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# A computative decision of pulse width in three-phase PWM inverter

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# A COMPUTATIVE DECISION OF PULSE WIDTH IN THREE-PHASE PWM INVERTER

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Abstract A new PWM technique is proposed for a three-phase sin-wave PWM inverter. The pulse width is decided to produce a sin-wave output voltage by numerical calculation based on a geometrical technique. The accuracy of output voltage and the harmonics in an output voltage waveform are discussed as compared with the conventional triangulation method. Further, a modified method is introduced for a PWM inverter with fluctuating input voltage. The validity of the proposed method for the PWM inverter is verified in an experiment using a microprocessor-based control system.

## INTRODUCTION

A PWM inverter is available for a variable voltage-variable frequency supply. It is desirable for the supply to eliminate lower-order harmonics from the output voltage waveform. A microprocessor-based control system is widely used for controlling power converters. The computing ability of the microprocessor should be fully utilized when deciding the pulse width in the PWM inverter. Y. H. Kim et al. proposed an algebraic method of pulse width modulation in a single-phase inverter suitable for a microprocessor-based control system [1].

The PWM inverter has generally three-phase loads in industrial applications. Therefore, it is desirable to develop a strategy of pulse width decision for a three-phase PWM inverter which is suitable for a microprocessor-based control system. Furthermore, a dc voltage supply for the PWM inverter is usually obtained by rectifying an ac voltage. Then, the control method of the PWM inverter with fluctuating input voltage is required to be developed in industrial applications [2,3].

In this paper, a new PWM technique is proposed for a three-phase sin-wave PWM inverter. In the proposed technique, the pulse width is computed to produce a sin-wave output voltage. This PWM technique is suitable for the microprocessor implementation because of the computative decision of pulse width. The accuracy of output voltage and the harmonics generated in the output voltage are discussed analytically. Further, a modified method is proposed for a PWM inverter with fluctuating input voltage. The validity of the proposed PWM technique is verified in an experiment using a microprocessor-based control system.

## DECISION OF PULSE WIDTH FOR PWM INVERTER

### PWM Inverter and Output Voltage Command

Fig. 1 shows a simplified model of a three-phase PWM inverter. The PWM inverter comprises a dc supply ($E_d$), six switches and a three-phase load. A pulse width of the PWM inverter is decided by numerical calculation in the proposed PWM technique in this paper. Then, the six switches operate in order to generate a three-phase voltage equal to an output voltage command.

Superimposing the 3rd harmonic to increase an output voltage of PWM inverter [4], the output voltage command of phase-A $v_{AO}^*(t)$ is obtained by

$$v_{AO}^*(t) = V^*\sin\omega_i t + V_3\sin 3\omega_i t \qquad (1)$$

where
$V^*$ amplitude of output voltage command

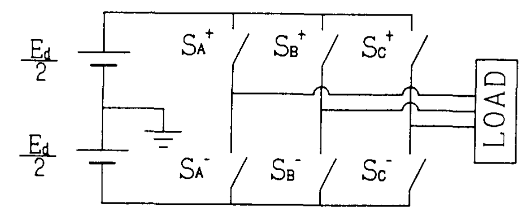


Fig. 1 Simplified model of three-phase PWM inverter

$V_3$ amplitude of the 3rd harmonic ($V_3=V^*/6$)
$\omega_i$ angular output frequency of PWM inverter

### Decision of Pulse Width

An output voltage command in phase-A and a switching behavior of the switches in the corresponding inverter leg are shown in Fig. 2. In this figure, the top and the center show the waveforms of the output voltage command and the dc supply, and the bottom indicates the switching sequence of the switches in the inverter leg. In the proposed PWM technique, the pulse width is numerically computed by using the areas $S_1$ and $S_2$ shown in Fig. 2.

