Physics

Electricity & Magnetism fields

Okayama University *Year* 1994

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On the Continuity of the Magnetizing Current Density in 3-D Magnetic Field Analysis with Edge Element

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Abstract **– The effects of the continuity of the magnetizing current density on the convergence of the incomplete Cholesky conjugate gradient method and the accuracy of the calculated flux densities are investigated by imposing different continuity conditions for both nodal and edge elements. It is shown that the continuity condition should be imposed precisely in the case of edge elements.**

I. Introduction

Recently, the edge element [1-5] has drawn the attention of many researchers. When applying the edge element to the analysis of magnetic fields in electrical machines, it was found accidentally that the ICCG method [6], which is an iterative solver for linear equations, is considerably more sensitive to the continuity of the magnetizing current density. In fact, the ICCG solver provides no convergent solution in the case of the edge element, if the current continuity is not sufficient [7].

In this paper, the effects of the continuity of the magnetizing current density on the convergence of the ICCG method and the accuracy of the calculated flux densities are investigated by imposing different continuity conditions. A method for imposing exactly the continuity condition in a winding of complicated shape, is also discussed, in which the current vector potential is introduced [8,9].

II. Significance of Continuity of Magnetizing Current Density

Fig. 1 shows the 3-D model used for investigation. The whole region is discretized into brick elements. In the straight parts except the corners shaded in the coil, the magnetizing current density $\boldsymbol{J_O}$ has only one (x- or y-) component, and satisfies the following continuity condition :

$$\mathrm{div}\,\boldsymbol{J_O} = 0\,. \tag{1}$$

In order to examine the effect of the continuity condition, the distribution of $\boldsymbol{J_O}$ is changed in the corners as shown in Table I, in which the cases (a) and (b) satisfy (1) while the cases (c)-(e) do not satisfy (1). When the edge element is used, the x-

Manuscript received July 6, 1994.

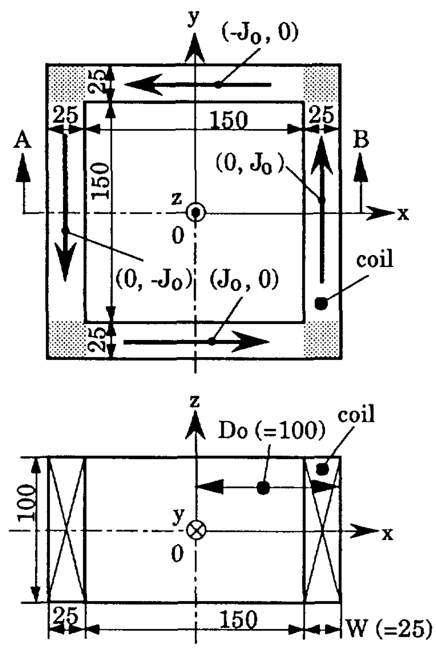
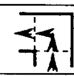
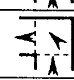


Fig.1 Model for investigation.

Table I Distribution of magnetizing current density in corner

case			J_{ox}	J_{oy}	div $\boldsymbol{J_O}$	current distribution
a			$-\dfrac{J_0(D_o-x)}{W}$	$\dfrac{J_0(D_o-y)}{W}$	= 0	
b	nodal	x < y	$-J_0$	0	= 0	
		x = y	$-J_0/2$	$J_0/2$		
		x > y	0	J_0		
	edge	x ≤ y	$-J_0$	0	= 0	
		x ≥ y	0	J_0		
c			$-\dfrac{J_0}{\sqrt{2}}$	$\dfrac{J_0}{\sqrt{2}}$	≠0	
d			$-\dfrac{J_0}{2}$	$\dfrac{J_0}{2}$	≠0	
e			0	0	≠0	

J_0 : magnitude of magnetizing current density in straight parts except corners shaded in Fig.1

and y-components of $\boldsymbol{J_o}$, J_{ox} and J_{oy}, are specified on edges parallel to the x- and y-axes respectively in all cases. When the nodal element is used, they are specified at each node in the cases (a) and (c)-(e). In the case (b) for the nodal element, they are specified at the center of gravity in each element. In the elements located on a diagonal line at $x=y$, J_{ox} and J_{oy} are equal to $-Jo/2$ and $Jo/2$ respectively, where J_o is the magnitude of $\boldsymbol{J_o}$ in the straight parts. The magnitude of $\boldsymbol{J_o}$ in those elements is equal to $Jo/\sqrt{2}$. In the other regions of $x>y$ and $x<y$, (J_{ox}, J_{oy}) is equal to $(0, J_o)$ and $(-J_o, 0)$ respectively. Such distributions of $\boldsymbol{J_o}$ can be obtained by introducing the current vector potential $\boldsymbol{T_o}$ [8]. A method for calculating the distribution of $\boldsymbol{J_o}$ from $\boldsymbol{T_o}$ is discussed later.

