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THREE-DIMENSIONAL EDDY CURRENT ANALYSIS BY THE BOUNDARY ELEMENT METHOD USING VECTOR POTENTIAL

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Abstract: A boundary element method using a magnetic vector potential for eddy current analysis is described. For three-dimensional problems, tangential and normal components of the vector potential, tangential components of the magnetic flux density and an electric scalar potential on conductor surfaces are chosen as unknown variables. When the approximation that the conductivity of conductor is very large in comparison with the conductivity of air is introduced, unknowns can be reduced. Futhermore, for axisymmetric models the scalar potential can be eliminated from unknown variables. Formulation of the boundary element method using the vector potential and the computation results by the proposed method are presented.

INTRODUCTION

For linear problem in three-dimensional eddy current analysis, boundary element methods are attractive in term of pre-processing and computer requirements. Several formulations of boundary integral equations were reported[1-4]. The boundary element method using unknown electric field vector and magnetic flux density vector has been developed for three-dimensional eddy current analysis[5], and can be applied to conducting, magnetic and dielectric regions.

In this paper, we propose a boundary element method using a magnetic vector potential as an unknown vector variable. For the conducting and magnetic regions in which fields are excited by current source, the eddy current analysis can be performed by using the magnetic vector potential and an electric scalar potential. When the approximation that the conductivity of conductor is very large in comparison with the conductivity of air is introduced, the normal component of the gradient of the scalar potential on the boundaries can be related to the normal component of the vector potential and unknown variables are reduced. Formulation of the proposed boundary element method and its computation results are described.

FORMULATION

Boundary element methods using vector variables are formulated by the vector Green's theorem[6]. In the proposed method, a magnetic vector potential is introduced as an unknown variable and the boundary integral equations are formulated.

A magnetic vector potential **A** and an electric scalar potential φ are generally introduced to solve the Maxwell's equations. The electric field **E** and the magnetic flux density **B** with sinusoidal time dependence are defined by using the potentials as follows:

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\varphi \tag{1}$$

$$\mathbf{B} = \nabla\times\mathbf{A} \tag{2}$$

The sources of the vector potential **A** and the scalar potential φ in free space are electric current and electric charge, respectively. Here, we consider the region including conductors and magnetic materials in which the fields are excited by electric currents and no displacement current arises. Therefore the source of the scalar potential appears on the boundary surfaces between conductor and air.

From Eqs. (1) and (2), the equations for **A** and φ to solve eddy current distributions which are governed by the Maxwell's equations are given by

$$\nabla\times\nabla\times\mathbf{A}-\omega^2\mu\varepsilon^*\mathbf{A}+j\omega\mu\varepsilon^*\nabla\varphi=\mu\mathbf{J}_0 \tag{3}$$

$$\nabla^2\varphi + j\omega\nabla\cdot\mathbf{A} = -\rho_0/\varepsilon^* \tag{4}$$

where $\mathbf{J}_0$ and ρ_0 are the source current density and the source charge density, respectively. Furthermore the Lorentz gauge is introduced as a gauge condition. That is given by

$$\nabla\cdot\mathbf{A} + j\omega\mu\varepsilon^*\varphi = 0 \tag{5}$$

where ε^* is the generalized complex permittivity. In the case of integral equation methods using the vector potential and the scalar potential, we can prove that the Lorentz gauge is equivalent to the following continuity equation.

$$\nabla\cdot\mathbf{J} + j\omega\rho = 0 \tag{6}$$

Using the vector Green's theorem for the vector potential **A** and the Green's theorem for the scalar potential φ, following boundary integral equations at computation point i are obtained from Eqs. (1)-(5).

$$\frac{\Omega_i}{4\pi}\mathbf{A}_i=\int_S\{-(\mathbf{A}\cdot\mathbf{n}')\nabla'\phi+(\mathbf{A}\times\mathbf{n}')\times\nabla'\phi-(\nabla'\times\mathbf{A})\times\mathbf{n}'\phi$$
$$-j\omega\mu\varepsilon^*\varphi\phi\mathbf{n}'\}\,dS + \mu\int_V\mathbf{J}_0\phi dv + \mathbf{A}_{0i} \tag{7}$$

