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Operator-Based Nonlinear Feedback Control Design Using Robust Right Coprime Factorization

Mingcong Deng, Akira Inoue, and Kazushi Ishikawa

Abstract—In this note, robust stabilization and tracking performance of operator based nonlinear feedback control systems are studied by using robust right coprime factorization. Specifically, a new condition of robust right coprime factorization of nonlinear systems with unknown bounded perturbations is derived. Using the new condition, a broader class of nonlinear plants can be controlled robustly. When the spaces of the nonlinear plant output and the reference input are different, a space change filter is designed, and in this case this note considers tracking controller design using the exponential iteration theorem.

Index Terms—Nonlinear feedback control, operator, robust right coprime factorization, tracking.

I. INTRODUCTION

Nonlinear control system design problems have been considered by many researchers in different fields. One of the approaches is based on coprime factorization [1], [4]–[9]. The concept of coprime factorization is first considered in linear feedback control systems and provides a convenient framework for researching input–output stability problems of feedback control systems. Then, the coprime factorization problem of nonlinear feedback control systems is also discussed for nonlinear analysis, design, stabilization, and control. Especially, right coprime factorization of nonlinear systems has attracted much attention due to its usefulness in stabilization of nonlinear plants. Recently, robust right coprime factorization of nonlinear plants under perturbations has been studied in [1], and output tracking problem of perturbed nonlinear plants [3] has been considered by extending the design scheme given in [1]. In this case, the nonlinear plant output and reference input share the same space and the Bezout identity is equal to the identity operator, where the perturbation is known. The above robust right coprime factorization leads to the robust stabilization of the entire feedback control system using an operator-theoretic approach, where the stability is based on the internal stability. However, the method only controls a class of nonlinear plants with bounded perturbations; the problem of checking the robust right coprime factorization condition for nonlinear plant with unknown bounded perturbations might be also difficult in practice; and the plant output tracking problem has not been considered for the case in which the nonlinear plant output and reference input is different.

The purpose of this note is concerned with robust stabilization and tracking performance of operator based nonlinear feedback control systems using robust right coprime factorization. That is, we develop a robust right coprime factorization condition and a tracking controller based on generalized Lipschitz operator theory [2]. The detailed explanation is given as follows. A new condition for the robust right coprime factorization of nonlinear plants with unknown bounded perturbations is given. The new condition is obtained by using the generalized Lipschitz operator theory and the definition of unimodular operator. Robust stabilization of the nonlinear feedback control system can be obtained

by using the proposed condition. Concerning the plant output tracking problem, in general the spaces of the plant output and reference input are different. In this case, a space change operator is designed, and we consider a tracking controller using the exponential iteration theorem [10], where the spaces of operators are defined by using the generalized Lipschitz norm. As a result, a broader class of nonlinear plants with bounded perturbations can be robustly controlled and satisfactory tracking performance can be obtained.

The outline of this note is as follows. This note begins with several definitions of operators, coprime factorization and internal stability. Useful references on these topics are [1], [2], and [6], which provide numerous relevant results. In Section III, the proposed robust right coprime factorization condition is given. Tracking design problems are discussed in Section IV. Finally, we draw some conclusions in Section V.

II. MATHEMATICAL PRELIMINARIES

In this section, we recall several definitions of operators, right coprime factorization and internal stability.

Consider a space U of time functions. U is said to be a vector space if it is closed under addition and scalar multiplication. The space U_s is said to be normed if each element x in U_s has a norm $\|x\|$ which can be defined in any way so long as the following three properties are fulfilled: 1) $\|x\|$ is real, positive number and is different from zero unless x is identically zero. 2) $\|ax\| = |a| \|x\|$. 3) $\|x_1 + x_2\| \leq \|x_1\| + \|x_2\|$ ($x_1, x_2 \in U_s$).

