An extension of generalized minimum variance control for multi-input multi-output systems using coprime factorization approach

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Abstract

This paper proposes a new generalized minimum-variance controller (GMVC) having new design parameters by using coprime factorization approach for multi-input multi-output (MIMO) case. The method is directly extended from a conventional GMVC and to construct the controller, it needs to solve only one Diophantine equation as in the conventional method. In this paper, by using double-coprime factorization, a simple formula for closed-loop system given by the parametrized controller is obtained and using the formula, it is proved that the closed-loop characteristic from reference signal to plant output is independent of the selection of the design parameters and the poles of the controller can be chosen by the design parameters without changing the closed-loop system.

1 Introduction

To control plants with uncertainty, Generalized Minimum Variance Control (GMVC) is widely applied in industry. GMVC is first proposed by Clarke and others[1] and GMVC design methods for multi-input multi-output (MIMO) systems have been given by several authors[3],[4],[8]. Using a generalized output, GMVC can be applied to a wider class of plants such as, unstable plants or non-minimum phase plants.

In designing GMVC, the generalized output is usually selected for the closed-loop system of the GMVC to be stable. Then the controller is designed to minimize the variance of the generalized output. And the poles of the controller can not be designed independently to the poles of the closed-loop system.

The authors have proposed a new GMVC design method for single-input single-output (SISO) systems[2]. The method have new design parameters introduced by using coprime factorization approach[7], In the method, the poles of the controller are designed by selecting the newly introduced parameters and are chosen independently to the poles of the closed-loop systems. This paper extends the GMVC design method having new design parameters to be applicable to MIMO systems by using coprime factorization approach and Youla Parametrization[7].

Kouvaritakis and others used Youla Parametrization to obtain an extended generalized predictive control (GPC) for SISO systems[5] and for MIMO systems[6]. Their multivariable GPC uses both of the right- and left-coprime factorization of the plant transfer function and also the double(dual)-coprime factorization and the solution of an additional Bezout identity to calculate the control law. Their method is based on the factorization of polynomial matrix transfer functions and their formulae are rather complicated. They focused their method on obtaining a new stable GPC and a robust GPC and did not extend directly a conventional method.

The self-tuning controller proposed in this paper is calculated in 2 steps. In the 1st step, weighting matrices in generalized output are designed to have a desired closed-loop characteristics using nominal values of transfer function matrix. After the weighting matrices are selected, the control law is calculated in each sampling period as the 2nd step. In the 1st step, this paper uses both of the right- and left-coprime factorization and the double-coprime factorization which are needed only at the start of control. But in the 2nd step, to calculate the control input at each sample period, the proposed method requires to solve only one Diophantine equation and no additional Bezout equation and uses only the left-coprime factorization of the given transfer function. Hence the amount of calculation at each sampling period is reduced. In self-tuning cases, it is important to reduce calculations of the control law in order to be completed in sampling period. The method in this paper is a direct extension of a conventional method and it includes the conventional one as a special case of the proposed method. In this paper, by using the coprime factorization of rational function, a simple formula of the closed-loop transfer function is derived.

2 The Problem Statement for MIMO GMVC

Consider a multi-input multi-output system having p inputs p outputs given by the following model,

$$\begin{aligned} \boldsymbol{A}(z^{-1})\boldsymbol{y}(t) &= z^{-k_m}\boldsymbol{B}(z^{-1})\boldsymbol{u}(t)+\boldsymbol{C}(z^{-1})\boldsymbol{\xi}(t) \quad (1) \\ t &= 0,\ 1,\ 2\cdots \end{aligned}$$

where $\boldsymbol{u}(t)$ is p-dimensional input vector, $\boldsymbol{y}(t)$ is p-dimensional output vector, k_m is the time delay, $\boldsymbol{\xi}(t)$ is a white Gaussian noise with zero mean. $\boldsymbol{A}(z^{-1})$, $\boldsymbol{B}(z^{-1})$ and $\boldsymbol{C}(z^{-1})$ are $p\times p$ polynomial matrices with degrees n, m and l.

On the system (1) we assume the followings;

[A.1] The degrees n, m, l of $\boldsymbol{A}(z^{-1}), \boldsymbol{B}(z^{-1}), \boldsymbol{C}(z^{-1})$ and the time delay k_m are known.

