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Takayoshi Nakata
Okayama University

N. Takahashi
Okayama University

K. Fujiwara
Okayama University

Y. Shiraki
Okayama University

COMPARISON OF DIFFERENT FINITE ELEMENTS FOR 3-D EDDY CURRENT ANALYSIS

T.Nakata, N.Takahashi, K.Fujiwara and Y.Shiraki
Department of Electrical Engineering,
Okayama University, Okayama 700, Japan

ABSTRACT

In order to evaluate the best type of element for the finite element analysis of 3-D eddy currents, a fundamental model is analyzed using the usual 1st-order tetrahedral, triangular prism and brick nodal elements and also the brick edge element. The effects of the types of elements on the flux and eddy current distributions are investigated using the A-ϕ method and the $\mathbb{T}$-Ω method.

It is concluded that the brick edge element is best from the viewpoints of accuracy and CPU time.

1. INTRODUCTION

There was a large difference in the calculated results of the FELIX brick model(Problem 4)[1,2] between our result using the 1st-order tetrahedral nodal element and the results using the other types of elements[3]. It is important to examine the reason why the difference occurred.

In order to illustrate the cause of the difference, a brick model(FELIX Workshop, Problem 4)[1] and an asymmetrical conductor model (TEAM Workshop, Problem 7)[4] are analyzed using the 1st-order tetrahedral, triangular prism and brick nodal elements and the brick edge element[5,6]. The A-ϕ and $\mathbb{T}$-Ω methods are used for the nodal element, and the A and $\mathbb{T}$-Ω methods are used for the edge element. The accuracy and the CPU time are compared with each other, and the effects of the types of elements on the flux and eddy current distributions are investigated quantitatively. Experimental verification is also carried out.

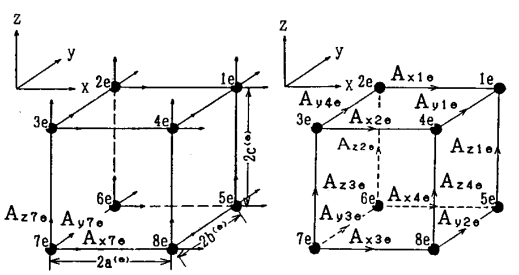
2. ANALYSIS

2.1 Types of Elements

The 1st-order tetrahedral, triangular prism and brick nodal elements and a brick edge element are examined.

In the usual nodal element, each component of the vector potential has the same interpolation function, and continuity condition of all three vector components between adjacent elements is imposed by setting nodal values of the adjacent elements to be identical. In the case of the 1st-order brick element of the A-ϕ method shown in Fig.1(a), the vector potential $A^{(e)}$ can be denoted as follows:

$$A^{(e)}=\sum_{k=1}^{8} N_{ke}A_{ke} \tag{1}$$



(a) nodal element (b) edge element

Fig.1 Types of elements.

Where N_{ke} is the interpolation function[7], and N_{1e}, for example, can be written as follows:

$$N_{1e}=\frac{1}{8}(1+\frac{x}{a^{(e)}})(1+\frac{y}{b^{(e)}})(1+\frac{z}{c^{(e)}}) \tag{2}$$

$2a^{(e)}$, $2b^{(e)}$ and $2c^{(e)}$ are the lengths of edges as shown in Fig.1(a).

In the edge element[5], the tangential projection of the vector potential on each edge is interpolated. In the case of the edge element in Fig.1(b), the x-component $Ax^{(e)}$ can be denoted as follows:

$$A_x^{(e)}=\sum_{k=1}^{4} N_{xke}A_{xke} \tag{3}$$

N_{x1e}, for example, can be written as follows[6]:

$$N_{x1e}=\frac{1}{4}(1+\frac{y}{b^{(e)}})(1+\frac{z}{c^{(e)}}) \tag{4}$$

The interpolation functions of the nodal element and the edge element are different from each other as denoted in Eqs.(2) and (4).

2.2 Descriptions of Models

Two kinds of models, namely transient eddy current and ac eddy current models are analyzed.

(1) Transient Eddy Current Model (Problem 4)

Figure 2 shows the FELIX brick to be analyzed. An aluminum brick with a hole is placed in a uniform magnetic field. The conductivity of the brick is 2.54×10^7 S/m. The applied magnetic field is perpendicular to the brick and decays exponentially with time as denoted in Fig.2.

