Attitude control system design of a helicopter experimental system

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***Abstract*-In this paper, we consider the problem of attitude control of a helicopter experimental system. We design a nonlinear controller which combines nonlinear adaptive robust control and nonlinear feedback controls. Simulation and experimental results show the effectiveness of the proposed method.**

I. Introduction

Attitude control for helicopters is an important control topic in nonlinear feedback design, due to the nonlinearity of the dynamics and strong interactions between variables. Concerning with this topic, many works have been developed (for example, [1],etc.). Recently, we gave a combined adaptive and non-adaptive attitude control method [2] based on adaptive sliding mode control method [4], [5] and some conventional control methods [3] for our helicopter experimental system. In [2], it is assumed that the structure of the uncertainty was known but the parameters were unknown. In this paper, we extend the result in [2] to more general case. The detailed explanation is shown as follows. In this paper, we give an MIMO nonlinear controller design method, where one part of uncertainty is in known structure with unknown parameter and another part is assumed that the structure is unknown with known upper-bound. Further, a nonlinear controller is added for controlling the known nonlinear dynamics obtained by estimation from experiment by using the result in [6]. As a result, the proposed controller is a combined controller, namely, combined nonlinear adaptive and nonlinear attitude controller for two kind of uncertainties based system parameters estimation. Further, the robust stability of the proposed control is also ensured. Finally, some numerical simulations and some experimental results are given to show the effectiveness of the proposed scheme by our 2 degrees of freedom nonlinear helicopter experimental system.

II. Experimental system modelling and parameters estimation

In research of the attitude control of a helicopter, 2 degree-of-freedom helicopter is used in many studies. Also in this research, proposed control scheme is verified by using 2 degree-of-freedom helicopter.

The experimental system is 2 input 2 output which attaches the motor for turning a main rotor and a tail rotor, and detects a pitch angle and a yaw angle with a rotary encoder. The equation of motion of 2 degree-of-freedom helicopter is shown as follows.

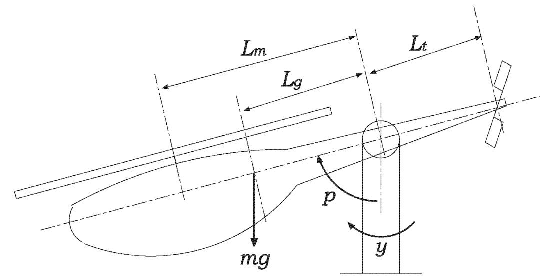

Fig. 1. 2 degree-of-freedom helicopter

(The direction of the pitch angle)

$$I_p\ddot{p} + D_p\dot{p} + mgL_g\sin p + h_tA_tL_t\omega_t^2 = A_mL_m\omega_m^2 \quad (1)$$

(The direction of the yaw angle)

$$I_y\ddot{y} + D_y\dot{y} + h_mA_mL_m\omega_m^2\sin p = A_tL_t\omega_t^2\sin p \quad (2)$$

where,

m : Weight of the system
g : Gravity acceleration
p : Pitch angle
y : Yaw angle
I_p : Moment of inertia(The direction of the pitch angle)
I_y : Moment of inertia(The direction of the yaw angle)
D_p : Coefficient of friction(The direction of the pitch angle)
D_y : Coefficient of friction(The direction of the yaw angle)
A_m : The multiplier by the configuration of the main rotor
A_t : The multiplier by the configuration of the tail rotor
h_m : The multiplier about the lateral force of the main rotor
h_t : The multiplier about the lateral force of the tail rotor
L_m : Distance from an axis to the main rotor
L_g : Distance from an axis to the center of gravity
L_t : Distance from an axis to the tail rotor
ω_m : The angular speed of the main rotor
ω_t : The angular speed of the tail rotor

m, g, I, D, A, h are constants and p, y, ω are variables. The direction of p and y are the directions of the arrows shown in Fig. 1.

Since the equation of motion is differential equation of second order for parameters estimation. Measurement of displacement, velocity and acceleration are required. In this research, the identification approach without an angular acceleration signal [6] is used. First, it is assumed that

$$\begin{pmatrix} \omega_m \\ \omega_t \end{pmatrix} = \begin{pmatrix} k_m & 0 \\ 0 & k_t \end{pmatrix} \begin{pmatrix} u_m \\ u_t \end{pmatrix} \quad (3)$$

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where, k_m and k_t are constants.

