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Acceleration of convergence characteristic of the ICCG method

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Acceleration of Convergence Characteristic of the ICCG Method

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Abstract – The effectiveness of renumbering for the incomplete Cholesky conjugate gradient solver, which is usually applied to direct solvers, is examined quantitatively by analyzing 3-D standard benchmark models. On an acceleration factor which is introduced to obtain convergence quickly, indices for determining the optimum value of the acceleration factor, which minimizes the number of iterations, are discussed. It is found that the renumbering is effective to use with the ICCG solver, and the solver using the acceleration factor gives a good convergence characteristic even in the case when the conventional solver fails to provide convergent solutions.

I. INTRODUCTION

In 3-D electromagnetic field analysis using the finite element method, the CPU time for solving linear equations is dominant in the total CPU time. Therefore, an acceleration of the ICCG solver [1,2] is significant in reducing the total CPU time. The acceleration can be obtained by reducing the number of iterations by improving preconditioning.

The improvement in preconditioning can be carried out by renumbering unknown variables [3]. It is often said that there is no total timesaving, especially in the 3-D computation which requires much CPU time, because the renumbering [4,5] is more time-consuming than the reduction in the solution time due to the renumbering. However, it has not been clarified quantitatively how the renumbering can reduce the number of iterations for the ICCG method, and how much time is required for the renumbering.

T.A.Manteuffel succeeded in improving the preconditioning by introducing an acceleration factor. However, the optimum value of the acceleration factor, which minimizes the number of iterations, was determined by trial and error [6].

In this paper, the effect of the renumbering on the number of iterations for the ICCG method is examined quantitatively by analyzing several standard benchmark models for 3-D magnetic field computations. Furthermore, a method for automatically determining the optimum acceleration factor is investigated, and various factors (a gauge condition, types of elements, etc.) affecting the acceleration factor are discussed.

II. IDEAS FOR IMPROVING CONVERGENCE CHARACTERISTIC

The number of iterations for the ICCG method can be reduced when a preconditioning matrix approaches the original coefficient matrix. Ideas for improving the preconditioning are explained.

A. Renumbering

When the incomplete Cholesky decomposition, which is available for a symmetric coefficient matrix, is applied, the preconditioning matrix LDL^T obtained is different from the original matrix H as follows:

$$LDL^T = H + E \qquad (1)$$

where E is a matrix which describes the difference between the incomplete and complete decompositions. All non-zero entries in H are decomposed.

If LDL^T approaches H by reducing E, the convergence characteristic can be accelerated. The following objective function W is defined as a useful index which shows the amplitude of E.

$$W = \sum_{i=1}^{n_u} \sum_{j=1}^{n_u} E_{ij}^2 \qquad (2)$$

where E_{ij} is an entry at a row number *i* and a column number *j* in E, and n_u is the number of unknown variables. The entries of $E_{ij} \neq 0$ appear at positions of fill-ins when an acceleration factor γ described later is equal to 1, and also at those of the diagonal entries H_{ii} of the matrix H when $\gamma \neq 1$. In the case of the complete decomposition, W is equal to zero because all E_{ij}'s are equal to zero. When the renumbering [5,6] is applied, the preconditioning matrix is improved because the number of fill-ins is reduced.

B. Acceleration Factor

As shown in (3), an acceleration factor γ is introduced into the incomplete decomposition [6-8].

$$L_{ij} = \begin{cases} \gamma H_{ii} - \sum_{k=1}^{i-1} L_{ik}^2 D_{kk} & (i = j) \qquad (3) \\ H_{ij} - \sum_{k=1}^{j-1} L_{ik} L_{jk} D_{kk} & (i > j) \qquad (4) \end{cases}$$

$$D_{ii} = 1 / L_{ii} \qquad (5)$$

where L_{ij} and D_{ii} are entries in L and D respectively. The case of $\gamma = 1$ corresponds to the conventional ICCG method.

From (1) and (3)-(5), it is noted that E_{ij} shown in (2) is a function of γ, and hence can be controlled by γ. Therefore, the acceleration factor can improve the preconditioning matrix.

III. CONVERGENCE CHARACTERISTICS

The convergence characteristics are examined by solving the basic models.

