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PERIODIC BOUNDARY CONDITION FOR 3-D MAGNETIC FIELD ANALYSIS AND ITS APPLICATIONS TO ELECTRICAL MACHINES

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ABSTRACT

A periodic boundary condition in 3-D analysis has been examined. The definition of 3-D periodic phenomena is clarified, and the periodic conditions for $A-\phi$ method are established. The usefulness of the periodic boundary conditions is shown by analyzing some models and observing that the CPU time is considerably decreased.

1. INTRODUCTION

The introduction of a periodic boundary condition in 3-D magnetic field analysis is especially important in order to reduce the computer storage and the CPU time[1]. The periodic boundary condition in 3-D analysis, however, has not been examined systematically until now. The reasons why it has not been examined are as follows:

(a) The definition of 3-D periodic phenomena has been obscure.

(b) Though the periodic phenomena are defined by the flux and current distributions, the boundary conditions for the periodic phenomena must be given by the vector potential A and the electric scalar potential ϕ or the current vector potential $\mathbb{T}$ and the magnetic scalar potential Ω.

(c) The periodic conditions of A and ϕ or $\mathbb{T}$ and Ω cannot be directly obtained from $\mathbb{B}$ and $\mathbb{J}$, because the A or $\mathbb{T}$ vector at a point cannot be directly obtained from $\mathbb{B}$ and $\mathbb{J}$ at the point, and the scalar quantity ϕ or Ω cannot be imagined from the vector quantities $\mathbb{B}$ and $\mathbb{J}$.

In this paper, firstly, the periodic boundary conditions in 3-D analysis using the $A-\phi$ method are clarified by investigating the above-mentioned problems. Secondly, the usefulness of the periodic boundary conditions is shown by examing the CPU time when the periodic conditions are applied. Lastly, the periodic conditions are applied to the analysis of an actual induction motor.

2. PERIODIC BOUNDARY

2.1 Definition of 3-D periodic phenomena

In usual rotating machines, the flux and current are distributed periodically in the 2-D plane, even if the construction of the machine and the flux and current distributions are 3-dimensional. In this case, as the flux and current distributions are point symmetric, there is a center line. The case when the center line is a straight line and is perpendicular to 2-D plane on which the phenomena are periodic is considered.

The definition of the periodic phenomena in 3-D magnetic field is as follows:
$\mathbb{B}$ and $\mathbb{J}$ vectors have three components and are distributed periodically on a flat plane which is perpendicular to the center line of symmetry.

Let us consider two points p and q on a plane Spq as shown in Fig.1. The line o-o_1 is perpendicular to the plane Spq and it is the center line of symmetry. The points p and q are chosen so that the distance between two points o and p is equal to that between o and q, and the angle between two lines o-p and o-q is equal to θ on the plane Spq. Next, let us consider the plane Sp including the point p and the center line and the plane Sq including the point q and the center line, which are perpendicular to the plane Spq. Sp coincides with Sq when Sp is rotated by the angle θ along the line o-o_1.

The definition of the periodic boundary in 3-D magnetic field analysis is as follows:
In order to define periodic boundaries, 1/2 period of the change of phenomena is considered. If the flux densities $\mathbb{B}p$ and $\mathbb{B}q$ and current densities $\mathbb{J}p$ and $\mathbb{J}q$ at the points p and q satisfy the following equations, the planes Sp and Sq are called the "periodic boundaries".

$$B_{px} = -B_{qx'} \tag{1}$$

$$B_{py} = -B_{qy'} \tag{2}$$

$$B_{pz} = -B_{qz'} \tag{3}$$

$$J_{px} = -J_{qx'} \tag{4}$$

$$J_{py} = -J_{qy'} \tag{5}$$

$$J_{pz} = -J_{qz'} \tag{6}$$

Where Bpx, Bpy, Bpz and Jpx, Jpy, Jpz are the x-, y-, and z-components of $\mathbb{B}p$ and $\mathbb{J}p$ respectively. Bqx', Bqy', Bqz' and Jqx', Jqy', Jqz' are the x'-, y'- and z'-components of $\mathbb{B}q$ and $\mathbb{J}q$ respectively. x', y' and z' are the local coordinates and the angle between x- and x'-axes is equal to θ.

The planes Sp and Sq may also be curved as shown in Fig.2. The points p' and q' which are on the plane Spq should satisfy the same conditions as those mentioned above for the points p and q.

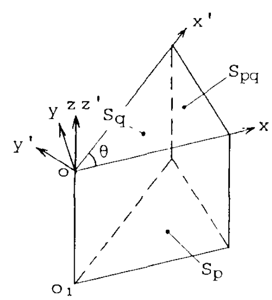

Fig.1 Periodic boundaries.