Then equations are

$$a_1\ddot{p} + a_2\dot{p} + a_3\sin p + a_4 u_2 = u_1 \quad (4)$$
$$b_1\ddot{y} + b_2\dot{y} + b_3 u_1 \sin p = u_2 \sin p \quad (5)$$

and

$$\frac{d}{dt}(a_1\dot{p}) = a_1\ddot{p}, \quad \frac{d}{dt}(b_1\dot{y}) = b_1\ddot{y} \quad (6)$$

are used in order to erase the angular acceleration($\ddot{p}$) of equation (4), and the angular acceleration($\ddot{y}$) of equation (5) (See Appendix A). Using a constant $\mu > 0$ equation (4) is rewritten as

$$\left(\frac{d}{dt} + \mu\right)(a_1\dot{p}) - \mu a_1\dot{p} + a_2\dot{p} + a_3\sin p + a_4 u_2 - u_1 = 0 \quad (7)$$

Using variables

$$\begin{aligned} z^0 &= u_1 \\ z^1 &= \dot{p} \\ z^2 &= \sin p \\ z^3 &= u_2 \end{aligned} \quad (8)$$

and filter variable ψ defined by

$$\left(\frac{d}{dt} + \mu\right)\psi^j = z^j \quad (9)$$

then equation (7) becomes

$$\left(\frac{d}{dt} + \mu\right)\left[a_1\dot{p} + (a_2 - \mu a_1)\psi^1 + a_3\psi^2 + a_4\psi^3 - \psi^0\right] = 0 \quad (10)$$

If this differential equation is solved, it will be set to

$$a_1\dot{p} + (a_2 - \mu a_1)\psi^1 + a_3\psi^2 + a_4\psi^3 = \psi^0 + Ce^{-\mu t} \quad (11)$$

C is a constant and for large t ($\mu > 0$), it can be assumed as $Ce^{-\mu t} \approx 0$. Equation (11) is rewritten as

$$a_1(\dot{p} - \mu\psi^1) + a_2\psi^1 + a_3\psi^2 + a_4\psi^3 = \psi^0 \quad (12)$$

Also in the similar way, equation (5) becomes

$$b_1(\dot{y} - \mu\psi^4) + b_2\psi^4 + b_3\psi^5 = \psi^6 \quad (13)$$

where

$$\begin{aligned} z^4 &= \dot{y} \\ z^5 &= u_1\sin p \\ z^6 &= u_2\sin p \end{aligned} \quad (14)$$

Using

$$\begin{aligned} \boldsymbol{\Phi}_2 &= \begin{pmatrix} \dot{p} - \mu\psi^1 & \psi^1 & \psi^2 & \psi^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot{y} - \mu\psi^4 & \psi^4 & \psi^5 \end{pmatrix} \\ \boldsymbol{a} &= \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & b_3 \end{pmatrix}^T \end{aligned} \quad (15)$$

equations (12) and (13) are given as

$$\boldsymbol{\Phi}_2(t)\boldsymbol{a} = \begin{pmatrix} \psi^0 \\ \psi^6 \end{pmatrix} \quad (16)$$

This model is linear, does not contain acceleration and is used to identify unknown parameters in $\boldsymbol{a}$.

III. Problem formulization

If $\bar{\boldsymbol{c}}$, $\bar{\boldsymbol{d}}$ are considered to be identified value and $\Delta\boldsymbol{c}$, $\Delta\boldsymbol{d}$ are considered to be identification error, the values $\boldsymbol{c}$ and $\boldsymbol{d}$ can be decomposed with

$$\begin{aligned} \boldsymbol{c} &= \bar{\boldsymbol{c}} + \Delta\boldsymbol{c} \\ \boldsymbol{d} &= \bar{\boldsymbol{d}} + \Delta\boldsymbol{d} \end{aligned} \quad (17)$$

where, it is referred to as $\bar{\boldsymbol{f}}(\boldsymbol{x}(t), t)$, $\bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)$ when the parameter of $\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\boldsymbol{g}(\boldsymbol{x}(t), t)$ are $\bar{\boldsymbol{c}}$, $\bar{\boldsymbol{d}}$. And, it referred to as $\Delta\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\Delta\boldsymbol{g}(\boldsymbol{x}(t), t)$ when the parameters of $\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\boldsymbol{g}(\boldsymbol{x}(t), t)$ are $\Delta\boldsymbol{c}$, $\Delta\boldsymbol{d}$ (See Appendix B). Then, $\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\boldsymbol{g}(\boldsymbol{x}(t), t)$ are

$$\begin{aligned} \boldsymbol{f}(\boldsymbol{x}(t), t) &= \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \Delta\boldsymbol{f}(\boldsymbol{x}(t), t) \\ \boldsymbol{g}(\boldsymbol{x}(t), t) &= \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t) + \Delta\boldsymbol{g}(\boldsymbol{x}(t), t) \end{aligned} \quad (18)$$

