Engineering

Industrial & Management Engineering fields

Okayama University *Year* 1994

A construction of multivariable MRACS with fixed compensator using coprime factorization approach

Shiro Masuda
Okayama University

Akira Inoue
Okayama University

Kenji Sugimoto
Okayama University

TA-7 11:20

Proceedings of the 33rd
Conference on Decision and Control
Lake Buena Vista, FL - December 1994

A Construction of Multivariable MRACS with Fixed Compensator Using Coprime Factorization Approach

S. MASUDA, A. INOUE and K. SUGIMOTO
Department of Information Technology,

Okayama University, Okayama, Japan, 700
siro@suri.it.okayama-u.ac.jp

Abstract

A multivariable model reference adaptive control system (MRACS) with a fixed compensator is proposed. First, a new two-degree-of-freedom (2 d.o.f.) compensator with disturbance estimator is derived. Using this structure, a multivariable MRACS with fixed compensator is constructed. Since the proposed method is based on the 2 d.o.f structure, the fixed compensator is chosen independently of specifications for reference commands. The boundedness of all signals in the closed-loop system and the convergence of the output error are proved. A design method of the fixed compensator for MRACS with low sensitivity is also given. Finally, numerical examples are illustrated in order to show the effectiveness of the proposed method.

1. Introduction

The model reference adaptive control system (MRACS) is one of the main design schemes for adaptive systems. The problem of constructing globally stable MRACS has been solved in the ideal case [15] and in the presence of perturbations such as disturbances and unmodelled dynamics [1]-[3]. But even if the robust stability of MRACS is ensured, the system often results in undesirable performances. For example large transient oscillations often occur and the system is sensitive to perturbations. Hence in recent years much concern has been given to improvement of the performances.

Ohmori et al. [4], [5] proposed the MRACS which has an additional feedback loop with a fixed compensator. The fixed compensator is driven by the error signal between the plant and the reference model output, and it directly compensates tracking error caused by disturbances and parametric uncertainties. Further in [5] the design scheme of the fixed compensator to get low sensitivity was given using the H_∞ sub-optimal design method. However, the fixed compensator should satisfy a certain existence conditions which depend on the reference model transfer matrix, and the fixed compensator cannot be designed independently of specifications for the reference model. These constraints cause a great complexity to the design scheme of the fixed compensator. Indeed, it was necessary to solve the problem of minimizing the H_∞ norm of a unit which satisfies interpolation conditions for the design of the fixed compensator. On the other hand, Sun[6] proposed a modified MRACS with an additional feedback compensator which is driven by an estimated error and showed that this modification was effective for the improvement of the transient response. In Sun's method there is no constraints on the fixed compensator and it was shown that the stability of the modified MRACS was ensured if the fixed compensator was a stable transfer function. Hence the design of the fixed compensator is more straightforward than the one in the Ohmori's method. However the direct performance analysis of the output error is difficult because the estimated error is included in additional feedback loop. This drawback becomes serious especially in the multivariable case because the analysis is still more indirect owing to complexity of the controller structure.

Motivated from the result of Sun [6], Data et al. [7] showed that an extended system from the modified MRACS was able to provide an arbitrarily improved transient performance. Further Pappadakis et al. [8] proposed an alternative structure of the controller based on the modified output error method (MOEM). Later Datta et al. [9] showed that these modifications of MRACS could be derived as linear feedbacks from the tracking error, and the design problem for the auxiliary input was equivalent to the problem of choosing a proper stable rational function by using Youla parametrization [10] which can parametrize the set of all stabilizing compensators. This approach is excellent because it is expected to simplify the design scheme of the fixed compensator and unify other design schemes. However in designing the fixed compensator a parametrization of the reference model transfer functions is not utilized, and the set of achievable fixed compensators is limited to some extent.

These researches on the modification of MRACS essentially implies the investigation on the structure of the MRACS for the purpose of improving performances. Therefore in order to eliminate the drawbacks of the conventional methods we should at first invent a new controller structure which can throughly utilize the degree of freedom of feedback systems and then should extend to the modified MRACS. This approach is especially effective and significant when we treat the multivariable plants because the structure of the controller is considerably more complex than the case of single-input single-output plants.

