

A Consideration on Physics of Saturated-Unsaturated Groundwater Motion

Makoto NISHIGAKI* and Ichiro KONO*

(Received January 10, 1980)

Synopsis

The purposes of this paper are primarily to re-search on behavior of groundwater flow in saturated and unsaturated zone, and to present the fundamentals of the theory of groundwater flow.

This paper discusses the physics of the saturated-unsaturated groundwater motion. Evaluations confirm the early belief that Darcy's law is of the nature of statistical result giving the empirical equivalent of Navier-Stokes equations. The governing equation of saturated-unsaturated flow in porous media is derived from the law of mass conservation and from the Darcy's law and Richard's equation of motion and is compared with the Klute's diffusion equation which has been widely used in the analysis of unsaturated flow. As a result; it is concluded that the governing equation has the advantage that can be applied for the whole flow region. Typical boundary conditions are enumerated.

1. Introduction

In solving a specific physical problem, such as the flow of a liquid through a specified porous medium domain, it is necessary to develop the fundamental equations describing the transport of fluid in a porous medium.

In this paper, firstly, Darcy's law is discussed as the equation of motion. The experiment of Darcy will not be uniquely defined, therefore, there is considerable ambiguity in postulating a differential

* Department of Civil Engineering

equation which would be equivalent to the results of the experiments. In fact, the differential equation which is now commonly called "Darcy's law" is not an equivalent expression for Darcy's finding, although these do follow from it. However, they would equally well follow from other types of differential equations. This is especially true if generalizations of Darcy's law to anisotropic compressible and unsaturated porous media are attempted. Some discussion will be devoted to this subject later in this paper. It is to be expected that Darcy's law will have limitations. Indeed, such limitations occur generally as high and low flow rates, as well as in relation to various other effects. The range of validity of Darcy's law and its limitations will also be discussed in this paper.

Secondly, Richard's equation is discussed as the law of continuity or conservation of mass. Coupled the equation of motion with the law of continuity, the governing equations for saturated-unsaturated flow will be derived and compared with the Klute's diffusion equation which was widely used in the analysis of unsaturated flow.

When a problem is described simultaneously by a number of dependent variables, the same number of equations is needed for a complete solution. Being a general description of an actual phenomena, it is obvious that the partial differential equation itself does not contain any information concerning the specific values of quantities characterizing a specific case of a phenomenon. Therefore, any partial differential equation has an infinite number of possible solutions, each of which corresponds to a particular case of the phenomenon. To obtain from this multitude of possible solutions corresponding to a certain specific problem of interest, it is necessary to provide supplementary to a certain specific problem of interest, it is necessary to provide supplementary information that is not contained in the partial differential equation. Finally section 5 will include a discussion of the initial and boundary conditions of flow of fluids through porous media.

2. Equation of Motion

In 1865, Henri Darcy published in an appendix to his book "Les Fontaines Publiques de la Ville de Dijon" the results of his experiments on the flow of water through granular material. Using a cylindrical sample, the direction of flow being along the cylinder, he found the discharge per unit area of cross-section to be proportional

to the gradient of piezometric head in the direction of flow, i.e.

$$\frac{Q}{A} = K \frac{\Delta h}{\Delta l} \quad (1)$$

Here, Q is the discharge through the sample, A is the gross area of the sample's cross-section, Δh is the head lost in a length Δl and K is constant for a given sample. This expression, and various rearrangements of it, have been named Darcy's law and it is the basic relationship in quantitative study of the flow of fluids through porous media.

Darcy's law originally was limited to one-dimensional flow in a steady state for a homogeneous incompressible fluid. When Darcy's law is extended to a formal generalization of the equation of motion, some problems arise as follows:

- (1) to three-dimensional flow
- (2) to the flow in an isotropic medium
- (3) to the flow in an anisotropic medium
- (4) to unsteady state flow

Several researchers have derived Darcy's law from the general Navier-Stokes equations for viscous flow to extend Darcy's law to above problems. Under unsteady state conditions the Navier-Stokes equation for low Reynolds numbers (neglecting the higher order inertial term) becomes

$$\rho \frac{\partial v_i}{\partial t} + \rho \frac{\partial \phi}{\partial x_i} = \mu \nabla^2 v_i \quad (2)$$