Substituting these variables into equation (63), then

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = \; & \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \Delta\boldsymbol{f}(\boldsymbol{x}(t), t) \\ & + \Delta\boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \end{aligned} \quad (19)$$

In order for $\Delta\boldsymbol{f}(\boldsymbol{x}(t), t) + \Delta\boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t)$ to fulfill matching conditions for sliding mode control, it is necessary to calculate $\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$ defined by

$$\bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) = \Delta\boldsymbol{f}(\boldsymbol{x}(t), t) + \Delta\boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \quad (20)$$

then

$$\dot{\boldsymbol{x}}(t) = \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \quad (21)$$

from equation (19). $\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$ is called the uncertainty of a definite part, and consists of known model configuration and unknown parameters [2]. In this paper, control law is designed by using

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = \; & \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \\ + \; & \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\Delta\boldsymbol{h}(\boldsymbol{x}(t), t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \end{aligned} \quad (22)$$

Moreover, $\Delta\boldsymbol{h}(\boldsymbol{x}(t), t)$ is called uncertainty of an indefinite part and is a unknown model. However, it is referred to as

$$\left\|\Delta\boldsymbol{h}(\boldsymbol{x}(t), t)\right\| \leq \eta(\boldsymbol{x}(t), t) \quad (23)$$

and upper-bound value function $\eta(\boldsymbol{x}(t), t)$ is considers as bounded and known.

IV. Attitude controller design

In this section, the proposed controller is shown. The controller has six parts. Further, the robust stability of the control system with the proposed controller is ensured.

A. The structure of the proposed controller

Set point vector

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ r_2 \\ 0 \\ 0 \end{pmatrix} \tag{24}$$

where, r_1 is the set point of x_1, and r_2 is the set point of x_2. A switching surface is designed as

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \boldsymbol{S}\Big(\boldsymbol{x}(t) - \boldsymbol{y}_r(t)\Big) \tag{25}$$

$$\boldsymbol{S} = \begin{pmatrix} \iota_1 & 0 & 1 & 0 \\ 0 & \iota_2 & 0 & 1 \end{pmatrix} \tag{26}$$

where $\boldsymbol{y}_r(t)$ is taken as first order filter output of a step response.

$$\boldsymbol{y}_r(t) = \begin{pmatrix} y_{r1} \\ y_{r2} \\ y_{r3} \\ y_{r4} \end{pmatrix} \tag{27}$$

$$\dot{\boldsymbol{y}}_r(t) = \begin{pmatrix} \frac{1}{T_1} & 0 & 0 & 0 \\ 0 & \frac{1}{T_2} & 0 & 0 \\ 0 & 0 & \frac{1}{T_3} & 0 \\ 0 & 0 & 0 & \frac{1}{T_4} \end{pmatrix} \Big(\boldsymbol{r} - \boldsymbol{y}_r(t)\Big) \tag{28}$$

The proposed controller includes the equivalent controller, linear feedback controller, relay controller, nonlinear controller for the uncertainty of a definite part, nonlinear controller for the uncertainty of an indefinite part and a controller for a set point variation. That is,

$$\boldsymbol{u}(t) = \boldsymbol{u}_{eq}(t) + \boldsymbol{u}_{lf}(t) + \boldsymbol{u}_{nl}(t) + \boldsymbol{u}_{ad}(t) + \boldsymbol{u}_{\eta}(t) + \boldsymbol{u}_r(t) \tag{29}$$

The detailed structure of these controllers are shown as follows.

