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Abstract

This paper proposes a new method for computing state deadbeat feedback gains from systems given in the staircase form. The proposed method uses only manipulations of given matrices and hence is more direct than the existing one which requires orthogonal transformations repeatedly. The paper also shows that the obtained gain is LQ optimal for some weighting matrices.

1. Introduction

As is widely recognized, it is by no means trivial to compute state deadbeat gains in a numerically stable way. Emami-Naemi et al. [2] have proposed a method based on the linear quadratic (LQ) control theory. Dooren [1] has computed, by repeating conversion of a given system, the deadbeat gain together with the resulting nilpotent state-transition matrix. Both of these methods have excellent numerical reliability.

In actual computation, however, the above methods are not straightforward in the following sense. In the LQ method [2], we need to solve the discrete-time Riccati equation for a specially chosen pair of weighting matrices. This is rather indirect and artificial because those weightings have a less physical meaning as a control performance. On the other hand, in the method of [1] we have to compute the nilpotent matrix and orthogonal matrices, which are not essential for the computation of the gain.

In this paper we propose a new method for computing the deadbeat gain more directly than [1]. As a result, this method reduces both computational time and theoretical complexity. Furthermore, it is shown that the obtained feedback gain is LQ optimal for some weighting matrices.

Thus the present approach has both of the advantages of the above two methods.

2. Main Result

Consider a reachable system

$$x(t+1) = F_o x(t) + G_o u(t), \quad t = 1, 2, \ldots \quad (1)$$

where $F_o \in \mathbb{R}^{n \times n}, G_o \in \mathbb{R}^{n \times m}$. No assumption is made on nonsingularity of F_o. As in [1] and [2], we start by converting (F_o, G_o) into the so-called staircase form:

$$G = \begin{bmatrix} \Delta_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1\mu} \\ \Delta_2 & F_{22} & & \vdots \\ & \ddots & \ddots & \\ 0 & & \Delta_\mu & F_{\mu\mu} \end{bmatrix} \quad (2)$$

where μ is the reachability index, Δ_p has full row rank r_p, the diagonal element matrices F_{pp} are $r_p \times r_p$, and the rest are of compatible sizes. It is well known that the form (2) can easily be obtained via orthogonal transformations.

Now we compute the deadbeat gain directly from (2). In order to explain the recursive nature of the algrorithm, we first define a sequence of smaller matrices

$$\widehat{G}_p := \begin{bmatrix} \Delta_p \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \widehat{F}_p := \begin{bmatrix} F_{pp} & & \cdots & F_{p\mu} \\ \Delta_{p+1} & \ddots & & \vdots \\ & & \ddots & \ddots & \\ 0 & & & \Delta_\mu & F_{\mu\mu} \end{bmatrix}$$

for $p = 1, \ldots, \mu$. Then, $F = \widehat{F}_1$, $G = \widehat{G}_1$, and

$$\widehat{F}_p = \begin{bmatrix} F_{pp} & * \\ \widehat{G}_{p+1} & \widehat{F}_{p+1} \end{bmatrix}, \quad p = 1, \ldots, \mu - 1 \quad (3)$$

Namely, $\widehat{F}_{p+1}$ is embedded into the preceding $\widehat{F}_p$.

With these notions, the deadbeat gain K is given by the simple formula:

$$K_\mu := \Delta_\mu^- \widehat{F}_\mu,$$
$$K_p := \Delta_p^-[I \quad K_{p+1}]\widehat{F}_p, \quad p = \mu - 1, \ldots, 1, \quad (4)$$
$$K := K_1$$

Here, A^- denotes any right inverse of the matrix A.

THEOREM 1. The feedback gain (4) achieves minimal time deadbeat control.

Outline of Proof. The closed-loop state-transition matrix is

$$F - GK = \begin{bmatrix} 0 & -K_2 \\ 0 & I \end{bmatrix} F \quad \text{(by (2), (4))}$$

$$\sim \begin{bmatrix} I & K_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} 0 & -K_2 \\ 0 & I \end{bmatrix} F \begin{bmatrix} I & -K_2 \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ \widehat{G}_2 & \widehat{F}_2 - \widehat{G}_2 K_2 \end{bmatrix} \quad (5)$$

with $\sim$ denoting similarity. Proceeding recursively, we finally obtain

$$F - GK \sim \begin{bmatrix} 0 & \cdots & 0 \\ & \ddots & \vdots \\ * & & 0 \end{bmatrix} \quad (6)$$

i.e., a block triangular form. Hence (4) achieves minimal time deadbeat control (see [1]).   □

Note that in actual computation, we do not have to perform similarity transformations (5) and (6). We need only to compute $K_\mu, \ldots, K_1$ according to (4).

3. LQ Optimality

Now let us show that the gain (4) is optimal for some weightings. To this end, we give a criterion for optimality in a more general form.

THEOREM 2. Consider the system (2), and suppose that G has full column rank. (Then Δ_1 is square and invertible.) If a stabilizing feedback gain K is written as

$$K = \Delta_1^{-1}[I \quad L]F \quad (7)$$

for some L, then K is LQ optimal for the index with weightings

$$Q := H^T H, \quad H := \Delta_1^{-1}[I \quad L], \quad (8)$$
$$R := 0$$

Outline of Proof. Define

$$Z(z) := (zI - F)^{-1}G$$

The return difference matrix for (7) is

$$W(z) := I + KZ(z)$$
$$= \Delta_1^{-1}[I \quad L]G + \Delta_1^{-1}[I \quad L]FZ(z)$$
$$= z\Delta_1^{-1}[I \quad L]Z(z) \quad (9)$$

Hence we have

$$W^T(z^{-1})W(z) = Z^T(z^{-1})QZ(z) \quad (10)$$

On the other hand, let $\overline{W}(z)$ be the return difference matrix of the LQ optimal gain $\overline{K}$ for Q and R. Then $\overline{K}$ satisfies the Kalman equation

$$\overline{W}^T(z^{-1})G^T \overline{\Pi} G\overline{W}(z) = Z^T(z^{-1})QZ(z) \quad (11)$$

where $\overline{\Pi}$ is the solution of the corresponding Riccati equation. In view of (10) and (11) we can readily show that $K = \overline{K}$ by slightly modifying the technique in [3].   □

The deadbeat gain (4) clearly satisfies (7) for $L := K_2$, and hence it is LQ optimal.

4. Conclusion

We have shown that the gain (4) achieves deadbeat control and is also LQ optimal. Although we do not have to solve Riccati equation for any weightings, our feedback gain belongs to the class of optimal control as in [2]

References

[1] P. Van Dooren, "Deadbeat Control: A Special Inverse Eigenvalue Problem," *BIT*, Vol. **24**, pp. 681–699, 1984.

[2] A. Emami-Naeini and G. F. Franklin, "Deadbeat Control and Tracking of Discrete-Time Systems," *IEEE Trans. Autom. Control*, Vol. **AC-27**, pp. 176–181, 1982.

[3] K. Sugimoto and Y. Yamamoto, "Solution to the Inverse Regulator Problem for Discrete-Time Systems," *Int. J. Control*, Vol. **48**, pp. 1285–1300, 1988.