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Abstract–**3-D optimization method using the combined experimental design method and direct search method is developed to apply to the optimal design of a permanent magnet type of retarder. It is shown that the braking torque is increased by using the optimization method. The CPU time can be considerably reduced by utilizing the initial values obtained by the experimental design method.**

Index terms–**Optimization, finite element method, direct search method, retarder, eddy current**

I. INTRODUCTION

In heavy vehicles, auxiliary braking systems such as permanent magnet type of retarders, which produce a braking torque by fluxes and eddy currents, are used sometimes[1,2]. In order to improve the performance of the retarder, the optimization combined with 3-D finite element method should be carried out taking into account the rotation with high speed. The 3-D finite element method using hexahedral elements is superior to that using other types of elements from the standpoints of accuracy and CPU time[3]. It is difficult, however, to apply hexahedral elements to the 3-D optimization, because the automatic mesh generation or modification for the hexahedral elements has not been developed.

In this paper, the 3-D optimization of the permanent magnet type of retarder is carried out in order to increase the braking torque. The eddy current analysis in moving conductor is carried out using the moving coordinate system[4], which can obtain stable results even if the conductor moves at high speed. The direct search method such as the Rosenbrock's method[5] is used for the optimization. As the convergence of the optimization process is affected by the initial values and constraints, the experimental design method (Taguchi's method[6]), is used for determination of initial values and constraints. A technique for modifying automatically the mesh of hexahedral elements at each iteration of optimization is also discussed.

II. MODEL OF RETARDER

Fig. 1 shows a model of permanent magnet type of retarder. The inner side having pole piece, permanent magnet and yoke is standstill and outer rotor rotates. When rotor rotates in the counterclockwise direction as shown in Fig. 1, eddy current flows in the rotor and electromagnetic force is induced due to the flux of permanent magnet and eddy current in the clockwise direction.

The angles and dimensions are denoted in Table I. Only 1/12 of the whole region is shown due to symmetry. The outer rotor rotates with a constant speed at 1,000 rpm. The outer rotor, pole piece and yoke are made of carbon steel (S15C) and the nonlinearity is taken into account. The permanent magnet (Sm_2Co_{17}) is assumed to be magnetized in parallel direction and the magnetization is equal to 1T. The conductivity of the outer rotor is equal to 7×10^6S/m. Table I shows actual value of the each parameter. Four design variables(θ_1, θ_2:angles of pole piece, L_1, L_2:width of yoke and length of magnet) are optimized. Table II shows initial values and constraints of those design variables.

III. FINITE ELEMENT ANALYSIS

In the moving conductor region of the retarder, the fundamental equations of the $A-\phi$ method (A: magnetic vector potential, ϕ: electric scalar potential) using a moving coordinate system[4] are given by

$$\mathrm{rot}(\nu\,\mathrm{rot}\boldsymbol{A}) = -\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right) \quad (1)$$

$$\mathrm{div}\left\{-\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right)\right\} = 0 \quad (2)$$

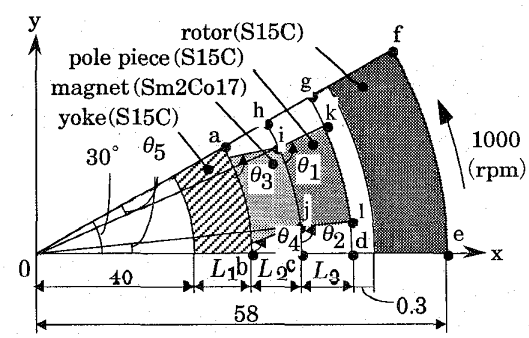

Fig. 1. Permanent magnet type of retarder (thickness: 16mm).

TABLE I
PARAMETERS

θ_1 (deg)	θ_2 (deg)	θ_3 (deg)	θ_4 (deg)	θ_5 (deg)	L_1 (mm)	L_2 (mm)	L_3 (mm)
75	105	75	105	6.55	6.3	3.4	3.0

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TABLE II
INITIAL VALUES AND CONSTRAINTS

design variable	initial value	constraint
L_1(mm)	6.3	$0 \leq L_1 \leq 7.3$
L_2(mm)	3.4	$0 \leq L_2 \leq 7.3$
θ_1(deg)	75	$25 \leq \theta_1 \leq 145$
θ_2(deg)	105	$35 \leq \theta_2 \leq 155$

where ν and σ are the reluctivity and the conductivity, respectively.

