

Urban Expressway Pricing under Constraint

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SYNOPSIS

Four radials-single ring urban expressway network is priced and, simultaneously, optimized in its spatial formation under the constraint that a balance must be kept of revenue and expenditure. The model consists of three sub-models; road system, car trip generation-attraction and traffic diversion. Network performance is assessed on two criteria; trip number criterion on which the aggregate number of car trips diverted onto expressway is maximized and travel hour criterion on which the travel hours of car trips integrated over the road system; surface and expressway, is minimized. Optimization is tried by numerical calculation for some sets of parameters in the model. The results are summarized as follows; (1) simultaneous optimization of price and spatial formation of the expressway network is possible on each of criteria, (2) trip number criterion produces lower pricing and smaller network while travel hour criterion does higher pricing and larger network, (3) optimum solution lies in a delicate relation of price and spatial network formation that comes from the balance constraint.

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1. INTRODUCTION

The first twoes of the authors have been concerned with a subject for urban expressway pricing and planning under constraints. Key constraint in the problem is a balance of revenue and expenditure that has been existing in urban expressway system in Japan. It comes from the balance constraint that pricing and planning urban expressway network are interdependent. Revenue is subject to pricing and the aggregate of car trips diverted onto expressway network, which depends on pricing and spatial formation of the network on which expenditure, it is assumed in the study, depends. The interdependence, symbolicaly, is represented by $R\{F, Q(F, S)\} = E(S)$, where R and E are revenue and expenditure, respectively, Q is the aggregate of diverted car trips, and F and S are toll rate and spatial network formation, respectively.

The subject has been studied in two field; welfare economics and transportation planning. Yamada [1], Myojin et al. [2], [3], [4] and [5] are those in the former, Sasaki et al. [6], [7] and Myojin et al. [8] in the latter. The present paper is a comparative study and the 3atter.

New criterion is introduced to compare with the one that was adopted in [8]. The model defined in [8] is modified in order to introduce new criterion. Major redefinitions are made of (1) speed functions that were used in traffic diversion submodel and (2) traffic capacity constraint that was placed on expressway network not to overflow with traffic flow. Traffic capacity constraint is omitted in the present paper because of impertinence to comparative study on two criteria.

2. MODEL DEFINITION

The present model is defined by three submodels with one constraint, as shown in Fig.1, those are called road system, trip generation-attraction, traffic diversion and equilibrium of revenue and expenditure, respectively, hereafter in the paper.

Authors are interested in comparison of two criteria, trip number criterion and travel hour criterion, on which the expressway system under study is optimized. On the former the system is decided so as to maximize the aggregate number of car trips diverted onto expressway, while on the latter, it is decided to minimize the travel hours

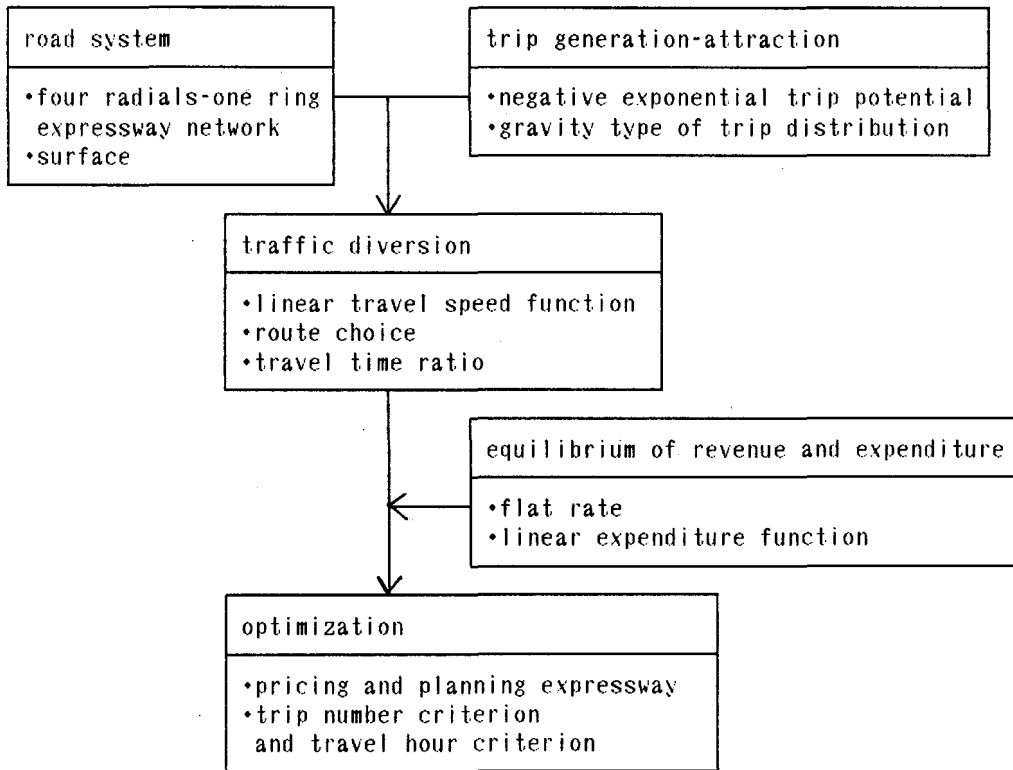


Fig. 1 Model definition

of car trips integrated over the whole road system. The model is described in the following.