Figure 3 illustrates the assigned boundary conditions which exploit the 8 fold symmetry of the problem[8]. The model is analyzed using the various types of nodal elements. The tetrahedral mesh used is shown in Fig.4. This mesh is finer than the mesh specified in the reference[1]. The numbers of elements and nodes are 15120 and 3135 respectively. The positions of nodes for all the other types of nodal elements are the same. This means that the number of unknown variables is the same in each mesh even if the type of element is different. The time interval Δt of the step-by-step method[9] is 1ms. In the $\mathbb{T}$-Ω method, the conductivity in the hole is assumed to 1S/m[10]. No gauge condition is imposed in either methods. The ICCG method is used to solve the set of linear equations.

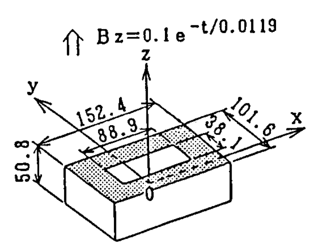


Fig.2 FELIX brick.

(2) AC Eddy Current Model (Problem 7)

Figure 5 shows the model of asymmetrical conductor with a hole (TEAM Workshop, Problem 7)[4]. A thick

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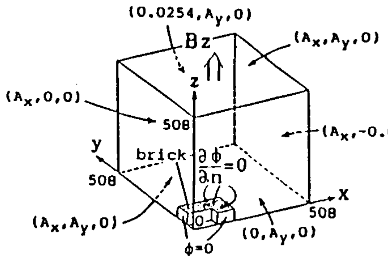
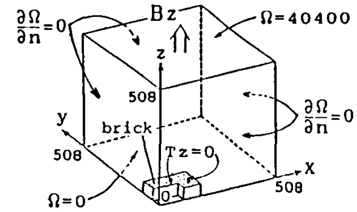
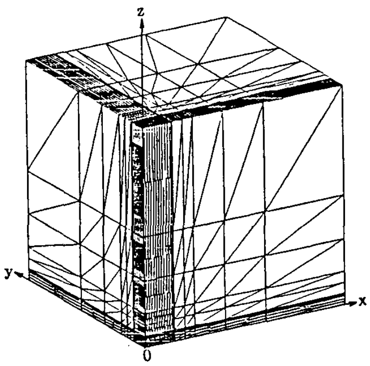


(a) A-ϕ method

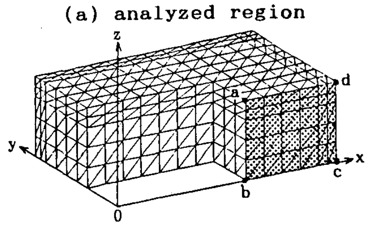


(b) $\mathbb{T}$-Ω method

Fig.3 Boundary conditions.



(a) analyzed region



(b) conductor region

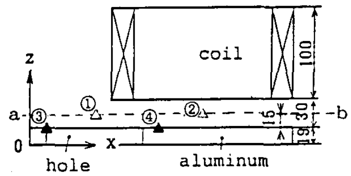
Fig.4 Finite element mesh.

aluminum plate with a hole, which is made eccentrically, is set unsymmetrically in a non-uniform magnetic field. The conductivity of the plate is 3.526×10^{7} S/m. The field is produced by the exciting current varying sinusoidally with time. The ampere turns of the coil are equal to 2742AT. The frequency is 50Hz.

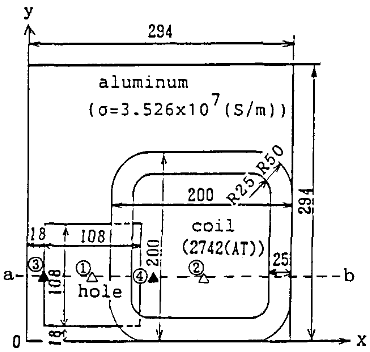
Figure 6 shows boundary conditions of the current vector potential $\mathbb{T}$e corresponding to the eddy current[11]. On the outermost boundary, the normal component of the flux density is set to be equal to zero in order that the flux is parallel to the boundary[4].

Because the corners of the coil are rounded as shown in Fig.5, the model cannot be subdivided only into brick elements. In order to compare the brick element with the other kinds of elements, the shape of the coil is modified so that the corner of the coil forms 90° edge. When the flux densities along the line a-b in Fig.5 are calculated in the cases of the square coil and the round(real) coil, the error due to the change of the shape is negligibly small (within 1%).

