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Identification of Electric Charge Distribution Using Dual Reciprocity Boundary Element Models

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Identification of unknown electric charges or sources distributed in space is made from the data observed over the field boundary using dual reciprocity boundary element models. The inhomogeneous term of the Poisson field can equivalently be expressed as the linear combination of the functions associated with the particular solutions to transform into Laplace equation. For the solution procedure, the variational formulation is employed, in which the regular boundary integral approach is incorporated to avoid the singularity. Numerical examples are presented to demonstrate the availability and the capability.

1 INTRODUCTION

The boundary element analysis of Poisson equation requires the cell integration for the domain. The discretization of the interior domain means loss of its "boundary only" nature. A great deal of effort has gone into eliminating the domain integrals. The most promising approach to this is the use of Dual Reciprocity Method[1].

The identification of the locations and magnitudes of the external forces in a domain using boundary element algorithm has been proposed by many investigators[2-4]. The methods in most cases based on the minimizing the sum of the squares of the relative errors evaluated over the boundary.

We proposed a procedure to be called simulated charge searching approach[5] in which the DRM is extended to the problems of the identification of electric charge distribution. In the present work, using the simulated charge searching approach, both the location and magnitude of the electric charge are identified without iterative calculation. When the charges are uniformly distributed, the shape of the domain can be determined by minimizing the sum of the squares of the relative errors evaluated over the boundary. The variational formulation is also employed in which with the regular boundary integral procedure[6] is incorporated to avoid the singular integrals.

2 FORMULATION FOR THE DRM BOUNDARY ELEMENT METHOD

Defining ψ is the electric scalar potential, and ρ is the space charge density distributed in domain Ω, the functional of hybrid form corresponding to the Poisson equation can be expressed as

$$\Pi(\psi,\tilde{\psi},\tilde{p})=\frac{1}{2}\int_{\Omega}(\nabla\psi)^2 d\Omega-\int_{\Gamma}(\psi-\tilde{\psi})\tilde{p}d\Gamma-\int_{\Gamma_2}\tilde{\psi}\,\hat{p}d\Gamma-\int_{\Omega}\frac{\rho}{\varepsilon}\psi\,d\Omega \tag{1}$$

where $\tilde{\psi}$, $\tilde{p}(=\partial\tilde{\psi}/\partial n)$ are the potential and flux on boundary Γ, and $\hat{p}(=\partial\hat{\psi}/\partial n)$ is the forcing term prescribed on boundary Γ_2. The expression allows the discrepancy between potential ψ closest to the boundary in the domain and potential $\tilde{\psi}$ on the boundary. Taking variation with respect to ψ, $\tilde{\psi}$ and $\tilde{p}$ respectively, we obtain the governing equation

$$\nabla^2\psi=-\frac{\rho}{\varepsilon}=b \qquad \text{in } \Omega \tag{2}$$

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and the boundary conditions

$$\left.\begin{aligned} \psi &= \tilde{\psi} && \text{on } \Gamma \\ \tilde{\psi} &= \hat{\psi} && \text{on } \Gamma_1 \\ p &= \tilde{p} && \text{on } \Gamma \\ \tilde{p} &= \hat{p} && \text{on } \Gamma_2 \end{aligned}\right\} \qquad (2)'$$

Potential ψ can be expressed as the sum of particular solution φ and fundamental solution ϕ, so that

$$\psi(x,y) = \varphi(x,y) + \phi(x,y) \qquad (3)$$

Dividing the boundary into elements Γ_k (k = 1,2,...M), and locating simulated charges α_j (j=1, 2, ..., L) inside the domain, and fictitious charges β_k (k=1, 2, ..., M) outside the domain, the particular solution and fundamental solution at arbitrary point i in the field is expressed in terms of the linear combination of the contributions from each source α_j and β_k, that is

$$\varphi_i = \sum_{j=1}^{L} \varphi_{ij}^{*} \alpha_j = \left\{\varphi^{*}\right\}_i^T \{\alpha\}$$

$$\phi_i = \sum_{k=1}^{M} \phi_{ik}^{*} \beta_k = \left\{\phi^{*}\right\}_i^T \{\beta\}$$

where

$$\left\{\varphi^{*}\right\}_i = \left\{\varphi_{i1}^{*}, \varphi_{i2}^{*}, \ldots\ldots, \varphi_{iL}^{*}\right\}, \quad \left\{\phi^{*}\right\}_i = \left\{\phi_{i1}^{*}, \phi_{i2}^{*}, \ldots\ldots, \phi_{iM}^{*}\right\}$$

$$\{\alpha\} = \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_L\}, \quad \{\beta\} = \{\beta_1, \beta_2, \ldots\ldots, \beta_M\}$$

and φ_{ij}^{*} is the particular solution of the Poisson equation $\nabla^2 \varphi_{ij}^{*} = f_{ij}$, where f_{ij} is an approximate function, the linear combination of which forms the forcing terms as one will see later. ϕ_{ik}^{*} is the fundamental solution of the Laplace equation whose value is evaluated at i for a unit source given at point k outside the domain.