In this paper at first a new parametrization of two-degree-of-freedom (2 d.o.f.) compensation is proposed and then using this controller structure we construct a multivariable MRACS with a fixed compensator. The proposed parametrization of 2 d.o.f. compensation includes a freely designed compensator for feedback loop which is designed independently of the feedforward compensator for the model matching. The compensator in the feedback loop is directly related with the free parameter of the Youla parametrization and have the role of the disturbance estimator since it only works in the presence of disturbances or parameter uncertainties. And the compensator works as a fixed compensator in the MRACS proposed in this paper. Hence it follows that the fixed compensator can be designed independently of specifications for the reference model and it can improve robustness to perturbations. Furthermore factorization approach [10] over the ring of proper stable rational functions ($\mathbf{RH}_\infty$) is utilized in the proposed MRACS. It gives the efficient design method for MRACS[11]-[13] in the case of both single-input, single-output(SISO)[11] and multi-input, multi-output(MIMO) systems[12][13]. Further factorization approach gives the unified scheme of designing controller structure of MRACS for both SISO and MIMO systems and the extension to multivariable systems can be easily done. Therefore the construction of the structure becomes simple and clear[11] and the extension of multivariable case, where the structure is very complex, becomes easier[12],[13].

This paper is organized as follows. Section 2 gives problem statements and *a priori* information. In section 3 a new parametrization of 2 d.o.f. compensator is derived and it is shown that the proposed parametrization have the controller structure with the disturbance estimator. In Section 4 a multivariable MRACS with fixed compensator is constructed based on the parametrization of 2 d.o.f.

0-7803-1968-0/94$4.00©1994 IEEE

compensator. Section 5 gives the design scheme of the fixed compensator with low sensitivity. In section 6 numerical examples of the proposed MRACS are illustrated in order to show the effectiveness of the design scheme of the fixed compensator in section 5. Section 7 concludes the paper. In appendix A the boundedness of all signals in the closed-loop system and the convergence of the output error are proved.

The following notations are used. $\mathbf{C}^+$ is the right half complex plane and $\mathbf{C}^-$ is the left half complex plane. $\mathbf{R}^n$ denotes an n dimensional Euclidean space. $\mathbf{RH}_\infty$represents the ring of proper stable rational functions of variable s with coefficients in the field of the real numbers ($\mathbf{R}$). The ring of polynomials with real coefficients is given by $\mathbf{R[s]}$. For the size of given matrix, the notations $\mathbf{RH}_\infty{}^{p\times l}$, $\mathbf{R(s)}^{p\times l}$ or $\mathbf{R[s]}^{p\times l}$ are used. Further, $\partial_{ci}[\cdot]$ and $\partial_{ri}[\cdot]$ express the column and raw degrees of a given polynomial matrix.

2. Problem Statements

Consider an unknown linear, time-invariant, finite dimensional, plant with m-inputs and m-outputs with a transfer matrix $P(s)$:

$$y_p(t) = P(s)(u(t) + d_1(t)) + d_2(t) \tag{2.1}$$

where $u(t) \in \mathbf{R}^m$, $y_p(t) \in \mathbf{R}^m$ are the plant input and output vectors, respectively. $d_1(t) \in \mathbf{R}^m$ are disturbances added to the plant input and $d_2(t) \in \mathbf{R}^m$ are noises in measuring the plant outputs. It is assumed that $P(s)$ is full rank (rank $P(s) = m$) and strictly proper. Let the right coprime factorization of $P(s)$ over $\mathbf{RH}_\infty$be $(N_p(s),\ D_p(s))$, that is, $N_p(s) \in \mathbf{RH}_\infty{}^{m\times m}$, $D_p(s) \in \mathbf{RH}_\infty{}^{m\times m}$ are relatively right coprime over $\mathbf{RH}_\infty$and

$$P(s) = N_p(s)D_p(s)^{-1} \tag{2.2}$$

is satisfied. In constructing MRACS this factorization of the given plant over $\mathbf{RH}_\infty$is not calculated. But these notations are used to parametrize reference model transfer matrices and derive the controller structure of MRACS.