The equivalent controller

The equivalent controller is expressed by the following equation.

$$\begin{aligned} \boldsymbol{u}_{eq}(t) &= \begin{pmatrix} u_{eq1} \\ u_{eq2} \end{pmatrix} \\ &= -\Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t), t\big)\Big)^{-1} \boldsymbol{S}\bar{\boldsymbol{f}}\big(\boldsymbol{x}(t), t\big) \\ &= \frac{1}{\bar{c}_4\bar{d}_3 - \bar{c}_1\bar{d}_1} \begin{pmatrix} \bar{d}_1 & \frac{\bar{c}_4 x_4}{\sin x_1} \\ \bar{d}_3 & \frac{\bar{c}_1 x_4}{\sin x_1} \end{pmatrix} \\ &\quad \begin{pmatrix} \iota_1 x_3 - \bar{c}_2 x_3 - \bar{c}_3 \sin x_1 \\ \iota_2 - \bar{d}_2 \end{pmatrix} \end{aligned} \tag{30}$$

Linear feedback controller

Linear feedback controller is expressed by the following equation.

$$\begin{aligned} \boldsymbol{u}_{lf}(t) &= \begin{pmatrix} u_{lf1} \\ u_{lf2} \end{pmatrix} \\ &= -\Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t), t\big)\Big)^{-1} \boldsymbol{K}\boldsymbol{\sigma} \\ &= \frac{1}{\bar{c}_4\bar{d}_3 - \bar{c}_1\bar{d}_1} \begin{pmatrix} \bar{d}_1 & \frac{\bar{c}_4}{\sin x_1} \\ \bar{d}_3 & \frac{\bar{c}_1}{\sin x_1} \end{pmatrix} \\ &\quad \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \end{aligned} \tag{31}$$

Here, constant matrix $\boldsymbol{K}$ is taken as a positive definite.

Relay controller

Relay controller is expressed by the following equation.

$$\begin{aligned} \boldsymbol{u}_{nl}(t) &= \begin{pmatrix} u_{nl1} \\ u_{nl2} \end{pmatrix} \\ &= -\Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t), t\big)\Big)^{-1} \boldsymbol{\kappa}\ \boldsymbol{\Lambda}(\boldsymbol{\sigma}) \end{aligned} \tag{32}$$

$$\boldsymbol{\Lambda}(\boldsymbol{\sigma}) = \begin{pmatrix} \text{sgn } (\sigma_1) \\ \text{sgn } (\sigma_2) \end{pmatrix} \tag{33}$$

$$\text{sgn } (x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases} \tag{34}$$

It is referred to as $\kappa > 0$.

Controller for the uncertainty of a definite part

The control input for compensating the right hand side 2nd term of the equation (22) which is the term of a presumed error is considered. It is referred to as

$$\boldsymbol{h}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big) = \boldsymbol{\Phi}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big)\boldsymbol{\theta} \tag{35}$$

equivalent controller. Where $\boldsymbol{\Phi}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big)$ is 2×7 matrix, and

$$\begin{aligned} \boldsymbol{\theta} &= \begin{pmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 \end{pmatrix}^T \\ &= \begin{pmatrix} \Delta\boldsymbol{c} \\ \Delta\boldsymbol{d} \end{pmatrix} \end{aligned} \tag{36}$$

Because $\boldsymbol{\theta}$ is unknown, it is calculated by on-line identification. If the estimate of $\boldsymbol{\theta}$ is set to

$$\hat{\boldsymbol{\theta}}(t) = \begin{pmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_3 & \hat{\theta}_4 & \hat{\theta}_5 & \hat{\theta}_6 & \hat{\theta}_7 \end{pmatrix}^T \tag{37}$$

$\hat{\boldsymbol{\theta}}(t)$ will be performed by

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}(t) &= \boldsymbol{\Gamma\Phi}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big)^T \Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t), t\big)\Big)^T \boldsymbol{\sigma} \\ &= \boldsymbol{\Gamma}\Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t), t\big)\boldsymbol{\Phi}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big)\Big)^T \boldsymbol{\sigma} \end{aligned} \tag{38}$$

where, $\boldsymbol{\Gamma}$ is a positive definite symmetrical matrix, and it is introduced in order to adjust the rate of identification. And the control input to the term of a presumed error is calculated by

$$\begin{aligned} \boldsymbol{u}_{ad}(t) &= \begin{pmatrix} u_{ad1} \\ u_{ad2} \end{pmatrix} = -\hat{\boldsymbol{h}}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big) \\ &= -\boldsymbol{\Phi}\big(\boldsymbol{x}(t), \boldsymbol{u}(t), t\big)\hat{\boldsymbol{\theta}}(t) \end{aligned} \tag{39}$$