The eddy current term $\boldsymbol{J_e}(=-\sigma(\partial \boldsymbol{A}/\partial t+\mathrm{grad}\phi))$ in (1) and (2) at a point p_2 at dc steady state can be discretized by the backward finite difference as follows[4]:

$$\boldsymbol{J}_e(p_2) = -\sigma\left\{\frac{\boldsymbol{A}^*(p_2)-\boldsymbol{A}^*(p_1)}{\Delta t} + \mathrm{grad}\phi^*(p_2)\right\} \tag{3}$$

where it is assumed that the point p_1 is moved to the point p_2 during the time interval Δt. Δt is chosen such that the rotor rotates at 0.1° during the period of Δt. The superscript (*) indicates the unknown variable. When both $\boldsymbol{A}(p_1)$ and $\boldsymbol{A}(p_2)$ are treated as the unknown variables, the dc steady state flux and eddy current distributions can be obtained without time iteration.

In the permanent magnet region (standstill) without eddy currents, the fundamental equation is given by

$$\mathrm{rot}(\nu_0 \mathrm{rot}\boldsymbol{A}) = \nu_0 \mathrm{rot}\boldsymbol{M} \tag{4}$$

where $\boldsymbol{M}$ is the magnetization vector. ν_o is the reluctivity in vacuum.

In order to reduce the analyzed region to 1/24 of the whole region, the periodic boundary condition[7] is introduced on the surfaces o-e and o-f. The analyzed region is subdivided into 3,456 1st-order hexahedral nodal elements.

The braking torque is calculated using the nodal force method[8]. In this method, the nodal force $\boldsymbol{f}_i$ on node i is calculated by the following equations:

$$f_i = -\int_\Omega T_{jk}\partial_j N_i dV \tag{5}$$

where T_{jk} is the Maxwell's stress tensor and N_i is the interpolation function of node i. The subscripts of j,k mean x, y and z components. ∂_x means $\partial/\partial x$.

IV. METHOD OF OPTIMIZATION

Although the simulated annealing method[9] is suitable to obtain the global minimum of the objective function, the number of iterations becomes huge and it is not applicable to 3-D optimization[10]. Therefore, the Rosenbrock's method[5], which is the direct search method, is used for the optimization from the standpoints of the CPU time. The experimental design method (Taguchi's method[6]) is used to determine appropriate initial values and constraints of design variables. In this case, constraints are divided into three levels (1: low, 2: medium, 3: high).

The braking torque, which has nine patterns, is calculated from orthogonal array shown in Table III.

TABLE III
ORTHOGONAL ARRAY

No.	design variable L_1	L_2	θ_1	θ_2
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	3	2
5	2	2	1	3
6	2	3	2	1
7	3	1	2	3
8	3	2	3	1
9	3	3	1	2

As the models No. 1~3 have a level 1 of L_1, the effective value of level 1 is the average value of models No. 1~3. The effective values of levels 2 and 3 are also the average values of models No. 4~6 and 7~9 respectively. The initial values and constraints can be determined by considering the effect of each design variable on the braking torque shown in Fig. 2. Those are determined as follows:

(a) Search area can be reduced by comparing the braking torque of each level. If the level 1 is largest and level 2 is smaller and level 3 is smallest as shown in CASE 2, it is regarded that the optimal value is less than level 2. In the same way, the constraint for CASE 3 is regarded as larger than level 1 and less than level 3. If design variables with less sensitivity, such as CASE1, search area can not be reduced.

(b) The level of design variable where the braking torque becomes maximum can be adopted as an initial value. Or the combination of the levels of design variables for maximum braking torque can be chosen.

V. TECHNIQUE FOR MODIFYING 3-D MESH

In this section, the method for modifying 3-D mesh is explained by an example of which the angles θ_1 and θ_2 of the pole piece are treated as design variables. The following automatic mesh modification for changing the shape of machine at each iteration is carried out:

(1) α, β, γ and δ regions, where nodes are moved in order to change the shape, are selected as shown in Fig. 3(a).

(2) The nodes in α, β and γ regions, of which the shapes are modified, are moved by the linear interpolation.

(3) In the δ region, where the nodes on the surfaces adjacent to the α, β and γ regions are moved by the above-mentioned modification(2), the internal nodes are also moved by the

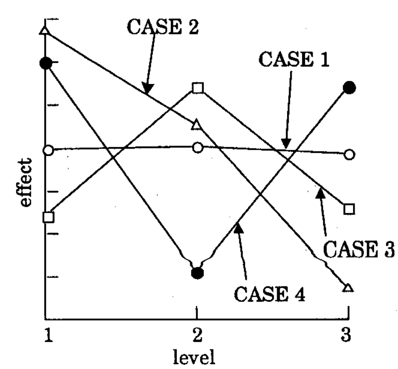

Fig. 2. Effect of design valiables.