The transfer matrix of the reference model is denoted as $P_M(s)$ which is strictly proper and asymptotically stable. And $r(t) \in \mathbf{R}^m$, piecewise continuous and uniformly bounded, are the reference input and $y_m(t) \in \mathbf{R}^m$ are model output vectors. $P_M(s)$ is assumed to be satisfied with the necessary and sufficient condition for the model matching. This condition is given as follows using the right coprime factorization of the plant.

$$P_M(s) = N_p(s)K(s), \quad K(s) \in \mathbf{RH}_\infty{}^{m\times m} \tag{2.3}$$

The following assumptions are made for the plant[14]

(A.1) The stable interactor matrix $L[s]$ is known, that is, a matrix $L[s] \in \mathbf{R[s]}^{m\times m}$ such that

$$\lim_{s\to\infty} L[s]P(s) = G_p \tag{2.4}$$

with G_p nonsingular. In this paper it is assumed to be $G_p = I$.

(A.2) The plant maximum observability index ν is known.

(A.3) The plant is minimum phase, that is, $N_p(s)$ is of full rank for any $s \in \mathbf{C}^+$.

According to the results of [12], [13], $N_p(s)$ can be derived using an interactor matrix of the plant, which is assumed to be known from the assumption (A.1). Hence the rational function matrix K satisfying the model matching condition (2.3) can be obtained.

Based on the description above, the control problem is stated as follows. Let a plant (2.1) be unknown except for the assumptions (A.1)-(A.3). Then determine a differentiator free controller which generates a bounded control input signal vector, so that all the signals in the closed loop system remain bounded and the following equation is satisfied.

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y_p(t) - y_m(t)\| = 0. \tag{2.5}$$

3. A Parametrization of 2 D.O.F Compensators

In this section we introduce a new parametrization of 2 d.o.f. compensator and show that the parametrization includes disturbance estimator. At first in addition to the factorization of the plant P in eq.(2.2) we give a doubly coprime factorization of the plant as follows.

$$P = N_pD_p^{-1} = \tilde{D}_p{}^{-1}\tilde{N}_p,$$

$$\begin{bmatrix} Y_p & X_p \\ -\tilde{N}_p & \tilde{D}_p \end{bmatrix} \begin{bmatrix} D_p & -\tilde{X}_p \\ N_p & \tilde{Y}_p \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{3.1}$$

$$N_p, D_p, \tilde{D}_p, \tilde{N}_p, X_p, Y_p, \tilde{X}_p, \tilde{Y}_p \in \mathbf{RH}_\infty{}^{m\times m}.$$

Using the doubly coprime factorization, the set of all 2 d.o.f compensators $C = [C_1, C_2]$ that stabilize P is given as follows[10].

$$\begin{aligned} u &= C_1 r - C_2 y_p \\ [C_1,\ C_2] &= (Y_p - Q\tilde{N}_p)^{-1}[K,\ X_p + Q\tilde{D}_p] \end{aligned} \tag{3.2}$$

where $K, Q \in \mathbf{RH}_\infty{}^{m\times m}$ is arbitrary [1]. In the next step we derive a new parametrization of 2 d.o.f. compensator by rewriting eq.(3.2). Multiplying eq.(3.2) by $(Y_p - Q\tilde{N}_p)^{-1}$ from the left hand side and rearranging the expression, eq.(3.2) becomes

$$u = (I - Y_p)u + Kr - X_py_p + Q\tilde{D}_py_p + Q\tilde{N}_pu. \tag{3.3}$$

Since the plant is strictly proper, $\tilde{Y}_p$ is invertible. Hence Q can be substituted by $Q\tilde{Y}_p^{-1}\tilde{Y}_p$. Then eq.(3.3) can be rewritten as follows.

$$u = (I - Y_p)u + Kr - X_py_p + Q\tilde{Y}_p^{-1}\tilde{Y}_p\tilde{D}_py_p + Q\tilde{Y}_p^{-1}\tilde{Y}_p\tilde{N}_pu. \tag{3.4}$$

Noting that eq.(3.1) is commutable it is shown that the following equations are satisfied.

$$N_pX_p + \tilde{Y}_p\tilde{D}_p = I, \quad \tilde{Y}_p\tilde{N}_p = N_pY_p. \tag{3.5}$$