Let U_s and Y_s be two normed linear spaces over the field of complex numbers, endowed, respectively, with norms $\|\cdot\|_{U_s}$ and $\|\cdot\|_{Y_s}$. Let $A : U_s \rightarrow Y_s$ be an operator mapping from U_s to Y_s and denote by $\mathcal{D}(A)$ and $\mathcal{R}(A)$, respectively, the *domain* and *range* of A . As mentioned above, if the operator $A : \mathcal{D}(A) \rightarrow Y_s$ satisfies

$$A : ax_1 + bx_2 \rightarrow aA(x_1) + bA(x_2)$$

for all $x_1, x_2 \in \mathcal{D}(A)$ and all $a, b \in \mathbb{C}$, then A is said to be linear; otherwise, it is said to be nonlinear. Let $\mathcal{N}(U_s, Y_s)$ be the family of all nonlinear operators mapping from $\mathcal{D}(A) \subseteq U_s$ into Y_s . Recall that $\mathcal{L}(U_s, Y_s)$ is used to denote the family of bounded linear operators from U_s to Y_s . Obviously, $\mathcal{L}(U_s, Y_s) \in \mathcal{N}(U_s, Y_s)$. In the case that $U_s = Y_s$, we use the notation $\mathcal{L}(U_s)$ and $\mathcal{N}(U_s)$, respectively, instead of $\mathcal{L}(U_s, Y_s)$ and $\mathcal{N}(U_s, Y_s)$ for simplicity.

Let D_s be a subset of U_s and $\mathcal{F}(D_s, Y_s)$ be the family of operators A in $\mathcal{N}(U_s, Y_s)$ with $\mathcal{D}(A) = D_s$. A (semi)-norm on (a subset of) $\mathcal{F}(D_s, Y_s)$ is denoted by

$$\|A\| := \sup_{\substack{x_1, x_2 \in D_s \\ x_1 \neq x_2}} \frac{\|A(x_1) - A(x_2)\|_{Y_s}}{\|x_1 - x_2\|_{U_s}} \quad (1)$$

if it is finite. In general, it is a seminorm in the sense that $\|A\| = 0$ does not necessarily imply $A = 0$. In fact, it can be easily seen that $\|A\| = 0$ if and only if A is a constant-operator (need not be zero) that maps all elements from D_s to the same element in Y_s .

Definition 1: Let $Lip(D_s, Y_s)$ be the subset of $\mathcal{F}(D_s, Y_s)$ with each element A satisfying $\|A\| < \infty$. Each $A \in Lip(D_s, Y_s)$ is called a Lipschitz operator mapping from D_s to Y_s , and the number $\|A\|$ is called the Lipschitz seminorm of the operator A on D_s .

In this note, we assume operators are of Lipschitz type and use seminorm of Lipschitz operators. It is clear that an element A of $\mathcal{F}(D_s, Y_s)$ is in $Lip(D_s, Y_s)$ if and only if there is a number $L \geq 0$ such that

$$\|A(x_1) - A(x_2)\|_{Y_s} \leq L \|x_1 - x_2\|_{D_s}$$

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for all $x_1, x_2 \in D_s$. The norm $\|A\|$ is the least such number L . It is also evident that a Lipschitz operator is both bounded and continuous on its domain. Basic theories of nonlinear Lipschitz operators are given in [2].

Definition 2: Let U^e and Y^e be two extended linear spaces, which are associated respectively with two given Banach spaces U_B and Y_B of measurable functions defined on the time domain $[0, \infty)$, where a Banach space is a complete vector space with a norm. Let D^e be a subset of U^e . A nonlinear operator $A : D^e \rightarrow Y^e$ is called a generalized Lipschitz operator on D^e if there exists a constant L such that

$$\| [A(x)]_T - [A(\tilde{x})]_T \|_{Y_B} \leq L \|x_T - \tilde{x}_T\|_{U_B} \quad (2)$$

for all $x, \tilde{x} \in D^e$ and for all $T \in [0, \infty)$.

Note that the least such constant L is given by

$$\|A\| := \sup_{T \in [0, \infty)} \sup_{\substack{x, \tilde{x} \in D^e \\ x_T \neq \tilde{x}_T}} \frac{\| [A(x)]_T - [A(\tilde{x})]_T \|_{Y_B}}{\|x_T - \tilde{x}_T\|_{U_B}} \quad (3)$$

which is a seminorm for general nonlinear operators and is the actual norm for linear A . The actual norm for a nonlinear operator A is given by

$$\begin{aligned} \|A\|_{Lip} &= \|A(x_0)\|_Y + \|A\| \\ &= \|A(x_0)\|_{Y_B} \\ &\quad + \sup_{T \in [0, \infty)} \sup_{\substack{x, \tilde{x} \in D \\ x_T \neq \tilde{x}_T}} \frac{\| [A(x)]_T - [A(\tilde{x})]_T \|_{Y_B}}{\|x_T - \tilde{x}_T\|_{U_B}} \end{aligned} \quad (4)$$

for any fixed $x_0 \in D^e$.

