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Continuous-Time Anti-Windup Generalized Predictive Control of Non-Minimum Phase Processes with Input Constraints

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Abstract—This paper deals with a design problem of a continuous-time anti-windup generalized predictive control system using coprime factorization approach for non-minimum phase processes with input constraints. Based on the proposed design scheme, a condition for stability of the closed-loop system with input constraints and a straight-forward method to improve the output response of the system with input constraints are given. Simulation results are presented to support the theoretical analysis.

I. INTRODUCTION

Generalized predictive control (GPC) is widely used in industry. In applying the control scheme to real processes in industry, process control systems must deal with some constraints such as pressure and temperature limit, and the control system must also avoid the unsafe operating regimes. Especially constraints on input variables and unsafe problem of controller are crucial in the case of control of process having unstable zeros, that is, non-minimum phase process. One of the features of GPC is that it can control non-minimum phase process. However, the control of the above process often needs an unstable controller to cancel unstable zeros, and the use of the unstable controller in a non-minimum phase process causes an excess input over constraints. This paper considers a design problem of GPC for non-minimum phase processes with input constraints.

So far, input constraints have been taken into account in two ways. In the first case, predictive controls of constrained continuous-time systems were considered by using quadratic programming [1], [2]. In the second case, two-step design paradigm was discussed [3]. The second method is simply stated as follows: design a linear controller ignoring control input constraint and then add anti-windup bumpless transfer compensation to minimize the adverse effects of any control constraints on the closed-loop performance. The algorithms of GPC for processes with input constraint to use quadratic programming are rather complicated. Many design methods

have been reported based on the second design paradigm (e.g. [4,5,6]). Recently, anti-windup two-degree-of-freedom control of invertible plants was also given [7]. Most of the methods are for non-GPC scheme. As for GPC, only the following result is obtained, that is, a connection between model predictive control with constraints and a controller with saturating actuators is given [8]. However, the connection is given under a restricted condition and the controller considered is also restricted.

This paper, based on the second approach, proposes a relatively simple design scheme of an anti-windup GPC for continuous-time non-minimum phase process. The design procedure has two steps: First, a strongly stable feedback controller for predictive control system is introduced by using coprime factorization representation [10]. The strongly stable is also called internally stabilizing controller in the case of discrete-time systems [11]. Second, the controller is extended to the case of processes with input constraints using coprime factorization representation and Youla parametrization for the controller of non-minimum phase processes. The controller has the following two characteristics: 1) By adding the anti-windup compensation, the control performance of the predictive control does not deteriorate during un-saturation period, and better tracking performance can be obtained during saturation. 2) A stability condition for the closed-loop system with input constraints is obtained.

Notations: When A is a function of s , $A[s]$ means a polynomial function of s , whereas, $A(s)$ is a rational function of s .

II. PROBLEM STATEMENT

Consider a single-input single-output time-invariant linear process described by the following transfer function:

$$Y_0(s) = \frac{B[s]}{A[s]}U_0(s) + \frac{C[s]}{A[s]}V_0(s) \quad (1)$$

where $A[s], B[s], C[s]$ are polynomials in the Laplace operator s . $Y_0(s), U_0(s)$ and $V_0(s)$ are the system output, control input and disturbance input respectively. $Y_0(s)$ and $U_0(s)$ are the Laplace transforms of $y(t)$ and $u(t)$. $A[s], B[s]$ are coprime polynomials of degree n and m respectively, and $A[s], B[s]$ are stable and unstable polynomials respectively. $C[s]$ is a designed Hurwitz polynomial of degree $n-1$, since no assumption is placed on the disturbance V_0 . $C[s]$ can also be designed by choosing $\deg(C[s]) = n$ when we would not wish to use the process output directly. In this paper, the degree of $C[s]$ will be assumed as $n-1$. The control input $u(t)$ is subject to the following constraint.

$$u_{min} \leq u(t) \leq u_{max}$$

This constraint is equivalently expressed as

$$u(t) = \sigma(u_1(t)) \quad (2)$$

$$\sigma(v) = \begin{cases} u_{max} & \text{if } v > u_{max} \geq 0 \\ v & \text{if } u_{min} \leq v \leq u_{max} \\ u_{min} & \text{if } v < u_{min} \leq 0 \end{cases} \quad (3)$$

The objective is to design a continuous-time anti-windup generalized predictive control system using coprime factorization representation and Youla parametrization for the above process.

