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3-D Time-Periodic Finite Element Analysis of Magnetic Field in Non-Oriented Materials Taking into Account Hysteresis Characteristics

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***Abstract* - Problems in analyzing 3-D stationary nonlinear magnetic field in non-oriented material taking into account hysteresis characteristics and eddy current, such as finite element formulation and convergence of nonlinear iteration, are discussed.**

I. INTRODUCTION

In order to calculate the exact waveform of flux in the electric machines, such as a transformer, 3-D analysis taking into account hysteresis characteristics and eddy current should be carried out. The method of analysis of magnetic field taking into account hysteresis characteristics in anisotropic materials, in which the direction of magnetization $\boldsymbol{M}$ is given, has been already proposed[1]. However, many problems occur in the analysis of stationary nonlinear magnetic field in non-oriented material using the time-periodic finite element method[2,3], because the direction of $\boldsymbol{M}$ and hysteresis loops used are changed during the nonlinear iterations and time stepping.

In this paper, the finite element formulation for analyzing 3-D nonlinear magnetic field in non-oriented material taking into account hysteresis characteristics and eddy current is described. The technique how to determine the position of flux density vector on hysteresis loop, the convergence characteristics, etc. are investigated. The flux waveform obtained using the proposed method is compared with that using an initial B-H curve.

II. METHOD OF ANALYSIS

A. Formulation

In the nonlinear magnetic field analysis using the initial B-H curve, the magnetic vector potential $\boldsymbol{A}$ and the reluctivity ν are usually treated as unknown variables. On the other hand, in the case when the hysteresis characteristics are taken into account, the magnetization $\boldsymbol{M}$ should be used instead of ν due to the discontinuity of ν at the point where the flux density $\boldsymbol{B}$ is equal to zero[1]. The fundamental equation is given by[1]:

$$\nu_0\{\mathrm{rot}(\mathrm{rot}\boldsymbol{A}-\boldsymbol{M})\}=\boldsymbol{J_o}-\sigma\frac{\partial \mathbf{A}}{\partial t} \tag{1}$$

where $\boldsymbol{J_o}$ is the current density vector in the magnetizing winding. ν_0 and σ are the reluctivity in vacuum and the conductivity, respectively.

In the finite element method, the residual G_i is represented as follows:

$$G_i=\iiint_V \mathrm{rot}\boldsymbol{N_i}\bullet\nu_0(\mathrm{rot}\boldsymbol{A}-\boldsymbol{M})dV \\ -\iiint_{Vc}\boldsymbol{N_i}\bullet\boldsymbol{J_o}dV+\iiint_{Ve}\boldsymbol{N_i}\sigma\frac{\partial \boldsymbol{A}}{\partial t}dV \tag{2}$$

where V, Vc and Ve denote the whole region, winding region and eddy current region, respectively. $\boldsymbol{N_i}$ is the interpolation function.

In the nonlinear analysis using the Newton-Raphson iteration technique, the increments δA_j of the unknown variables is obtained from the following equation:

$$\left[\frac{\partial G_i}{\partial A_j}\right]\{\delta A_j\}=\{-G_i\} \tag{3}$$

$[\partial G_i/\partial A_j]$ in (3) is the same as that of the conventional finite element method except the term relating to the magnetization $\boldsymbol{M}$. The term $\partial Mx/\partial A_j$, for example, is represented as follows:

$$\frac{\partial Mx}{\partial A_j}=\frac{\partial Mx}{\partial|\boldsymbol{B}|}\bullet\frac{\partial|\boldsymbol{B}|}{\partial A_j} \tag{4}$$

where $\partial Mx/\partial|\boldsymbol{B}|$ cannot be directly obtained from M-B loop because the direction of $\boldsymbol{M}$ is not given in M-B loop for non-oriented material. In this paper, $\partial Mx/\partial|\boldsymbol{B}|$ is determined as follows under the assumption that the direction of $\boldsymbol{M}$ is the same as that of $\boldsymbol{B}$:

$$\frac{\partial Mx}{\partial|\boldsymbol{B}|}=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\frac{\partial Mx}{\partial|\boldsymbol{M}|}=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\frac{\partial|\boldsymbol{M}|\cos\theta}{\partial|\boldsymbol{M}|} \\ =\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\cos\theta=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\frac{Bx}{|\boldsymbol{B}|} \tag{5}$$

where θ is the angle between the $\boldsymbol{M}$ vector and the x-axis. Bx is obtained directly from the FEM calculation.

