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NUMERICAL ANALYSIS OF ANTENNA BY A SURFACE PATCH MODELING

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Abstract- In this paper, the cylindrical dipole antenna is numerically analyzed by the moment method. Surface of the antenna is approximated by triangular patches and the electric field integral equation is used for direct calculation of the surface current distribution. Therefore we can treat the cylinder antenna in open or closed boundary form. The current expansion functions and the testing functions of the electric field boundary condition are triangular type. The surface integrals are numerically solved by a 33-point Gaussian quadrature approximation. The current distribution on a flat plate illuminated by a plane wave and the input admittance of a hollow cylindrical dipole as the near field quantities have been investigated. The convergence of the input admittance against the number of the triangular patches is presented and also the admittance solution is compared with the thin-wire approximation and theoretical results. Finally the CPU time and memory storage size for different number of patches are presented. Rapid admittance convergence and few required unknowns per square wavelength are the advantages of the surface patch modeling.

INTRODUCTION

The surface current distribution is the most important parameter in the antenna analysis. All the near or far field quantities may be derived from the current distribution. The input admittance or impedance reflects the current at feeding region.

The cylindrical antenna may be treated by a thin-wire approximation which is a solution for axial component of the current distribution. The surface patch and wire-grid techniques investigate all components of the current on the antenna surface. The wire-grid solution is more simple than the surface patch technique but its validity on the near field solution is still doubtful [1], [2].

Advantages of the surface patch approximation from view point of the antenna analysis are a) direct and simple relation between vectors of calculated current matrix and the current distribution on the antenna surface, b) no restriction for treating very thin surfaces and c) very few unknowns per wavelength square.

Recently several surface patch approximation techniques have been developed for modeling solid surfaces as an accurate and direct solution of the surface current and charge distributions. N.N. Wang et al. used piecewise-sinusoidal reaction technique and rectangular surface patches for treating scattering and radiation problem of arbitrary shaped conducting bodies [3]. J.J. Wang introduced a triangular surface patch technique to calculate the scattering characteristics of a three-dimensional arbitrary-shaped closed conducting body [4] and Rao et al. used the electric field integral equation and triangular patches to develop a simple and efficient numerical procedure for treating problems of scattering from conducting bodies of arbitrary shape [5].

In this work we have embarked upon to use triangular patches [5] for the antenna analysis. We have treated a plane wave illuminated flat square plate as a comparison with the result of [5]. Then the input admittance of a hollow cylinder dipole has been investigated for different number of the patches. The computed results have been compared with analytical and thin-wire approximation data. The required CPU time and memory storage size also have been presented.

In the next section the surface patch technique for the moment solution of the electric field integral equation is briefly presented. The numerical results and conclusion are presented in the next sections consecutively.

THEORY

Consider an arbitrary shaped perfectly conducting body (closed or open) illuminated by an incident electric field $\mathbf{E}^i$. A frequency domain expression for scattered field is given by

$$\mathbf{E}^s = -j\omega\mathbf{A} - \nabla\Phi \tag{1}$$

where the vector potential $\mathbf{A}$ and the scalar potential Φ are defined as

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\iint \mathbf{J}(\mathbf{r}')\frac{e^{-jkR}}{R}\,ds' \tag{2}$$

$$\Phi(\mathbf{r}) = \frac{-1}{j4\pi\omega\varepsilon}\iint \nabla\cdot\mathbf{J}(\mathbf{r}')\frac{e^{-jkR}}{R}\,ds' \tag{3}$$

in which $\mathbf{J}$ is the surface current distribution, k is the wave number given by $k=2\pi/\lambda$ and $R=|\mathbf{r}-\mathbf{r}'|$ is the distance between a field point $\mathbf{r}$ and a source point $\mathbf{r}'$, ω is the frequency in radians per second and ε and μ are the permittivity and permeability of the medium respectively. The electric field boundary condition results in an integrodifferential equation with unknown $\mathbf{J}$ as