Fig. 2 shows the distributions of the flux density vectors computed by the ungauged magnetic vector potential formulation [7]. The vectors on the x-z and y-z planes are plotted. Only the distributions for the cases (b) ($\mathrm{div}\boldsymbol{J_o}=0$) and (c) ($\mathrm{div}\boldsymbol{J_o}\neq 0$) are illustrated, because the distributions of the cases (a) and (b), and the cases (c)-(e) show a similar tendency in each group. When $\mathrm{div}\boldsymbol{J_o}\neq 0$, linear equations for the edge element are solved by Gaussian elimination instead of the ICCG method, because convergent solutions cannot be obtained by the ICCG method. In the case of nodal elements, however, the convergent solutions are obtainable for all cases by the ICCG method. When the nodal element is used, the distributions with $\mathrm{div}\boldsymbol{J_o}\neq 0$ are similar to those with $\mathrm{div}\boldsymbol{J_o}=0$. It is obvious that the distribution with $\mathrm{div}\boldsymbol{J_o}\neq 0$ obtained with edge elements has no physical meaning as shown in Fig. 2(ii).

Fig. 3 shows the z-component B_z of the flux density along the z-axis. B_z at z=0mm obtained with edge elements is normalized to unity. The results for cases (a) and (b) satisfying (1) are almost the same. The results for cases (c)-(e) ($\mathrm{div}\boldsymbol{J_o}\neq 0$) are different from those for cases (a) and (b). The discrepancy between the result with $\mathrm{div}\boldsymbol{J_o}=0$ and that with $\mathrm{div}\boldsymbol{J_o}\neq 0$ depends on the size of the discontinuous region.

III. Method for Calculating Current Distribution

A method for imposing exactly the continuity condition of the magnetizing current density, has been discussed in [8,9]. $\boldsymbol{J_o}$ can be written in terms of the current vector potential $\boldsymbol{T_o}$:

$$\boldsymbol{J_o} = \mathrm{rot}\, \boldsymbol{T_o} \,. \tag{2}$$

$\boldsymbol{T_o}$ is defined only in the region of the windings.

When the eddy current in the winding can be neglected, the electric field intensity $\boldsymbol{E_o}$ satisfies the following equation:

$$\mathrm{rot}\, \boldsymbol{E_o} = \boldsymbol{0} \tag{3}$$

The governing equation for $\boldsymbol{T_o}$ is obtained from (2), (3) and Ohm's law ($\boldsymbol{E_o}=\rho\boldsymbol{J_o}$, ρ : resistivity) as follows:

$$\mathrm{rot}\,(\,\rho\, \mathrm{rot}\, \boldsymbol{T_o}\,) = \boldsymbol{0}\,. \tag{4}$$

Equation (4) has the same expression as the governing equation for the $\boldsymbol{A}$ method, which uses the magnetic vector potential $\boldsymbol{A}$ and the reluctivity ν instead of $\boldsymbol{T_o}$ and ρ. Therefore, it is not necessary to develop a new code.

Dirichlet boundary conditions, which are required to solve (4), can be determined easily from the following equation:

$$I = \iint_S \boldsymbol{J_o} \bullet d\boldsymbol{S} = \oint_c \boldsymbol{T_o} \bullet d\boldsymbol{s}\,, \tag{5}$$

The current I passing through the area S surrounded by the closed loop c in the winding can be represented by a function of $\boldsymbol{T_o}$. When the scalar variable T_o is defined as a line integration of a projection of $\boldsymbol{T_o}$ on an edge [4,5], the current I can be written as the summation of T_o's. Equation (5) for the current $I_1^{(e)}$ shown in Fig. 4 is given by