Here ϕ is a force potential defined as

$$\phi = -x_i g_i + \frac{p}{\rho} \quad (3)$$

$g_i = (0, 0, -g)$ is the acceleration due to gravity, p , μ and ρ are respectively the pressure, viscosity and density of the fluid. The mass average velocity v_i^* [1] is introduced into Eq.(2).

$$\rho \frac{\partial v_i^*}{\partial t} + \rho \frac{\partial \phi}{\partial x_i} = \mu \nabla^2 v_i^* \quad (4)$$

In above equation $\mu v^2 v_i^*$ represents the density of the force due to the fluid's viscosity, which resists the motion. It is a viscous force per unit volume of fluid. Whese resistance forces depend upon the friction of fluid particles with soil particles. The forces of internal friction (fluid particles with fluid particles) are negligibly small in comparison with the forces of external friction. The resistance forces offered by a single sphere for the special case of slow viscous flow of a Newtonian fluid is given by Stokes law [2,3]. This law can be expressed in a generalized form as

$$f_p = \lambda \mu d v_i^* \quad (5)$$

in which f_p is the resistance or drag of a single particle, μ represents the dynamic viscosity of the fluid, d is the particle diameter; for irregular-shape nonuniform particles, d would be the characteristic length of the average-size particle and would have to be determined by some appropriate technique. v_i^* denotes the local average velocity of flow around the particle (i.e., the seepage velocity), and λ represents a coefficient that takes into account the effects of neighboring particles. The coefficient λ will depend upon the local streamline configuration around the particle and, hence, must be some function of the geometry of the pore system. More specifically λ will depend upon the porosity, the shape of the particles, and the distribution of the sizes of the particles.

The total resistance F_R offered by all of the particles in the element will thus be

$$F_R = N f_p \quad (6)$$

in which N represents the number of particles in the element. F_R is rewritten in the another form

$$F_R = (\mu/B) v_i^* \quad (7)$$

Considering that the coefficient B retains the same value for the unsteady movement as for the steady. Eq.(4) with the help of Eq.(7) becomes

$$\rho \frac{\partial v_i^*}{\partial t} + \rho \frac{\partial \Phi}{\partial x_i} = -(\mu/B) v_i^* \quad (8)$$

and this equation can be shown in the next form

$$v_i^* = -\left(\frac{B}{\mu}\right)\rho \frac{\partial \Phi}{\partial x_i} - \left(\frac{B}{\mu}\right)\rho \frac{\partial v_i^*}{\partial t} \quad (9)$$

Averaging over the cross-section [4] we obtain

$$v_i^* = -\frac{K_{ij}}{n\bar{\mu}} \bar{\rho} \frac{\partial \Phi^*}{\partial x_j} - \left(\frac{\bar{B}}{\bar{\nu}}\right) \frac{\partial v_i^*}{\partial t} \quad (10)$$

where $\bar{\nu}$ is the average kinematic viscosity ($\approx \bar{\mu}/\bar{\rho}$). Thus, the equation of motion derived in the form of Eq.(10) is valid for an inhomogeneous fluid in laminar flow through an anisotropic porous medium. The coefficient K_{ij} (the medium's permeability) will be discussed in detail in many researches. Eq.(10) is the generalized form of Darcy's law for nonsteady state flow and the form of the extension of Darcy's law to three-dimensional flow in anisotropic media.

Another derivations of Darcy's law from Navier-Stokes eq. are presented by Whitaker [4,5,6] and by Slattery [7,8,9]. They used the Slattery-Whitaker averaging theorem which discovered simultaneously and independently by Slattery [7] and by Whitaker [5]. This theorem enables one to express the volume averages of space derivatives in terms of space derivatives of volume averages, thereby making it possible to proceed with the integration of differential equations from one scale of measurement to another in a mathematically reigorous fashion.

The second term on the right-hand side of the motion equation Eq.(10) expresses the average acceleration. Since in practive Darcy's law is always expressed in the form neglecting the last term in Eq.(10), it is important to know under what conditions can one justify neglecting this term. For flows in which the local inertial forces can be neglected with respect to the viscous (resistance) forces, Polubarinova-Kochina [10] has indeed shown that the acceleration term in the equation of motion tends very rapidly (e.g., within a fraction of a second) to zero after the onset of flow. Hence, one is justified in deleting this term from the equation.

