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Analysis of Thyristor Phase Control Circuit with Parallel Resonance Elements

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Synopsis

The phase control characteristics in a thyristor phase control circuit with parallel resonance elements indicate very interesting phenomenon. Several extreme values appear on the phase control curve. The phenomenon is different from the step-up one in a thyristor phase control circuit with series *RLC* elements which is interpreted as series resonance. To comprehend the circuit performance with those loads, it is necessary that the phenomenon on extreme value is physically clarified from other viewpoints.

In this paper the performance in this circuit is studied from two viewpoints of a natural oscillation and a parallel resonance. Then, it is found that the performance depends on a natural frequency in thyristor conducting period and a parallel resonance frequency in thyristor non-conducting period. Therefore, the interesting phenomenon on extreme value is affected by the alternative of natural frequency or parallel resonance frequency.

1. Introduction

The thyristor phase control circuit is available in extensive fields of AC power control. Various configurations of load are

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connected to the thyristor power controller[1)2)]. We can sometimes observe very interesting phenomenon in that circuit performance, which occurs due to the mutual interference of a thyristor switching and a load property. For example, it is a step-up phenomenon in the phase control circuit with series *RLC* elements[3)4)]. The step-up mechanism can be explained theoretically and practically from a viewpoint of series resonance phenomenon[5)6)]. It is also reported that the phase control circuit with parallel *LC* elements indicates alternative interesting phenomenon[7)]. That is, several extreme values exist on the phase control curve. Therefore, to comprehend the circuit performance with those loads, the phenomenon on extreme value must be physically clarified from other viewpoints.

The circuit with series resonance elements acts only in thyristor conducting period. Therefore, the phenomenon is discussed taking notice of only the performance in thyristor conducting period. The circuit with parallel *LC* elements, however, is in live in not only thyristor conducting period but also thyristor non-conducting period. Accordingly the same technique as the above-mentioned cannot be employed. Then we study the circuit performance from two viewpoints of a natural oscillation and a parallel resonance. The circuit has two important factors; natural frequency and parallel resonance frequency, which participate the circuit performance. Namely the performance depends on a natural frequency in thyristor conducting period and a parallel resonance frequency in thyristor non-conducting period.

2. Analysis of Thyristor Phase Control Circuit with Parallel Resonance Elements

2.1 Thyristor Phase Control Circuit with Parallel Resonance Elements

A single-phase thyristor phase control circuit with parallel resonance elements is shown in Fig.1. Whenever the thyristor bursts into conduction every half cycle, a transient phenomenon appears. When the thyristor is open, the circuit acts in a closed-loop state of series *RLC* shown in Fig.2. The capacitor is discharged in this state.

Analyzing this circuit behavior, next assumptions are set up.

(1) The thyristor is an ideal switch, that is, there is no leakage current, and its turn-on and turn-off time are negligible.

(2) The thyristor is symmetrically fired every half cycle. The gate

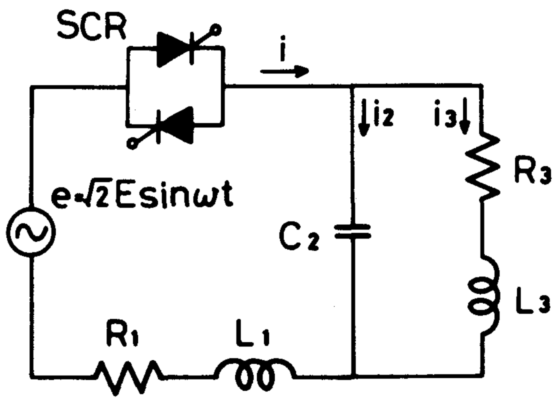


Fig.1. Thyristor Phase Control Circuit with Parallel Resonance Elements.