(1) Road system (Fig.2)

Road system consists of surface road and urban expressway. Surface roads are free and assumed to be supplied very densely in both radial and circumferential directions. Expressway is toll road of flat rate, having four radials with one ring. Radial expressways meet at right angles at the center of the city under study. Entrance and exit ramps of expressway are assumed to be located as densely as surface roads. The assumptions on the density of surface roads and expressway ramps are rather an expedient for numerical solutions. By the way toll system of flat rate is existing in Japanese urban expressway.

(2) Trip generation-attraction

Every car trip is generated and attracted in the city under study according to trip potential, whose function is assumed in common to

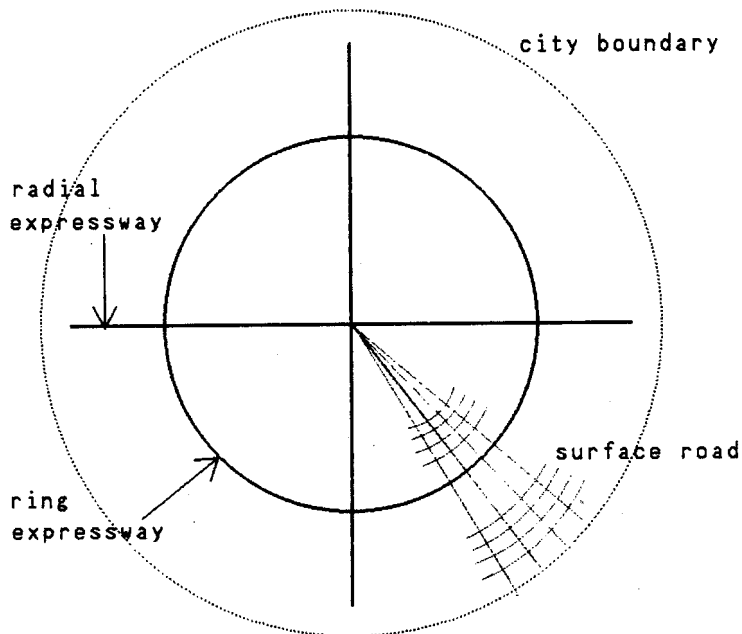


Fig.2 Road system

generation and attraction as

$$f(r) = \mu e^{-\lambda r} \quad (1)$$

where

r : air-line distance to the city center,
 μ and λ : parameters.

This function represents car-trip-generating and car-trip-attracting rates as measured, for example, in the daily number of trip ends per unit area. Accordingly, integration of the trip potential function over the area under study gives the daily number, for example, of car trips in the area. Note that the potential function is assumed to be homogeneous in circumferential direction.

Gravity type is applied to origin-destination distribution of trips.

$$g(r_1, \theta_1; r_2, \theta_2) = \frac{\alpha \{f(r_1) \times f(r_2)\}^p}{\{d(r_1, \theta_1; r_2, \theta_2)\}^F} \quad (2)$$

where

g ; the number of trips between the points (r_1, θ_1) and (r_2, θ_2)
 where r_i is the distance to the city center and
 $0 \leq \theta_i < 2\pi, i=1,2.$

$f(r_i)$; trip potential at r_i ,

d ; air-line distance between the points,

α, β and γ ; parameters.

No trip is assumed to travel along surface road in any other direction than in radial and circumferential directions, in which surface road is assumed to exist.

(3) Traffic diversion

This submodel is concerned with estimation of the number of car trips diverted to expressway. Four steps constitutes the submodel as shown in Fig.3 ; speed, route choice, diversion and convergence.