Controller for the uncertainty of an indefinite part
The control input to a unknown model design with

$$\begin{aligned} \boldsymbol{u}_\eta(t) &= \begin{pmatrix} u_{\eta 1} \\ u_{\eta 2} \end{pmatrix} \\ &= -\eta\big(\boldsymbol{x}(t),t\big)\boldsymbol{\chi}\big(\boldsymbol{x}(t),t\big) \end{aligned} \tag{40}$$

where upper-bound value function $\eta\big(\boldsymbol{x}(t),t\big)$ is known, and

$$\boldsymbol{\chi}\big(\boldsymbol{x}(t),t\big) = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \text{sgn } (\tau_1) \\ \text{sgn } (\tau_2) \end{pmatrix} \tag{41}$$

$$\begin{aligned} \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} &= \Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t),t\big)\Big)^T \boldsymbol{\sigma} \\ &= \Big(\boldsymbol{\sigma}^T \boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t),t\big)\Big)^T \end{aligned} \tag{42}$$

Controller for a set-point variation
$\boldsymbol{u}_r(t)$ is needed when a set point varies.

$$\begin{aligned} \boldsymbol{u}_r(t) &= \begin{pmatrix} u_{r1} \\ u_{r2} \end{pmatrix} \\ &= \Big(\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t),t\big)\Big)^{-1} \boldsymbol{S}\dot{\boldsymbol{y}}_r(t) \end{aligned} \tag{43}$$

where, $\dot{\boldsymbol{y}}_r(t)$ shall be bounded.

B. Stability of the control system

For ensuring control system stability, a brief explanation is shown as follows.
Lyapunov function

$$V = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma} + \frac{1}{2}\Big(\hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta}\Big)^T \boldsymbol{\Gamma}^{-1}\Big(\hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta}\Big) \tag{44}$$

From equation (25), we obtain

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{S}\Big(\dot{\boldsymbol{x}}(t) - \dot{\boldsymbol{y}}_r(t)\Big) \tag{45}$$

The derivative of V from equation (44) is

$$\dot{V} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} + \Big(\hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta}\Big)^T \boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}(t) \tag{46}$$

where, it is the Schwarz inequality

$$\begin{aligned} \dot{V} \leq\ & -\boldsymbol{\sigma}^T\boldsymbol{K}\boldsymbol{\sigma} - \boldsymbol{\sigma}^T\kappa\boldsymbol{\Lambda}(\boldsymbol{\sigma}) \\ & - \left\| \Big(\boldsymbol{\sigma}^T\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t),t\big)\Big)^T \right\| \eta\big(\boldsymbol{x}(t),t\big) \\ & + \left\| \Big(\boldsymbol{\sigma}^T\boldsymbol{S}\bar{\boldsymbol{g}}\big(\boldsymbol{x}(t),t\big)\Big)^T \right\| \cdot \parallel \Delta\boldsymbol{h}\big(\boldsymbol{x}(t),t\big) \parallel \\ \leq\ & -\boldsymbol{\sigma}^T\boldsymbol{K}\boldsymbol{\sigma} - \boldsymbol{\sigma}^T\kappa\boldsymbol{\Lambda}(\boldsymbol{\sigma}) \end{aligned} \tag{47}$$

Since $\boldsymbol{\sigma}$ is negative definite, the stability of the system is ensured.

V. Simulation and experiment

A. Simulation

Simulation for the case with the uncertainty $\Delta\boldsymbol{h}\big(\boldsymbol{x}(t),t\big)$ and the upper-bound value function $\eta\big(\boldsymbol{x}(t),t\big)$ is performed. The control parameters in (4) and (5) are obtained by the estimation method in Section 2.

$$\begin{aligned} & a_1 = 21.880226, \quad a_2 = 1.0211740 \\ & a_3 = 18.484688, \quad a_4 = 0.0709710 \\ & b_1 = 12.691833, \quad b_2 = 1.8394160 \\ & b_3 = 0.1147300 \end{aligned} \tag{48}$$

where, the identification errors are

$$\begin{aligned} & \theta_1 = 0.000249, \quad \theta_2 = 0.001217 \\ & \theta_3 = 0.117540, \quad \theta_4 = -0.001302 \\ & \theta_5 = -0.012118, \quad \theta_6 = -0.018707 \\ & \theta_7 = -0.009142 \end{aligned} \tag{49}$$