Applying eq.(3.5) to eq.(3.4) for elimination of $\tilde{Y}_p\tilde{D}_p$ and $\tilde{Y}_p\tilde{N}$, the next parametrization with fixed compensator is obtained. (see Fig.1)

$$u = (I - Y_p)u - X_py_p + Kr - Q\tilde{Y}_p^{-1}\{y_p - N_p(Y_pu + X_py_p)\}. \tag{3.6}$$

The derived control law (3.6) represents 2 d.o.f compensators that

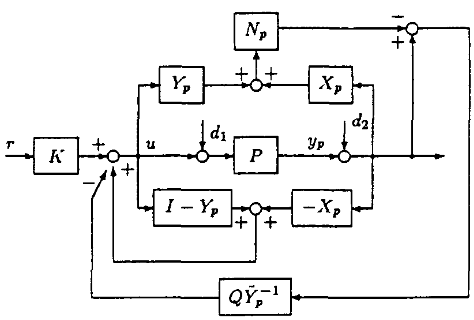

Fig.1 Parametrization of 2 d.o.f. compensators that stabilize a plant

stabilize the plant. In fact calculating transfer functions matrices from (r, d_1, d_2) to (u, y_p), we can get

$$\begin{bmatrix} D_pK & D_p(Y_p - QN_p) - I & -D_p(X_p + Q\tilde{D}_p) \\ N_pK & N_p(Y_p - QN_p) & -N_p(X_p + Q\tilde{D}_p) + I \end{bmatrix}. \tag{3.7}$$

Eq.(3.7) shows that the transfer matrix involve two independent parameters Q and K, and K determines the command response and Q *determines the feedback properties.* As is shown in the discussion above, the the control law (3.6) is equivalent to (3.2). In fact the next theorem is satisfied.

[1]In general the constraint that $\det(Y_p - Q\tilde{N}_p) \neq 0$ is necessary. But it can be removed since it is always satisfied when the plant is strictly proper.

Theorem 3.1 *The system (3.6) is internally stable if and only if $Q,\ K \in \mathbf{RH}_\infty{}^{m\times m}$ are satisfied.*

Proof: It is clear from the derivation of eq.(3.7). ∎

Now we examine features of the parametrization of 2 d.o.f. Multiply the following Bezout identity

$$Y_p D_p + X_p N_p = I \tag{3.8}$$

by $D_p^{-1}u$ from the right hand side, multiply by N_p from the left hand side and use plant dynamics (2.1). Then

$$N_p(Y_p u + X_p y_p) = y_p - N_p Y_p d_1 + (N_p X_p - I)d_2. \tag{3.9}$$

This equation represents plant dynamics in terms of X_p and Y_p which are solutions of Bezout identity. When there is neither deviation of plant parameters from the true values nor external disturbances, that is $d_1 = d_2 = 0$ in (3.9), the input of $Q\tilde{Y}_p^{-1}$ is zero. Hence $Q\tilde{Y}_p^{-1}$ only works in the presence of some perturbations or disturbances, and the output of $Q\tilde{Y}_p^{-1}$ can be regarded as estimated disturbances. These facts show that compensator $Q\tilde{Y}_p^{-1}$ has the role to improve robustness of the control system, and the proposed structure includes disturbance estimator. As shown in the subsequent section, the compensator $Q\tilde{Y}_p^{-1}$ works as a fixed compensator in the MRACS proposed in this paper. Therefore we can see that the fixed compensator has features which is similar to the compensator $Q\tilde{Y}_p^{-1}$.

4. Multivariable MRACS with the Fixed Compensator Based on the Coprime Factorization over $\mathbf{RH}_\infty$

In this section a multivariable MRACS with a fixed compensator is constructed based on the parametrization of 2 d.o.f. compensator given in the previous section. First we construct an exact model matching (EMM) system. From the transfer function matrices (3.7), it is clear that an EMM system can be constructed by selecting the feedforward compensator K which satisfies eq.(2.3). The compensator Q can be chosen arbitrarily, but here we substitute Q by $Q_1\tilde{Y}_p$, where $Q_1 \in \mathbf{RH}_\infty$ because it is difficult to derive a linear representation of control law including $\tilde{Y}_p^{-1}$. Then the EMM control law to follow P_M is given by

$$u = (I - Y_p)u - X_p y_p + Kr - Q_1\{y_p - N_p(Y_p u + X_p y_p)\} \tag{4.1}$$

In the problem of constructing MRACS, the only *a priori* information on the plant is given in the assumptions (A.1)-(A.3). Hence the identification model must be derived in order to estimate unknown parameters, and control law (4.1) must be rewritten into the form with adjustable parameter. Then the following results have the important role with the derivation [12].