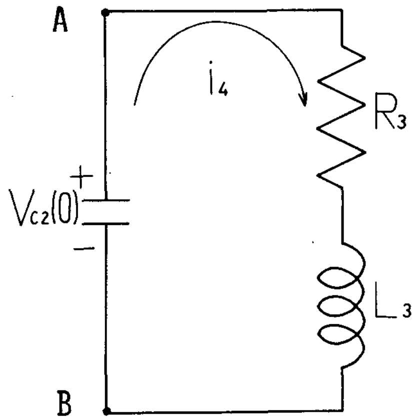


Fig.2. Equivalent Circuit in Thyristor Non-Conducting State.

signal of thyristor is a pulse waveform.
(3) The source impedance is negligible.

2.2 Circuit Equations

According to the assumptions, the equivalent circuit in thyristor conducting state is shown in Fig.3. The differential equation of this circuit is

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_2}\int i_2 dt = e \ , \qquad (1)$$

$$i_1 = i_2 + i_3 \ . \qquad (2)$$

This equation is rearranged in a matrix form, thus we obtain

$$\frac{dx(t)}{dt} = A \cdot x(t) \ , \qquad (3)$$

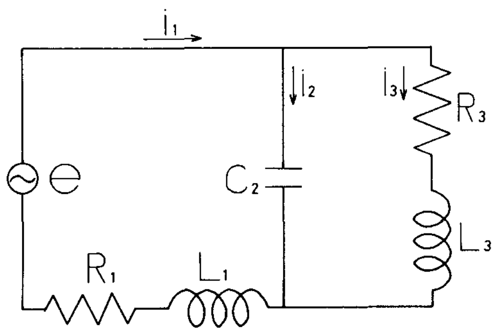


Fig.3. Equivalent Circuit in Thyristor Conducting State.

$$x(t) = \begin{bmatrix} e_1 \\ e_2 \\ i_1 \\ i_2 \\ i_3 \\ v_{c2} \end{bmatrix}, \tag{4}$$

$$A = \begin{bmatrix} 0 & \omega & 0 & 0 & 0 & 0 \\ -\omega & 0 & 0 & 0 & 0 & 0 \\ 1/L_1 & 0 & -R_1/L_1 & 0 & 0 & -1/L_1 \\ 1/L_1 & 0 & -R_1/L_1 & 0 & R_3/L_3 & -(L_1+L_3)/(L_1 L_3) \\ 0 & 0 & 0 & 0 & -R_3/L_3 & 1/L_3 \\ 0 & 0 & 0 & 1/C_2 & 0 & 0 \end{bmatrix}. \tag{5}$$

Where $t=0$ at the time when the thyristor bursts into conduction. And also,

$$e_1 = \sqrt{2}\ E\ \sin(\omega t+\alpha)\ ,$$
$$e_2 = \sqrt{2}\ E\ \cos(\omega t+\alpha)\ .$$

Resolving eq.(3), we obtain

$$x(t) = \phi(t-t_0)\cdot x(t_0)\ . \tag{6}$$

Where, $x(t_0)$ is the initial value at $t=t_0$.

$$\begin{aligned} \phi(t-t_0) &= e^{A\cdot(t-t_0)} \\ &= I + A\cdot(t-t_0) + \frac{A^2\cdot(t-t_0)^2}{2!} + \cdots \\ &\quad \cdots + \frac{A^n\cdot(t-t_0)^n}{n!} \cdots\ . \end{aligned} \tag{7}$$

Secondly, the equivalent circuit in thyristor non-conducting state shown in Fig.2 is considered. Provided that the time when the thyristor is extinguished is the origin, the next differential equation is derived.

$$L_3 \frac{di_4}{dt} + R_3 i_4 + \frac{1}{C_2}\int i_4 dt = 0\ . \tag{8}$$

Now, provided that the initial values of capacitor voltage and current through a reactor are $v_c(0)$ and $I(0)$ respectively, the solution of eq.(8) is in the case of oscillatory circuit as follows:

$$i_4(t) = I_3(0)\cdot e^{-\tau_1 t}\cdot(\cos\beta_1 t - \frac{\tau_1}{\beta_1}\sin\beta_1 t) - \frac{\tau_1{}^2+\beta_1{}^2}{\beta_1}\cdot C_2\cdot v_c(0)\cdot e^{-\tau_1 t}\sin\beta_1 t \ . \tag{9}$$