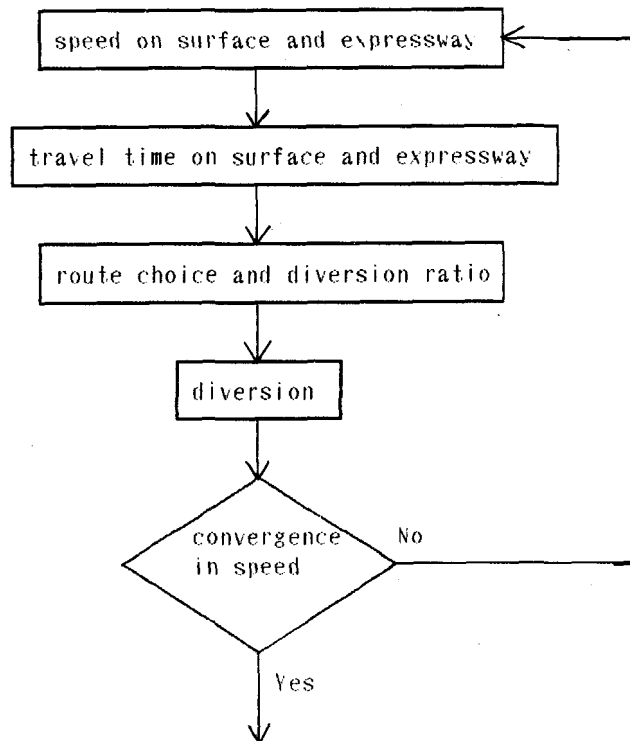


Fig. 3 Diversion of car trip from surface to expressway

1) Speed

Speed is in general a function of traffic flow. A linear function is assumed for the speed on surface road.

$$v = v_0 - kq \quad (3)$$

where

q ; traffic volume at a section on surface road,
 v_0 and k ; parameters.

What is called traffic volume above is the number of car trips traveling through the section.

Another type is assumed for the speed on expressway, as shown in Fig.4.

$$V = \begin{cases} V_1 - K_1 Q, & Q \leq Q_1 \\ V_2 - K_2 (Q - Q_1), & Q_1 \leq Q \leq Q_2 \\ V_3, & Q \geq Q_2 \end{cases} \quad (4)$$

where

Q ; traffic volume at a section on expressway,
 V_1 , K_j and Q_j ; parameters, $i = 1, 2, 3$, $j = 1, 2$.

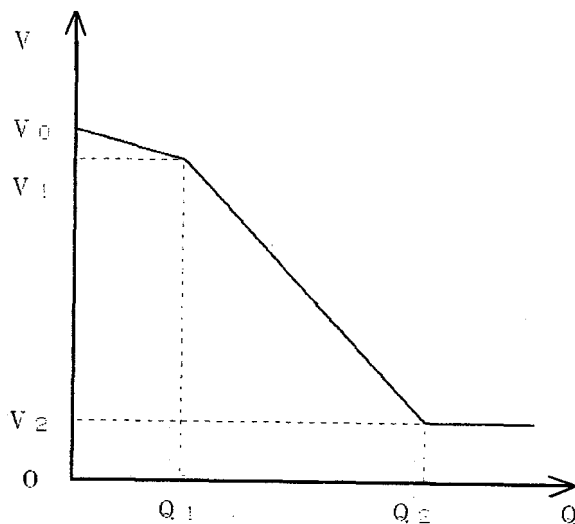


Fig.4 Speed on expressway

Both types, (3) and (4), are often used for road traffic assignment. The reason why different types of functions are assumed for surface road and expressway is that traffic flow is much less active on speed on surface road than on expressway. Conversely, traffic flow on expressway begins to show a rapid decrease in speed at a certain point of traffic volume, while it slows down inactively on surface road.

2) Route choice

This step includes routines through which a route of the shortest travel time is found from among innumerable number of routes between each pair of origin and destination. Two kinds of the routes of the shortest travel time have to be found between each pair of origin and destination in order to estimate trip diversion from surface to expressway as described later; the shortest surface route and the shortest expressway route. The shortest surface route consists of surface road alone and the shortest expressway one consists of expressway with surface access and egress.

The shortest surface route is assumed to exist among the three, S_1 , S_2 and S_3 as shown in Fig.5. S_1 consists of two radials alone passing through the city center, S_2 includes a ring whose starting point is at origin while S_3 includes a ring which ends in destination. The shortest surface route can be found from among the three by computing.

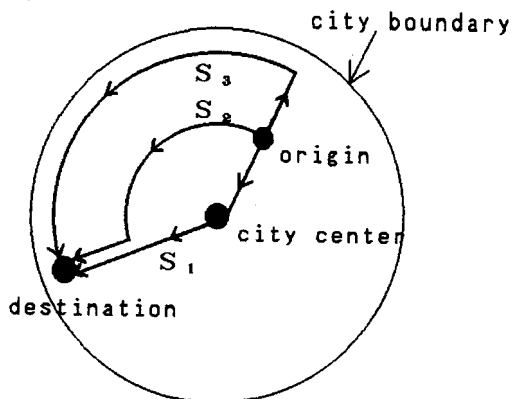


Fig.5 Alternative of the shortest travel time surface routes

The shortest expressway route has surface access and egress that are linked by expressway of the shortest travel time which is computed. A car trip is assumed to access to and egress out of its nearest expressways along surface radial and/or ring roads. Accordingly, study area is divided into, so called, expressways' territories. Conversely, each territory has its nearest expressway in itself. Fig.6 shows territories in part together with typical surface accesses and egresses.