And parameters are set with

$$\begin{aligned} r_2 &= \begin{cases} \dfrac{\pi}{2}, & \text{for } 50 < t \leq 150 \\ 0, & \text{for } t \leq 50,\ t > 150 \end{cases} \\ \iota_1 &= 0.2 \\ \iota_2 &= 0.1 \\ \eta &= 5 \\ \Delta\boldsymbol{h}\big(\boldsymbol{x}(t),t\big) &= \begin{pmatrix} 0.5\cos\Big(x_2 + 2t + \dfrac{\pi}{3}\Big) \\ 0.5x_1^2 \end{pmatrix} \\ \boldsymbol{K} &= \begin{pmatrix} 10 & 0 \\ 0 & 5 \end{pmatrix} \\ \kappa &= 0.03 \\ k_s &= 0.001 \\ T_1 &= 2 \\ T_2 &= 3 \end{aligned} \tag{50}$$

The sampling time is set to 1ms. A simulation result is shown in Fig. 2. Even if the uncertainty of an indefinite part existed, when the upper-bound value function was known, the control objective was attained and the effectiveness of this proposal approach has been verified.

B. Experiment

2 degree-of-freedom helicopter produced by this research is shown in Fig. 3. We experimented using this 2 degree of freedom helicopter. The set point was changed and the disturbance was given at near 40 seconds.

$$\begin{aligned} r_2 &= \begin{cases} \dfrac{\pi}{2}, & \text{for } t \leq 20 \\ 0, & \text{for } t > 20 \end{cases} \\ k_s &= 0.001 \end{aligned} \tag{51}$$

The other parameters are the same as (48) and (50).
An experimental result is shown in Fig. 4. Even if the set point varied, it has verified that it is robust to disturbance.

VI. Conclusion

In this paper, attitude control scheme for a helicopter experimental system is given. Simulation and experimental results confirm that the proposed method is effective.

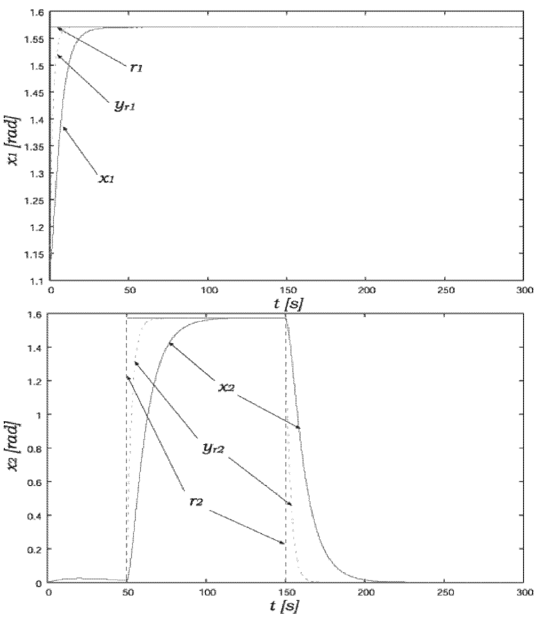

Fig. 2. The simulation result using the proposed control method

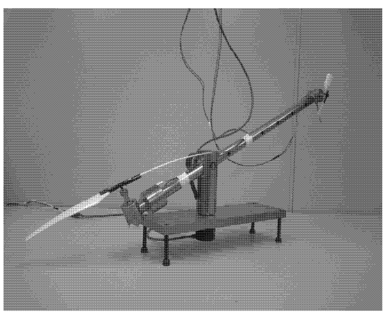

Fig. 3. 2 degree-of-freedom helicopter

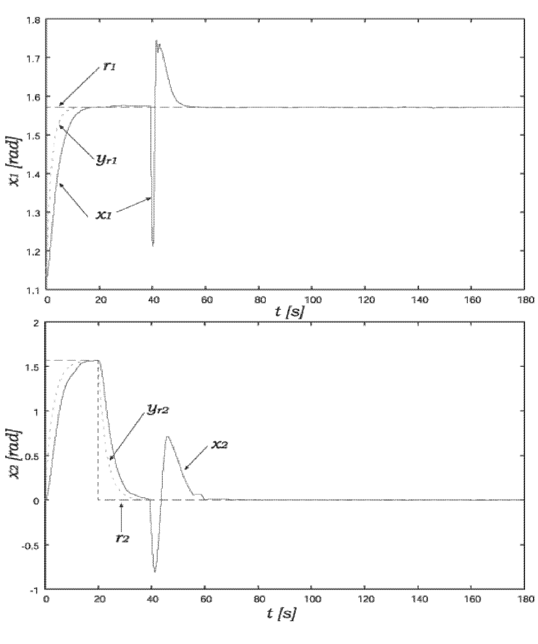

Fig. 4. The experiment result using the proposed control method

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Appendix A

From the equation (1), (2) and the relational expression of the angular speed of a motor and an input pressure (3), we have (4) and (5) experimental setup, where,