The same interesting way of looking at this problem has been suggested by Whitaker [6]. To use the author's own words " If a tube filled with fluid is subjected to a sudden change is the pressure drop, essentially steady flow occurs for times greater than to where to $\approx d^2/4\nu$. Here d is the tube diameter and ν is the kinematic viscosity. For the purpose of estimating microscopic transient times in porous media, a practical lower bound on ν is $10^{-2} \text{cm}^2/\text{sec}$, and an upper bound on d might be on the order of 10^{-1}cm .

This gives a microscopic transient time on the order of 1 sec, and if the transient time for the macroscopic process is much larger (say on the order of minutes), we should be treat the flow as quasi-stead ...". When the fluid is incompressible and the porous medium is rigid, all the transient effects are caused entirely by temporal change in the external boundary conditions. In practice the characteristic time of such changes is usually at least on the order of minutes, indicating that the time derivative in Eq.(10) can probably be neglected in many situations.

Then for a homogeneous incompressible fluid, $\bar{\rho}=\text{const}$, $\bar{\mu}=\text{const}$ and the motion Eq.(10) may be written in terms of the piezometric head $\bar{h}=z+\bar{p}/\bar{\gamma}$.

$$v_i^* = - (K_{ij} \bar{\gamma}/\bar{\mu}) \frac{\partial \bar{h}}{\partial x_j} \quad (11)$$

This equation is Darcy's experimental law.

Although upper evaluations do not contribute to the formulation of a new law, these confirm the early belief that Darcy's law is of the nature of statistical result giving the empirical equivalent of Navier-Stokes equations.

It is important to define a range of validity of Darcy's law, because Darcy's law is not universally valid for all conditions of liquid flow in porous media. In derivation of Darcy's law from Navier-Stokes equation, it is assumed that the flow is laminar (i.e., non turbulent slippage of parallel layers of the fluid one atop another, and inertial forces are negligible compared to viscos forces). Laminar flow prevails in silts and finer materials for any commonly occurring hydraulic gradients found in nature. In coarse sands and gravels, however, hydraulic gradients much in excess of unity may result in nonlaminar flow conditions, and Darcy's law may not always be applicable. In flow through conduits, the Reynolds number (R_e), a dimensionless number expressing the ratio of inertial to viscous forces, is used as a criterion to distinguish between laminar flow and turbulent flow. By analogy, a Reynolds number is defined also for flow through porous media:

$$R_e = v^*d/\nu \quad (12)$$

where v^* is the mean flow velocity, d the effective pore diameter, and ν is the kinematic viscosity of the fluid.

The limit can be found by plotting the dimensionless fanning friction factor f , used in hydraulics, against R_e . The factor f is defined by

$$f = \frac{d\Delta p}{2\rho Lv^*{}^2} \tag{13}$$

where Δp is the pressure difference over a length of porous media, L measured along the line of flow and other quantities are as defined above. Data from several investigations are plotted in Fig.1. Departures from a linear relationship appear when R_e reaches the range between about 1 and 10 [12], thus indicating an upper limit for the validity of Darcy's law.