There are three kinds of surface accesses and egresses; radial, ring and radial-ring. The first is found in the ring expressway's territory and the second and the third in the radial expressway's territory inside and outside, respectively, of the circle whose radius is equal to the length of a radial expressway. In this connection, the circle is a marginal end of the area where a flat toll rate is imposed on expressway users.

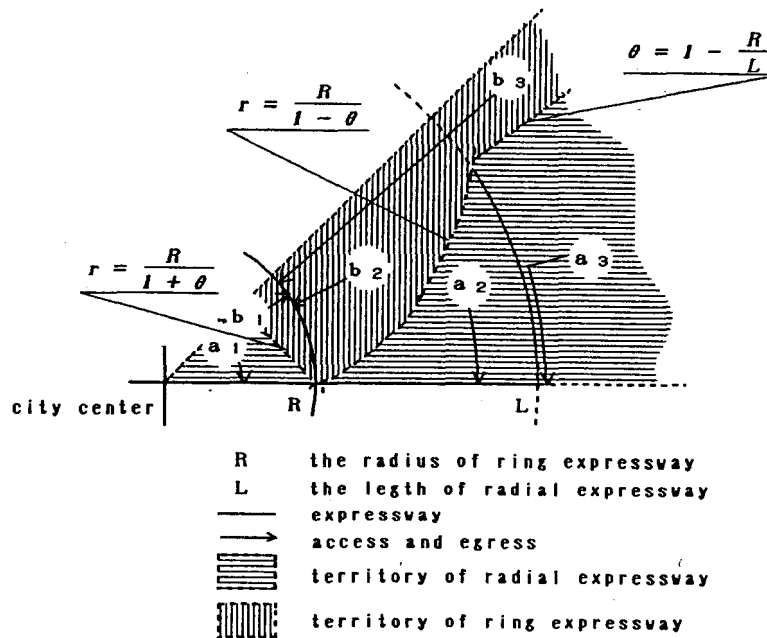


Fig.6 Territory, access and egress

3) Diversion

The number of car trips diverted onto expressway is given by multiplying the number of origin-destination trips, that is given by eq.(2) for each pair of origin and destination, by diversion ratio, which is assumed by

$$P = \frac{1}{1 + \eta^6} - 0.05 \quad (5)$$

where

P; diversion ratio that is defined as the ratios of the number of diverted trips of a certain origin-destination pair to the total number of trips of the same pair.

η ; travel time ratio that is defined as the ratios of the time necessary to travel along the shortest expressway route to the time along the shortest surface route between the same pair.

Above equation is one of so called diversion ratio functions by use of travel time ratio. That has been applied to traffic prediction in urban expressway in Japan [9].

In the present study, travel time ratio is defined by

$$\eta = \frac{T + F/\delta}{t} \quad (6)$$

where

T; time necessary to travel between above origin-destination pair along the shortest expressway route,

t; time necessary to travel between the same pair along the shortest surface route,

F; toll rate that is flatly imposed on diverted trips, and

δ ; time value.

The second term in the numerator on the right hand side of eq.(6) is so called time equivalent to toll rate. Flat toll rate, of course differentiated by car classification, is usually imposed in Japanese urban expressway system.

4) Convergence

Numerical calculation is repeated until surface road and expressway are found converged in speed in every section. After convergence in speed, further calculation is continued until expressway finds itself converged in the number of diverted trips. In this calculation, time T to travel along the shortest expressway route and time t along the shortest surface route are obtained by integrating the inverse of the speed V given by (4) and that of the speed v given by

(3) along the corresponding shortest route, respectively. Finally, converged value of travel time ratio is obtained for each origin-destination pair and gives, substituted for eq.(5), the number of diverted trips by multiplication mentioned previously.

(4) Equilibrium of revenue and expenditure

The expenditure is assumed to be proportionate simply to the total length of expressway, that is,

$$\begin{aligned} E &= cS \\ &= 4cL + 2\pi cR \end{aligned} \quad (7)$$

where

- E; expenditure,
- S; total length of expressway,
- L; length of a radial expressway from the central inter change to the marginal end,
- R; radius of ring expressway, and
- c; **proportional constant.**

The first and the second terms on the right hand side of the equation are expenditures for radial and ring expressway, respectively. L is called the radius of the flat rate area.

