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Dynamic Behavior of Sand Bed around Structure under Wave Motion

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SYNOPSIS

Under the attack of storm waves, there are many destructions of coastal structures due to scouring, sinking and sliding. These types of destructions are considered to be in close relation to the dynamic behavior of sand bed around the structures. In this study the characteristics of pore water pressure and stresses in the sand bed around a breakwater under the attack of superposed waves are treated theoretically. The results show that during the crest or the trough being in front of the structure the strength of sand bed around the structure decreases notably and the unstable zone will occur.

1. INTRODUCTION

Under the wave motion, the oscillating water pressure acts on the surface of the sand bed. The oscillating water pressure propagates into the sand bed accompanied with the damping in amplitude and with the phase lag. It is considered that this phenomenon is induced mainly by the existence of the small amount of air in the sand bed(its percentage is about 0.3%-1.0%). As the result, the excess pore water pressure in the sand bed increases. Then, the shear stress of the sand bed decreases. The destructions of coastal structures due to

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scouring, sliding and sinking are considered to be in close relation to the above mentioned dynamic behavior of the sand bed. From this point of view, we have investigated the fundamental characteristics of the pore water pressure and of the effective stresses of the sand bed under the oscillating water pressure with simple models[1,2,3].

In this study, as an approach to the more practical problem, we are taking up two-dimensional breakwater model under the attack of superposed wave, and investigate the effect of the wave motion on the dynamic behavior of the sand bed.

2. OUTLINE OF THEORETICAL ANALYSIS

2.1 Breakwater Model and Fundamental Equations

In this study, two-dimensional sand bed-structure model as shown in Fig.1 was chosen as a breakwater model. There is a superposed wave in front of the structure. The water behind the structure is not in motion.

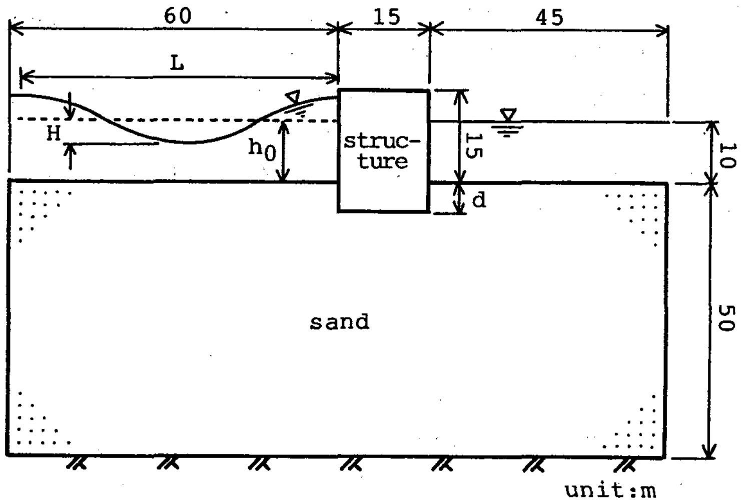

Fig.1 Definition sketch of the model

The motions of water and sand in the bed are analysed by the same method as for the ground water problem in the elastic sand bed [2,4,5]. Considering the small amount of air contained in the sand bed, following fundamental equations are obtained.

$$\rho g(\beta\lambda_w + \frac{\lambda a}{P_0+\rho gh})\frac{\partial h}{\partial t} + \frac{\partial}{\partial t}(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}) = k(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2}) \quad ---(1)$$

$$G(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial z^2}) + \frac{G}{1-2\nu}\frac{\partial}{\partial x}(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}) = \rho g\frac{\partial h}{\partial x} \quad ---(2)$$

$$G(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial z^2}) + \frac{G}{1-2\nu}\frac{\partial}{\partial z}(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}) = \rho g\frac{\partial h}{\partial z} \quad ---(3)$$

where, h : pore water pressure in head(variation from hydrostatic pressure due to mean water level),
u_x, u_z : displacement in x- and z- direction(variation from the value for initial state),
β : compressibility of the water,
g : acceleration due to gravity,
P_0 : atmospheric pressure,
k : permeability coefficient,
G : shear modulus,
ν : Poisson's ratio,
λ_w, λ_a : porosities of the water and the air

2.2 Boundary conditions and Method of Calculation

Water pressures on the sand surface are assumed as follows. The oscillating water pressure p derived from small amplitude wave theory acts on the sand surface in front of the structure. It is shown by the following equation.