If we take $v=0.018\text{cm}^2/\text{sec}$ (for water), then

$$v^*d < 0.018 \sim 0.18 \tag{14}$$

where v^* is expressed in cm/sec, d - in cm next, we take $d=0.1\text{cm}$, then

$$v^* < 0.18 \sim 1.8 \text{ (cm/sec)} \tag{15}$$

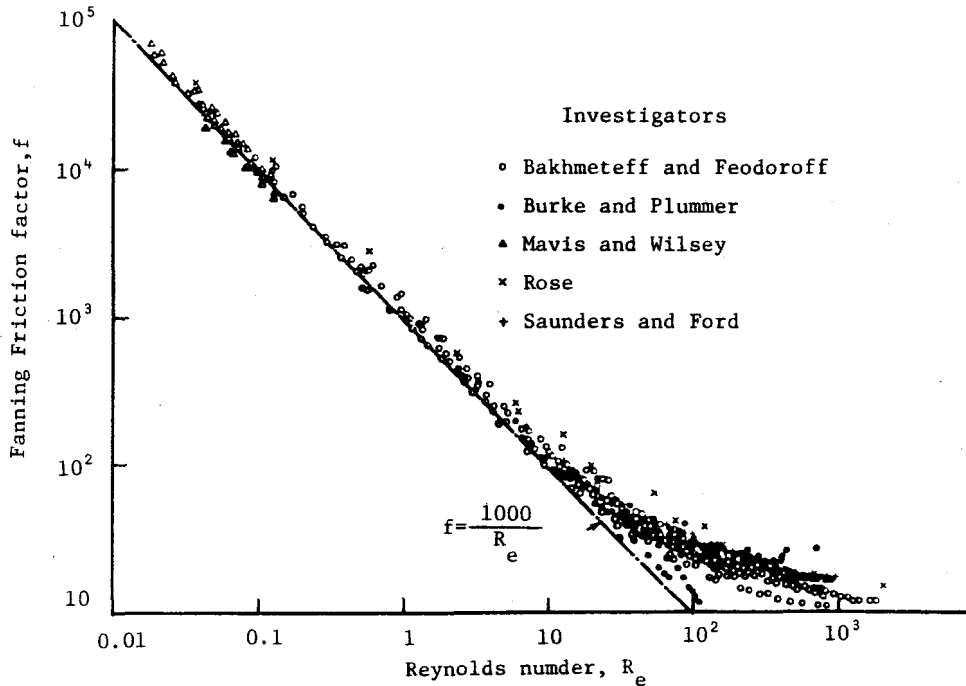


Fig.1 Relation of fanning friction factor to Reynolds number for flow through granular porous media [12]

this means used Darcy's law, then,

$$v^* = K \frac{\partial h}{\partial x} < 0.18 \sim 1.8 \text{ (cm/sec)} \quad (16)$$

when we take $K=1.0 \times 10^{-2}$ cm/sec

$$\frac{\partial h}{\partial x} < 18 \sim 180 \quad (17)$$

this values are very large hydraulic gradient, and Darcy's law can be employed in the vast majority of cases concerning the flow of water in soil.

On the contrary, for very low velocities, some investigators [13,14] have claimed that, in clayey soils, low hydraulic gradients may cause no flow or only low flow rates that are less than proportional to the gradient, while others have disputed some of these findings [15]. A possible reason for this anomaly is that the water in close proximity to the particles and subject to their adsorptive force field may be more rigid than ordinary water, and exhibit the properties of a "Bingham liquid" (having a yield value) rather than a "Newtonian liquid." The absorbed, or bound, water may have a quasicrystalline structure similar to that of ice, or even a totally different structure. Some soils may exhibit an apparent "threshold gradient," below which the flux is either zero (the water remaining apparently immobile), or at least lower than predicted by the Darcy relation, and only at gradients exceeding the threshold value does the flux become proportional to the gradient Fig.2.

Finally there seem to be a very important hydrological problem that may require the application of the Darcy's law to the flow in unsaturated soils. For the above problem, an experiment for the direct verification of Darcy's law in unsaturated materials has been carried out [16]. They devised a method whereby the moisture content and suction down a long column of porous conductor were uniform, the potential gradient being due solely to the gravitational component. Various magnitudes of potential gradient were imposed by suspending the column at various angles of inclination to the vertical. From the results it could be safely inferred that the rate of flow for a given degree of saturation was proportional to the potential gradient, as in the case of saturated materials. Thus it may be possible to assume that the Darcy's law is applicable to unsaturated flow in anisotropic media in the form:

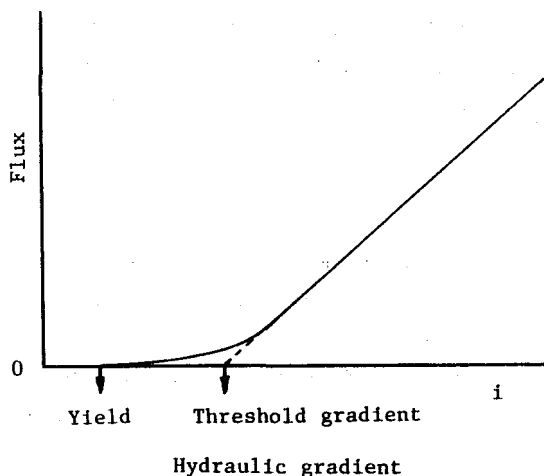


Fig.2 Possible deviations from Darcy's law at low gradients

$$v_i = -K_{ij}(\psi) \frac{\partial h}{\partial x_j} \quad (18)$$

where v_i is the (volumetric) flux (volume of water per unit area unit time). $\partial h / \partial x_i$ is the hydraulic head gradient, which may include both suction and gravitational components and K_{ij} is a function of the pressure head (ψ) or a function of the volumetric water content (θ), and not a constant, as in saturated flow. Eq.(18) is sometimes called the Richard's equation, because Richards has shown firstly that the modified Darcy's law applies to unsaturated soils [17]. Miller and Miller [18,19] point out an important difference that since the moisture characteristic is subject to hysteresis, the pressure head (or suction) (ψ) is not uniquely related to the volumetric moisture content (θ), for it depends also upon the history of wetting and drying by which that volumetric moisture content is reached.