An average value of the output voltage command $V_{AO}^*(k)$ in the k-th interval is calculated by integration in the next equation.

$$V_{AO}^*(k) = \int_{(k-1)\Delta x}^{k\Delta x} v_{AO}(x)dx/\Delta t \qquad (2)$$

where
x $\omega_i t$

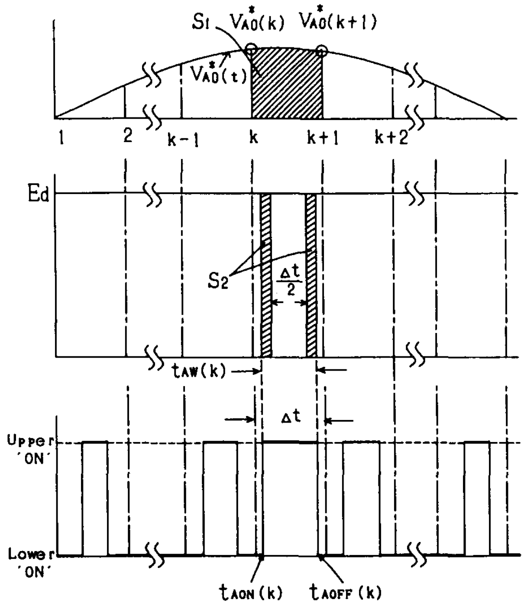


Fig. 2 Output voltage command and switching behavior

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$\Delta x$ $\omega_i \Delta t$
$\Delta t$ $=1/(n_p f_i)$ time of one interval
$n_p$ the number of pulses a cycle
$f_i$ output frequency of PWM inverter

The area $S_1$ is calculated by using (2).

$$S_1 = V_{AO}^*(k)\Delta t \tag{3}$$

Further, the area $S_2$ are given by

$$S_2 = E_d\left\{t_{AW}(k) - \Delta t/2\right\} \tag{4}$$

where
$t_{AW}(k)$ time of on-period of the switch in the upper leg in phase-A

Therefore, $t_{AW}(k)$ is decided by equaling the area $S_1$ to the area $S_2$.

$$V_{AO}^*(k)\Delta t = E_d\left\{t_{AW}(k) - \Delta t/2\right\} \tag{5}$$

Then, we get

$$t_{AW}(k) = \left\{\frac{V_{AO}^*(k)}{E_d} + \frac{1}{2}\right\}\Delta t \tag{6}$$

Therefore, an on- and off-time of the switch $S_A^+$ are computed as follows;

$$t_{AON}(k) = \frac{2k-1}{2}\Delta t - \frac{t_{AW}(k)}{2} \tag{7}$$

$$t_{AOFF}(k) = \frac{2k-1}{2}\Delta t + \frac{t_{AW}(k)}{2} \tag{8}$$

On the other hand, the switch $S_A^-$ behaves by contrast to the switch $S_A^+$. The on-periods in other legs of the PWM inverter are also calculated in the same procedure.

## OUTPUT VOLTAGE AND HARMONICS

### Output Voltage

The output voltage of the proposed PWM inverter is discussed in this section. We get the next expression from (2) and (6).

$$x_{AW}(k) = \omega_i t_{AW}(k)$$
$$= a\{\cos(k-1)\Delta x - \cos k\Delta x\} + \Delta x/2 \tag{9}$$

where
'a' $=2V^*/E_d$ modulation degree

The Fourier series of the output voltage waveform of phase-A is expressed by

$$v_A(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\{a_n \sin nx + b_n \cos nx\} \tag{10}$$

where

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_A(x)\sin nx\,dx \tag{11}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_A(x)\cos nx\,dx$$

And, $v_A(x)$ in the k-th interval is

$$v_A(x) = \frac{E_d}{2} \qquad x_{AON}(k) \le x < x_{AOFF}(k)$$

$$v_A(x) = -\frac{E_d}{2} \qquad \begin{array}{l}(k-1)\Delta x \le x < x_{AON}(k)\\ x_{AOFF}(k) \le x < k\Delta x\end{array} \tag{12}$$

Therefore, the fundamental component of the output voltage is derived as follows;

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} v_A(x)\sin x\,dx$$
$$\simeq \frac{E_d}{2\pi}\sum_{k=1}^{n_p}\left\{\cos k\Delta x - \cos(k-1)\Delta x\right.$$
$$\left. + 4\sin\frac{2k-1}{2}\Delta x \sin\frac{x_{AW}(k)}{2}\right\} \tag{13}$$

The term of $\sin(x_{AW}(k)/2)$ in (13) is expanded in the Taylor series as follows;

$$\sin\frac{x_{AW}(k)}{2} = \sin\frac{\Delta x}{4}$$
$$+ \cos\frac{\Delta x}{4}\cdot\frac{a}{4}\{\cos(k-1)\Delta x - \cos k\Delta x\}$$
$$+ \sin\frac{\Delta x}{4}\cdot\frac{a^2}{4}\{\cos(k-1)\Delta x - \cos k\Delta x\}^2$$
$$+ \cdot\cdot\cdot\cdot\cdot \tag{14}$$