With substitution of equation (3) into (2), the Poisson equation can be replaced by Laplace equation

$$\nabla^2 \phi = 0 \qquad \text{in } \Omega \qquad (4)$$

and the expression over the boundary is given as

$$\tilde{\phi}(x,y) = \tilde{\psi}(x,y) - \tilde{\varphi}(x,y) \qquad \text{on } \Gamma \qquad (5)$$

Defining ϕ in domain Ω, $\tilde{\phi}$, $\tilde{q}$ ($=\partial\tilde{\phi}/\partial n$) on the boundary Γ, equation (4) can also be expressed as the functional of hybrid form

$$\Pi(\phi, \tilde{\phi}, \tilde{q}) = \frac{1}{2}\int_{\Omega} \varepsilon (\nabla\phi)^2 d\Omega - \int_{\Gamma} \varepsilon (\phi - \tilde{\phi})\, \tilde{q}\, d\Gamma - \int_{\Gamma_2} \varepsilon \tilde{\phi}\, \hat{q}\, d\Gamma \qquad (6)$$

where $\hat{q}$ is the forcing term on the boundary Γ_2.

Integrating by parts the first term in equation (6) to eliminate the domain integration results in the expression with boundary integral only

$$\Pi(\phi, \tilde{\phi}, q, \tilde{q}) = \frac{1}{2}\int_{\Gamma} \varepsilon \phi\, q\, d\Gamma - \int_{\Gamma} \varepsilon (\phi - \tilde{\phi})\, \tilde{q}\, d\Gamma - \int_{\Gamma_2} \varepsilon \tilde{\phi}\, \hat{q}\, d\Gamma \qquad (7)$$

The variational boundary integral expression[7] of equation (7) is

$$[K]\left\{\tilde{\phi}\right\} - [G]\left\{\frac{\partial\tilde{\phi}}{\partial n}\right\} = \{0\} \qquad (8)$$

Applying the relation (5) into equation (8), one can write

$$[K]\left[\{\tilde{\psi}\} - \{\tilde{\varphi}\}\right] - [G]\left[\{\tilde{p}\} - \left\{\frac{\partial\tilde{\varphi}}{\partial n}\right\}\right] = [K]\left[\{\tilde{\psi}\} - \left\{\sum_{j=1}^{L} \varphi_{ij}^{*} \alpha_j\right\}\right] - [G]\left[\{\tilde{p}\} - \left\{\sum_{j=1}^{L} \frac{\partial\varphi_{ij}^{*}}{\partial n} \alpha_j\right\}\right] = \{0\} \qquad (9)$$

Finally, the discretized system equation becomes

$$[K]\{\tilde{\psi}\} - [G]\{\tilde{p}\} = \{[K][H] - [G][Q]\}\{\alpha\} = [S]\{\alpha\} \qquad (10)$$

where

$$[S] = [K][H] - [G][Q]$$

The components of the matrix [G], [H] and [Q] are

$$G_{ki} = \int_{\Gamma_i} \phi_{ki}^{*}\, d\Gamma_i, \quad H_{ij} = \varphi_{ij}^{*}, \quad Q_{ij} = \frac{\partial\varphi_{ij}^{*}}{\partial n} = q_{ij}^{*}$$

and

$$[K]=[E][R]$$

in which the component of $[E]$ is

$$E_{ki}=\frac{1}{2}\sum_{m=1}^{M}\int_{\Gamma_m}\phi_{km}^{*}\frac{\partial\phi_{im}^{*}}{\partial n}d\Gamma_m+\frac{1}{2}\sum_{m=1}^{M}\int_{\Gamma_m}\phi_{im}^{*}\frac{\partial\phi_{km}^{*}}{\partial n}d\Gamma_m$$

$$[R]=\left([G]^{-1}\right)^{T}[U] \qquad (k,i=1,2,\ldots,M,\ j=1,2,\ldots,L)$$

Here, in the present formulation, the constant elements are used. $[U]$ are diagonal matrix, the component of $[U]$ corresponds to the length of element Γ_i over the boundary Γ. Equation (10) is the result of the dual reciprocity expression for which regular boundary element approach is incorporated to avoid the singular integrals by locating fictitious charges β_k outside the domain.