However, the relation of hydraulic conductivity (sometimes called capillary conductivity) to volumetric moisture content $K_{ij}(\theta)$ is affected by hysteresis to a much lesser degree than is the $K_{ij}(\psi)$ function, at least in the media thus for examined [20]. Thus, Darcy's law for unsaturated soil can also be written as

$$v_i = -K_{ij}(\theta) \frac{\partial h}{\partial x_j} \quad (19)$$

which, however, still leaves one with the problem of dealing with the hysteresis between ψ and θ . A more complete review of this problem will have to be brought in future.

3. Equation of Continuity

In transient groundwater motion, the distribution of potential within the flow region undergoes continual change with time. The nature of the time-dependence of conservation of mass, subject to the constraints of the equations of motion and of state. The conservation law as applied to fluid or heat transfer is also known as the equation of continuity.

Consider a finite subregion of the flow region as depicted in Fig.3. Under transient conditions of flow, water enters and leaves the flow region at different rates at different parts of the enclosing surface (control surface). The amount and identity of matter in the control volume may change with time, but the shape and position of this volume remain fixed. The special case, when the inflow exactly equals the outflow so that there is neither a mass excess nor a mass deficiency, is the phenomenon of steady state flow.

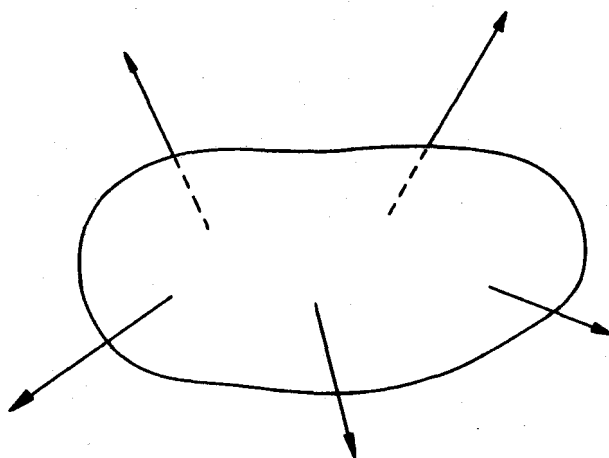


Fig.3 Fluxes crossing boundary surface of an arbitrary subregion of the flow region

Since mass can neither be created nor destroyed, the mass condition has to be absorbed into or released from the particular small part of the flow region under consideration. The equation of continuity, then, is a statement, equating the summation of the rates at which mass enters or leaves the control volume of the flow region and the rate at which mass is absorbed by the subregion. According to Sokolnikoff and Redheffer [21], the law of conservation of mass can be stated as

$$\int_S \vec{V} \cdot \vec{n} \, d\delta = \int_V \text{div } \vec{V} \, dV \quad (20)$$

where \vec{V} is the velocity of the fluid

\vec{n} is the unit normal

S is the control surface

and $d\delta$ is the differential surface area. Physically, the right hand side of Eq.(20) amounts to dividing a finite region S into a large number of small volume elements and summing up the rate of fluid increase in each element in order to obtain the overall fluid mass increase in V. The left hand side of Eq.(20) simply states that the fluid excess arising out of transient flow of fluid across control surface (left hand side of Eq.(20)) is accommodated by an equivalent increase in the fluid content within the element (right hand side of Eq.(20)). This is but a restatement of the conservation equation. Note also that the left hand side of Eq.(20) is the cause and the right hand side is the effect.