III. DEVELOPMENT OF THE PROPOSED ALGORITHM

The proposed controller is given by the following Youla parametrization (Fig.1), where w is reference input.

$$R(s) = (Y(s) - Q(s)N(s))^{-1}K(s) \quad (4)$$

$$U(s) = X(s) + Q(s)D(s) \quad (5)$$

$$V(s) = Y(s) - Q(s)N(s) \quad (6)$$

where $Q(s) \in RH_\infty$ is a design parameter for ensuring a strongly stable feedback controller [10]. $X(s) \in RH_\infty$, $Y(s) \in RH_\infty$ satisfy the following Bezout identity.

$$X(s)N(s) + Y(s)D(s) = 1 \quad (7)$$

where, the coprime factorization presentation $N(s)$ and $D(s)$ of the process can be chosen as follows.

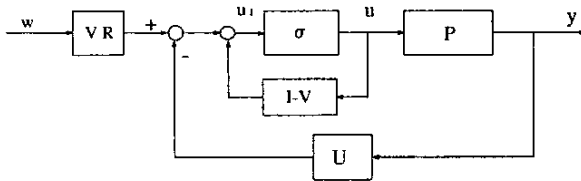


Fig. 1. The proposed control system

$$P(s) = \frac{N(s)}{D(s)}, N(s) \in RH_\infty, D(s) \in RH_\infty$$

Defining the stable closed-loop characteristic polynomial of the system without input constraint as T_0 , we choose $N(s)$ and $D(s)$ as

$$N(s) = \frac{B[s]}{T_0[s]} \quad (8)$$

$$D(s) = \frac{A[s]}{T_0[s]} \quad (9)$$

where $T_0[s] = A[s] + L_0[s] + gB[s]$, g and $L_0[s]$ are defined in (28) and (33) [9]. Since $C[s]$ is stable, we obtain as a solution of Bezout identity (7),

$$X(s) = \frac{gC[s] + F_0[s]}{C[s]} \quad (10)$$

$$Y(s) = \frac{C[s] + G_0[s]}{C[s]} \quad (11)$$

$$K(s) = g \quad (12)$$

where, $G_0[s]$ and $F_0[s]$ are defined in (31) and (32). Introduce two design polynomials $u_n[s], u_d[s]$ for $Q(s)$, that is,

$$Q(s) = \frac{u_n[s]}{u_d[s]} \quad (13)$$

Using the earlier work [9,10], $F_0(s), G_0(s), L_0(s), T_0$ and g can be obtained as follows.

The approximation $y^*(t+T)$ of the predicted output at $t+T$ can be given by writing the appropriate Maclaurin expansion of $y(t+T)$ about t and truncating this after N_y terms, where N_y is predictor order. Further, we can replace the values of the derivatives of $y(t)$ by their emulated values $y_k^*(t)$. Multiplying (1) by s^k and decomposing $C[s]/A[s]$ and $B[s]E_k[s]/C[s]$ into their strict proper part and remainder, we obtain

$$\frac{s^k C[s]}{A[s]} = \frac{F_k[s]}{A[s]} + E_k[s] \quad (14)$$

$$\frac{B[s]E_k[s]}{C[s]} = \frac{G_k[s]}{C[s]} + H_k[s] \quad (15)$$

where, the orders of $F_k[s], E_k[s], G_k[s]$ and $H_k[s]$ are $n-1, k-1, n-2$ and $k-\rho$ ($\rho = n-m$). Then, from (1) we have as a Laplace transformed form of Y_k^*

$$Y_k^*(s) = Y_k^0[s] + H_k[s]U_0(s) \quad (16)$$

$$Y_k^0(s) = \frac{F_k}{C[s]}Y_0(s) + \frac{G_k[s]}{C[s]}U_0(s) \quad (17)$$

Inverse Laplace transformation of (16) gives an emulated value of the k th derivative of $y(t)$ as