$$-\mathbf{E}^i_{tan} = (-j\omega\mathbf{A} - \nabla\Phi) \tag{4}$$

Referring to a triangulated conducting scatterer, every two triangles with a common side are supposed as a source-pair $T_n\pm$, or field-pair $T_m\pm$ as shown in Fig. 1. The electric current flows along radial direction $\rho_n\pm$ in triangles $T_n\pm$. In this literature the subscripts m and n mean a field and a source triangle-pair respectively and their superscript plus or minus sign mean a positive assumed current direction from plus triangle to minus triangle. Any point in the triangles can be defined either with respect to global origin O or with respect to the triangle vertices $O_m\pm$ and $O_n\pm$. Here a surface current expansion function associated with *n*th pair is given by

$$\mathbf{F}_n(\mathbf{r}) = \begin{cases} l_n\rho_n^+/2S_{n+} & ;\ \mathbf{r} \text{ on } T_n^+ \\ l_n\rho_n^-/2S_{n-} & ;\ \mathbf{r} \text{ on } T_n^- \\ 0 & ;\ \text{otherwise} \end{cases} \tag{5}$$

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and the divergence of the expansion function in the polar co-ordinates system may be given by

$$\nabla \cdot \mathbf{F}_n(\mathbf{r}) = \begin{cases} l_n / S_{n+} & ; \mathbf{r} \text{ on } T_n^+ \\ -l_n / S_{n-} & ; \mathbf{r} \text{ on } T_n^- \\ 0 & ; \text{ otherwise} \end{cases} \tag{6}$$

where l_n is the length of the common side of nth triangle-pair and $S_n\pm$ are the areas of triangles.

Let N represents the total number of pairs. Then

$$\mathbf{J} = \sum_{n=1}^{N} \mathbf{I}_n \mathbf{F}_n(\mathbf{r}) \tag{7}$$

where $\mathbf{I}_n$ is constant and unknown. In fact each $\mathbf{I}_n$ can be interpreted as the normal components of the current density to pass the common side of the nth pair.

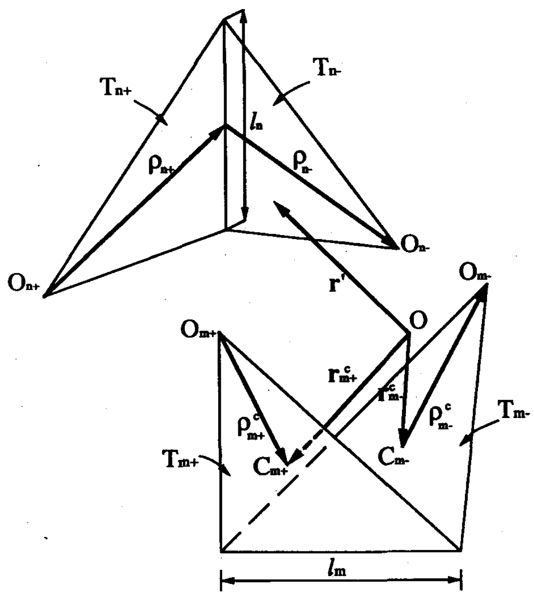

Fig. 1 Geometry representation of nth source-pair and mth field-pair.

In order to find the current coefficients, the electric field integral equation is tested with respect to testing functions $\mathbf{F}_m$. Testing functions $\mathbf{F}_m$ and expansion functions $\mathbf{F}_n$ are the same. Equation (4) is tested based on the following symmetric product

$$<\mathbf{E}, \mathbf{F}> \equiv \iint \mathbf{E} \cdot \mathbf{F} \, ds \tag{8}$$

Then, we obtain

$$<\mathbf{E}^i, \mathbf{F}_m> = j\omega <\mathbf{A}, \mathbf{F}_m> + <\nabla\Phi, \mathbf{F}_m> \tag{9}$$