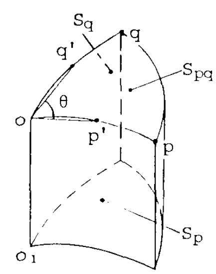

Fig.2 Curved periodic boundaries.

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2.2 Periodic conditions of A and ϕ

As the magnetic fields are determined from A and ϕ in the A-ϕ method, the boundary conditions for the periodic phenomena should be given by A and ϕ. Therefore, the periodic conditions of A and ϕ have to be examined.

As the vector potential A corresponds to the current density J [2], the following relationships between the x-, y- and z-components Apx, Apy and Apz at the point p and those Aqx', Aqy' and Aqz' at the point q can be obtained:

$$A_{px} = -A_{qx}' \tag{7}$$

$$A_{py} = -A_{qy}' \tag{8}$$

$$A_{pz} = -A_{qz}' \tag{9}$$

Je can be written using A and ϕ as follow:

$$Je = -\sigma \left(\frac{\partial A}{\partial t} + \mathrm{grad}\,\phi\right) \tag{10}$$

As the direction of Je is the same as that of A, the direction of grad ϕ should be also the same as that of A. Also, the direction of A vector at the point p is opposite to that at the point q as denoted by Eqs.(7)-(9). Therefore, the sign of gradϕ at the point p is different from that at the point q. As a result, the following relationship between ϕp at the point p and ϕq at the point q is obtained.

$$\phi_p = -\phi_q \tag{11}$$

3. APPLICATIONS AND DISCUSSIONS

3.1 Simple models

The usefulness of the boundary condition has been examined by applying it to simple models.

Figure 3 shows a cylinder with two slits at symmetrical positions. The magnetic field is uniformly applied in the y-direction and decays exponentially with time as follows:

$$By = 0.1e^{-t/0.0069}(T) \tag{12}$$

The flux distribution is point symmetric with respect to the line o_1-o_2 as shown in Fig.4. The angle θ is equal to 180(deg.) in this model. It corresponds to a 2-pole machine.

Figure 5 shows a model of 4-pole rotating machine. Both the stator and rotor windings carry currents, and the eddy current is neglected. In this case, as the machine has 4 poles, θ is equal to 90(deg.). Therefore, the transformation from the local coordinates to the ordinary coordinates is denoted as follows:

$$A_{qx} = -A_{qy}' \tag{13}$$

$$A_{qy} = A_{qx}' \tag{14}$$

$$A_{qz} = A_{qz}' \tag{15}$$

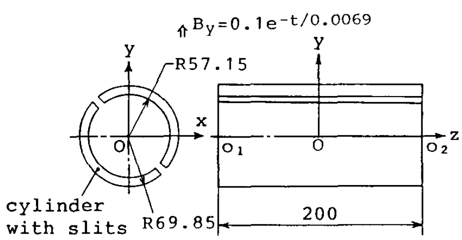


Fig.3 Analyzed model (2-pole).

Table 1 shows the comparison of the computer storage and the CPU time. When the 1/4 region of the 2-pole model shown in Fig.3 is analyzed by applying the periodic boundary conditions, the computer storage is reduced to about 1/2 of the analysis for the 1/2 region and the CPU time is reduced to about 1/3. As the number of iterations of the ICCG method[3] is decreased when the number of the unknown variables is small, the CPU time per one variable is rapidly decreased.

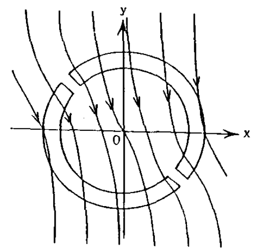
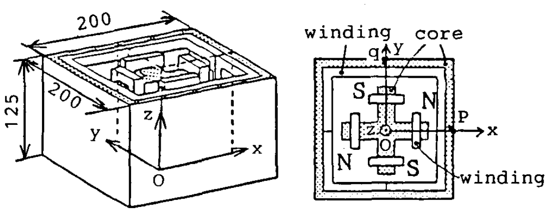


Fig.4 Flux distribution.



(a) bird's-eye view (b) cross section

Fig.5 Analyzed model (4-pole).