Here, it follows that for any $T \in [0, \infty)$

$$\begin{aligned} \| [A(x)]_T - [A(\tilde{x})]_T \|_{Y_B} &\leq \|A\| \|x_T - \tilde{x}_T\|_U \\ &\leq \|A\|_{Lip} \|x_T - \tilde{x}_T\|_{U_B} \end{aligned}$$

and let $Lip(D^e)$ denote the family of nonlinear generalized Lipschitz operators that map D^e to itself.

Remark 1: The family of standard Lipschitz operators and the family of generalized Lipschitz operators are not comparable since they have different domains and ranges. However, it can be easily verified that when the extended linear spaces become standard with all the subscript T dropped, generalized Lipschitz operators become standard ones. It can also be verified that many standard Lipschitz operators are also extended Lipschitz. In this note, we assume that $U_s = U^e$ and $Y_s = Y^e$.

III. PROPOSED ROBUST RIGHT COPRIME FACTORIZATION CONDITION

Let U and Y be linear spaces over the field of complex numbers, the normed linear subspaces U_s and Y_s are also called the stable subspaces of linear spaces U and Y , respectively. Let $Q : U \rightarrow Y$ be an operator. For any operator defined they are causal, but not necessarily linear or bounded (with a finite operator norm). We always assume that $\mathcal{D}(Q) = U$ with $\mathcal{R}(Q) \subseteq Y$. In this note, an operator $Q : U \rightarrow Y$ is said to be bounded input bounded output (BIBO) stable or simply, stable if $Q(U_s) \subseteq Y_s$.

Definition 3 [1]: Let $\mathcal{S}(U, Y)$ be the set of stable operators from U to Y . Then, $\mathcal{S}(U, Y)$ contains a subset defined by

$$\begin{aligned} \mathcal{U}(U, Y) \\ = \{M : M \in \mathcal{S}(U, Y), M \text{ is invertible with } M^{-1} \in \mathcal{S}(Y, U)\}. \end{aligned}$$

Elements of $\mathcal{U}(U, Y)$ are called unimodular operators.

For brevity, we omit the definitions of bounded-input-bounded-output (BIBO) stabilization (or simply stabilization), well-posedness,

internally stabilization and right coprime factorization given in [1]. We can use the following theorems of a right coprime factorization. We simply denote *rcf* instead of right coprime factorization.

In the following, the proposed robust *rcf* condition is described. Consider the nonlinear feedback system shown in Fig. 1 which is assumed to be well-posed.

Let the nominal plant and the plant perturbation be P and ΔP , respectively. The overall plant \tilde{P} is given as follows:

$$\tilde{P} = P + \Delta P \quad (5)$$

where \tilde{P} and P are nonlinear and unstable operators. The *rcf* of the nominal plant P and $P + \Delta P$ are

$$P = ND^{-1}, P + \Delta P = (N + \Delta N)D^{-1} \quad (6)$$

where N , ΔN , and D are stable operators and D is invertible. We assume that ΔN is unknown but the upper and lower bounds of ΔN are known.

Let the input space, output space, quasistate space be U , Y and W . N , ΔN and D are $N : W \rightarrow Y$, $\Delta N : W \rightarrow Y$, $D : W \rightarrow U$, respectively. A , B are the controller and stable operators and B is invertible. We can choose $W = U$, meaning U and W are the same linear space.

Then, we can get the Bezout identity

$$AN + BD = M, \quad \text{for some } M \in \mathcal{U}(W, U) \quad (7)$$

where $\mathcal{U}(W, U)$ is the set of unimodular operator. So, when M is a unimodular operator, it is said that operators A , N , B , D satisfy the Bezout identity. M of the Bezout identity is equal to the operator $M : W \rightarrow U$ of the overall system. If M is a unimodular operator, the operator $M^{-1} : U \rightarrow W$ of the overall system is stable. N is a stable operator because of *rcf*; finally the system is internally stable because all signals are bounded by the fact that input signal $u \in U$. $N + \Delta N$ is a nonlinear and stable operator, but the stability of the feedback system shown in Fig. 1 is unknown. Here, we provide the following condition. The equation of the system with the perturbation ΔN is

$$A(N + \Delta N) + BD = M, \quad M \text{ is unimodular.} \quad (8)$$

When N , D , A and B satisfy (8) and (7) is the Bezout identity of the nominal plant P , the system shown in Fig. 1 is stable [1]. It means $\mathcal{R}(\Delta N)$, the range of ΔN , is included in $\mathcal{N}(A)$, the null set of A , where ΔP is perturbation of the plant which can represent only ΔN . The reason is that ΔP is an additive uncertainty. However, it is difficult to check (8) if ΔN is unknown. Also, in some cases, (8) is not satisfied.