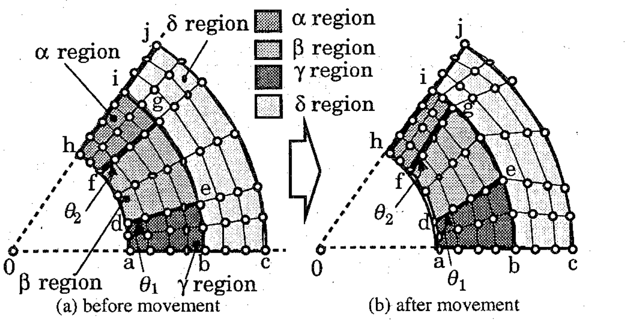

Fig. 3. Method for modifying mesh.

linear interpolation. This process is continued until there is no such region.

The mesh shown in Fig. 3(b) is obtained automatically from the above-mentioned process.

VI. Results and Discussion

A. Accuracy of Finite Element Analysis

The accuracy of the finite element analysis has been already investigated in [2]. The discrepancy of braking torque between analysis and measurement is within 10% when the number of elements is 18,564. The CPU time is 13.7 hours by using IBM3AT workstation(49.7MFLOPS). If such a finite element mesh is used in the optimization, CPU time of several weeks may be necessary, because more than fifty, for example, 3-D FEM calculations are necessary in the optimal design as shown in the following sections. Therefore, the shape of the model is simplified as shown in Fig.1. The actual shape of magnet and pole piece is more complicated, and it is a little troublesome to modify the shape at each iteration of optimization.

Table IV shows the effect of number of elements on the obtained design variables, braking torques, etc. For simplicity, the case of two design variables(θ_1 and θ_2) using the Rosenbrock's method is investigated. Fig.4 shows the meshes of 14,028 elements for initial and optimal shapes. As the CPU time is too long in the case of 14,028 elements, the mesh of 3,472 elements is used in the following investigation. Even if the model is simple and the mesh is not sufficiently fine, the obtained results can give useful suggestion for the optimal design.

B. Rosenbrock's Method

The angles θ_1 and θ_2 of pole piece and the width L_1 of yoke and the length L_2 of magnet are chosen as design variables. The optimization is carried out using the Rosenbrock's method. Figs. 5 and 6 show the meshes and flux distributions for initial and optimal shapes. The braking torque of the optimal shape is 1.39 times larger than that of the initial shape as shown in Table V.

C. Experimental Design Method + Rosenbrock's Method

Firstly, the initial values and constraints are determined using the experimental design method, and then the Rosenbrock's method is applied. The effectiveness using the obtained initial values and constraint is examined.

The levels of design variables L_1, L_2, θ_1, θ_2 are divided into three levels as shown in Table VI. The braking torque of nine patterns denoted in Table III are calculated. Fig. 7 shows the

TABLE IV
EFFECT OF NUMBER OF ELEMENTS

number of elements	shape	design variables		braking torque (N · m)	number of iterations	CPU time (h)
		θ_1(deg)	θ_2(deg)			
3456	initial	75	105	1.92	——	——
	optimal	30.0	140.6	2.18	23	8.2
14028	initial	75	105	1.64	——	——
	optimal	37.5	150	1.94	16	87.9

computer used: HPC160(140MFLOPS)

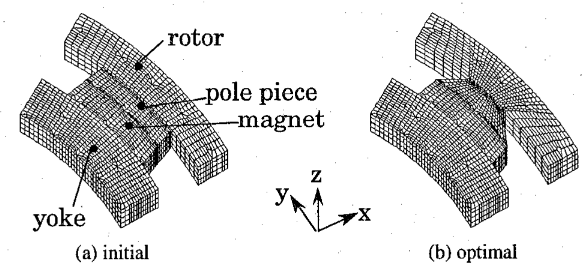

Fig. 4. Meshes(14,028 elements).

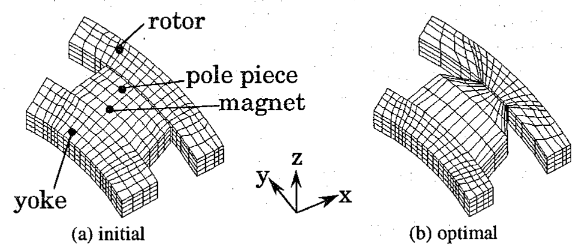

Fig. 5. Meshes(3,472 elements).