Figure 7 shows the finite element mesh for the tetrahedral nodal element. The numbers of elements and nodes for various types of elements are denoted in Table 1.



(a) front view



(b) plan view

Fig.5 Asymmetrical conductor with a hole.

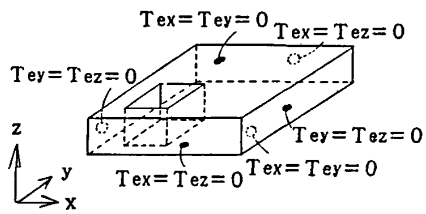


Fig.6 Boundary conditions for the current vector potential $\mathbb{T}$e.

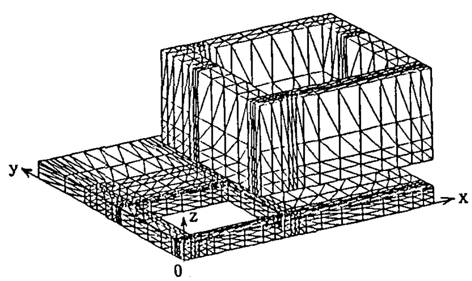


Fig.7 Finite element mesh.

3. RESULTS AND DISCUSSIONS

3.1 Transient Eddy Current Model (Problem 4)

Figure 8 shows time variations of total circulating currents at the cross section a-b-c-d-a hatched in Fig.4(b). The currents obtained by the A-ϕ method are larger than those by the $\mathbb{T}$-Ω method. The

Table 1 Numbers and CPU time for ac eddy current model

item	A−φ method		A method	T−Ω method		
	nodal		edge	nodal		edge
	tetra.	brick		tetra.	brick	
number of elements	61824	10304		61824	10304	
number of nodes	11832					
number of unknowns	28660		28059	15750		15303
number of iterations of ICCG method	1004	1189	689	1260	506	404
CPU time (s)	1864	3903	862	770	588	321

difference in the results of these methods using the brick element, is smaller than that using the tetrahedral or triangular prism element.

Table 2 denotes the CPU time for various nodal elements for the first 20ms. The CPU time of the T−Ω method is considerably shorter than that of the A−φ method.

Let us compare various nodal elements from the viewpoints of both the accuracy of currents and the CPU time. When the number of elements is increased, the amplitude Ia of the eddy current obtained by the A−φ method and the amplitude It obtained by the T−Ω method approach (Ia+It)/2 asymptotically[12]. Then, the error ε_I of each element may be defined by the following equation:

$$\varepsilon_I = \frac{I\,(cal) - I\,(true)}{I\,(true)} \times 100\ (\%) \qquad (5)$$

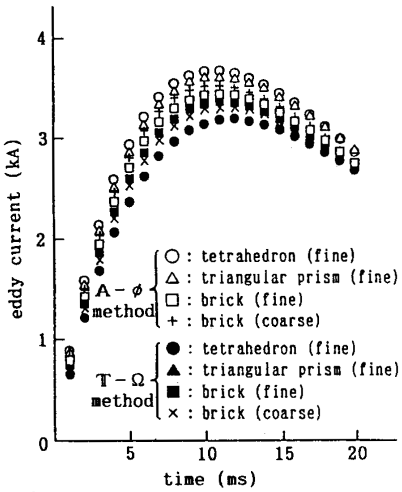


Fig.8 Time variations of total circulating currents (nodal elements).

Table 2 CPU time (S)

mesh	method	tetra.	triangular prism	brick
fine	A−φ	2472	3420	4151
	T−Ω	923	1262	1148
coarse	A−φ	558	654	794
	T−Ω	169	191	195

Used computer : NEC supercomputer SX-1E

where I(cal) denotes the current calculated, and I(true) denotes (Ia+It)/2 for the fine mesh of the brick. If ε_I is defined as Eq.(5), Fig.8 shows that ε_I for the brick element is smaller than those for the other elements, and ε_I for the brick is little affected by the number of unknown variables. On the other hand, Table 2 denotes that the CPU time for the coarse mesh of the brick is considerably shorter than that for the fine mesh of each element. Therefore, it may be concluded that the brick element is to be preferred.

3.2 AC Eddy Current Model (Problem 7)

The nodal element and the edge element are compared here. Because it is shown that the prism element is not favorable from the viewpoints of the accuracy and the CPU time compared with the brick element as shown in Section 3.1, the prism element is excluded from the comparison.