In Eq. (9) the products may be simplified by evaluating the vectors at the centroids of respect triangles ($\mathbf{r}^c{}_m\pm$) as follows

$$<\mathbf{E}^i, \mathbf{F}_m> \cong \mathbf{E}^i(\mathbf{r}^c{}_m\pm) \cdot \iint_{T_m\pm} \mathbf{F}_m(\mathbf{r}) \, ds \tag{10}$$

$$<\mathbf{A}, \mathbf{F}_m> \cong \mathbf{A}(\mathbf{r}^c{}_m\pm) \cdot \iint_{T_m\pm} \mathbf{F}_m(\mathbf{r}) \, ds \tag{11}$$

$$<\nabla\Phi, \mathbf{F}_m> \cong -\Phi(\mathbf{r}^c{}_m\pm) \cdot \iint_{T_m\pm} \nabla \cdot \mathbf{F}_m(\mathbf{r}) \, ds \tag{12}$$

Consequently Eq.(9) becomes

$$\begin{aligned} & l_m(\mathbf{E}^i(\mathbf{r}^c{}_{m+}).\rho^c{}_{m+}/2 + \mathbf{E}^i(\mathbf{r}^c{}_{m-}).\rho^c{}_{m-}/2) = \\ & j\omega l_m(\mathbf{A}(\mathbf{r}^c{}_{m+}).\rho^c{}_{m+}/2 + \mathbf{A}(\mathbf{r}^c{}_{m-}).\rho^c{}_{m-}/2) + \\ & l_m(\Phi(\mathbf{r}^c{}_{m-}) - \Phi(\mathbf{r}^c{}_{m+})) \end{aligned} \tag{13}$$

which Eq. (13) is the equation enforced at each triangle-pair, m=1, 2, ..., N. The expanded surface current Eq. (7) is now substituted into Eq.(13) to make an N×N system of linear equations which is written in matrix form as

$$[Z]\,[I] = [V] \tag{14}$$

where elements of [Z] and [V] are given by

$$Z_{mn} = l_m[\, j\omega(\mathbf{A}_{mn+}.\rho^c{}_{m+}/2 + \mathbf{A}_{mn-}.\rho^c{}_{m-}/2) + \Phi_{mn-} - \Phi_{mn+}\,] \tag{15}$$

$$V_{m,1} = l_m[\, \mathbf{E}_{m+}.\rho^c{}_{m+}/2 + \mathbf{E}_{m-}.\rho^c{}_{m-}/2\,] \tag{16}$$

$$\mathbf{A}_{m,n}\pm = \frac{\mu}{4\pi} \iint_{\mathbf{r}' \text{ on } T_n\pm} \mathbf{F}_n(\mathbf{r}') \frac{e^{-jkR_{mn}\pm}}{R_{mn}\pm} \, ds' \tag{17}$$

$$\Phi_{m,n}\pm = \frac{-1}{4\pi j\omega\varepsilon} \iint_{\mathbf{r}' \text{ on } T_n\pm} \nabla \cdot \mathbf{F}_n(\mathbf{r}') \frac{e^{-jkR_{mn}\pm}}{R_{mn}\pm} \, ds' \tag{18}$$

in which

$$\mathbf{E}_m\pm = \mathbf{E}^i(\mathbf{r}^c{}_m\pm) \tag{19}$$

$$R_{mn}\pm = |\mathbf{r}^c{}_m - \mathbf{r}'_n| \tag{20}$$

As Eqs. (17) and (18) show the vector potential $\mathbf{A}$ and the scalar potential Φ should be calculated at the centroid of triangles where for self impedances there are the singularities. It seems for numerical calculation of the integrals, at first, the singular points should be removed. Since only the centroid points are singular, high degree Gaussian quadrature formulas which have no sampling point at the centroid are capable to accomplish the singular integrals. In this work we used a 33-sampling point Gaussian quadrature formula [6] and no anomaly was observed.

Every triangle may be one part of at most three source-pairs and its field should be calculated at all triangles. One may compute the vector and scalar potentials of every individual triangle patch on the centroid of all the patches and then according to the membership of the patches in the source and field pairs the elements of Z_{mn} are computed.