The DRM implies an expansion for b(x, y) of the type

$$b_i=\sum_{j=1}^{L}f_{ij}\alpha_j=\sum_{j=1}^{L}(\nabla^2\varphi_{ij}^{*})\alpha_j \tag{11}$$

where L is the number of the node of simulated charges α_j, and φ_{ij}^{*} is the particular solution of the Poisson-type equation

$$\nabla^2\varphi_{ij}^{*}=f_{ij} \tag{11'}$$

where f_{ij} is an approximate expansion function for b. The coefficient as can be expressed in a matrix form as

$$\{\alpha\}=[f]^{-1}\{b\} \tag{12}$$

where

$$\{b\}=\{b_1,b_2,\ldots\ldots,b_L\},\quad b_i=b(x_i,y_i)$$

Equation (12) is substituted into equation (10) resulting in

$$[K]\{\tilde{\psi}\}-[G]\{\tilde{p}\}=[S][f]^{-1}\{b\} \tag{13}$$

In the forward analysis, b(x, y) is known, so that equation (13) arrives at the conventional boundary element expression.

3 SINGLE CHARGE IDENTIFICATION

3.1 Inversion

The inverse problem shown in Figure 1 is considered. The source term b is a function of (x, y) in domain Ω, bounded by Γ (=Γ_1+Γ_2). The potential $\tilde{\psi}$ and flux $\tilde{p}$ on Γ_2, and $\tilde{\psi}$ on Γ_1 are known or observed. We try to find the value $\tilde{p}$ on boundary Γ_1 and the source distribution b(x, y). We first allocate coefficients α, and then calculate the values of α, whose linear combination gives the approximate values for b(x, y) with equation (11). Boundary Γ_1 is divided into k elements, and Γ_2 is divided into M-k elements. Reordering the equations in such a way that all the unknown, coefficients α and potentials $\tilde{p}$ on Γ_1 are placed on the left hand side, and the known values on the right hand side. Therefore one can write equation (13) as

$$\begin{bmatrix} G_{11} & \cdots & G_{1k} & S_{11} & \cdots & S_{1L} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ G_{M1} & \cdots & G_{Mk} & S_{M1} & \cdots & S_{ML} \end{bmatrix}\begin{bmatrix} \tilde{p}_1 \\ \vdots \\ \tilde{p}_k \\ \alpha_1 \\ \vdots \\ \alpha_L \end{bmatrix}=\begin{bmatrix} K_{11} & \cdots & K_{1M} & -G_{1k+1} & \cdots & -G_{1M} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ K_{M1} & \cdots & K_{MM} & -G_{Mk+1} & \cdots & -G_{MM} \end{bmatrix}\begin{bmatrix} \tilde{\psi}_1 \\ \vdots \\ \tilde{\psi}_M \\ \tilde{p}_{k+1} \\ \vdots \\ \tilde{p}_M \end{bmatrix} \tag{14}$$

Assigning that the matrix on the left is [A], the unknown vector is {X} and the product of the matrix and vector on the right is $\{\hat{W}\}$, one can write equation (14) as

$$[A]\{X\}=\{\hat{W}\} \tag{15}$$

If the sum of the number of the known, $\tilde{\psi}$ and $\tilde{p}$ on Γ_2, and on Γ_1, is greater than the number of the unknown α and $\tilde{p}$ on Γ_1, then the order of equation (15) is greater than the number of the unknown variables. The equation can be solved in the least squares sense. Especially, when $\tilde{\psi}$ and $\tilde{p}$ are both specified on boundary Γ, the left hand side of equation (10) being replaced by $\{\hat{D}\}$, one has the expression