[A.2] In deterministic cases which are given in sections 3 and 4, the coefficients of $\boldsymbol{A}(z^{-1})$, $\boldsymbol{B}(z^{-1})$ and $\boldsymbol{C}(z^{-1})$ are supposed to be known. In adaptive case of section 5, the coefficients are unknown, but the nominal values of the coefficients are known.

[A.3] The polynomial matrices $\boldsymbol{A}(z^{-1})$ and $\boldsymbol{B}(z^{-1}), \boldsymbol{A}(z^{-1})$ and $\boldsymbol{C}(z^{-1})$ are coprime.

[A.4] The matrix polynomial $\boldsymbol{C}(z^{-1})$ is stable.

The control objective is to make the output $\boldsymbol{y}(t)$ follow the reference signal $\boldsymbol{w}(t)$. To achieve the objective, we minimize the following objective function J averaged over the noise $\boldsymbol{\xi}(t), \boldsymbol{\xi}(t-1), \cdots$;

$$\begin{aligned} \boldsymbol{\Phi}(t+k_m) &= \boldsymbol{P}(z^{-1})\boldsymbol{y}(t+k_m)+\boldsymbol{Q}(z^{-1})\boldsymbol{u}(t) \\ &\quad -\boldsymbol{R}(z^{-1})\boldsymbol{w}(t) \quad (2) \\ J &= E[\boldsymbol{\Phi}^2(t+k_m)] \quad (3) \end{aligned}$$

where $\boldsymbol{\Phi}(t+k_m)$ is called the generalized output. $\boldsymbol{P}(z^{-1})$, $\boldsymbol{Q}(z^{-1})$ and $\boldsymbol{R}(z^{-1})$ are polynomial matrices with the degrees of n_p, n_q and n_r.

For simplicity in writing polynomial matrices and signals, we will drop their arguments; z^{-1} or t. For example, we denote $\boldsymbol{A}(z^{-1})$ and $\boldsymbol{y}(t)$ as $\boldsymbol{A}$ and $\boldsymbol{y}$.

3 A Reformation of Conventional Multivariable GMVC

For p-inputs p-outputs system, Koivo[3] derived a generalized minimum variance control scheme using matrix transfer functions. In this section, we reform the conventional GMVC in order to extend the conventional GMVC in the next section.

First, for the left-coprime factorization of the transfer function (1), $\boldsymbol{G} = \boldsymbol{A}^{-1}\boldsymbol{B}$, we define a right-coprime factorization of $\boldsymbol{G}$ as

$$\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{A}}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{B} \quad (4)$$

and for the given polynomial matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$ in the generalized output, we define $\widetilde{\boldsymbol{E}}$, $\widetilde{\boldsymbol{F}}$, $\widetilde{\boldsymbol{G}}$, $\widetilde{\boldsymbol{T}}$, $\boldsymbol{E}$, $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{T}$ satisfying the following equations;

$$\begin{aligned} \boldsymbol{P} &= \widetilde{\boldsymbol{E}}\boldsymbol{A}+z^{-k_m}\widetilde{\boldsymbol{F}} \quad (5) \\ \widetilde{\boldsymbol{G}} &= \widetilde{\boldsymbol{E}}\boldsymbol{B}+\boldsymbol{Q} \quad (6) \\ \widetilde{\boldsymbol{T}} &= \boldsymbol{P}\widetilde{\boldsymbol{B}}+\boldsymbol{Q}\widetilde{\boldsymbol{A}} \quad (7) \\ \boldsymbol{PC} &= \widetilde{\boldsymbol{A}}\boldsymbol{E}+z^{-k_m}\boldsymbol{F} \quad (8) \\ \boldsymbol{G} &= \widetilde{\boldsymbol{B}}\boldsymbol{E}+\boldsymbol{QC} \quad (9) \\ \boldsymbol{T} &= \boldsymbol{BP}+\boldsymbol{AQ} \quad (10) \\ \widetilde{\boldsymbol{E}}\boldsymbol{C} &\triangleq \widetilde{\widetilde{\boldsymbol{E}}} \quad (11) \end{aligned}$$

To obtain these matrices, we need only to find the solutions $\widetilde{\boldsymbol{E}}$, $\widetilde{\boldsymbol{F}}$, $\boldsymbol{E}$ and $\boldsymbol{F}$ of the equations (5) and (8). Other polynomial matrices $\widetilde{\boldsymbol{G}}, \widetilde{\boldsymbol{T}}$, $\boldsymbol{G}$ and $\boldsymbol{T}$ are obtained by substituting the solutions into the equations (6), (7), (9) and (10).