$$p = \frac{\rho gH}{\cosh(mh_0)}\cos(mx)\cos\{n(t-\frac{T}{4})\} + \rho gh_0 \quad --- (4)$$

where, H : wave height,
L : wave length,
T : wave period,
m : wave number($m=2\pi/L$),
n : radian wave number($n=2\pi/T$),
h_0 : mean water level

Above equation is derived under the conditions that the sand bed is impermeable and rigid. Strictly speaking, in the case of the present problem, sand bed is permeable and is not rigid. That is, water pressure p calculated by the Eq.(4) is different from the value in real problem to some extent. But, for the simplicity, Eq.(4) was adopted as the first approximation. Behind the structure, the hydrostatic pressure due to mean water level h_0 acts on the sand surface.

Boundaries except the sand surface are assumed to be impermeable.

The conditions of the calculation are shown in Table 1. Run 1 is the basic model. Case 1 is a group of calculations for the variation of permeability coefficient of the sand bed k . Case 2 is for the depth of the footing d. Case 3 is for the wave period T and wave length L. Case 4 is for the wave height H. The constant values of other parameters in basic equations are as follows.

$\lambda_a = 0.003$, $\lambda_w = 0.4$, $\nu = 0.48$,

$\beta = 43.8\times10^{-10}m^2/kg(44.6\times10^{-11}m^2/N)$,

$G = 3.49\times10^7 kg/m^2(3.42\times10^8 N/m^2)$

Table 1 Conditions of calculation

		k	d	T	L	H
		(m/s)	(m)	(sec)	(m)	(m)
Case 1	Run 1	0.0015	0.0	10.0	92.3	4.0
	Run 2	0.015	〃	〃	〃	〃
	Run 3	0.00015	〃	〃	〃	〃
Case 2	Run 1	0.0015	0.0	10.0	92.3	4.0
	Run 4	〃	5.0	〃	〃	〃
	Run 5	〃	2.5	〃	〃	〃
Case 3	Run 1	0.0015	0.0	10.0	92.3	4.0
	Run 6	〃	〃	12.0	113.2	〃
	Run 7	〃	〃	8.0	70.9	〃
Case 4	Run 1	0.0015	0.0	10.0	92.3	4.0
	Run 8	〃	〃	〃	〃	3.0
	Run 9	〃	〃	〃	〃	2.0

Numerical calculations were carried out by using the Galerkin's finite element method which is one of weighted residual method(2). To evaluate the dynamic behavior of the sand bed, following method was adopted in this study. That is, the displacements u_x and u_z can be obtained by solving the fundamental equations. The normal effective stresses σ_x, σ_z and the shear stress τ_{zx} are calculated by using stress-strain relationship. Next, Mohr's stress circle can be drawn by these values, and the angle ϕ of the tangent to the instantaneous Mohr's stress circle from the origin is defined. This angle is called stress angle in this paper. As the stress angle becomes large, the sand bed becomes unstable. Then, we can evaluate the unstability of the sand bed by using this parameter.

The calculation is started from the initial stress state obtained by taking into account of the weight of the structures(its specific weight is 2.1).

3. RESULTS OF ANALYSES

We investigate the dynamic behavior of sand bed on the basis of the results of analyses on the displacement of sand bed, the pore water pressure and the stress angle. At first, results obtained for the basic model Run 1 are investigated. Then, the influences of the properties of sand bed and waves are investigated.

3.1 Fundamental Characteristics of Dynamic Behavior of Sand Bed

Fig. 2 shows the distribution of the displacement vectors. Fig. 3 shows the pore water pressure and the stress angle. These are the results for the basic model Run 1. From these figures, The dynamic behavior of sand bed are explained as follows.

a) Displacement of Sand Bed

When the wave crest is in front of the structure(t/T=0.25), displacement vectors of the sand bed draw circle lines around nodal point and their directions are toward the wave trough. Contrary to this, during the wave trough being in front of the structure(t/T=0.75), directions of displacement vectors are toward the bottom of the structure. At the latter state, the structure is lifted up. But the amount of rise of the structure is very small.