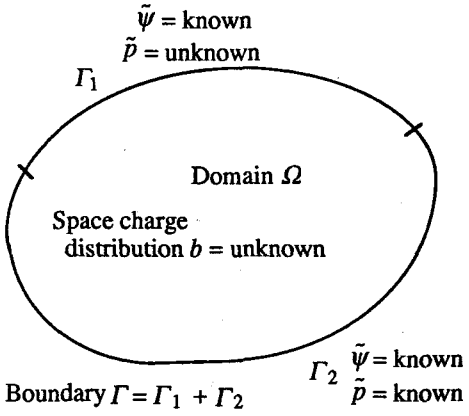


Fig.1 Field and boundary conditions for inverse problem

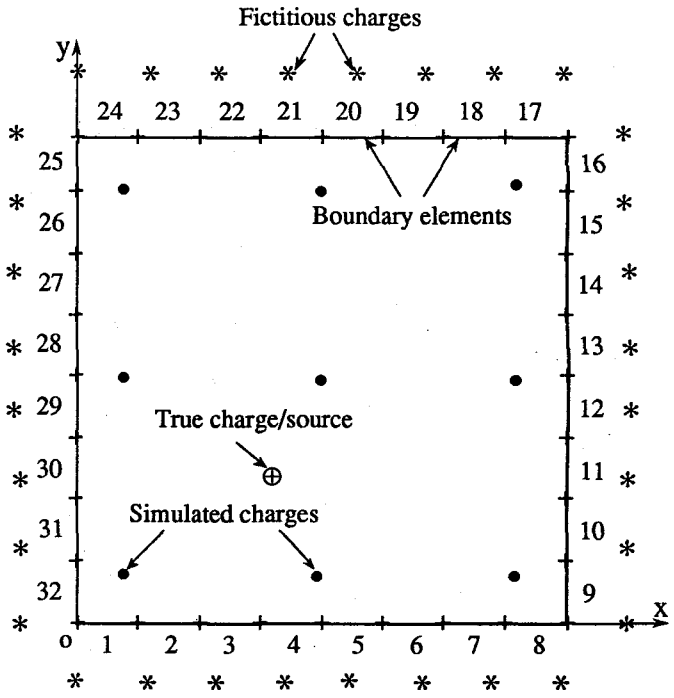


Fig.2 Boundary elements, simulated and fictitious charges

$$[S]\{\alpha\} = \{\hat{D}\} \tag{16}$$

Pre-multiplying $[S]^T$ on both sides of equation (16), one has

$$[S]^T[S]\{\alpha\} = [S]^T\{\hat{D}\} \tag{17}$$

which is solved for $\{\alpha\}$ by usual Gauss elimination method and the source distribution b(x, y) can readily be obtained by equation (11).

Now we consider the case when a positive source charge situated in the domain. The conventional strategy for the solution is that the location and magnitude of the charge to be identified is chosen so as to minimize the objective function or the sum of the squares of the relative error between the solution calculated and the observed at some reference points. Efficiency may depend on the choice of its initial location and magnitude.

Here we propose a procedure which could be called a simulated charge searching approach. Since term b(x, y) is expanded as the linear combination of simulated charges, that is

$$b_i \approx \sum_j^L \alpha_j \delta(x_i - x_j, y_i - y_j)$$

where, $\delta(x_i\text{-}x_j, y_i\text{-}y_j)$ is a delta function, which implies the approximate function

$$f_{ij} = \delta(x_i - x_j, y_i - y_j) = \delta_{ij}$$

The particular solution for $\nabla^2\varphi_{ij}^* = \delta_{ij}$ is therefore

$$\varphi_{ij}^* = -\frac{1}{2\pi}(\ln r_{ij})$$

$$r_{ij} = \sqrt{X^2 + Y^2}\ ,\ X = x_i - x_j\ ,\ Y = y_i - y_j$$

r_{ij} is the distance from node i on the boundary to node j where the simulated charges are allocated.

Allocating L number of simulated charges at locations (x_j, y_j), we can calculate the magnitudes α_j by solving equation (15). We estimate the best possible positions (x_c, y_c) of the charge by averaging the simulated charges obtained in such a way that

$$x_c = \sum_j^L \alpha_j x_j \Big/ \sum_j^L \alpha_j\ ,\ y_c = \sum_j^L \alpha_j y_j \Big/ \sum_j^L \alpha_j \tag{18}$$

and the best possible magnitude α_c by summing total charges up as

$$\alpha_c = \sum_j^L \alpha_j \tag{19}$$

3.2 Numerical Examples

A closed square domain, x = 0.0 ~ 1.0 and y = 0.0 ~ 1.0 is considered in which a single charge of magnitude 1.0 is placed at location (x_c, y_c) = (0.4, 0.3). We assume that the values of $\tilde{\psi}$ and $\tilde{p}$ are both observed or measured on boundary Γ, for which we employ the forward solution in the present simulation. Figure 2 shows the field and field boundary, and the locations of both simulated and fictitious charges, where the boundary is divided into 32 constant elements. In the inverse analysis the boundary is again divided into 32 constant elements for which the same number of the fictitious charges are allocated outside the domain along the boundary, and 9 simulated charges are allocated within the region. As the fictitious charges are arranged outside the domain, all of the boundary integrals do not involve singularity, which can easily be evaluated.

Each magnitude of the simulated charge or α_j is calculated by solving equation (17) and shown in Figure 3. The position of the charge is estimated to be at x_c=0.3977, y_c=0.2978 and the magnitude to be α_c=1.0003 with the help of equation (18) and (19), which is very close to magnitude 1.0 at location (0.4, 0.3) originally set. The potential distribution is shown in Figure 4. Convergence to the exact values is expected as the number of element division increases, which is given in Figure 5.