Theorem 4.1 *[12] From the assumption (A.1) the following equation is satisfied.*

$$\lim_{s\to\infty} L[s]P(s) = I. \tag{4.2}$$

Then the right coprime factorization of the plant transfer function matrix can be expressed in terms of $L[s]$ as follows:

$$N_p(s) = L[s]^{-1}, \quad D_p(s) = P(s)^{-1}L[s]^{-1}. \tag{4.3}$$

Theorem 4.2 *[12] Let the plant maximal observability index be denoted by ν, and let $\Xi[s] = \xi[s]I$ for some monic stable polynomial $\xi[s]$ of order $\nu - 1 + p$ $(p \geq 0)$. Then there exists a solution $X_p(s)$, $Y_p(s) \in \mathbf{RH}_\infty{}^{m\times m}$ of the Bezout identity (3.8) in the form:*

$$X_p(s) = \Xi[s]^{-1}Z_x[s], \quad Y_p(s) = \Xi[s]^{-1}Z_y[s]. \tag{4.4}$$

And the column degree of $Z_x[s]$ and $Z_y[s]$, $i = 1,\cdots,m$ are satisfied with

$$\partial_{ci}[Z_x[s]] \leq \nu - 1, \quad \partial_{ci}[Z_y[s]] \leq \nu + p - 1, \tag{4.5}$$

and the highest column degree coefficient matrix of $Z_y[s]$ is I.

According to Theorem 4.1, it follows that N_p and K can be assumed to be known from the assumption (A.1). The compensator Q_1 can be utilized as a fixed compensator in the adaptive system because it can be chosen arbitrarily irrespective of *a priori* information on the plant. Therefore X_p and Y_p are unknown matrices in the control law (4.1), and unknown parameters to be identified are coefficients matrix of $Z_x[s]$ and $Z_y[s]$ from Theorem 4.2.

In order to identify coefficients of $Z_x[s]$ and $Z_y[s]$ we shall derive an identification model described as a linear relation in terms of unknown parameters. The identification model can be derived from eq.(3.9). From eq.(3.9) the following equation is given.

$$y_p(t) = N_p(s)[Y_p(s)u(t) + X_p(s)y_p(t)] + \eta(t), \tag{4.6}$$

where

$$\eta(t) = -N_p(s)Y_p(s)d_1(t) + (N_p(s)X_p(s) - I)d_2(t) \tag{4.7}$$

Using above Theorem 4.1 and Theorem 4.2, eq.(4.6) can be rewritten as:

$$\begin{aligned} y_p(t) &= L[s]^{-1}[\Xi[s]^{-1}Z_y[s]u(t) + \Xi[s]^{-1}Z_x[s]y_p(t)] + \eta(t) \\ &= L[s]^{-1}\left[Z_y[s]\frac{1}{\xi[s]}u(t) + Z_x[s]\frac{1}{\xi[s]}y_p(t)\right] + \eta(t) \\ &= L[s]^{-1}\left[(Z_y[s] - \xi[s]I)\frac{1}{\xi[s]}u(t) + u(t)\right. \\ &\quad \left. + Z_x[s]\frac{1}{\xi[s]}y_p(t)\right] + \eta(t). \end{aligned} \tag{4.8}$$

Using results of Theorem 4.2, we can get the next equations.

$$\begin{aligned} Z_y[s] - \xi[s]I &= H_{\nu-2+p}s^{\nu-2+p} + \cdots + H_0, \\ Z_x[s] &= J_{\nu-1}s^{\nu-1} + \cdots + J_0. \end{aligned} \tag{4.9}$$