$$y_k^*(t) = y_k^0(t) + h_k \bar{u} \quad (18)$$

$$\bar{u} = [u(t) \ \bar{u}_1(t) \ \cdots \ \bar{u}_{k-\rho}(t)]^T \quad (19)$$

$$\bar{u}_i(t) = \frac{d^i u(t)}{dt^i}$$

where, h_k is a row vector comprising the coefficient of s^0, \dots, s^{N_u} in $H_k[s]$. N_u is the control horizon. The T-ahead prediction approximation $y^*(t+T)$ is given as

$$y^*(t+T) = T_{N_y}(H\bar{u} + Y^{r0})$$

where

$$T_{N_y} = \begin{bmatrix} 1 \\ T \\ \vdots \\ \frac{T^{N_y}}{N_y!} \end{bmatrix}^T, H = \begin{bmatrix} 0 \\ h_1 \\ \vdots \\ h_{N_y} \end{bmatrix}, Y^{r0} = \begin{bmatrix} y(t) \\ y_1^0(t) \\ \vdots \\ y_{N_y}^0(t) \end{bmatrix} \quad (20)$$

and $y_k^0(t)$ is inverse Laplace transformation of $Y_k^0(s)$. Consider the following cost function

$$J = \int_{T_1}^{T_2} (y_r^*(t, T) - w_r^*(t, T))^2 dT + \lambda \int_0^{T_2-T_1} u_r^*(t, T)^2 dT \quad (21)$$

where

$$y_r^*(t, T) = y^*(t+T) - y(t) \quad (22)$$

$$u_r^*(t, T) = T_{N_u} \hat{u} \quad (23)$$

$$T_{N_u} = \begin{bmatrix} 1 & T & \dots & \frac{T^{N_u}}{N_u!} \end{bmatrix} \quad (24)$$

$$\hat{u} = [u(t) \quad \bar{u}_1(t) \quad \dots \quad \bar{u}_{N_u}(t)]^T \quad (25)$$

The reference trajectory $w_r^*(t, T)$ will be taken as the output of a reference model $R_n[s]/R_d[s] \approx \sum_{i=0}^{N_y} r_i s^{-i}$, we obtain

$$w_r^*(t, T) = T_{N_y} r(w(t) - y(t))$$

where, r is a column vector which contains the Markov parameters of $R_n[s]/R_d[s]$, namely

$$r = [r_0 \quad r_1 \quad \dots \quad r_{N_y}]^T \quad (26)$$

The minimization of the cost function J results in

$$U_0(s) = \frac{gC[s]}{C[s] + G_0[s]} W(s) - \frac{gC[s] + F_0[s]}{C[s] + G_0[s]} Y_0(s) \quad (27)$$

$$k = [1 \ 0 \ \dots \ 0] (H^T T_y H + \lambda T_u)^{-1} H^T T_y \\ = [k_0 \ k_1 \ \dots \ k_{N_y}], g = kr \quad (28)$$

$$T_y = \int_{T_1}^{T_2} T_{N_y}^T T_{N_y} dT \quad (29)$$

$$T_u = \int_0^{T_2-T_1} T_{N_u}^T T_{N_u} dT \quad (30)$$

$$G_0[s] = \sum_{i=1}^{N_y} k_i G_i[s] \quad (31)$$

$$F_0[s] = \sum_{i=1}^{N_y} k_i F_i[s] \quad (32)$$

where, $L_0[s]$ is given by

$$L_0[s] = \sum_{i=1}^{N_y} k_i L_i[s] \quad (33)$$

$$\frac{s^k B[s]}{A[s]} = H_k[s] + \frac{L_k[s]}{A[s]} \quad (34)$$

Then, the closed-loop transfer function $G_{yw}(s)$ can be obtained as (Fig.2)

$$G_{yw}(s) = \frac{gB[s]}{A[s] + L_0[s] + gB[s]} \quad (35)$$

We can select λ so that $T_0[s] = A[s] + L_0[s] + gB[s]$ is stable.