In soil physics literature, Richards [17] was probably the earliest to express the equation of continuity for transient soil water movement in the form of a parabolic, partial differential equation. Familiarly known as the Richard's equation, it can be written in cartesian coordinates as,

$$-\text{div} \rho \vec{V} = -\vec{\nabla} \cdot \rho \vec{V} = -\frac{\partial}{\partial t} (\rho \theta) \quad (21)$$

where ρ is the density of water, \vec{V} is specific flux or Darcy velocity vector and θ is the volumetric moisture content, defined as the volume of water per unit volume of soil. In Eq.(21) the density of water ρ is assumed to be independent of space and time, Eq.(21) reduced to,

$$-\operatorname{div} \vec{V} = -\vec{V} \cdot \vec{V} = \frac{\partial \theta}{\partial t} \quad (22)$$

which is also an expression of the conservation of mass.

4. Governing Equation of Flow in Porous Media

In practical problems of fluid flow in porous media, the most easily measured physical parameter is the fluid potential. Also, in itself, the variation in fluid potential is a phenomenon of considerable interest in studying flow through porous media. Hence, in the practical use of Eq.(22), it is convenient to make fluid potential the dependent variable. Thus, substituting Eq.(19) into Eq.(22), assuming that x_3 is a constant during the time interval

$$\operatorname{div} K(\theta) \vec{V}_h = \frac{\partial}{\partial t}(\theta) \quad (23)$$

Eq.(23) is the governing equation for flow through porous media.

Rewriting Eq.(23) in the cartesian coordinate and in the tensor notation, one has

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left[D(\theta) \frac{\partial \theta}{\partial x_j} + K_{i3}(\theta) \right] \quad (24)$$

$$(i, j = 1, 2, 3)$$

which is the so-called diffusion equation derived by Klute [21], and

$$D(\theta) = K_{ij}(\theta) \frac{\partial \psi}{\partial \theta} \quad (25)$$

where $D(\theta)$ is the soil-moisture diffusivity in soil physics literature. It may be appreciated that $\partial \psi / \partial \theta$ is a measure of the storage properties of the geological material and is the first derivative of the pressure head, ψ , to the increment of the volumetric moisture content, θ . The functional relationship between ψ and θ (so-called water retention curve) is shown in Fig.4. As seen from Fig.4, the moisture content θ tends to become constant which is equal to porosity of soil, n , as soon as $\psi > \psi_{cr}$, i.e., when the suction is less than the air entry value (ψ_{cr}) and D becomes essentially discontinuous as $\partial \psi / \partial \theta \rightarrow \infty$. Therefore Eq.(24) cannot be used for the saturated zone. With this equation, since the movement of water in unsaturated zone must be distinguished

from the movement of water in saturated zone, it is quite inconvenient to solve the problem such as advancement of wetting front in the saturated-unsaturated soil media. Although Eq.(24) has been used quite often in the flow analysis [22,23,24], solutions are obtained only in the problems for unsaturated zone.

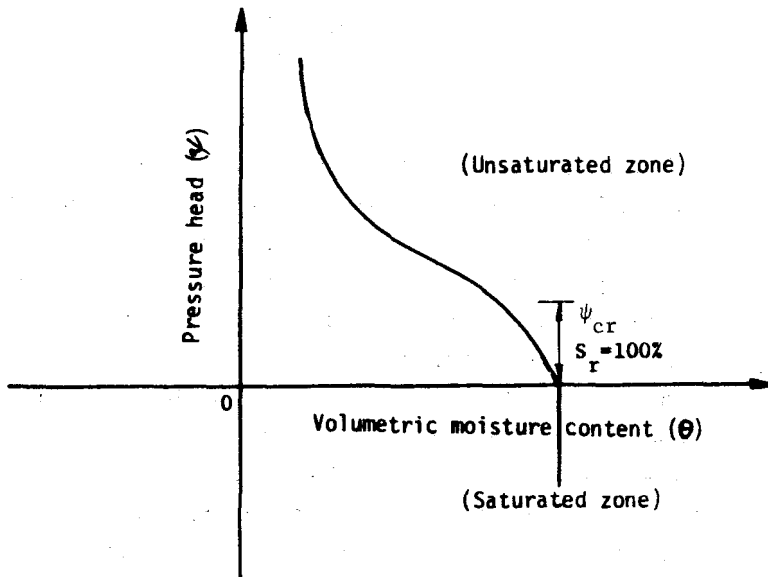


Fig.4 Variations in the moisture content

Meanwhile, it is known that the volumetric moisture content θ is expressed as

$$\theta = n S_w \tag{26}$$

where n is the porosity of soil and S_w ($0 \leq S_w \leq 1$) is the degree of saturation. Choosing ψ as only one dependent variable and applying the chain rule of differentiation to the time derivative on the right hand side of Eq.(23), Eq.(23) can be transformed to the following equation.