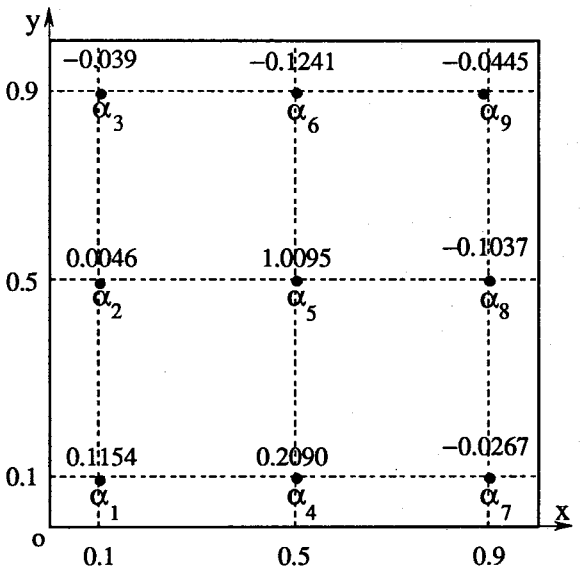
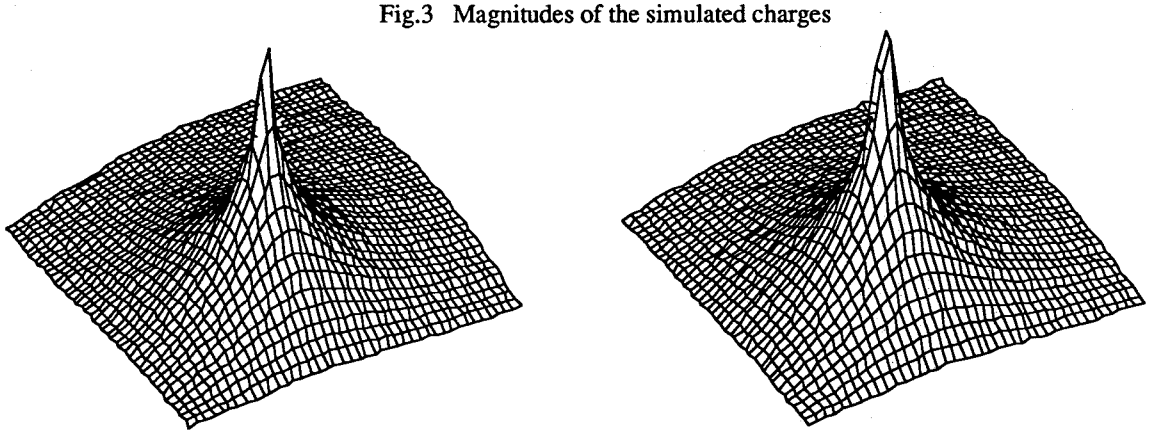


Fig.3 Magnitudes of the simulated charges



(a) Forward solution for a unit electric charge at position (x_c, y_c)=(0.4, 0.3) (32 boundary elements)

(b) Inverse solution obtained, magnitude α_c =1.0003 position (x_c, y_c)=(0.3977, 0.2978) (32 boundary elements)

Fig.4 Potential distribution

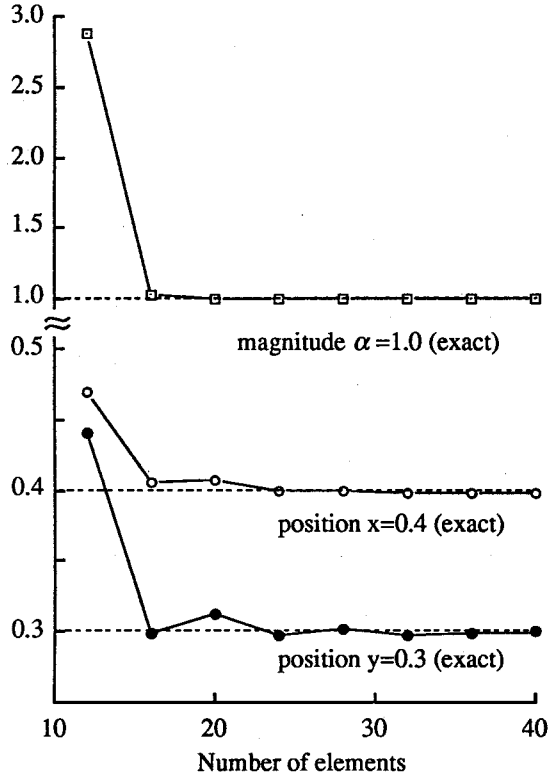


Fig.5 Identification capability depends on the element division

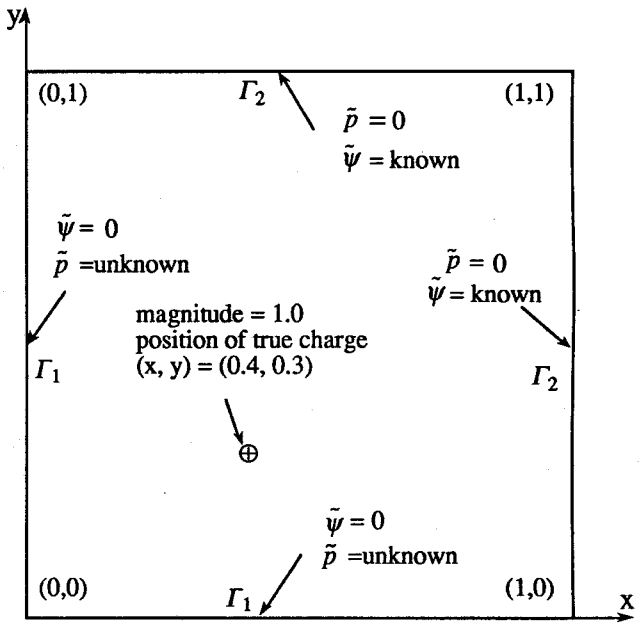


Fig.6 Boundary conditions for a closed square domain with a single elctric charge