Equilibrium constraint requires the expenditure to balance the revenue that is given by multiplying the aggregate number of diverted car trips by toll rate.

(5) Optimization

Toll rate F, the length L of a radial expressway and the radius R of ring expressway are numerically optimized on each of two criteria; trip number criterion and travel hour criterion.

3. PREPARATIONS FOR CALCULATION

(1) Calculation procedure

Calculation procedure is shown in Fig. 7. It is applied to every set of toll rate F, the length L of a radial expressway and the radius R of ring expressway that are given externally.

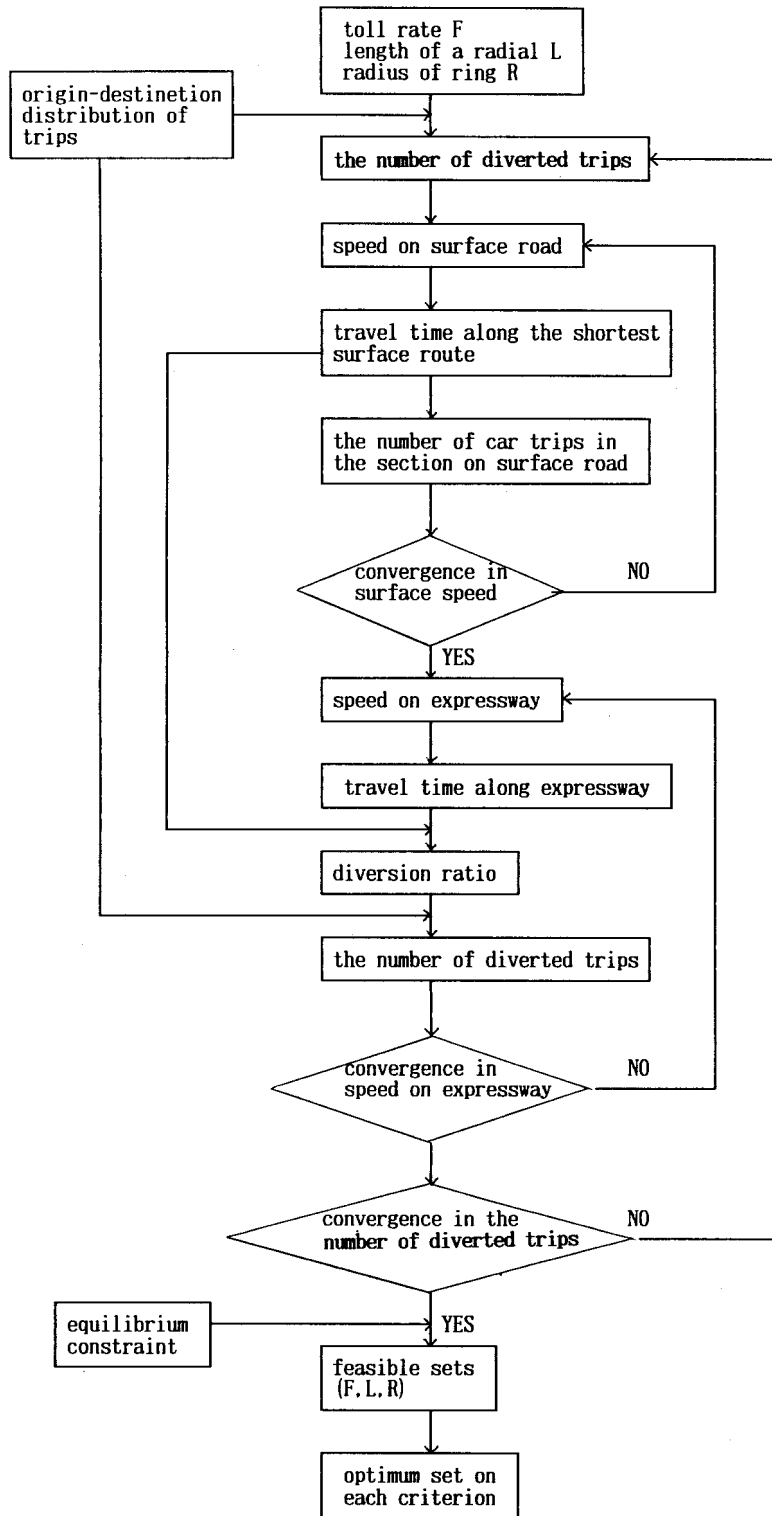


Fig. 7 Calculating procedure

Table 1 Values of the parameters

functions including parameters	values	remarks
$f(r) = \mu e^{-\lambda r}$	$\mu = 1.28 \times 10^4$ $\lambda = 0.16$	car trips/km ² /day* l/km *
$\frac{g(r_1, \theta_1; r_2, \theta_2)}{\alpha \{f(r_1) \times f(r_2)\}^\beta} = \frac{1}{\{d(r_1, \theta_1; r_2, \theta_2)\}^\gamma}$	$\alpha = 7.88 \times 10^{-8}$ $\beta = 1.0$ $\gamma = 1.0$	
$v = v_0 - kq$	$v_0 = 30$ $k = 1.25 \times 10^{-4}$	km/hr km/car trip
$\begin{aligned} V &= V_1 - K_1 Q, \quad Q \leq Q_1 \\ &= V_2 - K_2 (Q - Q_1), \\ &\quad Q_1 \leq Q \leq Q_2 \\ &= V_3, \quad Q \geq Q_2 \end{aligned}$	$V_1 = 70$ $K_1 = 1.25 \times 10^{-4}$ $V_2 = 62$ $K_2 = 4.0 \times 10^{-4}$ $V_3 = 5$	km/hr km/car trip km/hr km/car trip km/hr
$\eta = (T + F/\delta) / t$	$\delta = 25.8$	yen/trip·min
$E = 4cL + 2\pi cR$	$c = 1.36 \times 10^6$	yen/km *

*taken from [7].

(2) Parameters

The values of the parameters given for trial calculation of the model are shown in Table 1. The study area is divided into meshes of 2km by $\pi/20$ rad. in radial and circumferential directions, respectively. Every mesh is an origin and a destination. Car trip is supposed to be generated at and attracted to the center of the mesh, where surface radial and ring roads pass.

4. RESULTS AND DISCUSSION

(1) Existence of feasible area

Table 2 shows an example of feasible sets of the length L of a radial expressway and toll rate F for a fixed value of radius R=4km of ring expressway. Feasible sets are over the broken line except for the first row whose length is 6km that is too short relatively to given value of the radius to find solutions keeping the mesh-size as was decided in the preceding section.