$$-T_{o5e} - T_{o12e} + T_{o6e} + T_{o9e} = I_1^{(e)}. \tag{6}$$

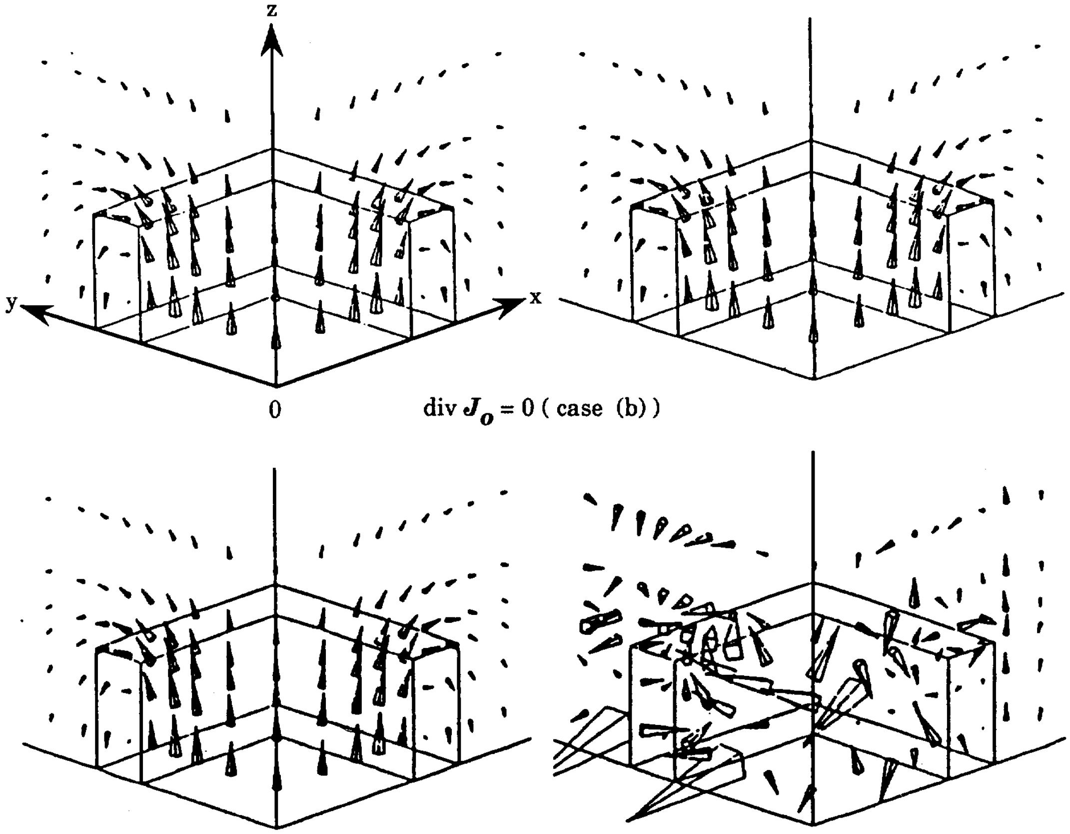
When $I_i^{(e)}$ passing through a facet i in an edge element (e) is specified, Dirichlet boundary conditions can be calculated.

If the currents in all facets are specified, all T_o's can be determined. In such a case, it is not necessary to solve (4).

IV. Example of Application

In order to illustrate the effectiveness of the method mentioned above, the current distribution in a practical model shown in Fig. 5 is calculated, which is used as a magnetizing winding in a flat motor. The thickness and the number of turns of the winding are 0.5mm and 30 respectively. The current densities are nearly uniform, because the winding is composed of thin conductors. As the driving torque is calculated from $\boldsymbol{J_o}\times\boldsymbol{B}$ ($\boldsymbol{J_o}$: magnetizing current density, $\boldsymbol{B}$: flux density), the current distribution should be imposed with high accuracy. As the winding is assumed to be a massive conductor, the current distribution is considerably affected by the electric path length.

Table II shows the boundary conditions. The boundaries are classified in two groups ($I\neq 0$ and



div $J_o \neq 0$ (case (c))

(i) nodal element (ii) edge element

Fig.2 Distributions of flux density vectors.

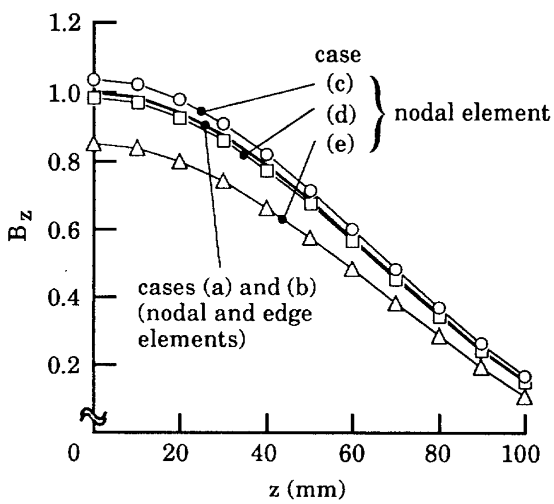


Fig.3 Z-component of flux density along z-axis.