Another case of a single charge of magnitude 2.0 at location (x_c, y_c)= (0.2, 0.2) is considered. The boundary conditions and element division are the same as in the previous case. The identified results of the position and magnitude of the charge are (x_c, y_c) = (0.1946, 0.19460) and α_c=2.0007 respectively, which are satisfactory.

Next is, as shown in Figure 6, the case when the values of $\tilde{\psi}$ and $\tilde{p}$ are both specified or observed on boundary Γ_2 but the value of $\tilde{\psi}$ is only known on Γ_1. The magnitude and position of a single charge are to be identified together with flux $\tilde{p}$ on Γ_1. Boundary Γ_2 is divided into 14 and boundary Γ_1 into 4 constant elements.

A charge of magnitude 1.0 is placed at location (0.4, 0.3). In inversion, the magnitude and the position obtained are α_c=1.0246, x_c=0.4306 and y_c=0.2920.

Another case is again single charge of magnitude 2.0 at location (x_c, y_c)= (0.2, 0.2). The identified results are α_c=2.2120 and (x_c, y_c)= (0.11692, 0.1754). The solution is not satisfactory, which could be improved if finer division or increased observation points are used.

4 UNIFORMLY DISTRIBUTED CHARGES OF ARBITRARY SHAPE

One can extend the discussion further to the case when the charges are uniformly distributed in circle. Figure 7 shows the Poisson field in which the boundary conditions are prescribed on the rectangular domain boundary. Electric charges are distributed uniformly within the circle. The center of the circle is at (x_c, y_c)= (2.0m, 2.0m), the radius is R = 0.8m and the charge density is known, ρ=100 (C/m^2).

Center of the circle and its radius are to be determined from the data observed on the boundary. The data observed on the boundary or the forward solution are given in the figure.

By solving equation (16), the circle's center is determined as (x_c, y_c)= (1.9874m, 1.9872m) and the total charge is α_c=198.2527(C). As the density is known, radius R can be calculated from $\alpha_c=\pi R^2\rho$. The results are given in table 1.

If uniformly distributed charges are of an arbitrary shape, the inversion process is not so simple as discussed in previous section and must be made in successive manner. We first calculate the central position of the distributed charges and the magnitude of the total charges by using the simulated charge search approach proposed in this paper, and then find the

equivalent radius as demonstrated above. Taking this as the initial boundary shape, true shape is searched by shifting the boundary so as to minimize the error function or objective function.

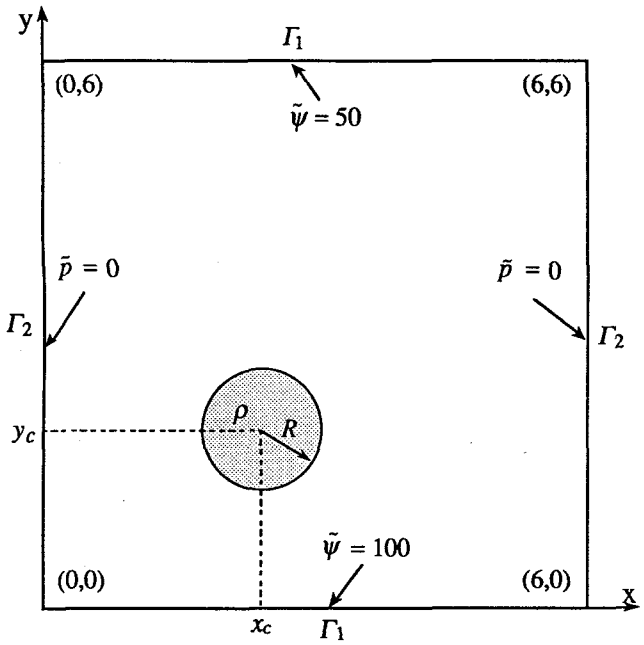


Fig.7 Boundary conditions for a closed square domain with circular charges uniformly distributed

Table 1 Identified results (32 boundary elements)

Examples	Exact	Identified
Case 1 ρ =50	x_c =3.0	x_c =2.9997
	y_c =2.0	y_c = 1.9924
	r =1.0	r =0.9932
Case 2 ρ =100	x_c = 2.0	x_c = 1.9874
	y_c =2.0	y_c = 1.9872
	r =0.8	r =0.7944