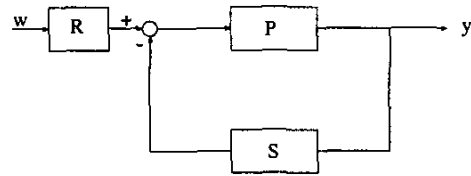


Fig. 2. The former control system ($S(s) = V^{-1}U$)

So far, the controller is designed without considering input constraint as step 1. Next, to obtain a stabilizing controller as design step 2, a stability condition will be derived by analyzing the stability of the proposed system in the presence of input constraints. The nonlinear system depicted in Fig.1 can be regarded as a system with perturbation depicted in Fig.3, where $\phi(\cdot)$ satisfies the following relation [6].

$$\phi(z) = z - 2\sigma(z) \quad (36)$$

The input constraint $\sigma(z)$ in equation (3) can be rewritten as

$$\sigma(z) = \frac{1}{2}(z - \phi(z)) \quad (37)$$

From the definition of σ , we have

$$|\phi(z)| \leq |z| \quad (38)$$

In the general framework of Fig.3

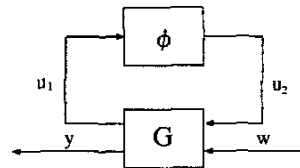


Fig. 3. The equivalent diagram of the proposed control system

$$u_2 = \phi(u_1) \quad (39)$$

$$u_1 = 2(I + V + UP)^{-1}VRw + (I + V + UP)^{-1}(V + UP - I)u_2 \quad (40)$$

$$y = (I + V + UP)^{-1}PVRw - (I + V + UP)^{-1}Pu_2 \quad (41)$$

Further, from (5), (6) and (7)

$$\begin{aligned} V + UP &= Y - QN + XND^{-1} + QDD^{-1}N \\ &= Y - (YD - I)D^{-1} \\ &= D^{-1} \end{aligned} \quad (42)$$

The input-output relations from (w, u_2) to (u_1, y) in Fig.2 are given by

$$\begin{bmatrix} u_1 \\ y \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_2 \\ w \end{bmatrix}$$

where

$$\begin{aligned} G_{11}(s) &= (I + V + UP)^{-1}(V + UP - I) \\ G_{12}(s) &= 2(I + V + UP)^{-1}VR \\ G_{21}(s) &= -(I + V + UP)^{-1}P \\ G_{22}(s) &= (I + V + UP)^{-1}PVR \end{aligned}$$

Let us review the following definition [12].

Definition:

(1) L_{2e} is the extended space of vector valued functions, $x(t)$, with the property

$$\|x(t)\|_T \equiv \left[\int_0^T (x^*(t)x(t))dt \right]^{\frac{1}{2}} < \infty, T \geq 0 \quad (43)$$

(2) Given $N: L_{2e} \rightarrow L_{2e}$ and LTI operators C and R , N is said to be inside $\text{Cone}(C, R)$ if

$$\|N(x) - Cx\|_T \leq \|Rx\|_T, T \geq 0, x \in L_{2e} \quad (44)$$

As stated above, in Fig.3 ϕ is an operator in the set ϕ_a

$$\phi_a \equiv \{\phi \mid \phi \in \text{Cone}(0, 1)\}$$

Using the result of [12], we have the following Lemma as a sufficient condition for nonlinear stability in the specified cone.

Lemma: In the case of G being an LTI operator with transfer function $G(s)$, the system in Fig.3 is stable for all $\phi \in \phi_a$ if

1. $G(s)$ is stable
2. $\inf_{T' \in \mathcal{T}} \|T'G_{11}(s)T'^{-1}\|_{\infty} \leq 1$

where

$$\mathcal{T} \equiv \{T' \mid T'\phi T'^{-1} \in \phi_a\}$$

For the simplification, instead of \mathcal{T} , the following set can be used.

$$\mathcal{T}' \equiv \{T' \mid T' \in \mathcal{T} \text{ and } T' \in C^{1 \times 1}\} \quad (45)$$

However, when Fig.3 is the equivalent diagram of Fig. 1, from (38) u_1 should be bounded. Then a strongly stable controller is necessary for ensuring the bounded controller output at the zero operator, $\sigma = 0$ and at $B(s) = 0$. Namely, for guaranteeing global stability of the system with non-minimum phase of Fig.1, we must have $S(s) = V^{-1}U$, P, V and R stable, where $S(s)$ is the controller for the process without input constraints (Fig.2).