Form a state variable filter

$$\omega(t)^T = [\omega_1^T(t), \omega_2^T(t)] \tag{4.10}$$

$$\omega_1(t)^T = \left[\frac{s^{\nu-2+p}}{\xi[s]}u(t)^T, \cdots, \frac{1}{\xi[s]}u(t)^T\right] \tag{4.11}$$

$$\omega_2(t)^T = \left[\frac{s^{\nu-1}}{\xi[s]}y_p(t)^T, \cdots, \frac{1}{\xi[s]}y_p(t)^T\right] \tag{4.12}$$

and define the unknown parameters matrix as

$$\Theta = [H_{\nu-2+p}, \cdots, H_0, J_{\nu-1}, \cdots, J_0]. \tag{4.13}$$

Using the above definitions equation (4.8) can be rewritten as

$$y_p(t) = L[s]^{-1}[\Theta\omega(t) + u(t)] + \eta(t). \tag{4.14}$$

Here we define filtered output error $e_f(t)$ as follows:

$$e_f(t) = \frac{1}{f[s]}L[s]\{y_p(t) - y_m(t)\}, \tag{4.15}$$

where $f[s]$ is stable polynomial and is chosen in such a manner that $\frac{1}{f[s]}L[s]$ belong to $\mathbf{RH}_\infty{}^{m\times m}$.

Using eq.(4.14) the filtered output error (4.15) can be rewritten by

$$e_f(t) = \Theta\zeta(t) + \nu(t) - y_{mf}(t) + \eta_f(t) \tag{4.16}$$

where

$$y_{mf}(t) = \frac{1}{f[s]}L[s]y_m(t), \quad \eta_f(t) = \frac{1}{f[s]}L[s]\eta(t) \tag{4.17}$$

$$\zeta(t) = \frac{1}{f[s]}\omega(t), \quad \nu(t) = \frac{1}{f[s]}u(t). \tag{4.18}$$

Eq.(4.16) gives a linear relation in terms of the unknown parameter matrix Θ. Therefore we can estimate the unknown parameter matrix Θ using eq.(4.16). In the next step, we shall give a method for the estimation of Θ. The estimated parameter of Θ is denoted by $\hat{\Theta}$. Then the estimated error model is given as

$$\hat{e}_f(t) = \hat{\Theta}(t)\zeta(t) + \nu(t) - y_{mf}(t). \tag{4.19}$$

From eqs.(4.16),(4.19) an estimated error is obtained by

$$\begin{aligned}\epsilon(t) &= \hat{e}_f(t) - e_f(t) \\ &= \Phi(t)\zeta(t) - \eta_f(t)\end{aligned} \tag{4.20}$$

where

$$\Phi(t) = \hat{\Theta}(t) - \Theta. \tag{4.21}$$

In order to estimate $\hat{\Theta}(t)$, the following gradient method is used in this paper

$$\frac{d}{dt}\hat{\Theta}(t) = -\gamma\frac{\epsilon(t)\zeta(t)^T}{\kappa + \zeta(t)^T\zeta(t)} \tag{4.22}$$

where γ, κ is positive constants.

Now we introduce the control law including the adjustable parameter $\hat{\Theta}(t)$. Using eqs.(4.9),(4.10),(4.13), the control law (4.1) can be rewritten into the following equation.

$$\begin{aligned}u(t) &= -\Theta\omega(t) + K(s)r(t) \\ &\quad -Q_1(s)\left\{y_p(t) - L[s]^{-1}(u(t) + \Theta\omega(t))\right\}. \end{aligned} \tag{4.23}$$

Replacing unknown parameter Θ by the adjustable parameter $\hat{\Theta}(t)$, the control law of MRACS can be obtained as follows:

$$\begin{aligned}u(t) &= -\hat{\Theta}(t)\omega(t) + K(s)r(t) \\ &\quad -Q_1(s)\left\{y_p(t) - L[s]^{-1}\left(u(t) + \hat{\Theta}(t)\omega(t)\right)\right\}.\end{aligned} \tag{4.24}$$

Thus MRACS proposed in this paper is established. The next theorem shows that the proposed multivariable MRACS assures the global stability of the system.