In Table 2, the total travel hours are minimized locally on the second column and sixth row, while the aggregate number of diverted trips find its maximum, as a matter of course, at the lowest pricing

to the longest radial expressway.

Feasible area is, on LF-plane, seemingly convex. Feasible optimization in Table 2 is found on pricing 330yen to a radial length 14km on travel hour criterion and on the lowest pricing 150yen to the shortest radial on trip number criterion.

(2) Effect of pricing to travel distance on expressway

Fig.8 shows the distributions of travel distance on expressway for a certain set of L and R. It implies a well-known effect of expressway pricing that higher pricing turns shorter trips out of expressway. Travel time ratio, including time equivalent to toll rate, increases more rapidly for shorter trips than for longer trips. Note that travel distance on expressway is roughly corresponding to trip length.

(3) Effect of pricing to total travel hours

Fig.9 shows an effect of pricing a certain set of L and R to travel hours on surface road and expressway. Total travel hours are

Table 2 Example of feasible sets (L, F)
R=4 km

L \ F	yen	210	270	330	390	450	510
6 km	150	*	*	*	*	*	*
8	1350 529	1349 446	1352 372	1357 306	1364 245	1371 194	1377 153
10	1346 520	1344 441	1345 371	1349 310	1355 254	1362 207	1368 165
12	1334 546	1332 465	1333 395	1337 333	1343 277	1350 229	1357 187
14	1326 555	1325 471	1325 400	1330 337	1336 283	1342 236	1350 196
16	1317 575	1315 488	1317 416	1321 353	1327 297	1334 248	1342 208

the upper=total travel hours (10³ car-trip hours)

the lower=aggregate number of diverted trips (10³ car trips)

*=no calculating because of impertinent mesh-size

approximately linear to toll rate, increasing and decreasing on surface road and expressway, respectively. In this connection, expressway utility is quantifiable by subtracting the sum of the travel hours from the absolute maximum travel hours, 1,403 (10^3 car-trip hours) in the present calculation, that will be required in case of no expressway.

Approximate linearity of total travel hours, whether on surface or on expressway, to toll rate is found to exist for the other sets of the values of L and R.