The time-periodic finite element method[3] is used for analyzing 3-D stationary nonlinear magnetic field. The flowchart is shown in Fig.1. Using this method, the time-periodic waveform can be obtained directly without transient calculation.

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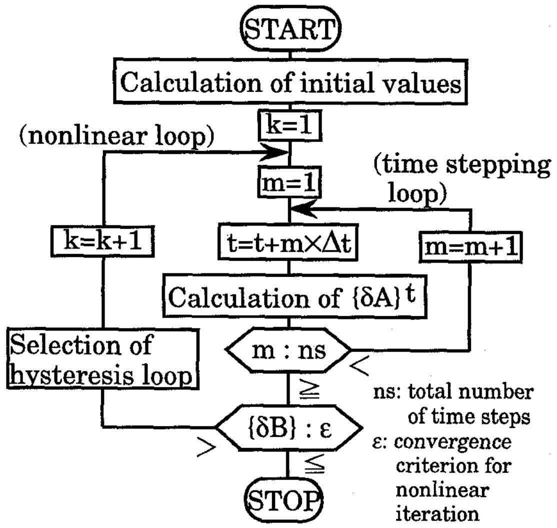


Fig.1 Flowchart of time-periodic finite element method taking into account hysteresis characteristics.

The relaxation factor[4] is introduced to improve the convergence characteristics of Newton-Raphson method. The 1-st order brick edge element is used in the 3-D analysis[5,6].

B. Representation of Hysteresis Loop

Typical dc hysteresis loops, which are measured, are stored in a computer as shown in Fig.2. A half region of loops is stored due to symmetry. The hysteresis loop used is interpolated by the loops stored as follows:

(1) The hysteresis loop "2" having the maximum flux density Bm2 shown in Fig.3 is interpolated from two nearest typical loops "1" and "3" satisfying the following relationship:

$$\frac{\overline{P'P}}{\overline{PP''}} = \frac{\overline{P_1P_2}}{\overline{P_2P_3}} \qquad (6)$$

(2) If the required hysteresis loop is smaller than the smallest loop which is stored, the required loop is obtained by scaling down the smallest one similarly.

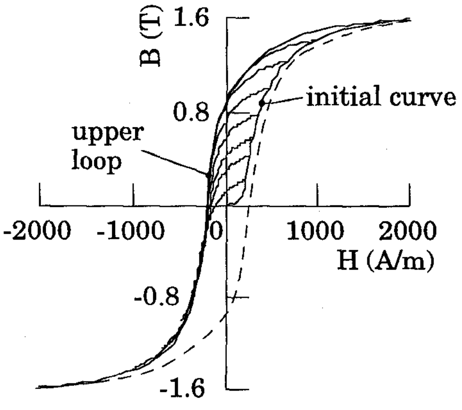


Fig.2 Data for hysteresis loop.

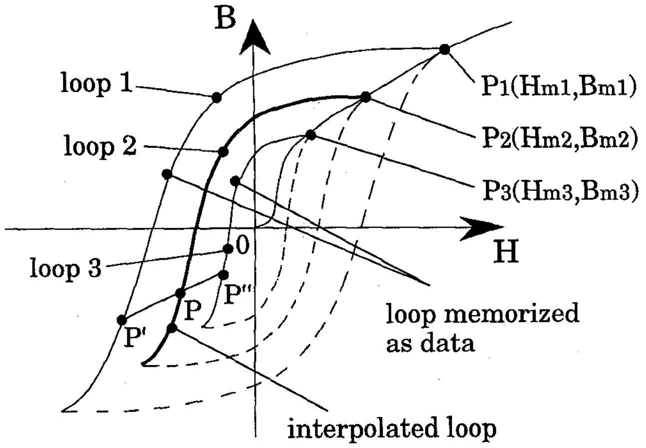


Fig.3 Interpolated loop.