Next, we will obtain the k_m-ahead optimally predicted value $\widehat{\boldsymbol{\Phi}}(t+k_m|t)$ of the generalized output $\boldsymbol{\Phi}(t+k_m)$. Multiplying Equation (1) by $z^{k_m}\widetilde{\boldsymbol{E}}$ from the left and substituting Equation (5) into the multiplied equation, we get

$$\boldsymbol{P}\boldsymbol{y}(t+k_m) = \widetilde{\boldsymbol{F}}\boldsymbol{y}+\widetilde{\boldsymbol{E}}\boldsymbol{B}\boldsymbol{u}+\widetilde{\boldsymbol{E}}\boldsymbol{C}\boldsymbol{\xi}(t+k_m) \quad (12)$$

From the stochastical independence of $\boldsymbol{y}$, $\boldsymbol{u}$ from $\boldsymbol{\xi}(t+k_m)$, k_m-ahead optimally predicted value $\boldsymbol{P}\widehat{\boldsymbol{y}}(t+k_m|t)$ of $\boldsymbol{P}\boldsymbol{y}(t+k_m)$ is given by

$$\boldsymbol{P}\widehat{\boldsymbol{y}}(t+k_m) = \widetilde{\boldsymbol{F}}\boldsymbol{y}+\widetilde{\boldsymbol{E}}\boldsymbol{B}\boldsymbol{u} \quad (13)$$

By substituting the above into the generalized output $\boldsymbol{\Phi}(t+k_m)$ of (2),

$$\begin{aligned} \boldsymbol{\Phi}(t+k_m) &= \widehat{\boldsymbol{\Phi}}(t+k_m|t)+z^{k_m}\widetilde{\widetilde{\boldsymbol{E}}}\boldsymbol{\xi}(t) \quad (14) \\ \widehat{\boldsymbol{\Phi}}(t+k_m|t) &= \boldsymbol{P}\widehat{\boldsymbol{y}}(t+k_m|t)+\boldsymbol{Q}\boldsymbol{u}(t)-\boldsymbol{R}\boldsymbol{w}(t) \quad (15) \\ &= [\widetilde{\boldsymbol{F}}\boldsymbol{y}(t)+\widetilde{\boldsymbol{G}}\boldsymbol{u}(t)]-\boldsymbol{R}\boldsymbol{w}(t) \quad (16) \end{aligned}$$

Since $E[\boldsymbol{\xi}(t)]=0$ and Equation (14), $E[\boldsymbol{\Phi}^2(t+k_m)] = \widehat{\boldsymbol{\Phi}}^2(t+k_m|t)$. If we determine the control input $\boldsymbol{u}(t)$ to satisfy

$$\widehat{\boldsymbol{\Phi}}(t+k_m|t) = \boldsymbol{0} \quad (17)$$

then $E[\boldsymbol{\Phi}^2(t+k_m)]$ becomes minimal. So the control input is determined to hold Equation (17), that is,

$$\boldsymbol{u} = \widetilde{\boldsymbol{G}}^{-1}[\boldsymbol{R}\boldsymbol{w}(t)-\widetilde{\boldsymbol{F}}\boldsymbol{y}(t)] \quad (18)$$

To calculate the input $\boldsymbol{u}$ through (18), only $\widetilde{\boldsymbol{E}}$, $\widetilde{\boldsymbol{F}}$ and $\widetilde{\boldsymbol{G}}$ of Equations (5) and (6) are required to be calculated and $\tilde{A}$, $\tilde{B}$ of (4) and E, F, G, T of (8)-(10) are not needed.

By substituting the equation (18) into the equation (1), we get the closed loop system;

$$y(t) = z^{-k_m}\tilde{B}\tilde{T}^{-1}Rw(t) + \tilde{B}\tilde{T}^{-1}\tilde{G}B^{-1}C\xi(t) \tag{19}$$

Usually the coefficient matrices P, Q and R of the generalized output Φ are determined by designers to make the closed-loop characteristics, $\tilde{T}$ stable. Once matrices P, Q and R are determined, then the poles of controller, that is, the roots of det $\tilde{G} = 0$ are uniquely determined and can not be designed independently to $\tilde{T}$.

In the next section we extend the generalized minimum variance control by using coprime factorization and we introduce new design parameters. Selecting the newly introduced design parameters, we can design $\tilde{G}$ without changing $\tilde{T}$.

4 An extension of the GMVC

This section concerns non-adaptive case, that is, it is assumed that the coefficients of $A(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$ are supposed to be known.