In this note, we have the following theorem to guarantee the stability of the nonlinear feedback control system with perturbation.

Theorem 1: Let D^e be a linear subspace of the extended linear space U^e associated with a given Banach space U_B , and let $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$. Let the Bezout identity of the nominal plant and the exact plant be $AN + BD = M \in \mathcal{U}(W, U)$, $A(N + \Delta N) + BD = \tilde{M}$, respectively. Under the condition of controller A to satisfy (8), if

$$\| [A(N + \Delta N) - AN]M^{-1} \| < 1 \quad (9)$$

the system shown in Fig. 1 is stable, where $\|\cdot\|$ is defined in (3).

Proof: M is unimodular operator, then M is invertible based on Definition 3. From $AN + BD = M$, $A(N + \Delta N) + BD = \tilde{M}$, we have

$$\tilde{M} = M + [A(N + \Delta N) - AN].$$

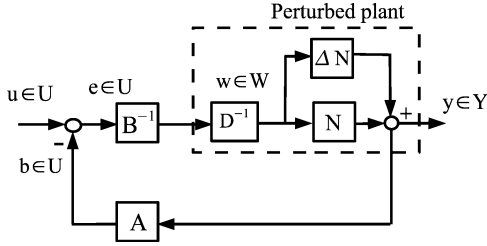


Fig. 1. Nonlinear feedback system with perturbation.

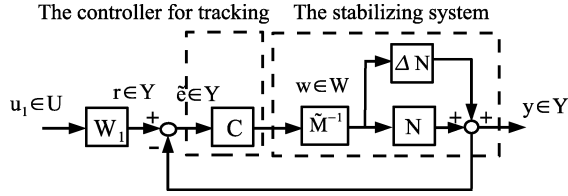


Fig. 2. Nonlinear feedback system for tracking.

Since

$$\tilde{M} = M + [A(N + \Delta N) - AN] = [I + (A(N + \Delta N) - AN)M^{-1}]M$$

and $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$, $I + (A(N + \Delta N) - AN)M^{-1}$ is invertible based on (9) [2], where I is the identity operator. Consequently, we have $\tilde{M}^{-1} = M^{-1}[I + (A(N + \Delta N) - AN)M^{-1}]^{-1}$. Meanwhile, since $\tilde{M} = M + [A(N + \Delta N) - AN]$, $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$, and $M \in \mathcal{U}(W, U)$, we have $\tilde{M} \in \mathcal{U}(W, U)$ provided that the system shown in Fig. 1 is well-posed. As a result, for any $u \in U_s$ we have $w = \tilde{M}^{-1}u \in W_s$. Further, since $y = (N + \Delta N)w$, $e = BDw$, and $b = A(N + \Delta N)w$, the stability of A , B , N , ΔN and D implies that $y \in Y_s$, $e \in U_s$ and $b \in U_s$. Then, the system is overall stable [1].

The main difference between the above condition and the condition of [1] lies in that, the proposed condition is in an inequality, and the condition of [1] is $A(N + \Delta N) - AN = 0$ for $A(N + \Delta N) + BD = AN + BD$. This shows that the proposed one includes more sets for designing controllers. That is, (9) includes the condition $A(N + \Delta N) - AN = 0$. Also, if $\| [A(N + \Delta N) - AN]M^{-1} \|$ of (9) can be obtained by using bounded information of ΔN , the detailed ΔN is not necessary. In the following, plant output tracking performance will be considered.

IV. PROPOSED TRACKING DESIGN SCHEME

It has been shown that the system in Fig. 1 is stable, but we have not considered the plant output tracking performance yet. In this section, we discuss the plant output tracking problems for the stabilizing system described in Fig. 1, where we assume that the spaces of the nonlinear plant output and reference input are different, namely, $U \neq Y$. First, a space change operator is designed. Next, we consider a tracking controller based on the exponential iteration theorem.