Theorem: The closed-loop system described in Fig.3 is stable in the presence of ϕ , if

1. $S(s)$ is stable
2. $G_{11}(s) \in RH_{\infty}$
3. $MD(s)M^{-1}$ is strictly positive real

where M is positive definite diagonal matrix.

Proof: From Fig.1, when no input constraint occurs, we have

$$\begin{aligned} u_1 &= VRw - Uy + (I - V)u \\ &= u \end{aligned} \quad (46)$$

$$u = Rw - V^{-1}Uy = Rw - Sy \quad (47)$$

The system (Fig.2) is stable [10]. That is, $R(s)$ and $S(s)$ are stable and u_1 is bounded. For the case of non-minimum phase processes with input constraint, from Fig.1, u is bounded by input constraint, $P(B(s) \neq 0)$, U , V and R are stable, we get that u_1 is bounded. Especially, at $B(s) = 0$, V and R are stable, we also get that u_1 is bounded. Further, we can show that if $MD(s)M^{-1}$ is strictly positive real and $G_{11}(s) \in RH_{\infty}$,

$$\begin{aligned} MG_{11}(s)M^{-1} &= M(I - D)(I + D)^{-1}M^{-1} \\ &= (I - MDM^{-1})(I + MDM^{-1})^{-1} \end{aligned}$$

That is,

$$\|MG_{11}M^{-1}\|_{\infty} < 1 \quad (48)$$

This fact leads to the desired result based on the Lemma. Remark: This paper considers the case of SISO. However, the above proof can be extended to the MIMO case.

IV. PERFORMANCE ANALYSIS OF THE PROPOSED SYSTEM

In this section, the analysis of input constraint for control performance is discussed. First, when no input constraint occurs, we have that the system is strongly stable. Then,

$$y = \frac{Pg}{D^{-1}}w \quad (49)$$

Next, in the process with input constraint, we obtain the following input-output relations for the former design method [10] and the proposed method.

1) Former method with input constraint

$$y = \frac{PR}{2I + V^{-1}PU}w - \frac{P}{2I + V^{-1}PU}u_2$$

$$= \frac{Pg}{V + D^{-1}}w - \frac{PV}{V + D^{-1}}u_2 \quad (50)$$

From (6), the design parameter $Q(s)$ in $V(s)$ affects the system performance. For reducing the influence of $V(s)$, we have to reduce the gain of $V(s)$. However, from the definition of $S(s)$, this may give a high gain controller and may reduce the robustness of the control system in some cases. The selection of $Q(s)$ is also limited for ensuring the stability of the zero of $V(s)$.

2) The proposed method with input constraint

$$y = \frac{PVR}{I + V + PU}w - \frac{P}{I + V + PU}u_2$$

$$= \frac{Pg}{I + D^{-1}}w - \frac{P}{I + D^{-1}}u_2 \quad (51)$$

If only D^{-1} is large by selecting adequate coprime factorization in a sense, that is, the gain of the closed-loop characteristic polynomial is large, the output under input constraint can be expected to track the output in (49).

For adjusting parameter $Q(s)$, a simple and quantitative design method is given as follows. The parameter $Q(s) = q$ is chosen such that the following polynomial is a Hurwitz polynomial.

$$T[s](C[s] + G_0[s]) - qB[s]C[s] = 0 \quad (52)$$

V. NUMERICAL EXAMPLE

The purpose of this simulation is to compare the former and the proposed methods and to demonstrate the benefit of the proposed method when input constraint exists. Simulation study is conducted using the following non-minimum phase process.

$$A[s] = s^2 + 0.7s + 0.1 \quad (53)$$

$$B[s] = -0.2s + 1 \quad (54)$$

$$C[s] = s + 0.2 \quad (55)$$

The design parameters of the former method are shown in Table 1. From (10) and (11),

$$X(s) = \frac{0.0892s + 0.0178}{s + 0.2} \quad (56)$$

$$Y(s) = \frac{s - 0.0354}{s + 0.2} \quad (57)$$

The control law of the former method is given by

$$R(s) = \frac{0.07104s^3 + 0.04595s^2 + 0.01143s + 0.001016}{s^3 + 0.2115s^2 + 1.016s + 0.1975}$$

$$S(s) = \frac{-0.9108s^3 - 0.8423s^2 - 0.2257s - 0.01873}{s^3 + 0.2115s^2 + 1.016s + 0.1975}$$

By selecting q so that (52) is stable, we obtain $-1.0816 < q < -0.0125$.