For coprime factorization approach, the family of stable rational functions are considered;

$$RH_\infty = \{G = \{g_{ij}\}, g_{ij} = \frac{g_{ij_n}}{g_{ij_d}}, g_{ij_d} : \text{stable polynomial}\} \tag{20}$$

Transfer functions are given in the form of a ratio of rational functions in RH_∞,

$$G = ND^{-1} = \tilde{D}^{-1}\tilde{N} \tag{21}$$

where N and D, $\tilde{D}$ and $\tilde{N}$ are rational functions in RH_∞ and are coprime to each other.

And we denote $X, Y, \tilde{X}, \tilde{Y} \in RH_\infty$ as the solutions of the following Bezout Identifies;

$$XN + YD = I \tag{22}$$

$$\tilde{N}\tilde{X} + \tilde{D}\tilde{Y} = I \tag{23}$$

and define

$$\triangle \triangleq -Y\tilde{X} + X\tilde{Y} \tag{24}$$

$$\tilde{\tilde{X}} \triangleq \tilde{X} + D\triangle, \quad \tilde{\tilde{Y}} \triangleq \tilde{Y} - N\triangle \tag{25}$$

Then the doubly coprime factorization is given as;

$$\begin{bmatrix} Y & X \\ -\tilde{N} & \tilde{D} \end{bmatrix}\begin{bmatrix} D & -\tilde{\tilde{X}} \\ N & \tilde{\tilde{Y}} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{26}$$

All of the stabilizing two-degree-of-freedom compensator is given in Youla Parametrization [7];

$$u(t) = C_1 w(t) - C_2 y(t) \tag{27}$$

$$\begin{aligned} C_1 &= (Y - U\tilde{N})^{-1}K \\ &= (D + C_2 N)K \end{aligned} \tag{28}$$

$$\begin{aligned} C_2 &= (Y - U\tilde{N})^{-1}(X + U\tilde{D}) \\ &= (\tilde{\tilde{X}} + DU)(\tilde{\tilde{Y}} - NU)^{-1} \end{aligned} \tag{29}$$

We compare the transfer function (21), Youla Parametrization (27)-(29) and Bezout identities in the equations (22) and (23) with the plant (1), the control law (18) and the equations (5)-(10) respectively. If P, Q and R are selected such that T and $\tilde{T}$ are stable, we can choose the rational function matrices as

$$N = z^{-k_m}\tilde{B}\tilde{T}^{-1}, \quad D = \tilde{A}\tilde{T}^{-1} \tag{30}$$

$$X = \tilde{F}, \quad Y = \tilde{G} \tag{31}$$

$$\tilde{N} = z^{-k_m}T^{-1}B, \quad \tilde{D} = T^{-1}A \tag{32}$$

$$\tilde{X} = FC^{-1}, \quad \tilde{Y} = GC^{-1} \tag{33}$$

These X, Y, $\tilde{X}$ and $\tilde{Y}$ satisfy Bezout identities of the equations (22) and (23).

If we choose K and U as

$$K = R, \quad U = 0 \tag{34}$$

then the compensator (27)-(29) coincides with the control law (18).

In order to extend the controller (18), instead of choosing $U(z^{-1})$ as $U(z^{-1}) = 0$, we use $U(z^{-1})$ as a newly introduced design parameter for the controller (18). To simplify the form of the controller, using new two design polynomial matrices U_n and U_d, we use the form of U as

$$U = U_d^{-1}U_n\tilde{E}T \tag{35}$$

Then substituting the equations (30)-(33) and (35) into (27)-(29), a newly extended multivariable GMVC is obtained;

$$\begin{aligned} &(U_d\tilde{G} - U_n\tilde{E}z^{-k_m}B)u(t) = \\ &\qquad U_dRw(t) - (U_d\tilde{F} + U_n\tilde{E}A)y(t) \end{aligned} \tag{36}$$

This control law satisfies the following theorems;

<u>**Theorem 1.**</u> Using the doubly coprime factorization (26), the closed loop system is given by

$$y(t) = NRw(t) + (\tilde{\tilde{Y}} - NU)T^{-1}C\xi(t) \tag{37}$$

$$u(t) = DRw(t) - (\tilde{\tilde{X}} + DU)T^{-1}C\xi(t) \tag{38}$$

This theorem shows that the poles of the transfer functions from the reference signal $w(t)$ to the output $y(t)$

and the input $u(t)$ are independent of the selection of U, that is, they are decided only by the roots of det $\widetilde{T} = 0$. And the poles of the transfer functions from the noise $\xi(t)$ to the output $y(t)$ and the input $u(t)$ can be changed through U.