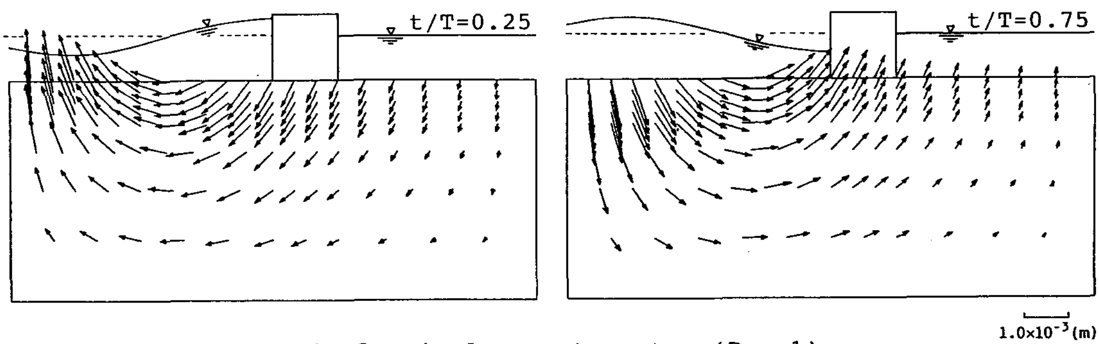

Fig.2 Displacement vector (Run 1)

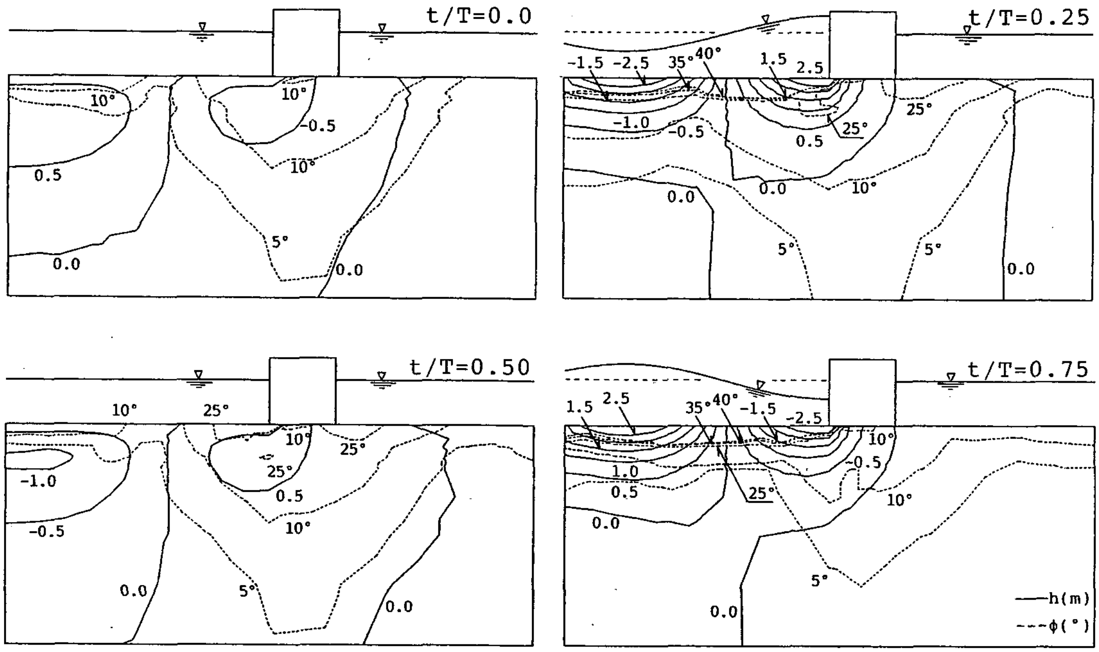

Fig.3 Pore water pressure and stress angle (Run 1)

b) Distribution of Pore Water Pressure

The pore water pressure in the sand bed increases in the region under the wave crest and decreases under the wave trough. Then, the seapage flow seems to move from the region under the crest to the region under the trough. Under the structure, the space of equipotential lines becomes slightly narrow. This means that the damping of the pore water pressure in this region becomes larger than in other region. In this region the rather large seapage force will occur. As the results, it is considered that the lateral movement of sand particles will be induced. Behind of the structure, pore water pressure varies little and seapage flow hardly occurs.

c) Distribution of Stress Angle

At the stages of the water surface being in level (t/T=0.0, t/T=0.50), there is no region of high stress angle in the sand bed around the structure. On the other hand, at the stages of the wave crest or the wave trough being in front of the structure, it is found that the unstable region of high stress angle occur in front of and under the structure.