$$\text{div } K(\psi) \vec{\nabla}(\psi + x_3) = -\frac{\partial}{\partial t} (n S_w)$$

$$\begin{aligned}
 &= \frac{d}{d\psi} (n S_w) \frac{\partial \psi}{\partial t} \\
 &= \left[S_w \frac{dn}{d\psi} + n \frac{dS_w}{d\psi} \right] \frac{\partial \psi}{\partial t}
 \end{aligned}$$

(27)

The two terms on the right hand side of Eq.(27) denote two distinct physical phenomena [25]. The first term represents the deformability of the soil skeleton which is analogous to the consolidation problem expressed by Biot's equation [26]. This term is usually ignored in the problems of flow through porous media by assuming that the porous media is rigid. Here we consider that the soil medium is slightly compressible and also includes the last term which represents the desaturation of the pores, that is, the capacity of the soil to absorb or release moisture due to saturation changes.

Assuming that the porosity n does not change due to the variation of pressure ψ in unsaturated zone, Eq.(27) may be reduced to:

$$\operatorname{div} K(\psi) \vec{\nabla} (\psi + x_3) = (C(\psi) + \beta S_s) \frac{\partial \psi}{\partial t} + S$$

(28)

where

$$\beta = \begin{cases} 0 & : \text{Unsaturated zone} \\ 1 & : \text{Saturated zone} \end{cases}$$

where S_s denotes the specific storage (defined as the volume of water instantaneously released from storage per unit bulk volume of saturated soil when ψ is lowered by one unit), $C(\psi)$ is the specific moisture capacity (defined as $d\theta/d\psi$) and S is a sink or source term. In Eq.(28), the advantage being that $C(\psi)$ remains finite throughout the range of flow.

$C(\psi)$ attains a maximum value in coarse sand, near the air entry value (ψ_{cr}), where large changes in θ occur at small changes of ψ . It vanishes at suction smaller than the air entry value or for $\psi > \psi_{cr}$ (loosely called, "at saturated soil").

Clearly Eq.(28) has the advantage over Eq.(24) that is applied for the whole flow region, including saturated and unsaturated flow. So Eq.(28) is called "the governing equation for saturated-unsaturated flow through porous media."

5. Initial and Boundary Conditions

The supplementary information that, together with the partial differential equation, defines an individual problem should include specifications of:

- (a) the geometry of the domain in which the phenomenon being considered takes place,
- (b) all physical coefficients and parameters that affect the phenomenon considered (e.g., medium and fluid parameters),
- (c) initial conditions which describe the initial state of the system considered,
- (d) the interaction of the system under consideration with surrounding systems, i.e., conditions on the boundaries of the domain in question.

5.1 Initial condition

Let us now consider the surface integral in Eq.(28). At any instant of time t_0 , there is an initial distribution of ψ_0 within the flow region boundary by the surface S . This region may be a part or all of the flow region. This distribution of ψ_0 forms the initial condition for the transient fluid flow problem.

$$\psi(x_i, 0) = \psi_0(x_i) \quad (29)$$

5.2 Boundary conditions

The flow region as a whole communicates across its boundaries with its surroundings. The nature of this communication is reflected in the conditions that exist on the boundary of the flow region. The natures of these conditions are termed boundary conditions and, in general, these are four types. In addition, there many exist "mixed boundary conditions" of special types, which will be omitted for the present.

5.2.1 No-flow boundary

When no fluid enters or leaves the flow region across its boundary, the boundary is called a no-flow boundary. In fluid flow, an impermeable barrier or a plane of symmetry of flow is an impermeable boundary. Mathematically, this is identical to the condition that there is no gradient in potential across this boundary and hence,

$$\frac{\partial h}{\partial \vec{n}} = (K_{ij}(\psi) \frac{\partial \psi}{\partial x_j} + K_{i3}) n_i = 0 \quad (30)$$

in which \vec{n} denotes the normal to the boundary.