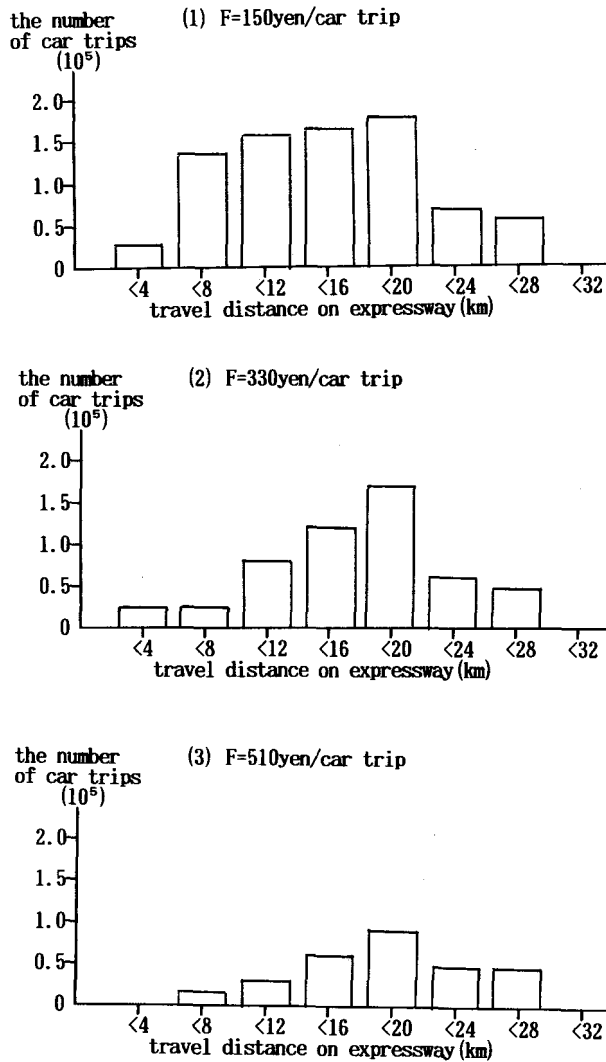


Fig. 8 Effect of pricing to travel distance on expressway (L=14km, R=4km)

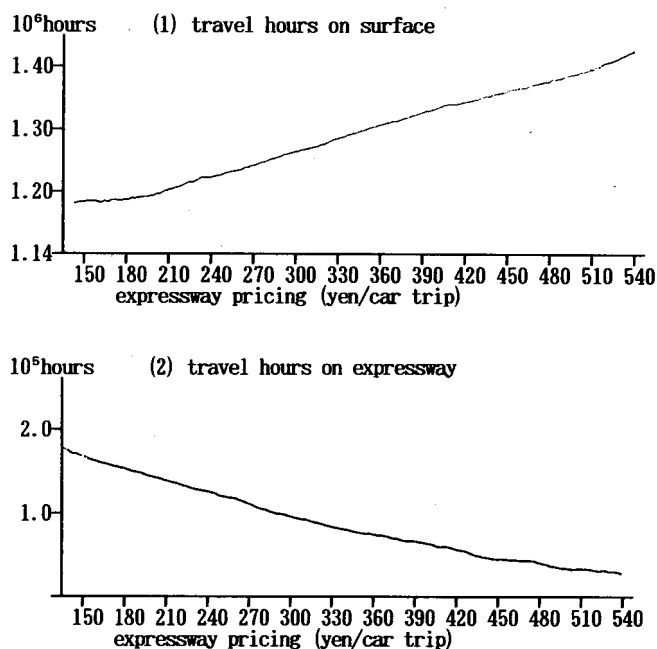


Fig. 9 Travel hours vs. expressway pricing

(4) Pricing the sets of L and R on two criteria

Table 3 (1) and (2) show the results of pricing on travel hour criterion and trip number criterion, respectively. Feasible sets of L and R are, as a matter of course, are common to both.

The values filled in the sets (L,R), where $L \geq 12\text{km}$ and R is arbitrary, are common to both tables, while those in the rest of the sets are not. It implies that both criteria are equivalent in pricing larger expressway network. This, however, does not mean that both criteria leads to the same result.

Optimization on travel hour criterion is, though a matter of course, found in the set $L=14\text{km}$ and $R=8\text{km}$, while that on trip number criterion in the set $L=8\text{km}$ and $R=6\text{km}$. The two optimum sets are picked up into Table 4 to show the characteristics. In a word, trip number criterion leads to cheaper and smaller expressway network, while travel hour criterion to more expensive and larger one.

Another aspect of the difference in the effect of two criteria is shown in Fig.10, representing the distribution of travel distance on expressway. The difference comes from a well-known fact on expressway

Table 3 (1) Pricing the sets (L,R) on travel hour criterion

L \ R	2km	4	6	8	10
4km	※	※ ※	※ ※	※ ※	※ ※
6	¥ 260	※	※ ※	※ ※	※ ※
	1388 257				
8	¥ 300	¥ 190	¥ 220	※ ※	※ ※
	1380 226	1345 472	1329 498		
10	¥ 260	¥ 210	¥ 220	¥ 250	※ ※
	1369 286	1344 441	1320 509	1305 507	
12	¥ 260	¥ 220	¥ 230	¥ 280	※ ※
	1356 317	1333 452	1309 514	1298 482	
14	¥ 320	¥ 300	¥ 270	¥ 340	※ ※
	1345 291	1327 368	1302 474	1295 425	
16	※	※	¥ 340	※	※
	※ ※	※ ※	1300 407	※ ※	※ ※
18	※ ※	※ ※	※ ※	※ ※	※ ※