Consider the nonlinear feedback system shown in Fig. 1. We design a tracking system given in Fig. 2. The stabilizing system as a part of Fig. 2 is equal to the system in Fig. 1 stabilized by the proposed method in Section III. $u_1 \in U$ is the reference input. W_1 is the space change operator to transform the reference input signal $u_1 \in U$ into the real reference input signal $r \in Y$. C is the designed tracking controller.

First, we design a space change operator W_1 for making real reference input signal r in space Y so that one of conditions of the exponential iteration theorem is satisfied. That is, the spaces of r and y are the same. In general, if $r \neq u_1$, W_1 is designed such that $W_1(u_1) - u_1$

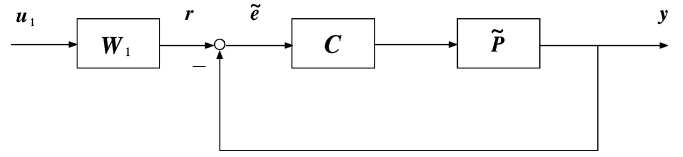


Fig. 3. Equivalent block-diagram of Fig. 2.

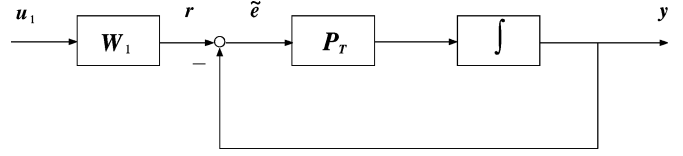


Fig. 4. Equivalent block-diagram of Fig. 3.

can be made arbitrarily small by making T large enough. Fig. 2 can be further re-expressed in the form of Fig. 3.

The error signal \tilde{e} is shown in the following equation:

$$\tilde{e} = (I + \tilde{P}C)^{-1}(r). \quad (10)$$

It is obvious that the operator $(I + \tilde{P}C)^{-1}$ is mapping Y to Y from Fig. 4. Hence, the relationship in the reference signal r and the error signal \tilde{e} is in linear space. Then, one of conditions of the exponential iteration theorem is also satisfied, namely, the spaces of \tilde{e} and y are the same.

Next, the controller C is designed so that the open loop $\tilde{P}C$ of a feedback system in Fig. 3 consists of an integrator in cascade with a system P_T (Fig. 4) and satisfies the following conditions.

- 1) For all t in $[0, T]$, C is stable, and $P_T(r) \geq K_1 > 0$ as $T \geq t \geq t_1 \geq 0$, $r > 0$.
- 2) $\tilde{P}C(0) = 0$.
- 3) $\| \tilde{P}C(x) - \tilde{P}C(y), t \| \leq h \int_0^t \| x - y, t_1 \| dt_1$ for all x, y in Y_s and for all t in $[0, T]$, h is any constant and is the gain of P_T in the first norm, where the norm of x restricted to any interval $[0, T]$ will be denoted by $\| x, t \|$.

In this note, the gain of P_T is the generalized Lipschitz operator norm defined in Definition 2. Since C and \tilde{P} are stable, the existence of h is ensured. Defining an operator from r to y as \tilde{G} , we have $\tilde{G} = PC * (I - \tilde{G})$ as the feedback equation, where the cascade $PC * (I - \tilde{G})$ means the operator PC following the operator $I - \tilde{G}$. Then, we summarized the exponential iteration theorem in Lemma 1.

Lemma 1 (Exponential Iteration Theorem [10]): The feedback equation $\tilde{G} = PC * (I - \tilde{G})$, in which all operators map the Banach space Y_B into itself, has a unique solution for \tilde{G} , which converges uniformly on $[0, T]$, provided that conditions 2) and 3) are satisfied. The plant output is bounded [10].

Lemma 1 means since Y_s is complete the sequence is uniformly convergent on $[0, T]$. It may be established that $\tilde{G} - PC * (I - \tilde{G}) = 0$ and it is unique. Then the plant output is bounded [10]. Further, $(I + \tilde{P}C)^{-1}(r)(t)$ exists.

Theorem 2: The error signal \tilde{e} with the controller C can be made arbitrarily small. That is, $y(t) - r(t)$ can be made arbitrarily small by $t \leq T$ large enough.