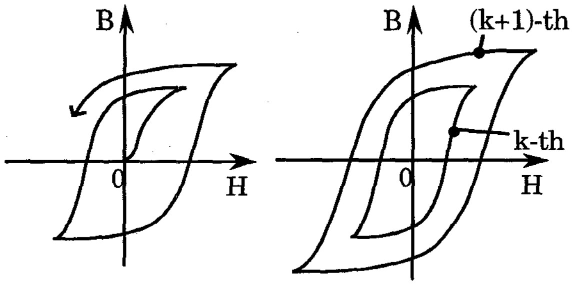
(3) If the required flux density is larger than that of the largest loop, the initial loop is added for high flux density region.

C. Selection of Hysteresis Loop

When the stationary nonlinear magnetic field is calculated using the step-by-step method[7], the position of the flux density on hysteresis loop is varied continuously as shown in Fig.4(a). On the other hand, in the time-periodic finite element method, the hysteresis loop used is changed in each element at each nonlinear iteration as shown in Fig.4(b). In the case of the time-periodic analysis, the hysteresis loop which is used for obtaining $\boldsymbol{M}$ and $\partial Mx/\partial|\boldsymbol{B}|$ for (k+1)-th nonlinear iteration is selected using the maximum flux density obtained at k-th nonlinear iteration as shown in Fig.5.

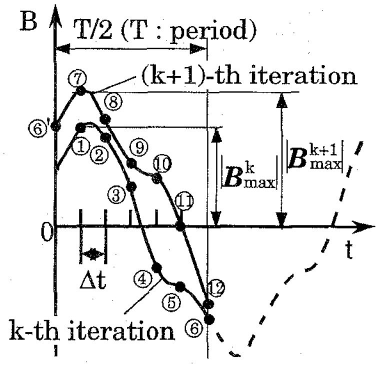
D. Position of Flux Density on Hysteresis Loop

When the flux density $\boldsymbol{B}_t{}^k$ at the instant t of the k-th nonlinear iteration is obtained, the position of the flux density $\boldsymbol{B}_t{}^k$ on hysteresis loop cannot be determined uniquely (point α or β) from only the absolute value $|\boldsymbol{B}_t{}^k|$ as shown in Fig.6(a). Then, the following methods for determining the position of $\boldsymbol{B}_t{}^k$ are proposed:

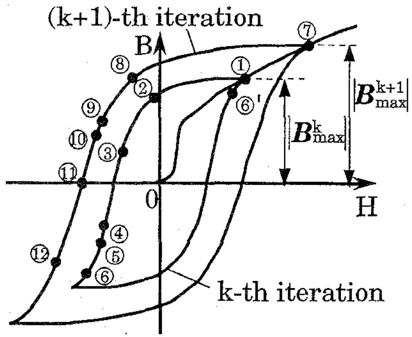


(a) step-by-step method
(b) time-periodic finite element method

Fig.4 Hysteresis loop.

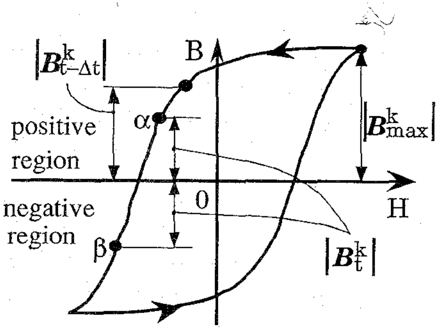


(a) waveform of flux density

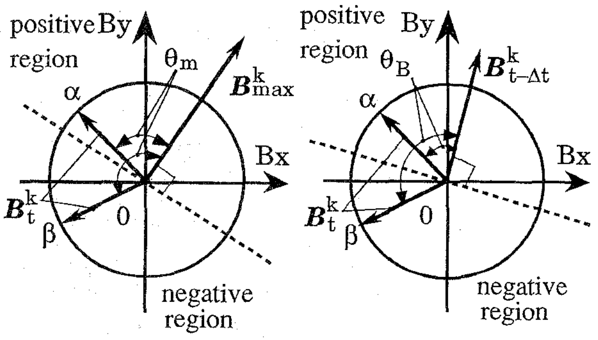


(b) positions on hysteresis loop at each step

Fig.5 Selection of hysteresis loop.