3.2 Influence of Sand Bed Properties

Figs. 3 to 7 show the distributions of the pore water pressure h and the stress angle ϕ, that are represented in these figures by solid lines and broken lines respectively. Fig. 3 shows them for the fundamental model Run 1. Other figures show the results for the variation of permeability coefficient(see Fig. 4 and Fig. 5) and those for the variation of the depth of footing(see Fig. 6 and Fig. 7). From these figures, the influences of the properties of sand bed are explained as follows.

a) Permeability Coefficient

As the permeability coefficient becomes small, the space of equipotential lines becomes narrow. That is, hydraulic gradient in the sand bed becomes large. But, there is little variation in the distribution of the stress angle against the change of permeability coefficient.

b) Depth of Footing

The propagation of pore water pressure toward the back of the structure seems to be interrupted gradually according as the depth of footing becomes deeper. The variations of the distribution of stress angle with the depth of footing are recognized in the regions of both sides of the structure. But, under the structure, the variation is considered to be negligible.

3.3 Influence of Wave Properties

Fig. 3 and Figs. 8 to 11 show the distributions of pore water pressure and stress angle obtained for the various wave periods, wave lengths and wave heights. From these figures the influences of wave properties are explained as follows.

a) Wave Period and Wave Length

According as the wave period becomes shorter, that is, the wave length becomes shorter, the propagation of the pore water pressure into the sand bed becomes harder. But the distributions of stress angle are almost same for the change of these wave parameters.

b) Wave Height

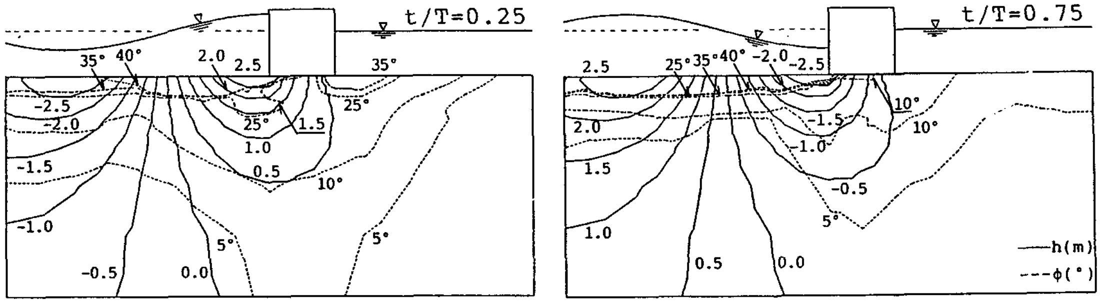

Fig.4 Pore water pressure and stress angle (Run 2 , k=0.015)

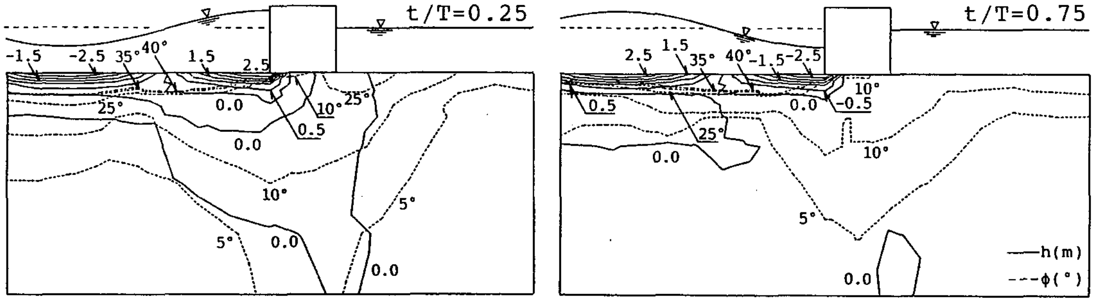

Fig.5 Pore water pressure and stress angle (Run 3 , k=0.00015)