A. Models Analyzed and Method of Analysis

The effects of the renumbering and the acceleration factor γ on the number of iterations for the ICCG method are investigated by analyzing several 3-D standard benchmark models i.e. the linear magnetostatic and ac steady state linear

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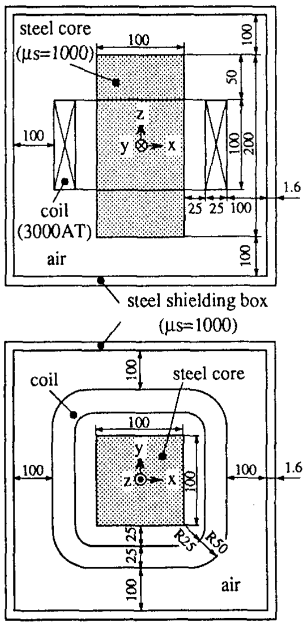
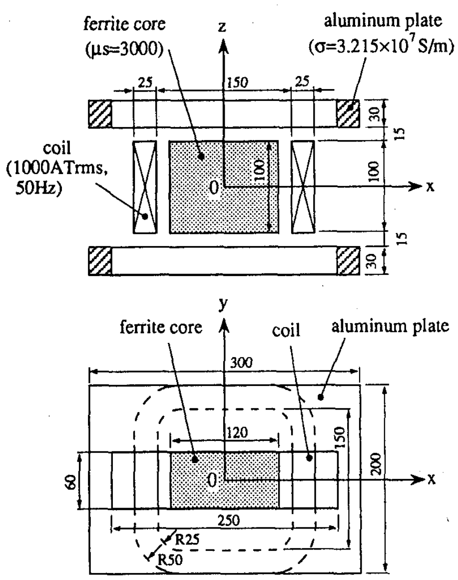
eddy current models [9,10] proposed by the Institute of Electrical Engineers of Japan (IEEJ) and shown in Fig.1. The coefficient matrices of the respective models are real and complex. The finite element method using the magnetic vector potential is applied. Both edge and nodal elements are investigated. All meshes used are regularly subdivided. The calculations are performed in double precision using the NEC SX-1E supercomputer whose maximum speed is 285 MFLOPS.



(a) magnetostatic model
(real coefficient matrix)



(b) eddy current model
(complex coefficient matrix)

Fig.1 IEEJ standard benchmark models.

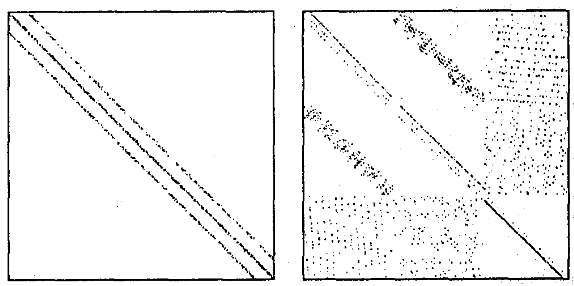
Table I shows the discretization data for the case using the brick element and without the gauge condition. The number of unknowns for the edge element is nearly the same as that for the nodal element. However, the number of non-zero entries in the original coefficient matrix H for the edge element is much less than that for the nodal element. The number of non-zero entries in E with $\gamma = 1$ defined in (1) and the size of profile are also shown in Table I. They are discussed in the next section.

B. Investigation of Renumbering Method

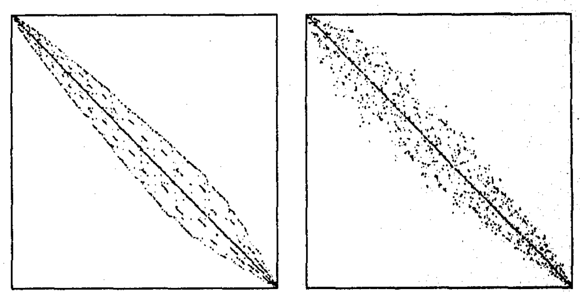
1) Distribution of non-zero entries in coefficient matrix: Fig.2 shows the distribution of non-zero entries obtained by various renumbering methods for the magnetostatic model discretized by the brick edge element. A non-zero entry is illustrated by a dot (·) symbol. REG and RAN mean that the unknown variables are renumbered regularly and at random. REG is available for only the regularly subdivided mesh. RCM and GPS mean the reverse Cuthill-McKee and Gibbs-Poole-Stockmeyer renumbering algorithms [4,5] were used

Table I Discretization data

model		magnetostatic		eddy current	
element		edge	nodal	edge	nodal
number of elements		13,750		14,400	
number of nodes		15,548		16,275	
number of unknowns		39,072	38,496	41,060	42,885
number of non-zero entries in H		621,852	1,434,642	653,718	1,734,684
number of non-zero entries in E with $\gamma = 1$	RAN	1,427,148	3,700,806	1,493,876	4,608,958
	REG	799,950	1,760,485	842,514	2,151,768
	RCM	1,093,633	1,894,520	1,150,437	2,274,856
	GPS	875,701	1,721,295	921,045	2,080,604
size of profile	RAN	547,755,119	506,771,474	584,806,256	645,176,955
	REG	46,766,858	68,560,372	56,574,742	94,497,537
	RCM	83,517,245	97,494,507	78,214,869	95,579,749
	GPS	71,821,938	100,994,751	67,279,953	90,836,142



