristor Control Circuit with Series *RLC* Elements", IEEE, Vol.IA-11, No.5, P531 (1975).
4) H. Ishida;"Study on General Analytical Program of Thyristor Phase-Control Circuit", MASc. Thesis Okayama University, 1976.
5) D. M. Himmelblau;"Applied Nonlinear Programming", McGRAW-HILL(1972).
6) T. Okuyama;"Study on Application of CAD to Discharge Lamp Operating Circuit", MASc. Thesis Okayama University, 1976.