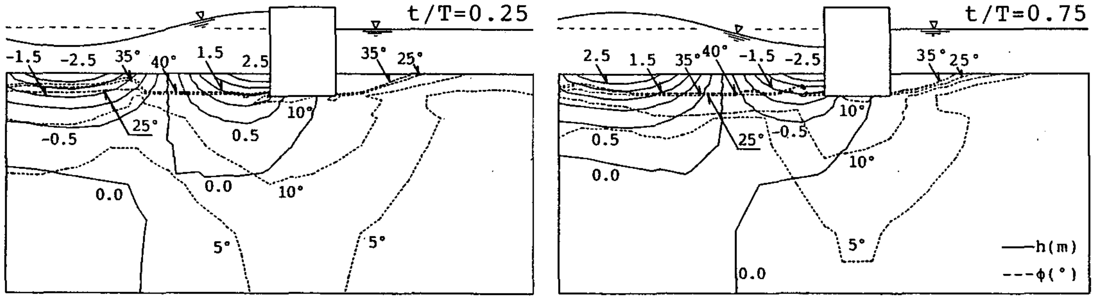

Fig.6 Pore water pressure and stress angle (Run 4 , d=5.0)

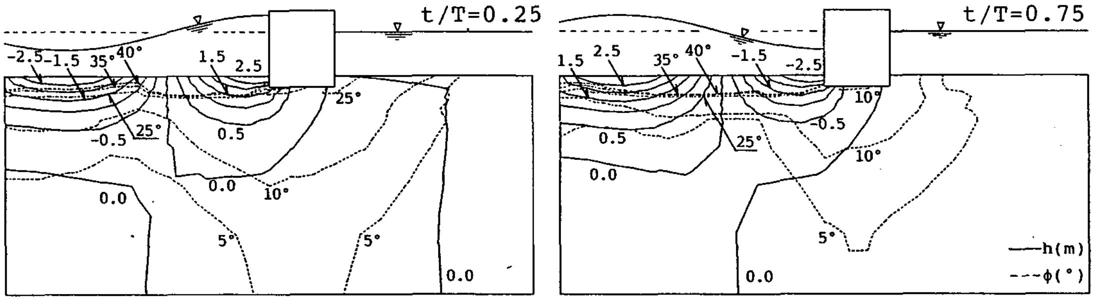

Fig.7 Pore water pressure and stress angle (Run 5 , d=2.5)

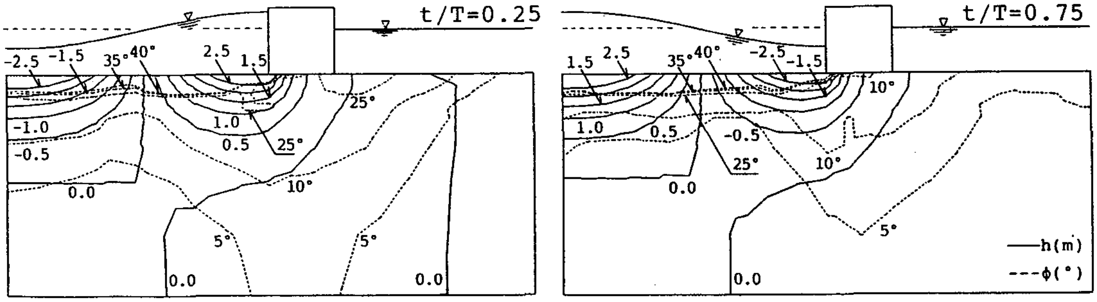

Fig.8 Pore water pressure and stress angle (Run 6 , T=12.0 , L=113.2)

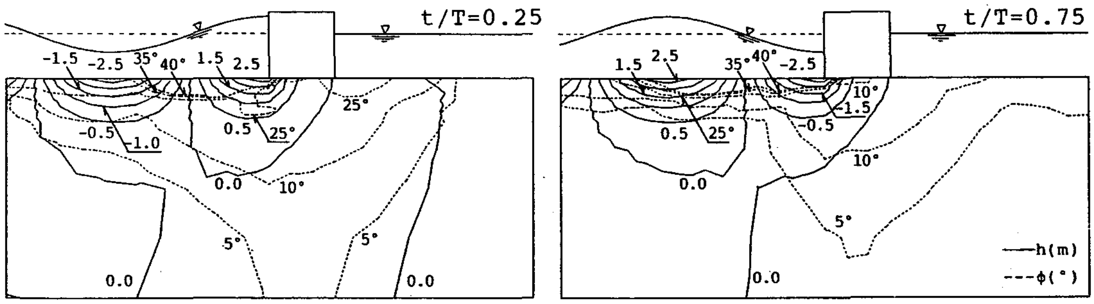

Fig.9 Pore water pressure and stress angle (Run 7 , T=8.0 , L=70.9)