$$\frac{\Omega_i}{4\pi}\varphi_i=\int_S(\phi\nabla'\varphi\cdot\mathbf{n}'-\varphi\nabla'\phi\cdot\mathbf{n}')\,dS + \frac{1}{\varepsilon}\int_V\rho_0\phi dv + \varphi_{0i} \tag{8}$$

where Ω_i is the solid angle subtended by S at i, $\mathbf{A}_{0i}$ and φ_{0i} are potentials which are induced by the external sources, and ϕ is the fundamental solution given as

$$\phi=\frac{e^{-jkr}}{4\pi r}$$
$$k=\omega\sqrt{\mu\varepsilon^*}$$

The boundary conditions between the region 1 and the region 2 are given by the continuity conditions of the potentials and the tangential components of the magnetic field and the normal component of the electric flux density on the boundaries as follows:

$$\mathbf{A}_{i1}=\mathbf{A}_{i2} \tag{9}$$

$$\varphi_{i1}=\varphi_{i2} \tag{10}$$

$$(\nabla'\times\mathbf{A})_{i1}\times\mathbf{n}_i'/\mu_1=(\nabla'\times\mathbf{A})_{i2}\times\mathbf{n}_i'/\mu_2 \tag{11}$$

$$\varepsilon_1^*\,(-j\omega\mathbf{A}_{i1}-\nabla'\varphi_{i1})\cdot\mathbf{n}_i'=\varepsilon_2^*\,(-j\omega\mathbf{A}_{i2}-\nabla'\varphi_{i2})\cdot\mathbf{n}_i' \tag{12}$$

After applying Eqs. (7) and (8) to the both side of the boundary surfaces, we can obtain the boundary integral equations which is expressed by the unknown variables in region 1 as follows:

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$$\frac{1}{2}\mathbf{A}_{i1} = \int_S [-(\mathbf{A}\cdot\mathbf{n}')_1\nabla'\phi + (\mathbf{A}\times\mathbf{n}')_1\times\nabla'\phi - \{(\nabla'\times\mathbf{A})\times\mathbf{n}'\}_1\phi - j\omega\mu_1\varepsilon_1{}^*\varphi_1\phi\mathbf{n}']\,dS \tag{13}$$

$$\frac{1}{2}\mathbf{A}_{i1} = \int_S [(\mathbf{A}\cdot\mathbf{n}')_1\nabla'\phi - (\mathbf{A}\times\mathbf{n}')_1\times\nabla'\phi + \frac{\mu_1}{\mu_2}\{(\nabla'\times\mathbf{A})\times\mathbf{n}'\}_1\phi + j\omega\mu_2\varepsilon_2{}^*\varphi_1\phi\mathbf{n}']\,dS \tag{14}$$

$$\frac{1}{2}\varphi_{i1} = \int_S \{\phi(\nabla'\varphi\cdot\mathbf{n}')_1 - \varphi_1\nabla'\phi\cdot\mathbf{n}'\}\,dS \tag{15}$$

$$\frac{1}{2}\varphi_{i1} = \int_S [-\phi\{j\omega(\frac{\varepsilon_1{}^*}{\varepsilon_2{}^*} - 1)(\mathbf{A}\cdot\mathbf{n}')_1 + \frac{\varepsilon_1{}^*}{\varepsilon_2{}^*}(\nabla'\varphi\cdot\mathbf{n}')_1\} + \varphi_1\nabla'\phi\cdot\mathbf{n}']\,dS \tag{16}$$

When the approximation that the conductivity of conductor is very large in comparison with the conductivity of air can be introduced, the term, $\nabla'\varphi\cdot\mathbf{n}'$, can be related to $\mathbf{A}\cdot\mathbf{n}'$ by Eq. (12) because the normal component of the electric field is equal to zero. The relation between $\nabla'\varphi\cdot\mathbf{n}'$ and $\mathbf{A}\cdot\mathbf{n}'$ is given by

$$\nabla'\varphi\cdot\mathbf{n}' = -j\omega(\mathbf{A}\cdot\mathbf{n}') \tag{17}$$

Therefore the interaction between the simultaneous equations for $\mathbf{A}$ and that for φ can be removed, and the final simultaneous equations which consist of only Eqs. (13) and (14) are solved for $\mathbf{A}$, $(\nabla'\times\mathbf{A})\times\mathbf{n}$ and φ. In this case, $\nabla'\varphi\cdot\mathbf{n}'$ is removed from the unknowns.