Then, the capacitor voltage is

$$v_c(t) = \{v_c(0)\cdot\cos\beta_1 t + \frac{1}{\beta_1}(\frac{1}{C_2}\cdot I(0)+\tau_1\cdot v_c(0))\cdot\sin\beta_1 t\}\cdot e^{-\tau_1 t} \ . \tag{10}$$

Where,

$$\delta_1 = \frac{R_3}{2}\sqrt{\frac{C_2}{L_3}}\ , \tag{11}$$

$$\tau_1 = \frac{R_3}{2L_3}\ , \tag{12}$$

$$\omega_0 = \sqrt{\frac{1}{L_3C_2} - \frac{R_3{}^2}{L_3{}^2}}\ , \tag{13}$$

$$\beta_1 = \omega_0\sqrt{\delta_1{}^2-1}\ . \tag{14}$$

2.3 Natural Frequency and Parallel Resonance Frequency

Two factors of circuit play an important part in the circuit behavior. Therefore, it is necessary to derive them.

At first, we suppose that a unit-step voltage is impressed on the circuit shown in Fig.3. Then, applying the Laplace transformation to the circuit equation thus gains

$$I(S) = \frac{L_3C_2\cdot S^2 + R_3C_2\cdot S + 1}{S\{L_1L_3C_2\cdot S^3 + C_2(R_1L_3+R_3L_1)\cdot S^2 + (C_2R_1R_3+L_1+L_3)\cdot S + C_2(R_1+R_3)\}}$$

$$= \frac{K(S+Z_1)(S+Z_2)}{S(S+P_1)(S+P_2)(S+P_3)}\ . \tag{15}$$

Where, K is a constant and $i(t)=L^{-1}(I(S))$.
Defining that P_1 is a real root and P_2, $P_3=-\alpha\pm j\omega_n$, ω_n is a natural angular frequency. Then, $f_{cn}(=\omega_n/2\pi)$ is termed as "natural frequency".

Secondly, the admittance Y observed between A and B in Fig.2 is

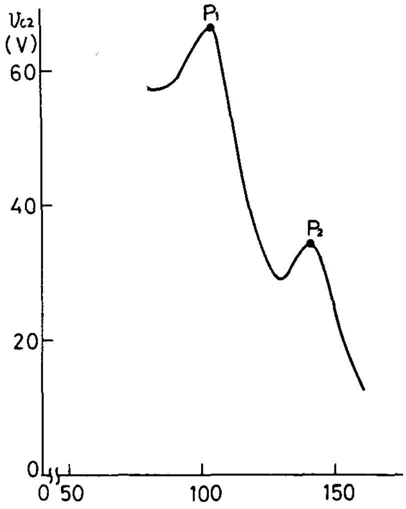
$$Y = \frac{R_3}{R_3{}^2+\omega^2 L_3{}^2} + j\left(\omega C_2 - \frac{\omega L_3}{R_3{}^2+\omega^2 L_3{}^2}\right). \tag{16}$$

Setting the imaginary part of Y in eq.(16) to zero, the resultant ω is equal to ω_0 in eq.(13). Then, $f_0(=\omega_0/2\pi)$ is termed as "parallel resonance frequency".

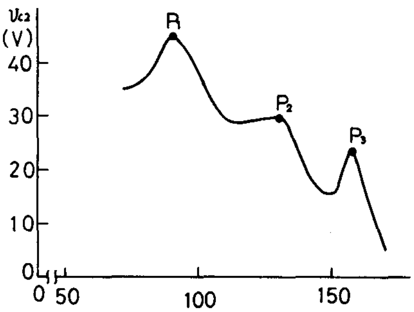
3. Characteristics

3.1 Phase Control Characteristics

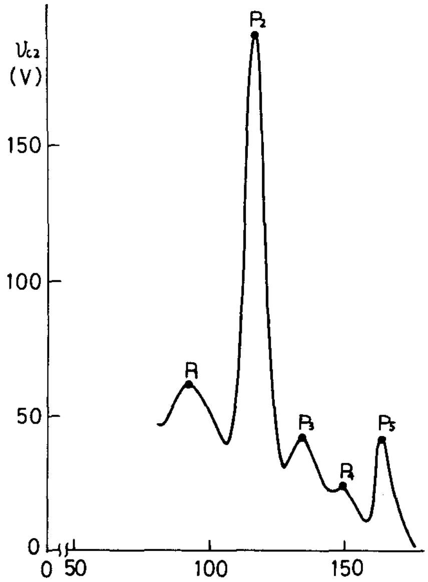
The calculated phase control curve is shown in Fig.4. These figures (a)~(c) are correspond to