Figure 9 shows the comparison of the maximum value of the z-component of the flux density along the line of y=72mm and z=34mm. In the case of the tetrahedral element, the flux density obtained by the T−Ω method is very much larger than that by the A−φ method contrary to the case of Section 3.1. Figure 9 shows that the error for the brick element is smaller than that using the tetrahedral element.

Figure 10 shows the comparison of the maximum values of the y-components of the eddy current densities.

Tables 3 and 4 show the comparisons of the calculated and measured values at the points denoted in Fig.5. The flux density is measured using a small search coil(diameter: φ3mm, height:0.6mm, 20turns). The eddy current density is measured using an improved probe method[13]. The discrepancies between the calculated and measured values may be due to the insufficient number of elements, the positioning and the setting errors of the sensor and the inhomogeneity of plate material. The figures in the parenthesis in the tables denote errors. The error ε_B of the flux density shown in Table 3 is defined as follows:

$$\varepsilon_B = \frac{B\,(cal) - B\,(mea)}{B\,(mea)} \times 100\ (\%) \qquad (6)$$

where B(cal) and B(mea) denote the flux densities which are calculated and measured respectively. Table 3 and 4 show that the accuracy for the edge element is not so bad compared with those for the other elements. Moreover, Table 1 shows that the CPU time for the edge

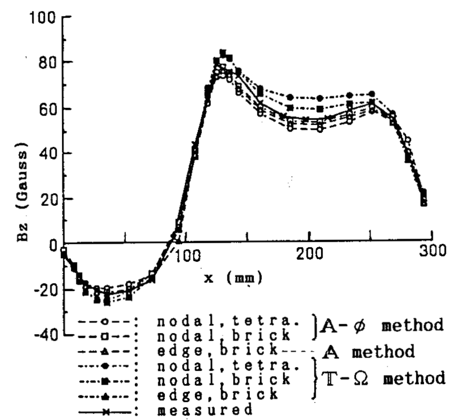


Fig.9 Comparison of the maximum values of the z-component of flux density (f=50Hz, y=72, z=34).

Table 3 Comparison of flux densities

examined point	coordinate (mm)			flux density (Gauss)						
				calculated						measured
				A−φ method		A method	T−Ω method			
				nodal		edge	nodal		edge	
	x	y	z	tetra.	brick	brick	tetra.	brick	brick	
①	72	72	34	-14.3 (-10.7)	-13.7 (-14.2)	-15.0 (-6.68)	-13.4 (-16.1)	-15.0 (-6.1)	-15.1 (-6.1)	-16.0
②	186	72	34	50.6 (-8.4)	53.7 (-2.8)	52.6 (-4.9)	63.6 (15.2)	59.3 (7.3)	59.2 (7.3)	55.2

Table 4 Comparison of eddy current densities

examined point	coordinate (mm)			eddy current density ($\times 10^6 A/m^2$)						
				calculated						measured
				A−φ method		A method	T−Ω method			
				nodal		edge	nodal		edge	
	x	y	z	tetra.	brick	brick	tetra.	brick	brick	
③	18	72	19	0.97 (8.99)	0.82 (-7.87)	0.88 (-1.12)	0.97 (8.99)	0.86 (-3.37)	0.87 (-2.25)	0.89
④	144	72	19	0.85 (-9.57)	0.84 (-10.64)	0.77 (-18.09)	0.73 (-22.34)	0.81 (-13.83)	0.82 (-12.77)	0.94

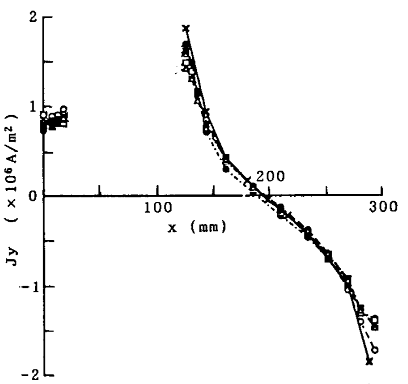


Fig.10 Comparison of the maximum values of the y-component of eddy current density (f=50Hz, y=72, z=19).

element is considerably shorter than that for the other elements. Therefore, it may be concluded that the brick edge element is to be preferred.

4. CONCLUSIONS

The accuracy and the CPU time are compared quantitatively by analyzing the fundamental models using the different types of elements. A big difference is found among those results. It may be concluded that the brick edge element is to be preferred for 3-D eddy current analysis.

The results obtained for the nonlinear model(Problem 10)[4] analyzed by using different elements are presented in reference[14].

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