4.1 Formulation

If the charges are uniformly distributed in the domain Ω^B within field Ω^A shown in Figure 8, the term b(x, y) of the Poisson's equation is constant b_0 and equation (11) can be expressed as

$$\sum_{j=1}^{L} f_{ij}\alpha_j = \alpha_k = b_0 \qquad \text{in } \Omega^B \tag{20}$$

Only one simulated charge could be used, that is, α_js are entirely zeros except at position k where $\alpha_k = b_0$. Here we allocate the simulated charge α_k in the center of domain Ω^B. With function f_{ik} =1 adopted, the particular solution φ^*_{ik} and its normal derivative are

$$\varphi^*_{ik} = \frac{r^2_{ik}}{4}, \quad \frac{\partial \varphi^*_{ik}}{\partial n} = q^*_{ik} = \frac{1}{2}(n_x X + n_y Y)$$

where

$$r_{ik} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}$$

n_x and n_y are the directional cosines of n with respect to x and y axes. In this case, equation (10) becomes

$$[K]\{\tilde{\psi}\} - [G]\{\tilde{p}\} = \left[[K]\{\varphi^*\} - [G]\{q^*\}\right] b_0 \tag{21}$$

In region Ω^A, since there is no charge, the governing equation is

$$\nabla^2 \tilde{\psi}^A = 0$$

and in region Ω^B in which the charges are uniformly distributed, it is

$$\nabla^2 \tilde{\psi}^B = -\frac{\rho}{\varepsilon} = b_0$$

The boundary $\Gamma^A = \Gamma_O + \Gamma_I$ of domain Ω^A is divided into $M_O + M_I$ elements and the boundary $\Gamma^B = \Gamma_I$ of domain Ω^B is divided into M_I elements. The discretized boundary element expressions can be expressed for each region as

$$\left[[K^A_O]\ [K^A_I]\right] \begin{bmatrix} \{\tilde{\psi}^A_O\} \\ \{\tilde{\psi}^A_I\} \end{bmatrix} = \left[[G^A_O]\ [G^A_I]\right] \begin{bmatrix} \{\tilde{p}^A_O\} \\ \{\tilde{p}^A_I\} \end{bmatrix} \tag{22}$$

$$\left[K_I^B\right]\left\{\tilde{\psi}_I^B\right\}-\left[G_I^B\right]\left\{\tilde{p}_I^B\right\}=\left\{d^B\right\} \tag{23}$$

where

$$\left\{d^B\right\}=\left[\left[K_I^B\right]\left\{\varphi^*\right\}-\left[G_I^B\right]\left\{q^*\right\}\right]b_0$$

where superscript A or B indicates the boundary of each domain, and I indicates the interface boundary. Along the interface, both ψ and p are unknown but compatible, so that equilibrium conditions are

$$\tilde{\psi}_I^A=\tilde{\psi}_I^B=\tilde{\psi}_I \tag{24}$$

$$\tilde{p}_I^B=-\tilde{p}_I^A=-\tilde{p}_I \tag{25}$$

Equations (22) and (23) are now combined to form

$$\begin{bmatrix}\left[K_O^A\right] & \left[K_I^A\right] & \left[-G_I^A\right] \\ [0] & \left[K_I^B\right] & \left[G_I^B\right]\end{bmatrix}\begin{bmatrix}\left\{\tilde{\psi}_O^A\right\} \\ \left\{\tilde{\psi}_I\right\} \\ \left\{\tilde{p}_I\right\}\end{bmatrix}-\begin{bmatrix}\left[G_O^A\right] & [0] \\ [0] & [0]\end{bmatrix}\begin{bmatrix}\left\{\tilde{p}_O^A\right\} \\ \{0\}\end{bmatrix}=\begin{bmatrix}\{0\} \\ \left\{d^B\right\}\end{bmatrix} \tag{26}$$

Equation (26) can be rearranged to provide the known variables on the right-hand side and to obtain the same form as equation (15). The size of the system equation obtained is M_O+2M_I. The sizes of the matrices are

$$\left[K_O^A\right]_{(M_o+M_I)\times M_o},\ \left[G_O^A\right]_{(M_o+M_I)\times M_o},$$

$$\left[K_I^A\right]_{(M_o+M_I)\times M_I},\ \left[G_I^A\right]_{(M_o+M_I)\times M_I},$$

$$\left[K_I^B\right]_{M_I\times M_I},\ \left[G_I^B\right]_{M_I\times M_I},\ \left\{\varphi^*\right\}_{M_I},\ \left\{q^*\right\}_{M_I}$$

4.2 A Numerical Example

The task of the inverse problem considered here is to determine the boundary Γ_I of the rectangular domain Ω^B in which the charges are uniformly distributed as shown in Figure 8. In the inverse analysis, first, by using equation (16) we obtained the equivalent circular radius, which is used as the initial boundary shape. The circumference is radially divided into 16 (R_i, i = 1, 2, ..., 16).