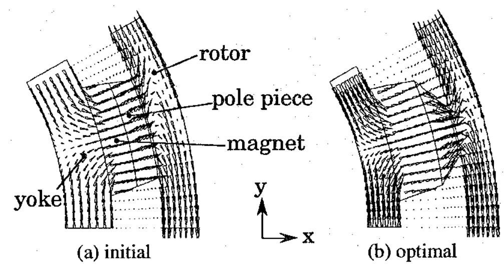

Fig. 6. Flux distributions (x-y plane, center at z-direction).

TABLE V
DESIGN VARIABLES AND BRAKING TORQUE

shape	design variable				braking torque (N · m)
	L_1 (mm)	L_2 (mm)	θ_1(deg)	θ_2 (deg)	
initilal	6.3	3.4	75	105	1.92(1.0)
optimal	4.3	6.3	25.3	146.3	2.66(1.39)

computer used: HPC160(140MFLOPS)
convergence criterion : 0.01 mm, 0.01 deg
number of iterations: 84
CPU time: 31.5 hours

effect of design variables. The ordinate is the average value of three levels of braking torque as denoted in Section IV. The obtained initial values and constraints are shown in Table VII. In order to examine the effectiveness of the experimental design method, four cases shown in Table VIII are investigated. Table IX shows the comparison of obtained results. In CASE C, the braking torque is increased 40% compared with that of initial shape, and CPU time is reduced 15% compared with that of CASE A. The braking torque becomes maximum when only the obtained initial value is utilized(CASE C). As it is not clear whether both obtained initial values and constraints should be used or not in the combined experimental design method and Rosenbrock's method, this should be investigated in future. At least, it can be understood that the braking torque can be increased and CPU time can be reduced by utilizing the obtained initial values.

TABLE VI
DESIGN VARIABLES AND LEVELS

design variable	level 1	level 2	level 3
L_1(mm)	1.8	3.7	5.5
L_2(mm)	1.8	3.7	5.5
θ_1(deg)	55	85	115
θ_2(deg)	65	95	125

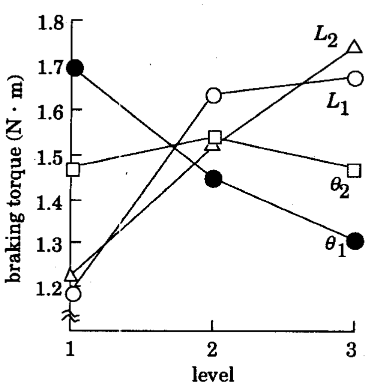

Fig. 7. Effect of design variables.

TABLE VII
INITIAL VALUES AND CONSTRAINTS

design variable	initial value	constraint
L_1(mm)	5.5	$3.7 \leqq L_1 \leqq 7.3$
L_2(mm)	5.5	$3.7 \leqq L_2 \leqq 7.3$
θ_1(deg)	55	$25 \leqq \theta_1 \leqq 85$
θ_2(deg)	95	$65 \leqq \theta_2 \leqq 125$

TABLE VIII
DESIGN METHOD WITH AND WITHOUT EXPERIMENTAL

CASE	initial value	constraint
A	×	×
B	×	○
C	○	×
D	○	○

○ : with experimental design method
× : without experimantal design method

TABLE IX
COMPARISON OF OBTAINED RESULTS

CASE	design variable L_1 (mm)	L_2 (mm)	θ_1 (deg)	θ_2 (deg)	braking torque (N · m)	number of iterations	CPU time (h)
initial	6.3	3.4	75.0	105.0	1.92(1)	-	-
A	4.3	6.3	25.3	146.3	2.66(1.39)	98(1)	33.5(1)
B	3.9	6.6	25.1	125.0	2.63(1.37)	105(1.07)	36.9(1.10)
C	4.1	6.6	28.1	152.5	2.68(1.40)	83(0.85)	27.8(0.83)
D	3.8	6.6	25.0	125.0	2.63(1.37)	71(0.72)	24.3(0.73)

computer used : HPC160(140MFLOPS)

VII. CONCLUSIONS

The 3-D optimal design of permanent magnet type of retarder is carried out by using experimental design method and the Rosenbrock's method. It is shown that the CPU time can be reduced by utilizing the initial values obtained by experimental design method compared with that by using only the Rosenbrock's method.

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