The 3rd and further terms in (14) can be ignored as they are smaller than the first and 2nd terms. Furthermore, from the relations of

$$\sum_{k=1}^{n_p}\{\cos k\Delta x - \cos(k-1)\Delta x\} = 0 \tag{15}$$

and $\cos\alpha=1$ and $\sin\alpha=\alpha$ when $\alpha$ is small, (13) is rewritten in the next equation.

$$a_1 = \frac{aE_d}{\pi}\frac{\Delta x}{2}\sum_{k=1}^{n_p}\sin^2\frac{2k-1}{2}\Delta x$$
$$\simeq \frac{1}{\pi}\int_0^{2\pi} aE_d \sin x \sin x\,dx$$
$$= \frac{aE_d}{2} \tag{16}$$

In the same manner, $b_1$ is derived as

$$b_1 = \frac{1}{2\pi}\int_0^{2\pi} v_A(x)\cos x\,dx$$
$$\simeq \frac{aE_d}{\pi}\cos\frac{\Delta x}{4}\sin\frac{\Delta x}{2}$$
$$\times\sum_{k=1}^{n_p}\cos\frac{2k-1}{2}\Delta x \sin\frac{2k-1}{2}\Delta x$$
$$= 0 \tag{17}$$

Therefore, the r.m.s. value of the line-to-line voltage is given by

$$V_{ab1} = \frac{aE_d}{2}\cdot\frac{\sqrt{3}}{\sqrt{2}} \tag{18}$$

The equations (16), (17) and (18) show that the PWM inverter based on the proposed PWM technique produces the output voltage following the output voltage command.

However, the equations (16), (17) and (18) apply under the assumptions which are introduced in the derivation of these equations. Therefore, there is a deviation between the output voltage of the PWM inverter and the output voltage command in practice. The error is defined by the next expression.

$$\varepsilon_v = \frac{V_0 - \sqrt{3}V^*}{\sqrt{3}V^*} \times 100 \qquad (19)$$

Table 1 shows the errors of output voltage vs. the modulation degree 'a' and the number of pulses $n_p$. The error decreases as the number of pulses increases. If we get the error less than 0.5%, $n_p$ must be selected more than 15. As the modulation degree increases, the error slightly increases in the same number of pulses.

Table 1 Errors of output voltage

| $n_p$ a | 0.2 | 0.4 | 0.6 | 0.7 |
|---|---|---|---|---|
| 3 | -7.8 | -7.9 | -8.0 | -8.1 |
| 6 | -2.0 | -2.0 | -2.1 | -2.1 |
| 9 | -0.9 | -0.9 | -0.9 | -0.9 |
| 12 | -0.5 | -0.5 | -0.5 | -0.5 |
| 15 | -0.3 | -0.3 | -0.3 | -0.3 |
| 18 | -0.2 | -0.2 | -0.2 | -0.2 |
| 21 | -0.2 | -0.2 | -0.2 | -0.2 |
| 24 | -0.1 | -0.1 | -0.1 | -0.1 |
| 27 | -0.1 | -0.1 | -0.1 | -0.1 |

## Harmonics

In the same manner as the preceding section, the harmonics of the output voltage waveform in the proposed PWM inverter are obtained by the next expression.

$$\begin{aligned} a_n &= \frac{1}{\pi}\int_0^{2\pi} v_A(x)\sin nx\,dx \\ &\simeq \frac{E_d}{2\pi}\,\frac{1}{n}\sum_{k=1}^{n_p}\Big\{\cos n\Delta x - \cos n(k-1)\Delta x \\ &\quad + 4\sin n\frac{2k-1}{2}\Delta x\,\sin\frac{n}{2}x_{AW}(k)\Big\} \\ &= \frac{E_d}{\pi}\,\frac{1}{n}\sum_{k=1}^{n_p}\sin n\frac{2k-1}{2}\Delta x\,\sin\frac{n}{2}x_{AW}(k) \end{aligned} \qquad (20)$$

$$\begin{aligned} b_n &= \frac{1}{\pi}\int_0^{2\pi} v_A(x)\cos nx\,dx \\ &\simeq \frac{E_d}{2\pi}\,\frac{1}{n}\sum_{k=1}^{n_p}\Big\{\sin n(k-1)\Delta x - \sin n\Delta x \\ &\quad + 4\cos n\frac{2k-1}{2}\Delta x\,\sin\frac{n}{2}x_{AW}(k)\Big\} \\ &= \frac{E_d}{\pi}\,\frac{2}{n}\sum_{k=1}^{n_p}\cos n\frac{2k-1}{2}\Delta x\,\sin\frac{n}{2}x_{AW}(k) \end{aligned} \qquad (21)$$