$$\begin{array}{lllllll} a_1 & = & \dfrac{I_p}{A_m L_m k_m^2} & & b_1 & = & \dfrac{I_y}{A_t L_t k_t^2} \\ a_2 & = & \dfrac{D_p}{A_m L_m k_m^2} & & b_2 & = & \dfrac{D_y}{A_t L_t k_t^2} \\ a_3 & = & \dfrac{mgL_g}{A_m L_m k_m^2} & & b_3 & = & \dfrac{h_m A_m L_m k_m^2}{A_t L_t k_t^2} \\ a_4 & = & \dfrac{h_t A_t L_t k_t^2}{A_m L_m k_m^2} & & & & \\ u_1 & = & u_m^2 & & u_2 & = & u_t^2 \end{array} \quad (52)$$

The parameter vector $\boldsymbol{a}$ to identify is set to

$$\boldsymbol{a} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & b_3 \end{pmatrix}^T \quad (53)$$

T is taken as a transposed matrix.

For equations (4) and (5), an equation is set to

$$\boldsymbol{\Phi}_1(t)\boldsymbol{a} = \boldsymbol{u}(t) \quad (54)$$

where,

$$\boldsymbol{\Phi}_1(t) = \begin{pmatrix} \ddot{p} & \dot{p} & \sin p & u_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddot{y} & \dot{y} & u_1 \sin p \end{pmatrix} \quad (55)$$

$$\boldsymbol{u}(t) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_m^2 \\ u_t^2 \end{pmatrix} \quad (56)$$

When the values of u_m and u_t are negative in experiment, we select that

$$\boldsymbol{u}(t) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -u_m^2 \\ -u_t^2 \end{pmatrix}$$

Appendix B

If $\bar{\boldsymbol{c}}$, $\bar{\boldsymbol{d}}$ are considered to be identification values and $\Delta\boldsymbol{c}$, $\Delta\boldsymbol{d}$ are considered to be the presumed errors, the real values $\boldsymbol{c}$, $\boldsymbol{d}$ can be decomposed with

$$\begin{array}{lll} \boldsymbol{c} & = & \bar{\boldsymbol{c}} + \Delta\boldsymbol{c} \\ \boldsymbol{d} & = & \bar{\boldsymbol{d}} + \Delta\boldsymbol{d} \end{array} \quad (57)$$

where,

$$
\begin{aligned}
\bar{\boldsymbol{c}} &= \begin{pmatrix} \bar{c}_1 & \bar{c}_2 & \bar{c}_3 & \bar{c}_4 \end{pmatrix}^T \\
\bar{\boldsymbol{d}} &= \begin{pmatrix} \bar{d}_1 & \bar{d}_2 & \bar{d}_3 \end{pmatrix}^T \\
\Delta \boldsymbol{c} &= \begin{pmatrix} \Delta c_1 & \Delta c_2 & \Delta c_3 & \Delta c_4 \end{pmatrix}^T \\
\Delta \boldsymbol{d} &= \begin{pmatrix} \Delta d_1 & \Delta d_2 & \Delta d_3 \end{pmatrix}^T
\end{aligned}
\tag{58}
$$

In this research, $\Delta c_i (i = 1, 2, 3, 4)$, $\Delta d_j (j = 1, 2, 3)$ are constant

$$
\begin{aligned}
\frac{d}{dt}\left(\Delta c_i\right) &\cong 0 \\
\frac{d}{dt}\left(\Delta d_j\right) &\cong 0
\end{aligned}
\tag{59}
$$

are filled and it shall vary too slowly.
It is referred to as $\bar{\boldsymbol{f}}(\boldsymbol{x}(t), t)$, $\bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)$ when the parameter of $\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\boldsymbol{g}(\boldsymbol{x}(t), t)$ are $\bar{\boldsymbol{c}}$, $\bar{\boldsymbol{d}}$. That is, it becomes to

$$
\begin{aligned}
\bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) &= \begin{pmatrix} x_3 \\ x_4 \\ -\bar{c}_2 x_3 - \bar{c}_3 \sin x_1 \\ -\bar{d}_2 x_4 \end{pmatrix} \\
\bar{\boldsymbol{g}}(\boldsymbol{x}(t), t) &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \bar{c}_1 & -\bar{c}_4 \\ -\bar{d}_3 \sin x_1 & \bar{d}_1 \sin x_1 \end{pmatrix}
\end{aligned}
\tag{60}
$$