(a) regular (REG) (b) random (RAN)



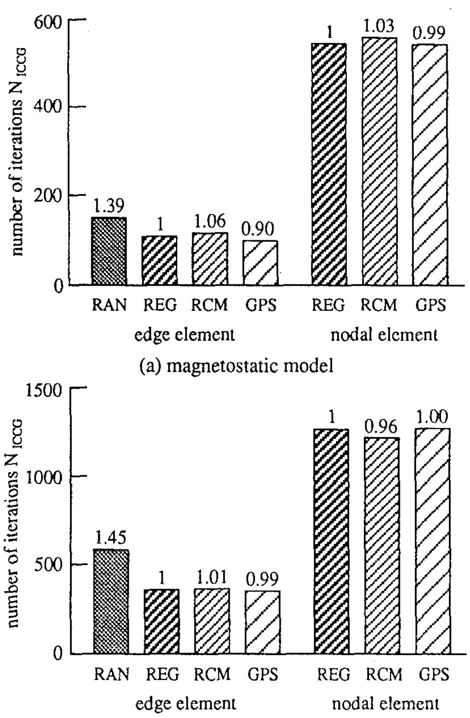
(c) reverse Cuthill-McKee (RCM) (d) Gibbs-Poole-Stockmeyer (GPS)

Fig.2 Positions of non-zero entries in coefficient matrix (magnetostatic model, edge element).

respectively. When RCM and GPS are applied, the non-zero entries are concentrated around diagonal entries. In the case of the eddy current model, a similar distribution is obtained. The nodal element also gives a similar distribution.

The number of non-zero entries in the original coefficient matrix H shown in Table I is identical for all renumbering methods. The size of the profile, which is defined as the summation of the half bandwidth of each row, is different for each renumbering method. When a direct solver such as the Gaussian elimination method is applied, the size of the matrix H is approximately equal to that of the profile. Therefore, the direct solver requires an extremely large memory space compared with the ICCG solver.

2) Effect of renumbering methods on convergence characteristic: Fig.3 shows the effect of the renumbering methods on the number of iterations for the ICCG method. The acceleration factor γ of 1.2 is used, which is discussed in the next section. The number of iterations using RCM and GPS is reduced by about 40% compared with that using RAN, and are nearly the same as that using REG. The reason is, as shown in Table I, the number of non-zero entries in the matrix E, which describes the difference between the incomplete and complete decompositions and corresponds to the number of fill-ins, are much less for REG, RCM and GPS than for RAN.



(a) magnetostatic model

(b) eddy current model

Fig.3 Effect of renumbering method (γ = 1.2).

Table II CPU time (s) for renumbering and ICCG method (γ = 1.2)

model		magnetostatic		eddy current	
element		edge	nodal	edge	nodal
renumbering	RCM	3.9	8.6	4.1	9.2
	GPS	4.8	9.6	5.1	10.3
ICCG	RCM	87.1	686.5	542.4	3420.7
	GPS	77.2	679.7	531.3	3605.7

Table II shows the CPU times for the renumbering method and the ICCG solver. As much CPU time is required to find a starting node in the original RCM [4], a technique developed in GPS [5] is adopted to find the starting node quickly. The CPU time for the renumbering method is negligibly small compared with that for the ICCG solver. Therefore, it may be concluded that the renumbering is effective to use with the ICCG solver.

C. Investigation of Acceleration Factor

1) Determination of acceleration factor: Fig.4 shows the number of iterations N_{ICCG} for the ICCG method. The optimum value γ_{opt} of γ, which minimizes N_{ICCG}, strongly depends on the type of element used. When γ is less than γ_{opt}, N_{ICCG} is greatly increased. Therefore, the conventional incomplete decomposition (γ=1) should not be used.

Fig.5 shows the objective function W defined in (2). The W-γ curves have forms similar to the N_{ICCG}-γ curves shown in Fig.4. The W-γ curves, however, are not enough to estimate γ_{opt}, because in the case of the nodal element, a great number of iterations is needed for γ which minimizes W as shown in Fig.4.

The maximum value $(L_{ii})_{max}$ of the diagonal entries in L is shown in Fig.6. The γ which minimizes $(L_{ii})_{max}$ is nearly the same as γ_{opt}. Therefore, γ_{opt} can easily be determined by finding the minimum value of $(L_{ii})_{max}$.