Theorem 2. From the equation (36) the generalized output $\Phi(t + k_m)$ is given by

$$\Phi(t + k_m) = (z^{k_m}\widetilde{E} - UT^{-1})C\xi(t) \tag{39}$$

The optimally predicted value of $\Phi(t + k_m)$ is

$$\widehat{\Phi}(t + k_m|t) = 0 \tag{40}$$

It shows that the control objective (12) is achieved for any selection of U.

Theorem 3. From the equation (36) the controller's poles are given by

$$\det\,(U_d\widetilde{G} - U_n\widetilde{E}B) = 0 \tag{41}$$

We can design them by the choices of U_d, U_n without changing the poles of the closed-loop system.

Hence the poles of the closed-loop system and the poles of the controller are designed sequentially, that is, first, the coefficients P, Q, R of the generalized output is selected so that $\widetilde{T}$ and T are stable, that is, the poles of the closed-loop system are stable, second, the newly introduced parameters U_d, U_n are designed such that the poles of controller, the roots of (41) are stable.

5 Self-tuning Controller

This section considers the case that the coefficients of the matrices A_i, B_j, C_k are assumed to be unknown. In the case we can apply an explicit self-tuning control, which calculates the control law (36) by using the identified values of the coefficients and solving the matrix equation (5). Parameter identification is obtained by the least-square method[9].

6 Example

Consider the next system with 2 inputs and 2 outputs

$$y(t) + A_1y(t-1) = B_0u(t-2) + \xi(t) \tag{42}$$

$$A_1 = \begin{bmatrix} -0.99101 & 8.80512 \times 10^{-3} \\ -0.80610 & -0.77089 \end{bmatrix}$$
$$B_0 = \begin{bmatrix} 0.89889 & -0.409329 \\ -0.56 & 0.88052 \end{bmatrix}$$

Controller is obtained to minimize the averaged value of the square of the following generalized output

$$\Phi(t+2) = P(z^{-1})y(t) + Q(z^{-1})u(t) - R(z^{-1})w(t)$$

$$\begin{aligned} P(z^{-1}) &= I, \quad Q(z^{-1}) = (1 + 1.1z^{-1})I \\ R(z^{-1}) &= I + Q(1)B^{-1}(1)A(1) \\ &= \begin{bmatrix} 1.0100 & 0.0210 \\ -2.1438 & 1.0840 \end{bmatrix} \end{aligned}$$

Reference signals $w(t) = [w_1(t)\ w_2(t)]^T$ are rectangular waves with amplitude 1.0 and periods of 50 and 40 steps respectively.

The control law (18) by conventional GMVC is

$$\begin{aligned} u(t) &= \\ &\begin{bmatrix} 1.8989 + 1.9957z^{-1} & -0.4093 - 0.4134z^{-1} \\ -0.56 + 0.2929z^{-1} & 1.8805 + 1.4488z^{-1} \end{bmatrix}^{-1} \\ &\left(\begin{bmatrix} -0.2028 & 0.3792 \\ -2.6875 & 1.7876 \end{bmatrix} w(t) \right. \\ &\left. - \begin{bmatrix} 0.975 & -0.0155 \\ 1.4203 & 0.5872 \end{bmatrix} y(t) \right) \end{aligned} \tag{43}$$

The poles of the closed-loop system are $z^{-1} = 1/0.9309$, $1/-0.9324$, $1/-0.6469$ and $1/0.4972$ and are stable. The poles of the controller are $z^{-1} = 1/-1.0721$ and $1/(-0.8409)$ and one of the poles is unstable.