Table 1 Comparison of computer storage and CPU time

model	2-pole model (Fig.3)		4-pole model (Fig.4)	
analyzed region	1/2	1/4	1/2	1/8
number of elements	36360	18180	12000	3000
number of nodes	6672	3616	2646	726
number of unknown variables	18388	9168	5054	1260
computer storage(MB)	1.6	0.8	0.4	0.1
number of iterations of ICCG method	398	241	81	71
CPU time(sec)	311	112	25	6

computer:SX-1E(NEC supercomputer)

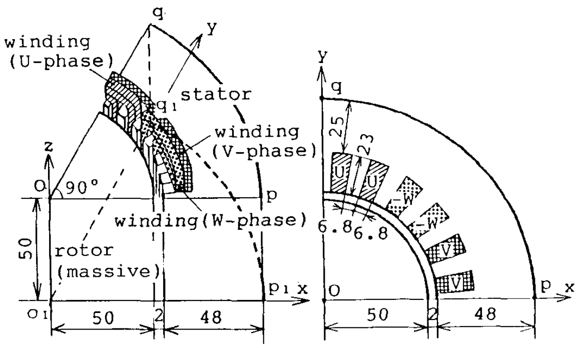
3.2 Induction motor

The flux distribution and the starting current of a 3-phase 4-pole induction motor with a massive rotor shown in Fig.6 are analyzed using 3-D finite element method, and compared with those of 2-D analysis. It is assumed that the relative permeabilities of the rotor and stator cores are constant(linear), and each is assumed to be 1000. The conductivity of the rotor core is 1.0×10^{7}(S/m). The m.m.f. per one slot is 250(AT). The frequency is 50(Hz). In 3-D model(Fig.6(a)), the planes p-o-o_1-p_1-p and q-o-o_1-q_1-q are the periodic boundaries. The vector potential Az on the z-axis is equal to zero. It is assumed that the flux is parallel to the plane p-q-q_1-

p_1-p, namely A is equal to zero on the plane. In 2-D model(Fig.6(b)), the lines p-o and q-o are the periodic boundaries, and Az at the point o is zero. The curved line p-q is the Dirichlet boundary.

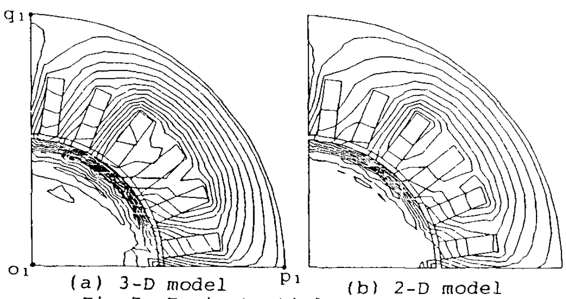
Figure 7 shows the equipotential contour plots of Az on the plane z=0 in 3-D and 2-D analyses. The Figures denote that the flux in 3-D model is larger than that in 2-D model. This is because the eddy current density in 3-D model is smaller than that in 2-D model due to the longer eddy current path length in 3-D model as shown in Fig.8.

Starting currents of induction motor are calculated by assuming that the applied voltages (=200(V)) to 3-D and 2-D models are the same. The calculated starting current of 3-D model is smaller than that of 2-D model by 25(%).



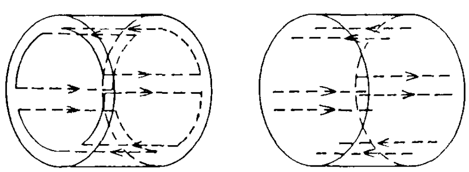
(a) bird's-eye view (b) cross section

Fig.6 3-phase induction motor.



(a) 3-D model (b) 2-D model

Fig.7 Equipotential contour plots.



(a) 3-D model (b) 2-D model

Fig.8 Eddy current paths in rotor.

4. CONCLUSIONS

The periodic boundary condition for 3-D A-ϕ method has been examined, and the usefulness of the condition is demonstrated.

The obtained results can be summarized as follows:

(a) The definition of 3-D periodic phenomena has been clarified.

(b) The periodic conditions of A and ϕ is obtained.

(c) The CPU time can be considerably decreased by using the periodic condition.

Although the periodic boundary conditions for A-ϕ method is examined, the periodic conditions for other methods, such as $\mathbb{T}$-Ω method, can also be treated in the same way. The boundary condition which appear, for example, in the linear induction motor[4] is different from that examined in this paper. This will be reported in an another paper later.

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REFERENCES

[1] J.R.Brauer, G.A.Zimmerlee, T.A.Bush, R.J.Sandel and R.D.Schultz: "3D Finite Element Analysis of Automotive Alternators under Any Load", IEEE Trans. on Magnetics, MAG-24, 1, 500 (1988).

[2] T.Nakata and N.Takahashi: "Finite Element Method in Electrical Engineering", (book), Morikita Publishing Co., Tokyo (1987).

[3] D.S.Kershaw: "The Incomplete Cholesky-Conjugate Gradient Method for the Iterative Solution of Systems of Linear Equations", Journal of Computational Physics, 26, 43 (1987).

[4] T.Narita, M.Kawabe and H.Nakagawa: "Performance Analysis of Linear Pulse Motor by Finite Element Method", Shinko Electric Journal, 29, 2, 18 (1984).