In axisymmetric models, the normal component of the vector potential and the scalar potential become zero because the eddy current has no normal component and the divergence of the vector potential is zero in all places. As the result, the number of unknowns can be reduced.

The magnetic flux density $\mathbf{B}_i$ and the electric field $\mathbf{E}_i$ at the field point i in the region to be analyzed are given by using the rotation of Eq. (7) and the gradient of Eq. (8) as follows:

$$\mathbf{B}_i = (\nabla\times\mathbf{A})_i = \int_S [-\{(\mathbf{A}\times\mathbf{n}')\cdot\nabla\}\nabla'\phi + k^2(\mathbf{A}\times\mathbf{n}')\phi + \nabla\phi\times\{(\nabla'\times\mathbf{A})\times\mathbf{n}'\} - j\omega\mu\varepsilon^*\varphi\nabla\phi\times\mathbf{n}']\,dS + \mu\int_V \mathbf{J}_0\times\nabla\phi\,dv + \mathbf{B}_{0i} \tag{19}$$

$$\mathbf{E}_i = -j\omega\mathbf{A}_i - (\nabla\varphi)_i = \int_S \{\varphi\omega(\mathbf{A}\cdot\mathbf{n}')\nabla'\phi - j\omega(\mathbf{A}\times\mathbf{n}')\times\nabla'\phi + j\omega(\nabla'\times\mathbf{A})\times\mathbf{n}'\phi + k^2\varphi\phi\mathbf{n}' - \nabla\phi(\nabla'\varphi\cdot\mathbf{n}') + \varphi(\mathbf{n}'\cdot\nabla)\nabla'\phi\}\,dS - \frac{1}{\varepsilon}\int_V \rho_0\nabla\phi\,dv + \mathbf{E}_{0i} \tag{20}$$

COMPUTATION RESULTS

In order to verify the applicability of the proposed boundary element method, a conducting sphere model in an uniform alternating magnetic field as shown in Fig. 1 was chosen as a computation model which can be solved theoretically. The conducting sphere model is an axisymmetric model but the computation of the model was performed as a three-dimensional model. The number of triangular elements is 64 on one eighth part of the sphere surface and unknown variables are defined to be constant on each element.

Figure 2 shows the distributions of the vector potential along x-axis for 50(Hz). Computation results agree with theoretical values[4]. Figure 3 and 4 show the distributions of the vector potential and the tangential component of magnetic flux density which are unknowns on the boundary surface, respectively. In the conducting sphere model, the external potentials $\mathbf{A}_{0i}$ and φ_{0i} were defined by

$$\mathbf{A}_{0i} = -\frac{y_i}{2}\mathbf{i} + \frac{x_i}{2}\mathbf{j} \tag{21}$$

$$\varphi_{0i} = 0 \tag{22}$$

Eqs. (21) and (22) give the external magnetic flux density $\mathbf{B}_{0i}=\mathbf{k}$ and the external electric field $\mathbf{E}_{0i}=j\omega(y_i/2\mathbf{i}-x_i/2\mathbf{j})$ which arise from a large circular loop current.

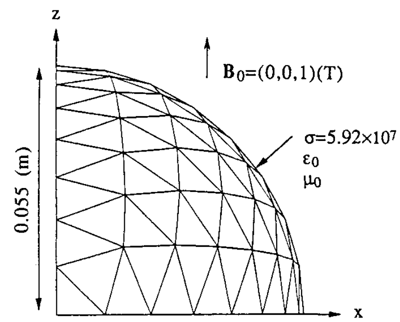

Fig. 1 Conducting sphere model.