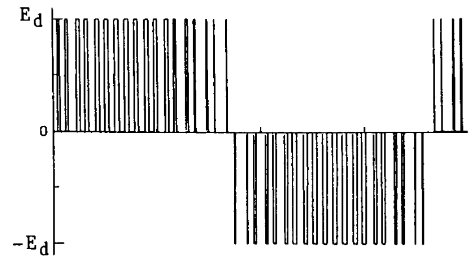
Then, the harmonics of the line-to-line output voltage are calculated by

$$V_{abn} = \begin{cases} \sqrt{a_n^2 + b_n^2}\cdot\sqrt{3/2} & n \neq 3M \\ 0 & n = 3M \end{cases} \qquad (22)$$

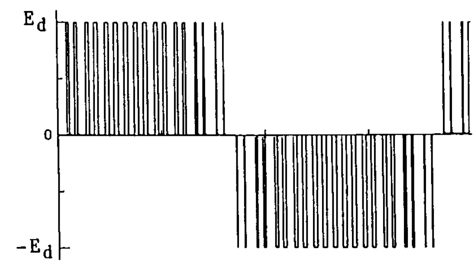
where

M natural number

The number of pulses $n_p$ equals 3N, where N is a natural number.

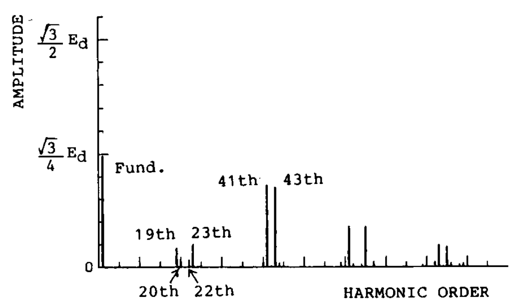


(a) proposed PWM technique

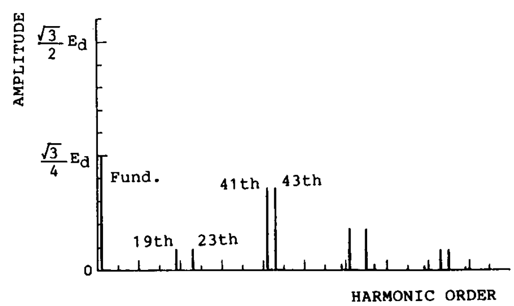


(b) conventional triangulation method

Fig. 3 Waveforms of line-to-line voltage



(a) proposed PWM technique



(b) conventional triangulation method

Fig. 4 Harmonics in line-to-line voltage

Fig. 3 shows the waveforms of the line-to-line voltage in the proposed PWM technique and the conventional triangulation method in which the modulation degree 'a' is 0.5 and the number of pulses $n_p$ is 21. Fig. 4 shows the harmonics of the line-to-line voltage. The odd harmonics exclusive of the triplen harmonics appear in the triangulation method, e.g. the 19th and 23th harmonics. On the other hand, the even harmonics also appear together with the odd harmonics in the proposed PWM inverter. For example, the generating odd harmonics are the 20th and 22th ones. The triplen harmonics do not generate as well as in the triangulation method. This results can be seen from the waveforms of the line-to-line voltage which is asymmetrical in one cycle.

## DECISION OF PULSE WIDTH WITH FLUCTUATING INPUT VOLTAGE

In practical applications, a dc voltage for a PWM inverter is obtained by rectifying an ac supply voltage using a diode bridge. The rectified voltage is a dc one with a fluctuating component. In this Chapter, the method of pulse width decision is described for the PWM inverter with fluctuating input voltage.