(a) Position of flux density



(b) method 1 (θm) (c) method 2 (θ_B)

Fig.6 Position of flux density on hysteresis loop.

Method 1: If the angle θm between $B_{max}{}^k$ and $B_t{}^k$ is less than 90°, the position of $B_t{}^k$ is regarded as α in the positive region as shown in Fig.6(b). If θm is larger than 90°, $B_t{}^k$ is β in the negative region.

Method 2: If the angle θ_B between $B_{t\text{-}\Delta t}{}^k$ and $B_t{}^k$ is less than 90°, the position of $B_t{}^k$ is regarded as α in the positive region as shown in Fig.6(c). If θ_B is larger than 90°, $B_t{}^k$ is β in the negative region.

Method 3: The following condition is added to Method 2 so that the position of flux density can be moved from the positive region to the negative region (vice versa) even if there is some numerical error: When $|B_t{}^k|$ is larger than $|B_{t\text{-}\Delta t}{}^k|$, the position is moved to the negative region.

E. Initial Value

The convergence characteristics of nonlinear iteration is affected by initial values. The following initial values are investigated in Section III.

Case 1: The magnetic vector potential A is set to zero.

Case 2: The result of magnetostatic analysis using the initial curve is used.

Case 3: The result of ac steady state analysis (time-periodic FEM) using the initial curve is used.

III. RESULTS AND DISCUSSION

A. Analyzed Model and Flux Density Waveforms

Fig.7 shows the analyzed model. The core, of which the hysteresis loop is shown in Fig.2, is laminated infinitely in the z-direction. The current density in the coil is 3.1×10^4A/m^2 (ac, 50Hz). The analyzed region is subdivided into the 1-st order brick edge elements. The number of elements is 400. Half a period of the waveform is divided into 6 steps.

The waveforms of the average flux densities in the core obtained from the calculations using the initial curve and the hysteresis loop are shown in Fig.8. The effect of hysteresis characteristics is remarkable in this model.

B. Convergence Characteristics

(1) Position of flux density

The effect of the methods for determining the position of flux density on the hysteresis loop,

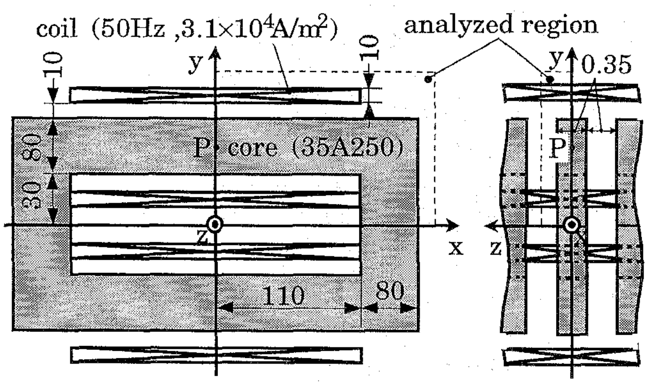


Fig.7 Analyzed model.

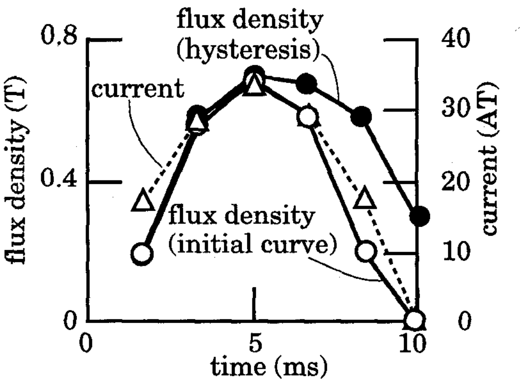


Fig.8 Waveform of flux density.

which are denoted in Section II D, on the convergence characteristics is shown in Fig.9 and Table I. Fig.8 shows the position of the flux density at the point P shown in Fig.7 on the hysteresis loop when the current is the maximum. The variation of the position in the case of the method 2 is suprious, because the positive and negative regions cannot be evaluated correctly due to a numerical error near the $\boldsymbol{B}=\boldsymbol{0}$ region. Therefore, the method 3 is preferable because the direction of flux density $\boldsymbol{B}_t^k$ should be estimated by comparing with $\boldsymbol{B}_{t-\Delta t}^k$ at previous step.