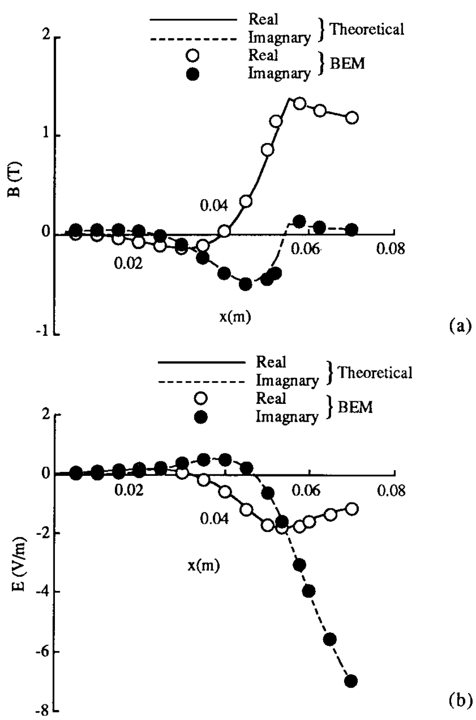

Fig. 2 Distributions of the magnetic flux density and the electric field along x-axis, (a) magnetic flux density, (b) electric field.

Figure 5 shows a conducting cube model in uniform alternating magnetic field which is a truly three-dimensional model. The external potentials were given by Eqs. (21) and (22). The number of triangular elements is 216 on one eighth part of the cube surface. Figure 6 shows the distribution of the eddy current density on the x-y, y-z, x-z planes for 50(Hz). Figure 7 shows the distribution of the magnetic flux density on the y-z plane. The computation results agree with those of the boundary element method using magnetic flux density and electric field as unknowns[5].

Equi-value lines of the normal component of the vector potential on the boundary between the conductor and air are shown in Fig. 8. Large values appear at the locations where the normal components of the external vector potential are large. Therefore the scalar potential is induced so that the normal component of the electric field which is expressed by $-j\omega\mathbf{A}\cdot\mathbf{n}-\nabla\varphi\cdot\mathbf{n}$ becomes zero.

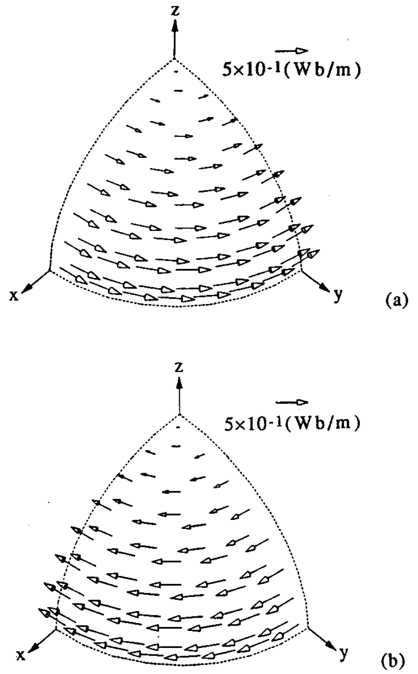

Fig. 3 Computation results of the vector potential on the boundary surface, (a)real part, (b)imaginary part.

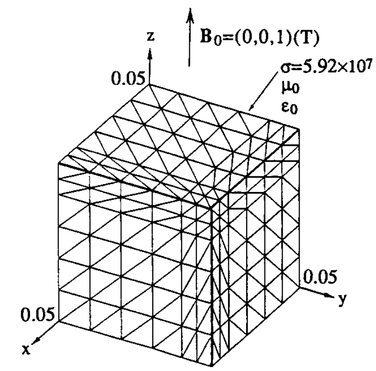

Fig. 5 Conducting cube model.

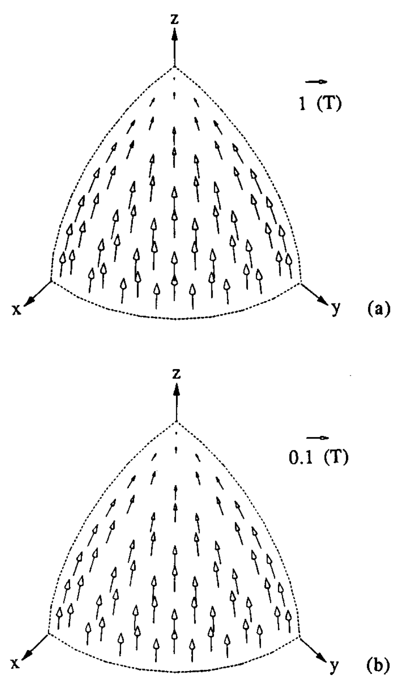

Fig. 4 Computation results of the tangential component of the magnetic flux density, (a)real part, (b)imaginary part.