CGPC	Reference input: $w = 1$ Predictor order: $N_y = 6$ Control horizon: $N_u = 1$ Control weighting: $\lambda = 0.1$ Min. prediction horizon: $T_1 = 0$ Max. prediction horizon: $T_2 = 10$ Design parameters: $u_n = -1; u_d = 1$
Str. stable	Closed-loop characteristic poly.: $T_0(s) = s^2 + 0.4468s + 0.0715$ The unstable pole of the CGPC [9]: 0.0354 The poles of the former method [10] $-0.0082 \pm 1.0062i$ -0.1950

Table 1 Design parameters

The control law proposed in this paper is given by

$$U(s) = \frac{0.0892s + 0.0178}{s + 0.2} - \frac{s^2 + 0.7s + 0.1}{s^2 + 0.4468s + 0.0715}$$

$$V(s) = \frac{s - 0.0354}{s + 0.2} + \frac{-0.2s + 1}{s^2 + 0.4468s + 0.0715}$$

In the case of input constraint being present ($u_{max} = 0.1; u_{min} = 0$), Fig.4 shows the process output (dashed line) of the former method under $Q(s) = -1$ and the process output (solid line) for the same conditions using the proposed method. Meanwhile, Fig.5 shows the process input (dashed line) of the former method and the process input (solid line) of the proposed method, Fig.6 shows the process input (dashed line) of the former method prior to input constraint part and the process input (solid line) for the same conditions using the proposed method. Comparing the preceding simulation results in Fig.4, the proposed control algorithm shows a better tracking performance. Also, better tracking performance is obtained on same conditions for the case of $Q(s) = -0.5$ (Fig.7). From the above simulations, the design parameter $Q(s)$ does not affect the proposed system performance.

VI. CONCLUSIONS

In this paper, a design problem of a continuous-time anti-windup generalized predictive control system using coprime factorization approach for non-minimum phase processes with input constraints was considered. Under the existence of input constraint, a condition for the closed-loop stability and a straightforward method to improve the output performance of the system are given. The effectiveness of the proposed method is also

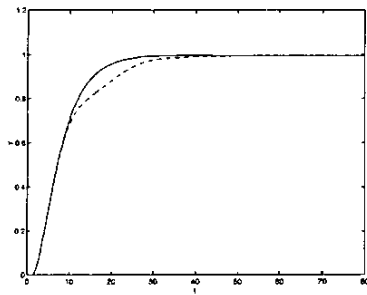


Fig. 4. Process output

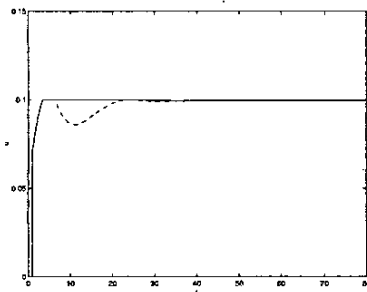


Fig. 5. Process input

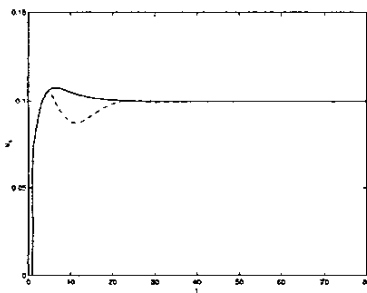


Fig. 6. Process input prior to input constraint part

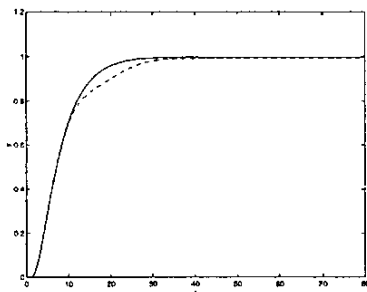


Fig. 7. Process output

confirmed through simulations. The further work will be on the control system design for non-minimum phase uncertain processes with input constraints. Meanwhile, disturbance rejection will be discussed.

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