Proof: From Figs. 3 and 4, we have

$$y(t) = r(t) - \tilde{e} \quad (11)$$

where

$$\tilde{P}C = \int_0^t P_T dt_2. \quad (12)$$

From (10) and (11), we have

$$y(t) = r(t) - (I + \tilde{P}C)^{-1}(r)(t). \quad (13)$$

Since I is the identity operator, namely, $I(r) = r$ [1], [10], from (12)

$$\begin{aligned} y(t) &= r(t) - (r(t) + \tilde{P}C(r(t)))^{-1} \\ &= r(t) - \left(r(t) + \int_0^t P_T(r(t))dt \right)^{-1}. \end{aligned} \quad (14)$$

Considering Condition 1) of the controller design, namely, $P_T(r) \geq K_1 > 0$ as $T \geq t \geq t_1 \geq 0$, we obtain

$$\int_0^t P_T(r(t))dt_2 \geq \int_0^{t_1} P_T(r(t))dt_2 + K_1 \int_{t_1}^t dt_2. \quad (15)$$

$K_1 \int_{t_1}^t dt_2$ can be made arbitrarily large by making $t \leq T$ large enough. Then, $(r(t) + \int_0^t P_T(r(t))dt_2)^{-1}$ becomes arbitrarily small. From (14), $y(t) - r(t)$ can be made arbitrarily small. This fact leads to the desired result.

It is noted that when the spaces of plant output and reference input are same, instead of space change filter, linear filter is required and the tracking controller proposed in this note still works. Also, the method proposed in [3] is difficult to use for the case in this note, because the method requires the detailed information of ΔN .

V. CONCLUSION

Based on the operator theory, the condition of robust stabilization for nonlinear feedback control system with unknown bounded perturbations has been given by using robust right coprime factorization; and tracking control using the exponential iteration theorem has also been considered.

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Wiener–Hammerstein Modeling of Nonlinear Effects in Bilinear Systems

Ai Hui Tan

Abstract—The modeling of nonlinear effects in bilinear systems using Wiener–Hammerstein models is considered. Such models are chosen based on the block-oriented structure of bilinear systems and the shapes of their Volterra kernels. Theoretical analysis is given for first-order bilinear systems, and simulation results are presented for first- and second-order systems. While the nonlinearity is only approximately Wiener–Hammerstein, the models are able to capture a significant part of the nonlinear dynamics. In order to reduce the complexity of optimizing the model parameters, the linear subsystems are estimated using the technique of linear interpolation in the frequency domain.

Index Terms—Bilinear systems, block-oriented models, multisine signals, system identification, Wiener–Hammerstein models.

I. INTRODUCTION

A continuous single-input–single-output bilinear system may be represented in the extended phase variable state–space canonical form [1] as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + u\mathbf{D}\mathbf{x}, \quad y = \mathbf{c}^T\mathbf{x} \quad (1)$$

where u and y represent the input and output, respectively; \mathbf{x} is the system state vector

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \cdot & & & & \cdot \\ \cdot & & \ddots & & \cdot \\ \cdot & & & \ddots & 0 \\ 0 & \dots & & 0 & 1 \\ -\alpha_0 & \dots & \dots & \dots & -\alpha_{n-1} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

$$\mathbf{c}^T = [\beta_0 \quad \beta_1 \quad \dots \quad \beta_m \quad 0 \quad \dots \quad 0] \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} 0 & \dots & 0 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ 0 & \dots & 0 \\ \rho_0 & \dots & \rho_{n-1} \end{bmatrix}.$$

Note that $\alpha_0, \alpha_1, \dots, \alpha_{n-1}, \beta_0, \beta_1, \dots, \beta_m, \rho_0, \rho_1, \dots, \rho_{n-1}$ are constants and that $n > m$. The discretization of such systems is discussed in [1].

Systems in a range of industries are well modeled by bilinear systems, for example, gas-fired furnaces [2], polymerization reactors [3] and papermaking processes [4]. While such a system possesses many similarities to linear systems, the effects of nonlinearity, caused by the multiplicative term involving the product of the control input and system states, are often not negligible. As such, considerable effort has been made in the past to model the effects of the nonlinearity, for example, using Volterra approximation approaches [5] and the concept of related linear dynamics [6]. Ironically, while these effects may pose problems to the effective control of the system, they often represent only a small percentage of the total output power. Taking the above into consideration, there is therefore a need for a fast and simple method to

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