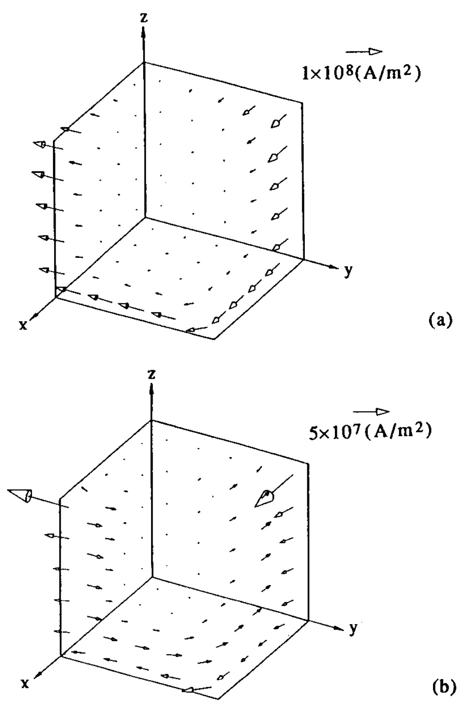

Fig. 6 Eddy current distributions on the x-y, y-z, x-z planes, (a) real part, (b) imaginary part.

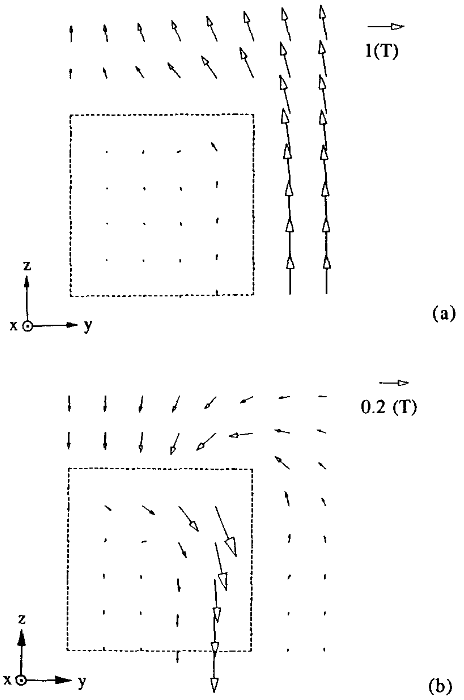

Fig. 7 Magnetic flux density distributions on the y-z plane, (a) real part, (b) imaginary part.

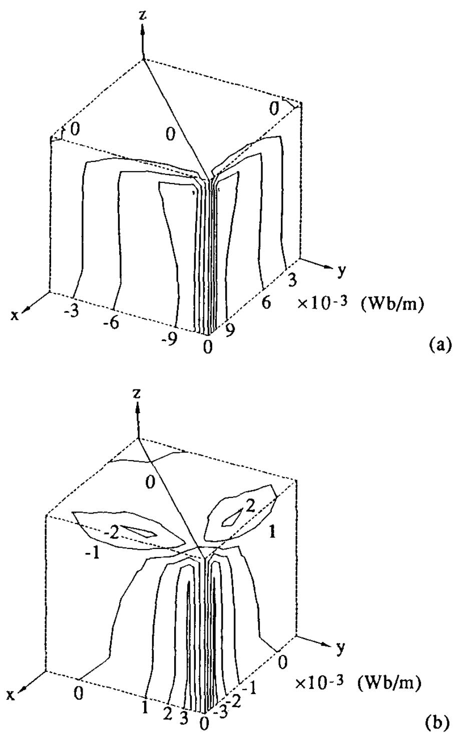

Fig. 8 Equipotential lines of the normal component of the vector potential on the boundary, (a) real part, (b) imaginary part.

CONCLUSION

The boundary element method using the magnetic vector potential was proposed, and the formulation and the computation results were described. The conclusions can be summarized as follows:

1) For the three-dimensional problems, the tangential and normal components of the vector potential, the tangential components of the magnetic flux density which is given by the curl of the vector potential and the scalar potential are defined as unknown variables on the boundaries.

2) Using the approximation that the conductivity of conductor is very large in comparison with the conductivity of air, the normal component of the gradient of the scalar potential on the boundaries can be related to the normal component of the vector potential and unknown variables can be reduced. In this case, the scalar potential is induced so that the normal component of the electric field becomes zero.

3) By using the computation results, the applicability of the proposed method were verified.

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