Theorem 4.3 *It is assumed that $d_1(t)$ and $d_2(t)$ belong to $L^2 \cap L^\infty$ and $\dot{d}_2(t)$ belong to L^∞. Then all internal signals in the system which consists of the plant (2.1), the reference model (2.3), the control law (4.24), the estimation error (4.20) and the adaptive adjusting law (4.22) are uniformly bounded, and the control objective (2.5) is achieved for any $Q_1(s) \in \mathbf{RH}_\infty{}^{m\times m}$.*

Proof: See Appendix A. ■

5. Robust Design of the Fixed Compensator

In this section we obtain a design scheme of the fixed compensator to reduce the effect of disturbances on the output. First, we derive the following error equation.

$$\begin{aligned}e(t) &= y_p(t) - y_m(t) \\ &= -N_p(s)(I - Q_1(s)N_p(s))\,\Phi(t)\omega(t) \\ &+ N_p(s)(I - Q_1(s)N_p(s))\,Y_p(s)d_1(t) \\ &+ \left\{I - N_p(s)\left(X_p(s) + Q_1(s)\tilde{Y}_p(s)\tilde{D}_p(s)\right)\right\}d_2(t).\end{aligned} \tag{5.1}$$

The second term and the third term of the right hand in eq.(5.1) represent the effect of the disturbances at the control input and the effect of measurement noises. This equation shows that the effect of disturbances on the output can be improved by choosing the fixed compensator.

Now we consider a way to design the fixed compensator $Q_1(s)$ so that the influence of $d_1(t)$ and parameter uncertainties could be reduced when measurement noises $d_2(t)$ is small enough.

Both the first term and second term of the right hand side in eq.(5.1) have the following filter

$$N_p(s)(I - Q_1(s)N_p(s)). \tag{5.2}$$

Hence if the gain of $N_p(s)(I - Q_1(s)N_p(s))$ could be reduced, the influence of disturbance $d_1(t)$ and parametric uncertainties would be reduced.

The design just mentioned can be easily done because $N_p(s)$ is known and it doesn't have zeros in $\mathbf{C}^+$. we may design $Q_1(s)$

$$Q_1(s) = \frac{1}{(\delta s + 1)^k}L[s] \tag{5.3}$$

where k is chosen so as to be $Q_1(s) \in \mathbf{RH}_\infty{}^{m\times m}$.

The H_∞ norm of the filter (5.2) is obtained by the next theorem. The theorem is derived by extending Sun's approach [6] to the multivariable case.

Theorem 5.1 *There exists a positive constant α such that the next equation is satisfied.*

$$\|N_p(j\omega)(I - Q_1(j\omega)N_p(j\omega))\|_\infty \le \alpha\delta. \tag{5.4}$$

From this theorem the H_∞ norm of $N_p(s)(I - Q_1(s)N_p(s))$ can be reduced arbitrarily. Therefore it follows that we can reduce the influence of disturbances and parametric uncertainties by choosing a small δ in eq.(5.3).

6. Numerical Examples

In this section numerical examples of the proposed MRACS are shown in order to illustrate effectiveness of the proposed method. Two-input two-output plant with transfer function

$$P(s) = \begin{bmatrix} \dfrac{s+1}{s(s-1)} & \dfrac{-2}{s(s-1)} \\ \dfrac{2}{s(s-1)} & \dfrac{s^2 - s - 4}{s(s-1)(s+1)} \end{bmatrix} \tag{6.5}$$

is studied. The plant is assumed to be unknown except for the following *a priori* information.

1) The highest frequency gain G_p is known($G_p = I$).
2) The maximal observability index ν is known($\nu = 2$).
3) The interactor matrix $L[s]$ is known. ($L[s] = \mathrm{diag}\,(s+3, s+3)$).

The control objective is to follow the reference model $P_M(s) = L[s]^{-1}$ and the boundedness of all the signals is assured. The reference signal is rectangular wave with period of 4 and amplitude of 0.5. The simulation condition in Fig.2 and Fig.3 is given in table 1. The simulation results of MRACS both without and

$d_1(t)$	$2.0\sin(0.1t), 8 \le t \le 13$
$d_2(t)$	0
$\xi[s]$	$s+2$
$f[s]$	$s+2$
γ	10
κ	0.001

Table 1: Simulation conditions

with fixed compensator are given in Fig.2 and Fig.3. The only first output is illustrated in these figures. In Fig.3 the design parameters of the fixed compensator is chosen as $k = 1$ and $\delta = 0.1$.