Provided that the average value of input voltage is obtained in the interval, the proposed PWM technique mentioned in the foregoing chapter may be applied to the PWM inverter with fluctuating input voltage. Then, the average value of input voltage $V_i(k)$ in the k-th interval is preestimated by

$$V_i(k) = \frac{3v_i(k-1) + v_i(k-2) - 2v_i(k-3)}{2} \quad (23)$$

where

$v_i(k-3)$, $v_i(k-2)$ and $v_i(k-1)$ sampled values of input voltage $v_i(t)$ at points (k-3), (k-2) and (k-1)

Then, the pulse width is computed by replacing $E_d$ with $V_i(k)$ in (6).

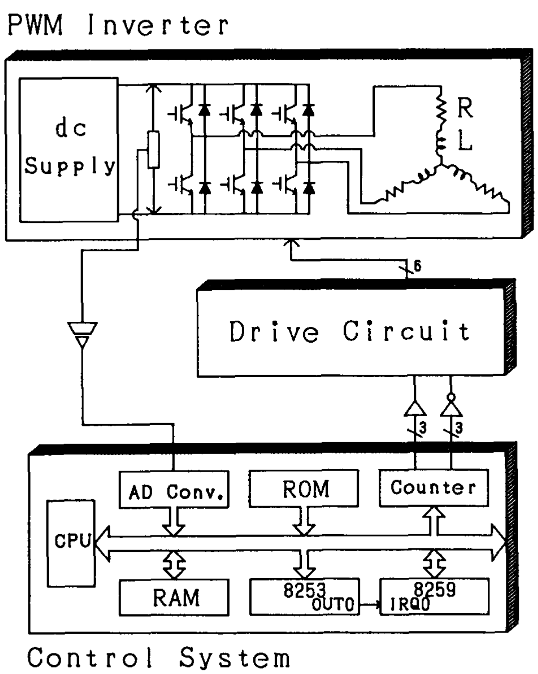
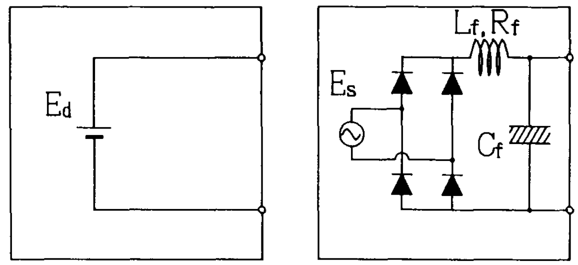



Fig. 5 Diagram of experimental system




(a) dc voltage supply without fluctuation (b) rectified voltage supply

Fig. 6 Dc voltage supply

## EXPERIMENT

### Experimental System

Fig. 5 shows a diagram of an experimental system. The PWM inverter is fed by a dc voltage supply shown in Fig. 6. The PWM inverter has a three-phase inductive load which is a series R-L one. Fig. 6(a) shows the dc voltage supply without fluctuation and Fig. 6(b) shows the rectified voltage supply by using a diode bridge. The switching devices used in the PWM inverter are BIMOSs.

A CPU used in the control system is a 16-bit microprocessor 8086. The input voltage is fed back to the control system through the isolation amplifier. The feedback loop of the input voltage is unnecessary when the PWM inverter is fed by the dc voltage supply without fluctuation. The pulse width is computed by the control system and output to the drive circuit through the 16-bit counter.

### Experimental Results

Fig. 7 shows the waveforms of output voltage and load current in the PWM inverter fed by a dc voltage supply without fluctuation. Table 2 shows the circuit constants and experimental condition. The harmonics of the output voltage waveform are shown in Fig. 8. The lower-order harmonics generating in this experiment are the 13th, 14th, 16th and 17th ones. Further, the other lower-order harmonics generate a little, e.g. the 5th and 7th harmonics. That is because the three-phase load is unbalanced in practice and there is the voltage drop in power semiconductors. Consequently, the experimental results agree well with the concluding remarks obtained in the theoretical analysis.

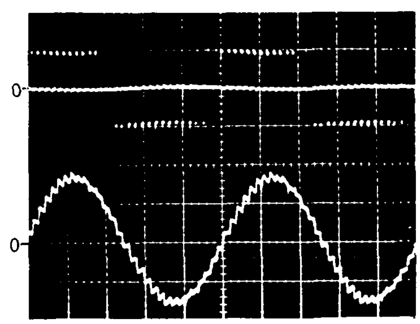



Fig. 7 Waveforms of voltage and current in PWM inverter fed by dc supply without fluctuation
Upper : output voltage 100V/div Lower : Load current 6A/div Horizontal : 5ms/div