(a) f_0=300Hz



(b) f_0=420Hz



(c) f_0=540Hz

Fig.4. Thyristor Phase Control Curve.

the circuit constants of (a)~(c) in Table 1. There are two extreme values in Fig.4 (a), and then the parallel resonance frequency f_0 is five times as many as a source frequency f(=60Hz). There are also three ones and f_0 is seven times in Fig.4 (b). Further there are five ones and f_0 is nine times in Fig.4 (c). Therefore, from these results the following relation is gained.

$$n = 2m + 1. \qquad (17)$$

Where, $n=f_0/f$, m;the number of extreme value.

Thereby, eq.(17) is valid in Fig.4 (a) and (b). Though the number of extreme value obtained from eq.(17) is four, there are five ones in Fig.4 (c). The cause of increase in extreme value is considered that the circuit has a natural frequency f_{cn} of 733.4(Hz). Therefore, the number of extreme value in the phase control curve is dependent on not only a parallel resonance frequency but also a natural frequency

3.2 Analysis of Condenser Current

The harmonics analysis of current i_2 with the circuit constants in Table 1 is shown in Fig.5 by using Fourier series expression. The current is a periodical non-sinusoidal waveform so that it has no even-component of harmonics.
In this figure the following facts are found out. In Fig.5 (a) the amplitude of fifth harmonics is larger than the others and is maximum at $\alpha=90^0$. In this figure (b) and (c), the seventh and the eleventh harmonics are larger than the others and are maximum at $\alpha=80^0$ and 115^0, respectively.

If we calculate the natural frequency f_{cn} as the same way of the previous section, f_{cn} is 327(Hz) in Fig.5 (a), 450(Hz) in Fig.5 (b). Thus each value is near the parallel resonance frequency f_0 in

Table 1. Circuit Constants.

(a) f_0=300Hz

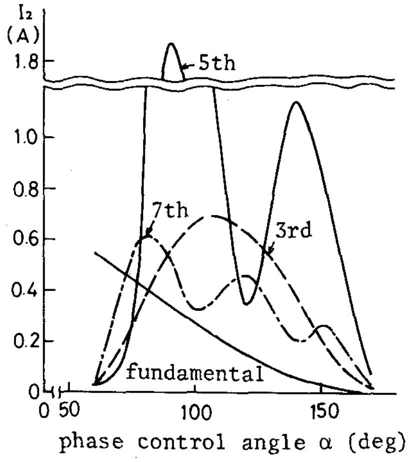
E(V)	R_1 (Ω)	L_1 (mH)	C_2 (μF)	R_3 (Ω)	L_3 (mH)
100	5	33.7	30	5	10.1

(b) f_0=420Hz

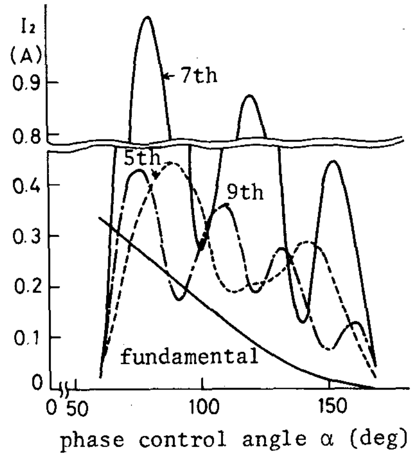
E(V)	R_1 (Ω)	L_1 (mH)	C_2 (μF)	R_3 (Ω)	L_3 (mH)
100	5	33.7	20	5	7.6