NUMERICAL RESULTS

Fig. 2 shows the surface current distribution on the square flat plate illuminated by a normally incident plane wave. For comparison the result of [5] is also presented.

To demonstrate the antenna numerical analysis applicability of the procedure numerical results are presented for the input admittance of a hollow cylindrical dipole as a thin and open surface. Fig. 3 shows the input admittance of a rectangular dipole for different number of the patches. For modeling the surface of dipole 24 patches at least is required. The length of the dipole is divided into three sections. The middle one is supposed as the feeding area and only two other sections carry the current distribution. The result, as shown in Fig. 3, for kh<3 is very close to the higher patch densities and this is a very sensitive test for the divergence of the surface patch technique.

Fig. 4 shows the input admittances of a hexagonal prism dipole derived by the method, the thin-wire approximation (point matching based on the Pocklingtin's integral equation) [7] and the King-Middleton second order solution [8].

Table 1 presents the CPU time and memory storage size required for different number of the patches on a NEC ACOS-2010 mainframe. Note that no advantage of inherent symmetry was made.

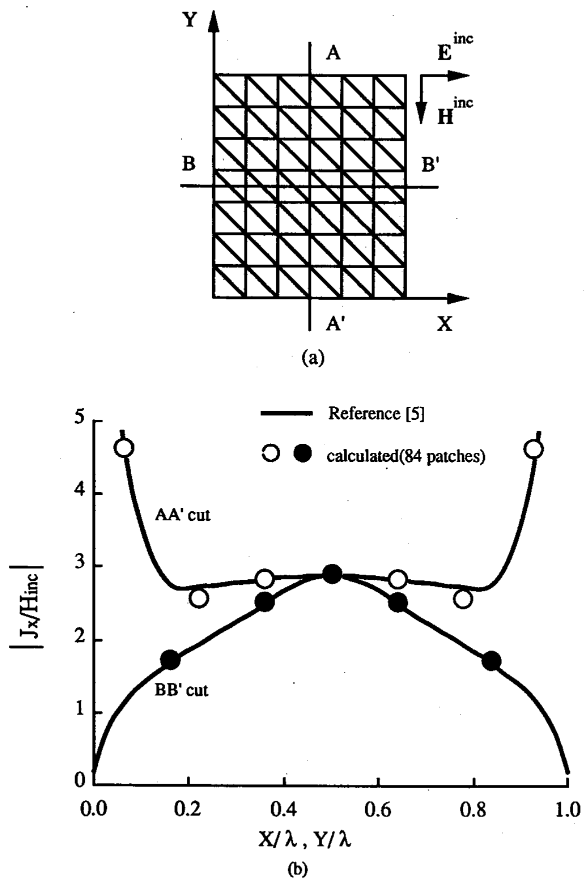

Fig. 2 (a) A triangulated 1λ square flat plate illuminated by a plane wave,
(b) Dominant component of current distribution on the plate.