	— pricing
	— minimum total travel hours (10 ³ car-trip hours)
	— aggregate number of diverted trips (10 ³ car trips)

※ = no calculating because of impertinent network
 ※ ※ = no network is assumed
 ※ = unfeasible set
 ※ ※

Table 3 (2) Pricing the sets (L,R) on trip number criterion

L \ R	2km	4	6	8	10
4km	*	* *	* *	* *	* *
6	¥ 130	*	* *	* *	* *
	1390 393				
8	¥ 190	¥ 150	¥ 170	* *	* *
	1383 311	1350 529	1330 567		
10	¥ 230	¥ 190	¥ 200	¥ 250	* *
	1369 312	1344 466	1321 534	1305 507	
12	¥ 260	¥ 220	¥ 230	¥ 280	* * *
	1356 317	1333 452	1309 514	1298 482	
14	¥ 320	¥ 300	¥ 270	¥ 340	* * *
	1345 291	1327 368	1302 474	1295 425	
16	*	*	¥ 340	*	*
	* *	* *	1300 407	* *	* *
18	* * *	* * *	* * *	* * *	* * *

	—pricing
	—total travel hours(10^3 car-trips hours)
	—maximum aggregate number of diverted trips(10^3 car trips)

*
*, * *, * * = the same as in(l)

pricing mentioned before. An interesting problem is left to be analyzed on what effects the parameters λ and γ , included in trip potential function and gravity type distribution of trip length, respectively, have to optimization upon two criteria.

Table 4 Optimization on two criteria

pricing and planning	criteria	
	trip number	travel hour
pricing F(yen)	1 7 0	3 4 0
length L(km)	8	1 4
radius R(km)	6	8
total travel hours (10^3 car-trip hours)	1, 3 3 0	1, 2 9 5
aggregate number of diverted trips (10^3 car trips)	5 6 7	4 2 5

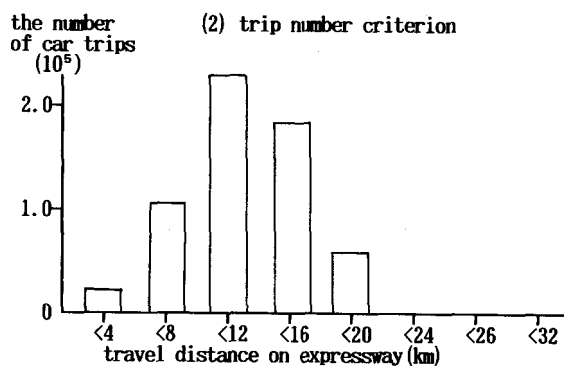
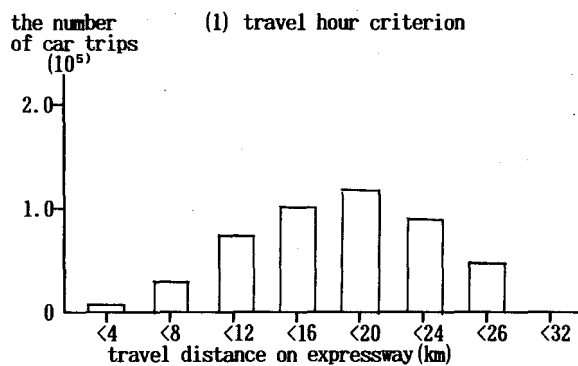


Fig. 10 Distribution of travel distance on expressway

5. CONCLUDING REMARKS

On LF-plane, there is certain pricing where the total travel hours are locally minimized, while the lowest pricing, as a matter of course, maximizes the aggregate number of diverted trips. Feasible sets on LF-plane are seemingly convex. A well-known effect of expressway flat pricing is also shown; higher pricing turns more shorter trips than longer trips out of expressway.

Total travel hours on each of surface road and expressway are approximately linear functions of expressway pricing, increasing and decreasing, respectively.

The two criteria are equivalent in pricing larger expressway network in that optimized values of the total travel hours and the aggregate number of diverted trips are equal to each other, respectively. It is a matter of further investigation why the two criteria are equivalent on larger expressway network.

A remarkable difference between the criteria is that trip number criterion brings cheaper and smaller expressway network, while the other criterion more expensive and larger one. Further investigation will be continued on the effect of the parameters λ and γ to the difference.

Some of the effects of the parameters included in the model to the optimum solutions will be presented later somewhere else.

6. ACKNOWLEDGEMENT

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