(c) f_0=540Hz

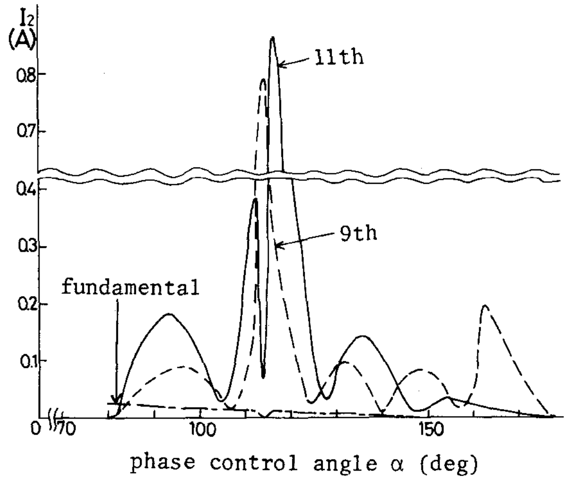
E(V)	R_1 (Ω)	L_1 (mH)	C_2 (μF)	R_3 (Ω)	L_3 (mH)
100	5	100	1	5	89



(a) f_0=300Hz



(b) f_0=420Hz



(c) f_0=540Hz

Fig.5. Analysis of Harmonics in i_2.

each case, which is equal to the frequency of harmonics with maximum amplitude. In this figure (c), however, f_0 is 540(Hz) and f_{cn} is 733.4(Hz), then the eleventh harmonics has a maximum amplitude. Consequently, the performance of this circuit is influenced by not only a parallel resonance frequency but also a natural frequency.

3.3 Waveforms of Current and Voltage

Fig.6 shows the waveforms of condenser current i_2 and condenser voltage v_{c2} in case of the circuit constants in Table 1 (c). Then, (a)~(d) in this figure indicate the waveforms of i_2 and v_{c2} at the Peak points P_1-P_4 of phase control curve in Fig.4 (c). The period indicated by arrows in this figure is a thyristor conducting period. The number of extreme value in conducting period and the natural frequency are shown in Table 2 (a). The number of extreme value in non-conducting period and the parallel resonance frequency are also shown in Table 2 (b).

y_0 and y_n in this table are expressed as follows:

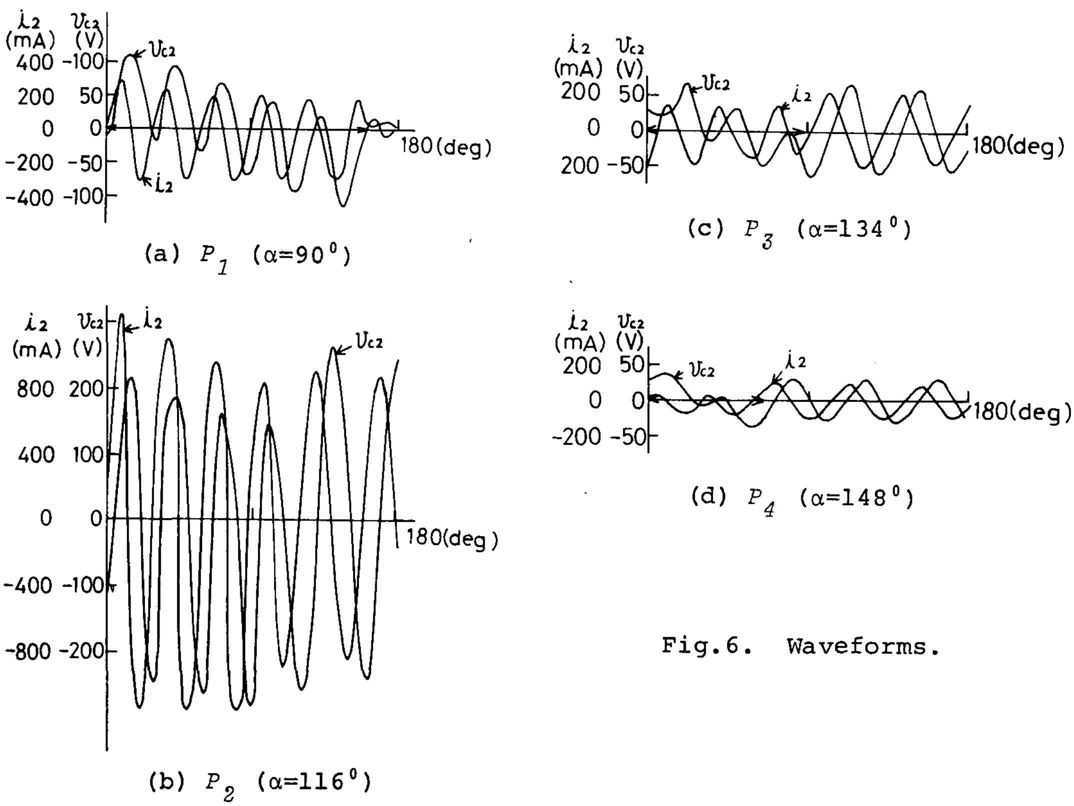
$$y_0 = 180/(f_0/f) \ , \tag{18}$$

$$y_n = 180/(f_{cn}/f) \ . \tag{19}$$

Then we get in this case

$$y_0 = 20^0 \ ,$$

$$y_n = 14.7^0 \ .$$



(a) P_1 ($\alpha=90^0$)

(b) P_2 ($\alpha=116^0$)

(c) P_3 ($\alpha=134^0$)

(d) P_4 ($\alpha=148^0$)

Fig.6. Waveforms.

Table 2. Relation between the Number of Extreme Value and Conduction Angle.

(a) Thyristor Conducting Period

α (deg)	1 $y_n \times m$	2 conduction angle	3 $\frac{2-1}{2}$ (%)
90	161.99	161.69	-0.19
100	145.79	145.67	-0.08
110	129.59	130.46	0.67
120	104.56	105.75	1.13
130	95.72	95.75	-0.28
140	76.58	75.70	-1.16
148	60.38	61.96	2.55
154	51.54	49.67	3.79

(b) Thyristor Non-Conducting Period

α (deg)	1 $y_0 \times m$	2 non-conduction angle	3 $\frac{2-1}{2}$ (%)
90	20.00	18.31	-9.23
100	34.00	34.33	0.96
110	48.00	49.54	3.11
120	76.00	74.25	2.36
130	84.00	84.55	0.65
140	106.00	104.30	1.63
148	118.00	118.04	0.03
154	138.00	130.33	0.59

Consequently, it is found from this table that the product of y_n by the number of extreme value in conducting period is nearly equal to the conducting angle and then the product of y_0 by the number of extreme value in non-conducting period is also nearly equal to the non-conducting angle. Therefore, the circuit perofrmance is dependent on the natural frequency in conducting period and also dependent on the parallel resonance frequency in non-conducting period.

3.4 Discussion of Phase Control Characteristics from a Viewpoint of Parallel Resonance Phenomenon

The performance of thyristor phase control circuit with series RLC elements has been discussed from a viewpoint of series resonance phenomenon. In this section, the relation between the phase control curve and the parallel resonance curve in the circuit is described.

From the r.m.s. values I_1, I_2, I_3 of currents i_1, i_2, i_3 with thyristor closed, the following relations are gained.

$$\frac{I_2}{I_1} = \frac{\sqrt{(\omega/\omega_0)^4(C_2R_3{}^2/L_3-1) + (\omega/\omega_0)^2(C_2R_3{}^2/L_3-C_2{}^2R_3{}^4/L_3{}^2)}}{\sqrt{(\omega/\omega_0)^4(C_2R_3{}^2/L_3-1)^2 + (\omega/\omega_0)^2(3C_2R_3{}^2/L_3-C_2{}^2R_3{}^4/L_3{}^2-2) + 1}}, \tag{20}$$