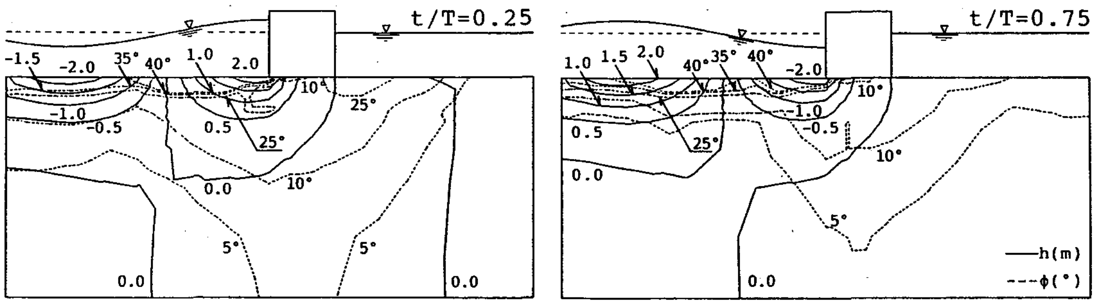

Fig.10 Pore water pressure and stress angle (Run 8 , H=3.0)

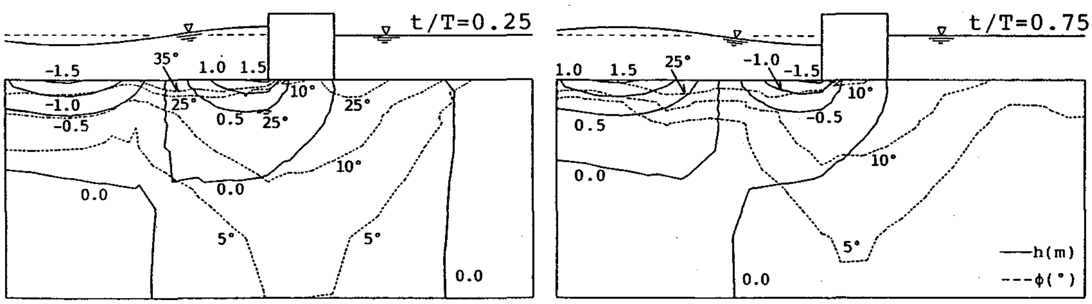

Fig.11 Pore water pressure and stress angle (Run 9 , H=2.0)

As the wave height becomes large, the amplitude of the oscillation of pore water pressure near the sand surface becomes large. But the pattern of the distribution of pore water pressure remains almost same. With respect to the stress angle, the unstable region with large stress angle spread in front of and under the structure with the increasing wave height. This means that the stability of the structure decrease according as the wave height becomes larger.

4. CONCLUDING REMARKS

In this paper, the dynamic behavior of the sand bed under the attack of superposed wave to the structure like a breakwater was analysed theoretically by using two-dimensional model. From the results of analyses, it is clarified that the notable unstability of sand bed around the structure occurs during the wave crest or the wave trough being in front of the structure, and that the wave height is the most dominant factor influencing on the unstability among the various wave parameters and sand bed properties.

In future studies, on the basis of the present results, it is necessary to investigate the more practical problems for the engineering use.

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REFERENCES

(1) H. Nagō : Liquefaction of Highly Saturated Sand Layer under Oscillating Water Pressure, Memoirs of the School of Engineering, Okayama Univ., Vol.16, No.1, 1981, pp.91-104.

(2) H. Nagō and S. Maeno : Pore Water Pressure in Sand Bed under Oscillating Water Pressure, Memoirs of the School of Engineering, Okayama Univ., Vol.19, No.1, 1984, pp.13-32.

(3) H. Nagō and S. Maeno : Dynamic Behavior of Sand Bed Under Oscil-

lating Water Pressure, Memoirs of the School of Engineering, Okayama Univ., Vol.20, No.2, 1986, pp.35-45.

(4) Biot, M.A. : General Theory of Three-dimensional Consolidation, J. Appl. Phys. 12, 1941, pp.155-164.

(5) De Wiest, R. J. M. : Flow Through Porous Media, Acad. Press, New York and London, 1969, pp.337-344.