Selecting the design parameters in the extended multivariable GMVC controller (35) as

$$\begin{aligned} U_d(z^{-1}) &= \begin{bmatrix} 1 & -0.06 \\ 0.2 & 1 \end{bmatrix} \\ U_n(z^{-1}) &= \begin{bmatrix} -1.5 & -0.85 \\ -0.6 & -1.24 \end{bmatrix} \end{aligned}$$

the controller is

$$\begin{aligned} u(t) &= \\ &\begin{bmatrix} 1.9325 + 1.9782z^{-1} + 0.8723z^{-2} + 1.5926z^{-3} \\ -0.1802 + 0.692z^{-1} - 0.1551z^{-2} + 0.9006z^{-3} \end{bmatrix. \\ &\quad \left. \begin{matrix} -0.5222 - 0.5003z^{-1} + 0.1344z^{-2} - 0.3236z^{-3} \\ 1.7987 + 1.3661z^{-1} + 0.8462z^{-2} + 0.1845z^{-3} \end{matrix} \right]^{-1} \\ &\left(\begin{bmatrix} -0.0415 & 0.272 \\ -2.728 & 1.8634 \end{bmatrix} w(t) - \left[\begin{matrix} -0.6102 + 2.6697z^{-2} \\ 1.0153 + 2.3461z^{-2} \end{matrix} \right. \right. \\ &\left. \left. \begin{matrix} -0.9007 - 0.4758z^{-2} \\ -0.6559 + 0.7188z^{-2} \end{matrix} \right] y(t) \right) \end{aligned} \tag{44}$$

The poles of the closed-loop system from $w(t)$ to $y(t)$ by this controller are same to ones by the controller (43) and the poles of the controller (44) are $z^{-1} = 1/-0.9892$, $1/-0.9625$, $1/(0.0755 \pm 0.8454i)$ and $1/(-0.0561 \pm 0.4992i)$. The absolute values of the poles are $|z^{-1}| = 1/0.9892$, $1/0.9625$, $1/0.8487$ and $1/0.5023$. The poles are improved to be stable.

Assuming the coefficients of the plant (42) are unknown, computer simulations of self-tuning GMVC are conducted. In the simulations, the variance of noise is

$\sigma = 0.1^2$, the initial values of identified coefficients are set to be equal to 0.8 of the true values and coefficients are identified by the least square method[9] with forgetting factor $\lambda_1 = \lambda_2 = 1$.

Simulated outputs are shown in Fig.1 using the controller (43) by conventional GMVC and in Fig.2 using the controller (44) proposed in this paper. In Fig.1 and Fig.2 the dotted lines are the reference signals.

In the simulations, the solid lines give the output responses of the case with noise and that at step $t = 60$, the feedback loop is cut. Fig.1 shows the output by the controller (43) is divergent, whereas, the outputs in Fig.2 by the controller (44) stay bounded.

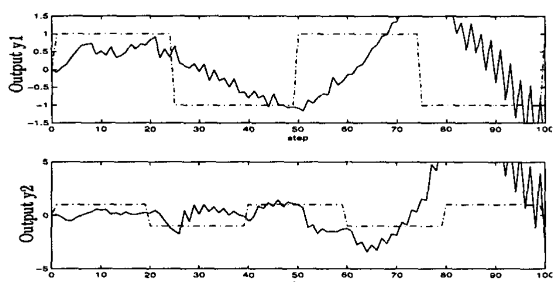

Figure 1: Outputs y_1 and y_2 by conventional GMVC

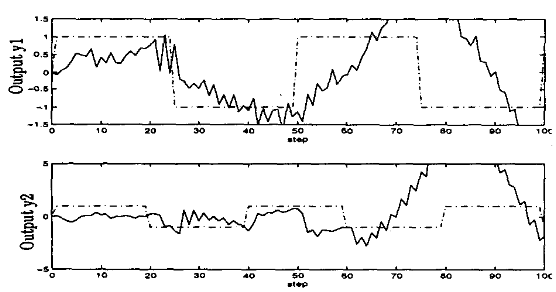

Figure 2: Outputs y_1 and y_2 by proposed GMVC

7 Conclusion

In this paper, the generalized minimum variance control (GMVC)[2] having new design parameters for single-input single-output (SISO) systems is extended for multi-input multi-output (MIMO) systems.

The extension is based on the design method given by Koivo[3] for MIMO systems. In order to apply the coprime factorization approach, the method by Koivo[3] is reformed in using both of the left- and the right-coprime factorization. Then the method is compared to Youla Parametrization and is extended directly to have new parameters.

To calculate the control law needs to solve only one Diophantine equation as in the method by Koivo and does not include the right-coprime factorization of the given system.

It is shown that the poles of the closed-loop system are independent of the selection of the newly introduced parameters. Also shown that the poles of the controller are changed by selecting the new design parameters without changing the poles of the closed-loop system. Hence the poles of the closed-loop system and the poles of the controllers are designed independently and sequentially. First the coefficients of the generalized output is designed to make the closed-loop system stable. Second, the newly introduced parameters are selected to place the poles of the controller stable.

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