Table 2 Circuit constants and experimental condition

| | | | | | |
|---|---|---|---|---|---|
| $E_d$ | 100 | V | $V^*$ | 33.1 | V |
| $f_i$ | 40 | Hz | L | 9 | mH |
| $n_p$ | 15 | | R | 1.4 | Ω |

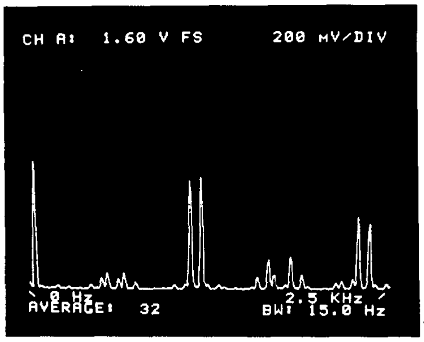
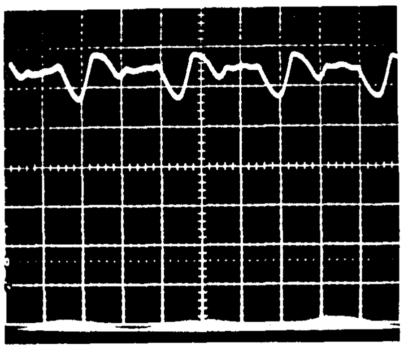
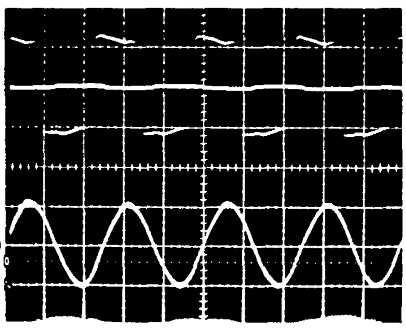



Fig. 8 Harmonics of output voltage in Fig. 7



(a) input voltage 20V/div
Horizontal : 10ms/div



(b) output voltage and load current
Upper : output voltage 108V/div Lower : load current 12A/div Horizontal : 10ms/div

Fig. 9 Waveforms of voltage and current in PWM inverter fed by rectified voltage supply

Table 3 Circuit constants and experimental condition

| | | | | | |
|---|---|---|---|---|---|
| $E_s$ | 100 | V | $f_i$ | 40 | Hz |
| $f_s$ | 60 | Hz | $n_p$ | 39 | |
| $L_f$ | 3.5 | mH | $V^*$ | 38.3 | V |
| $R_f$ | 0.08 | Ω | L | 9 | mH |
| $C_f$ | 1000 | μF | R | 1.4 | Ω |

Fig. 9 shows the waveforms of input voltage, output voltage and load current in the PWM inverter fed by the rectified voltage supply. The harmonics in the output voltage waveform are shown in Fig. 10. The circuit constants and experimental condition are shown in Table 3. As shown in Fig. 9(a), the input voltage, capacitor voltage, is a dc voltage with a fluctuating component due to the behavior of the rectifier and the PWM inverter. The influence of fluctuating input voltage appears in the amplitude of output voltage waveform in Fig. 9(b). However, the lower-order harmonics due to fluctuation in the input voltage waveform do not appear in the output waveform of PWM inverter. Namely, the lower-order harmonics due to fluctuation in the input voltage cannot be found out in Fig. 10. It is understood from the results that the proposed PWM technique is also valuable for the PWM inverter with fluctuating input voltage.

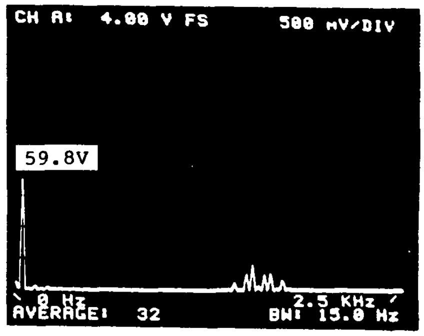



Fig. 10 Harmonics of output voltage in Fig. 9

## CONCLUSIONS

The new method of pulse width decision is proposed for a three-phase sin-wave PWM inverter. The validity of the proposed PWM technique is theoretically made clear with respect to the output voltage. It is found that a sufficient accuracy is obtained from the analytical results of the errors in the output voltage vs. the number of pulses. Furthermore, the method of pulse width decision with fluctuating input voltage is proposed. Finally, the proposed PWM technique is proved effective by the experiment using the microprocessor-based control system.

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