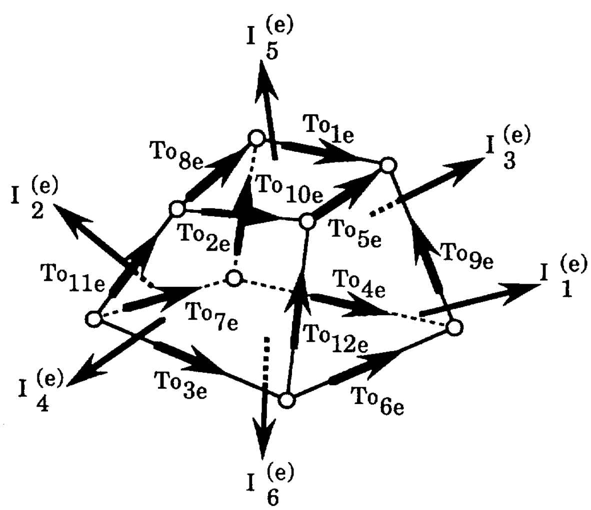


Fig.4 Definitions of T_0 and I.

I = 0) by the current passing through the boundaries. $\boldsymbol{J_o}$ is parallel to the boundaries with I = 0. T_0's on all boundaries are prescribed from (6). Under the boundary condition B, T_0's on the internal boundaries (α_2-β_2, α_3-β_3, α_4-β_4, α_6-β_5 and α_6-β_6) are given so that the current flow can approach to the actual pattern.

Fig. 6 shows the current distribution. The distribution for the boundary condition B is much more uniform than that for the condition A because the condition B puts the additional Dirichlet boundaries inside the model as shown in Fig. 5 and Table II.

V. CONCLUSIONS

The results obtained can be summarized as follows :

(1) When edge elements are used, the continuity condition should be imposed rigorously. Otherwise, the ICCG method cannot provide a convergent solution. If Gaussian elimination method is applied to such a case, the obtained magnetic field distribution has no physical meaning.

(2) The nodal element is not sensitive to the continuity condition. Even in the case when the continuity of the magnetizing current density is not sufficient, a convergent solution can be obtained. Of course, if the discontinuous region is wide, the solution has no meaning.

(3) The uniform current distribution can be obtained easily by putting Dirichlet boundaries at several cross sections inside the winding of complicated shape.

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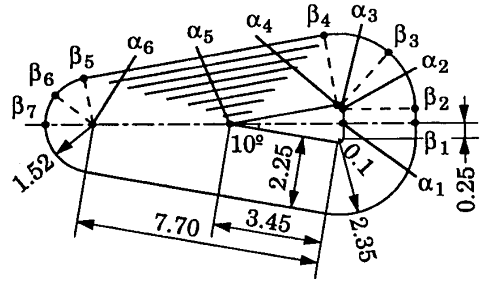
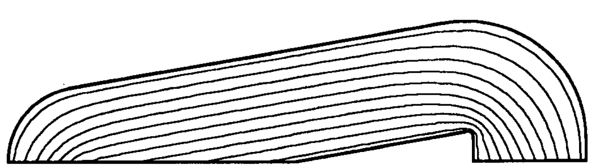


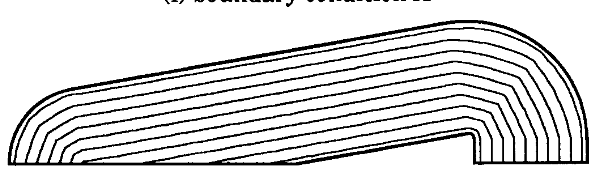
Fig.5 Winding used in flat motor.

Table II Boundary conditions

boundary condition	current passing through boundary	
	I ≠ 0	I = 0
A	$\alpha_1-\beta_1$, $\alpha_5-\alpha_6$, $\alpha_6-\beta_7$	$\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5$, $\beta_1-\beta_2-\beta_3-\beta_4-\beta_5-\beta_6-\beta_7$
B	$\alpha_1-\beta_1$, $\alpha_2-\beta_2$, $\alpha_3-\beta_3$, $\alpha_4-\beta_4$, $\alpha_5-\alpha_6$, $\alpha_6-\beta_5$, $\alpha_6-\beta_6$, $\alpha_6-\beta_7$	



(i) boundary condition A



(ii) boundary condition B

Fig.6 Distributions of magnetizing current.