And, it referred to as $\Delta \boldsymbol{f}(\boldsymbol{x}(t), t)$, $\Delta \boldsymbol{g}(\boldsymbol{x}(t), t)$ when the parameter of $\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\boldsymbol{g}(\boldsymbol{x}(t), t)$ are $\Delta \boldsymbol{c}$, $\Delta \boldsymbol{d}$. That is, it becomes to

$$
\begin{aligned}
\Delta \boldsymbol{f}(\boldsymbol{x}(t), t) &= \begin{pmatrix} 0 \\ 0 \\ -\Delta c_2 x_3 - \Delta c_3 \sin x_1 \\ -\Delta d_2 x_4 \end{pmatrix} \\
\Delta \boldsymbol{g}(\boldsymbol{x}(t), t) &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \Delta c_1 & -\Delta c_4 \\ -\Delta d_3 \sin x_1 & \Delta d_1 \sin x_1 \end{pmatrix}
\end{aligned}
\tag{61}
$$

Then, $\boldsymbol{f}(\boldsymbol{x}(t), t)$, $\boldsymbol{g}(\boldsymbol{x}(t), t)$ are

$$
\begin{aligned}
\boldsymbol{f}(\boldsymbol{x}(t), t) &= \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \Delta \boldsymbol{f}(\boldsymbol{x}(t), t) \\
\boldsymbol{g}(\boldsymbol{x}(t), t) &= \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t) + \Delta \boldsymbol{g}(\boldsymbol{x}(t), t)
\end{aligned}
\tag{62}
$$

If it substitutes for

$$
\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t) + \boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \tag{63}
$$

, an equation of state will be set to

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \Delta \boldsymbol{f}(\boldsymbol{x}(t), t) \\
&\quad + \Delta \boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{u}(t)
\end{aligned}
\tag{64}
$$

A right side 2 and 3 term are an uncertain term, and is

$$
\Delta \boldsymbol{f}(\boldsymbol{x}(t), t) + \Delta \boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) = \begin{pmatrix} 0 \\ 0 \\ \Delta c_1 u_1 - \Delta c_2 x_3 - \Delta c_3 \sin x_1 - \Delta c_4 u_2 \\ \Delta d_1 u_2 \sin x_1 - \Delta d_2 x_4 - \Delta d_3 u_1 \sin x_1 \end{pmatrix} \tag{65}
$$

In order for $\Delta \boldsymbol{f}(\boldsymbol{x}(t), t) + \Delta \boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t)$ to fulfill matching conditions, it is necessary to calculate $\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$ used as

$$
\bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) = \Delta \boldsymbol{f}(\boldsymbol{x}(t), t) + \Delta \boldsymbol{g}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \tag{66}
$$

Although equation (66) is the matrix of four lines, all of the component of 1st and 2nd lines are 0. Therefore, only 3rd and 4th lines are considered and it is set to

$$
\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) = \begin{pmatrix} \bar{c}_1 & -\bar{c}_4 \\ -\bar{d}_3 \sin x_1 & \bar{d}_1 \sin x_1 \end{pmatrix}^{-1} \begin{pmatrix} \Delta c_1 u_1 - \Delta c_2 x_3 - \Delta c_3 \sin x_1 - \Delta c_4 u_2 \\ \Delta d_1 u_2 \sin x_1 - \Delta d_2 x_4 - \Delta d_3 u_1 \sin x_1 \end{pmatrix} \tag{67}
$$

So, an equation of state is set to

$$
\dot{\boldsymbol{x}}(t) = \bar{\boldsymbol{f}}(\boldsymbol{x}(t), t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) + \bar{\boldsymbol{g}}(\boldsymbol{x}(t), t)\boldsymbol{u}(t) \tag{68}
$$

from equation (66).