2) Effect of acceleration factor on convergence characteristic: Fig.7 shows the effect of the gauge conditions on the number of iterations. The magnetostatic model is analyzed using RAN and REG methods, shown in Fig.2. In the case without gauge, γ=1 with RAN gives the converged result shown in Fig.7(a). In the case with gauge, however, both renumbering methods fail to provide convergent solutions at γ=1, as shown in Fig.7(b).

Fig.8 shows the effect of the types of elements on the number of iterations. Tetrahedral and brick nodal elements are

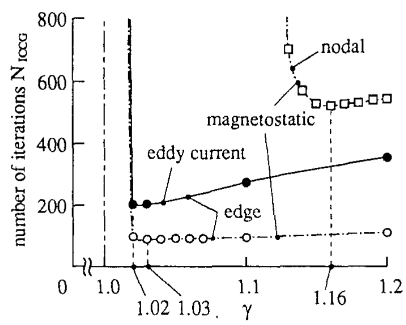


Fig.4 Number of iterations for ICCG method (REG).

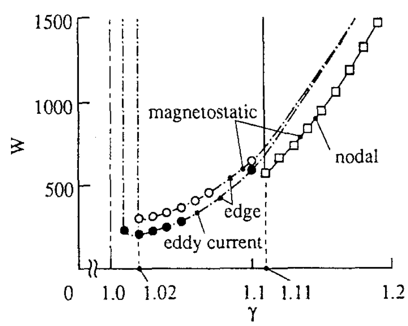


Fig.5 Error index W (REG).

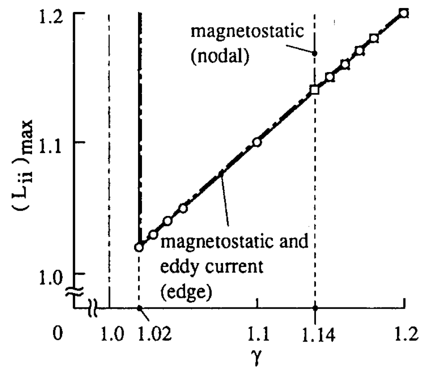
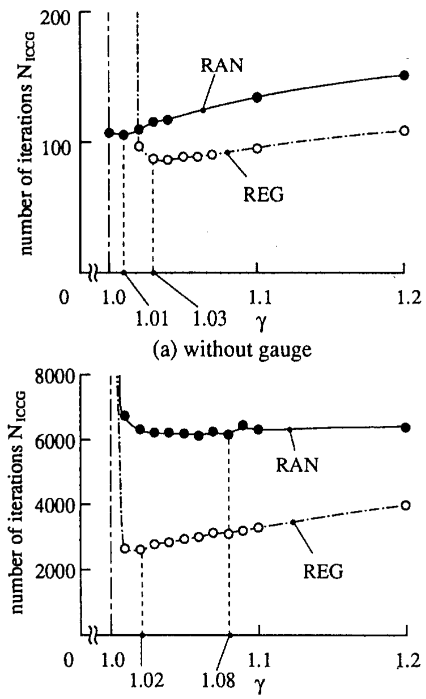


Fig.6 Maximum value among diagonal entries in lower triangular matrix (REG).



(a) without gauge

(b) with gauge

Fig.7 Effect of gauge conditions on number of iterations for ICCG method (magnetostatic model, edge element).

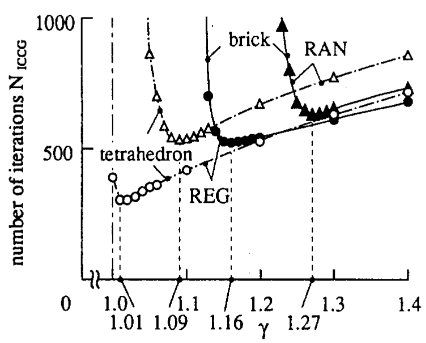


Fig.8 Effect of types of elements on number of iterations for ICCG method (magnetostatic model, nodal element).

compared. The unknown variables for both elements are the same because the tetrahedral mesh is generated by subdividing a brick into six tetrahedrons. γ_{opt} depends on the types of elements and the renumbering methods.

IV. CONCLUSIONS

In order to accelerate the convergence characteristic of the ICCG solver, improvements by renumbering and the acceleration factor were investigated. The results obtained are summarized as follows:

(1) As the renumbering can decrease the solution time, it is recommended to execute the renumbering before solving problems.

(2) The acceleration factor is robust and gives a good convergence characteristic, and its optimum value can be easily determined.

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