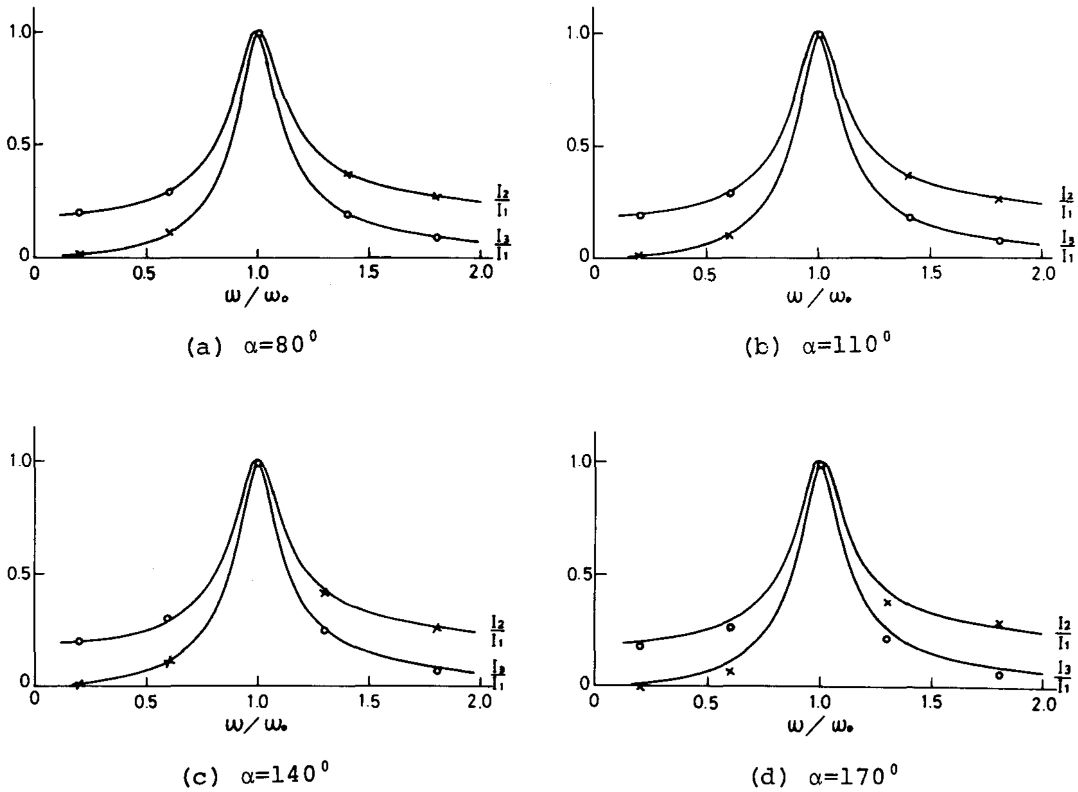


Fig.7. Phase Control Curve and Parallel Resonance Curve.

$$\frac{I_3}{I_1} = \frac{1}{\sqrt{(\omega/\omega_0)^4 (C_2 R_3{}^2/L_3 - 1)^2 + (\omega/\omega_0)^2 (3C_2 R_3{}^2/L_3 - C_2{}^2 R_3{}^4/L_3{}^2 - 2) + 1}} \ . \tag{21}$$

These relations are plotted with full line in Fig.7. The o and × points in this figure are gained from Fourier series of i_1, i_2, and i_3 in the phase control circuit with circuit constants in Table 1 (c).

Both agree well as observed from this figure, so that the performance of phase control circuit with parallel resonance elements is explained by parallel resonance phenomenon as well as one with series resonance elements.

4. Conclusions

The very interesting phenomenon in the thyristor phase control circuit with parallel *LC* elements is pointed out. Then, the phenomenon is investigated from a viewpoint of a parallel resonance and a natural oscillation. Thus, we conclude as follows:

(1) Several extreme values exist on the phase control curve of thyristor phase control circuit with parallel resonance elements.

(2) The circuit performance can be elucidated by not only a parallel resonance but also a natural oscillation. Then, the performance in thyristor conducting period depends on a natural frequency, while the performance depends on a parallel resonance frequency in thyristor non-conducting period.

(3) Consequently the phenomenon must be treated taking account of the interactions between natural oscillation and parallel resonance in case of the thyristor phase control circuit with parallel *LC* elements.

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