5.2.2 Prescribed flux boundary

It may so happen that the flow region receives fluid from or discharges fluid to its surroundings at a known or prescribed rate. For example, a soil may be receiving rainfall infiltration at a constant rate or a well may be discharging the flow region at a constant or variable rate. Such a condition is known as a prescribed flux boundary condition. Mathematically, this amounts to saying that, for a unit surface area of the boundary, $\partial h / \partial \vec{n}$ is known, and hence,

$$(K_{ij}(\psi) \frac{\partial \psi}{\partial x_j} + K_{i3}) n_i = -v(x_i, t) \quad (31)$$

Such a condition is called a Neumann problem in potential theory. Indeed, the no flow boundary condition in section 5.2.1, can be considered as a special case of Neumann problem.

5.2.3 Prescribed potential boundary

The third type of boundary condition arises when the flow region interacts with a very large reservoir of the fluid and receives fluid from or discharges fluid into this practically infinite reservoir. In this case, the potential on the boundary will be determined by the fluid potential in the infinite reservoir, which may fluctuate in time in a known, hence,

$$\psi(x_i, t) = \psi_b(x_i, t) \quad (32)$$

This type of boundary condition is called a Dirichlet problem in potential theory.

In most problems of practical interest different parts of the boundary of a flow region may experience different types of boundary conditions and thus these problems are mixed initial-boundary-value problems.

5.2.4 Seepage faces

A seepage face is an external boundary of the saturated zone where water leaves the system and ψ is uniformly zero as shown in Fig.5. Under transient conditions, the length of the seepage face varies with time in a manner that cannot be predicted a priori. If one treats the seepage face as a prescribed pressure head boundary with $\psi=0$, the length of this face remains fixed, and this is contrary to the reality of transient flow. On the other hand, the seepage face cannot be treated as a prescribed flux boundary because the values of Q are generally unknown there.

Along the seepage face, water emerges from the porous medium into the external space. The emerging water usually trickles down along the seepage face. (Fig.5). The seepage face is part of the boundary of the phreatic flow domain. Its geometry is generally known, except for its upper limit, which is also lying on the (a priori) unknown free surface. The location of this point is, therefore, part of the required solution.

The seepage face is exposed to the atmospheric pressure (neglecting the thin layer of water flowing above it). Actually, in order for water to emerge from the porous medium domain, the pressure just inside the boundary should be somewhat higher than atmospheric.

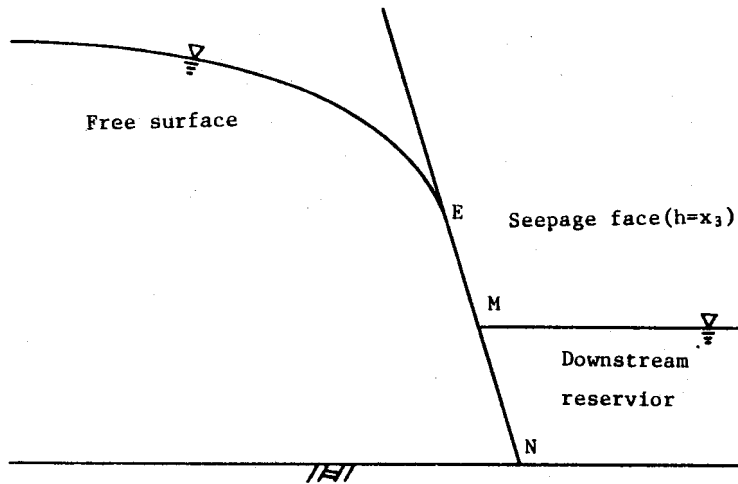


Fig.5 Seepage face

With $\psi=0$ along the seepage face, the boundary condition is described as

$$h = x_3 \quad \text{on } S_e \quad (33)$$

5.3 Sources and Sinks

In addition to the movement of fluid that is caused by the initial and boundary conditions, fluid may be arbitrarily extracted from or added to the flow region at one or more locations. Such locations, usually of very small or infinitesimal spatial extent are called sinks or sources. Sources and sinks materially affect the mass balance expressed by Eq.(28). Hence their effects would have to be duly incorporated into Eq.(28), as will be done subsequently.

6. Conclusion

In this paper, the physics of the saturated-unsaturated groundwater motion was discussed. The governing equation of saturated-unsaturated flow in porous media was derived from the law of mass conservation and

the Darcy's law. The governing equation was compared with the Klute's diffusion equation which has been widely used in the analysis of flow in unsaturated region. As a result, it is concluded that the governing equation has the advantage that can be applied for the whole flow region, including saturated and unsaturated flow. Typical boundary and initial conditions were enumerated.

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