(2) Initial value

The effect of initial values, which are denoted in Section II E, on the convergence characteristics is shown in Table II. The convergence is considerably improved by using the obtained values by static analysis and so on as initial values as shown in the cases 2 and 3. As the total CPU time for the case 3 is large and the convergence characteristics of cases 2 and 3 are nearly the same, the case 2 is preferable.

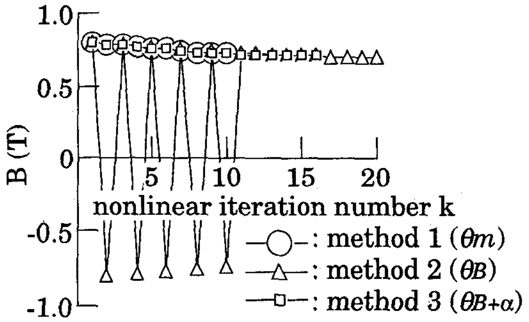


Fig.9 Position of flux density when current is the maximum.

Table I Effect of methods for determining position of flux density

method	method 1 (θm)	method 2 (θB)	method 3 ($\theta B+\alpha$)
total CPU time(s)	68 (10)	149 (20)	120 (16)

( ) : number of nonlinear iterations
Convergence criterion for Newton-Raphson method : 0.01T
Workstation: IBM3AT (49.7MFLOPS)

Table II Effect of initial values

case	case 1 ($\boldsymbol{A}=\boldsymbol{0}$)	case 2 (static analysis)	case 3 (ac steady state analysis)
CPU time for initial value (s)	—	8 (12)	89 (13)
CPU time for hysteresis (s)	120 (16)	65 (10)	69 (10)
total CPU time (s)	120	73	158

( ) : number of nonlinear iterations
Convergence criterion for Newton-Raphson Method : 0.01T
Workstation: IBM3AT (49.7MFLOPS)

IV. CONCLUSIONS

The results obtained are summarized as follows:

(1) The formulation for the ac steady state analysis of magnetic field, taking into account hysteresis characteristics in non-oriented material, is shown.
(2) The technique for determining the position of flux density on hysteresis loop of non-oriented material under time-periodic condition is investigated.
(3) The results of magnetostatic analysis should be used for initial value of nonlinear iteration from the viewpoint of CPU time.

The method proposed in this paper should be expanded to the analysis taking into account hysteresis loops of anisotropic materials. The analysis of iron loss taking into account eddy current loss and hysteresis loss should also be investigated.

REFERENCES

[1] T.Nakata, N.Takahashi and Y.Kawase, "Finite element analysis of magnetic fields taking into account hysteresis characteristics," *IEEE Trans. on Magn.*, vol.21. no.5, pp.1856-1858, September 1985.
[2] T.Hara, T.Naito and J.Umoto, "Field analysis of corona shield region in high voltage rotating machines by time-periodic finite element method," *Trans. of IEE of Japan*, vol. 102-B, no. 7, pp. 423-430, July 1982.
[3] T.Nakata, N.Takahashi, K.Fujiwara, K.Muramatsu, H.Ohashi and H.L.Zhu, "Practical analysis of 3-D dynamic nonlinear magnetic field using time-periodic finite element method," *IEEE Trans. on Magn.*, vol.31, no.3, pp.1416-1419, May 1995.
[4] K.Fujiwara, T.Nakata, N.Takahashi, N.Okamoto and K.Muramatsu, "Method for Determining Relaxation Factor for Modified Newton-Raphson Method," *ibid.*, vol.29, no.2, pp. 1962-1965, March 1993.
[5] A.Kameari, "Calculation of transient 3D eddy current using edge-elements," ibid., vol.26, no.2, pp.466-469, March 1990.
[6] K.Fujiwara, "3-D magnetic field computation using edge element (invited)," *Proceedings of the 5th IGTE Symposium on Numerical Field Calculation in Electrical Engineering*, pp.185-212, September 1992.
[7] T.Nakata and Y.Kawase, "Numerical analysis of nonlinear transient magnetic field using the finite element method", *Trans. of IEE of Japan*, vol.104-B, no.6, pp.380-386, June 1984.