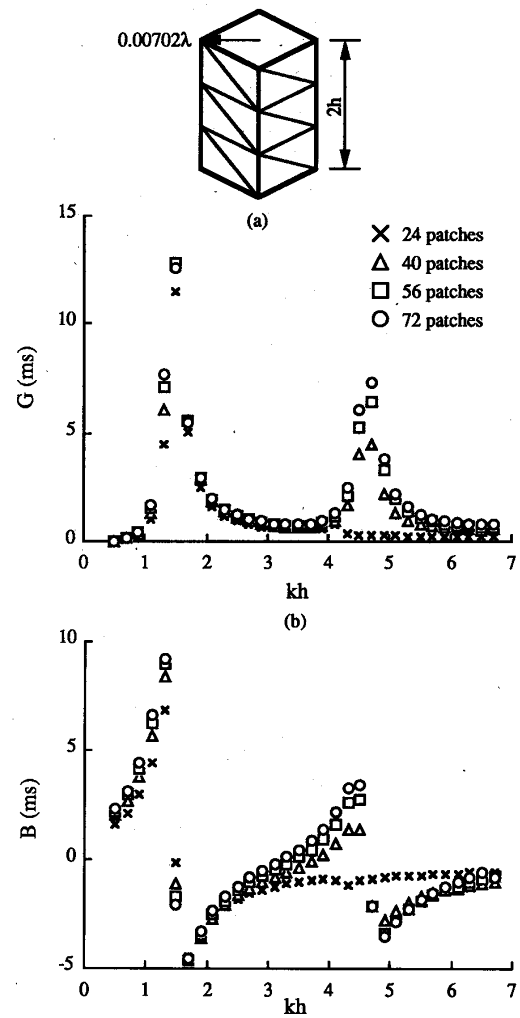

Fig. 3 Input admittance of a center feed rectangular prism for different number of patches:
(a) triangulated rectangular prism,
(b) input conductance,
(c) input susceptance.

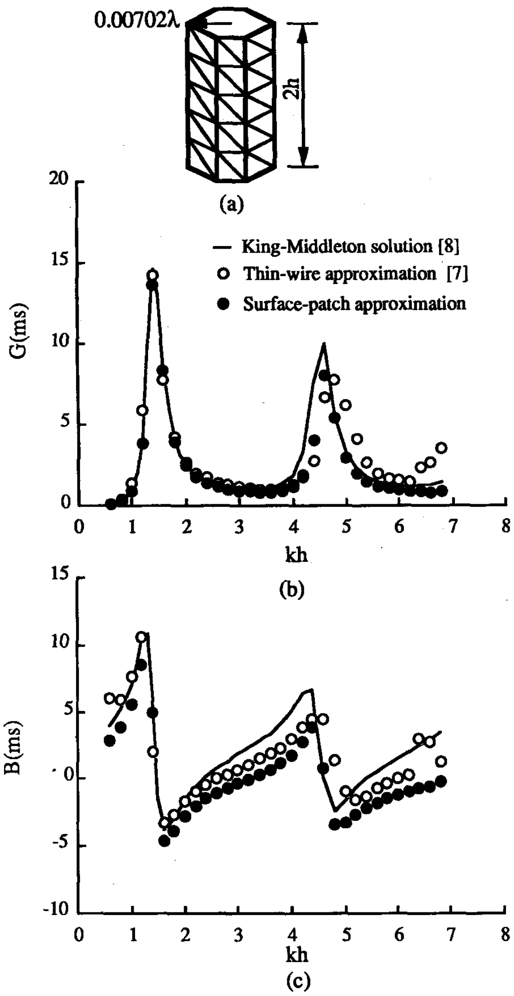

Fig. 4 Input admittance comparison between patch, thin wire and analytical solutions of center feed dipole:
(a) triangulated hexagonal prism (60 patches),
(b) input conductance,
(c) input susceptance.

Table 1 Computation time and storage size for the rectangular prism dipole on a NEC ACOS-2010 mainframe computer.

Number of patches	24	40	56	72
Total CPU time for each point (ms)	15.3	55.2	131.2	254.3
Total memory size (kW)	55	61	71	84

CONCLUSION

The numerical analysis of antenna was investigated by the moment method based on the surface-patch modeling and the electric field integral equation. Surface of the antenna was modeled by triangular patches. Identical triangular type current expansion functions and electric field boundary condition testing functions were used for developing a set of simultaneous equations for calculation of the surface current distribution. The main attention was spent to evaluation of divergence and cost compared with the thin-wire approximation and the wire-grid modeling.

The input admittance of a cylindrical dipole was computed for different number of the patches for the antenna surface. The input admittance convergence of a rectangular dipole antenna also was investigated.

The numerical results show that the surface patch modeling for calculation of the near field quantities, contrary to the wire-grid approximation, is explicit, the input admittance convergence based on the number of patches per square wavelength is very rapid (this can be observed clearly in the computed input admittance of the cylinder dipole with only 24 patches), and required memory storage size is lower than for the wire-grid technique.

Beside the above advantages there are complicated mathematics and higher computation time cost.

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