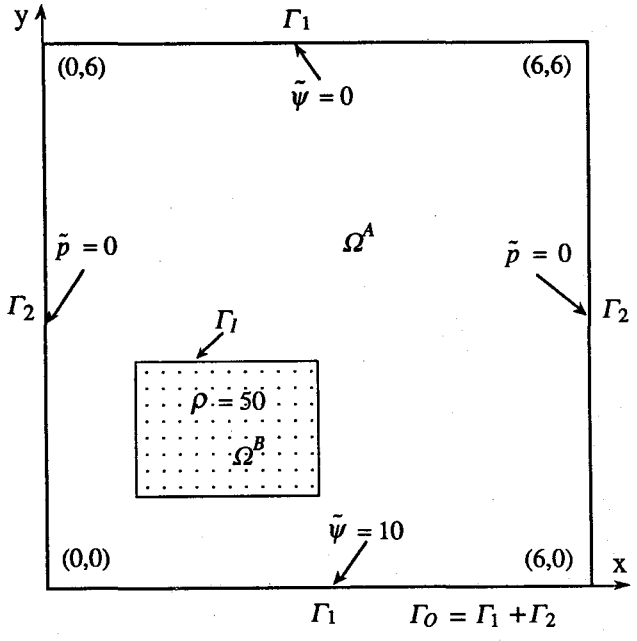
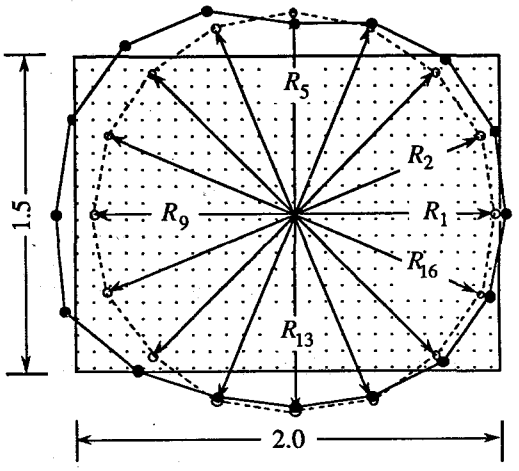


Fig.8 Boundary conditions for a closed square domain with rectangular charges uniformly distributed



· · · Charge distribution to be identified
o Initially obtained circle
• Identified or final shape

Fig.9 Inversion

The unknown interface boundary can be determined by varying the radial length R_i to minimize the objective function

$$W = \sum_{n=1}^{M_o} \psi^2$$

which is chosen as a square sum of the potentials evaluated at the nodes selected on the boundary Γ_O. The potentials are computed by equation (26) for the position of the interface boundary. Davidon-Fletcher-Powell (DFP) method[8] is employed for minimizing the objective function to determine the interface boundary shape. The determined shape is shown in Figure 9 and the convergence of the objective function is shown in Figure 10 against the number of iteration. The rectangular shape is not properly reconstructed. This is expected as the resolution would be within the length of the element on which the variables are assumed to be constant.

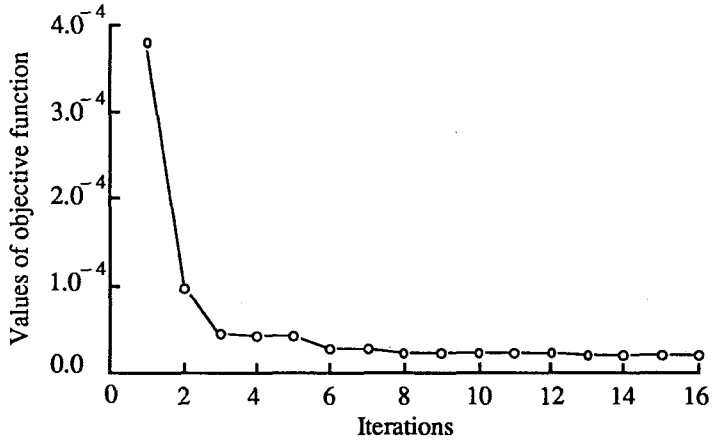


Fig.10 Convergence property of the objective function

5 CONCLUDING REMARKS

The use of DRM boundary element models was proposed to identify the distribution of the electric charges. The method incorporated a simulated charge approach to avoid the singular integral. In the case when a single charge is located in the domain, the position and magnitude are well identified by the direct inversion. This method could be used to give the first guess of the shape for the problem to determine the shape of the arbitrary domain of uniformly distributed. For the last case, the usual iterative method is inevitable.

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