Fig.3 shows that the influence of disturbances and parameter uncertainties on the plant output is reduced by the fixed compensator in both transient response and steady state comparing the results of Fig.2 without a fixed compensator. We can see that the proposed method is more robust to disturbances and parameter uncertainties than one without a fix compensator.

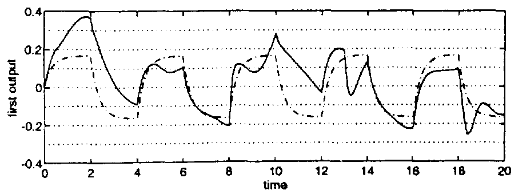

Fig.2 Plant and model output without the fixed compensator

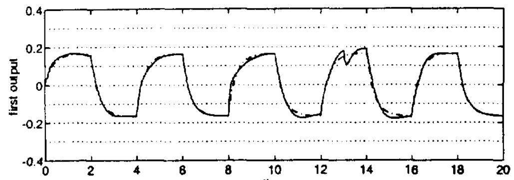

Fig.3 Plant and model output with the fixed compensator

7. Conclusion

In this paper we propose a new structure of model reference adaptive control system with a fixed compensator for a multi-input multi-output linear time invariant system. The boundedness of all signals of the system is proved. The structure is based on a parametrization of stabilizing compensator obtained from 2 d.o.f. compensator scheme, and the fixed compensator corresponds to a free parameter specifying feedback properties which can be designed independently of tracking properties. The development of a optimal design method of the fixed compensator Q_1 remains as a future research.

References

[1] B. B. Peterson and K. S. Narendra, "Bounded Error Adaptive Control", *IEEE Trans. on Automat. Contr.*, Vol.27, No.6, pp.1161-1168, 1982.

[2] P. A. Ioannou and P. V. Kokotovic, *Adaptive Systems with Reduced Models*, New York, Springer-Verlag, 1983.

[3] K. S. Narendra and A. M. Annaswamy, "A New Adaptive Law for Robust Adaptive Control without Persistent Excitation", *IEEE Trans. on Automat. Contr.*, Vol-32, No.2, pp.134-145, 1987.

[4] H. Ohmori and A. Sano, "Model Reference Adaptive Control System Including Compensator for Unknown Deterministic Disturbances", *Trans. of the Society of Instrument and Control Engineers*, Vol.20, No.10, pp.919-pp.925, 1984 (*In Japanese*).

[5] H. Ohmori and A. Sano, "Design of Robust Adaptive Control System with a Fixed Compensator", *Robust Control*, Springer-Verlag, pp.146-pp.153, 1991.

[6] J. Sun, "A Modified Model Reference Adaptive Control Scheme for Improved Transient Performance", *Proc. 1991 American Contr. Conf.*, pp.150-pp.155, 1991.

[7] A. Datta and P. A. Ioannou, "Performance Improvement Versus Robust Stability in Model Reference Adaptive Control", *Proc. 30th IEEE Conf. Dec. Contr.*, pp.748-753, 1991.

[8] I. N. M. Pappadakis and S. C. A. Thomopoulous, "Modified Output Error Method with Parameter Mismatch Compensation for Improved Performance Model Reference Adaptive Control", *Proc. 31st IEEE Conf. Dec. Contr.*, pp.3249-3254, 1992.

[9] A. Datta and M. Ho, "On Modifying Model Reference Adaptive Control Schemes for Performance Improvement", *Proc. 32nd IEEE Conf. Dec. Contr.*, pp.1093-1094, 1993.

[10] M. Vidyasagar, *Control System Synthesis, A Factorization Approach*, The MIT Press, 1985.

[11] A. Inoue, V. T. Kroumov and S. Masuda, "A unified design method for model reference adaptive control systems using the coprime factorization approach", *Proc. IFAC World Congress*, Sidney, Vol.2, pp.83-88, 1993.

[12] S. Masuda, V. T. Kroumov, A. Inoue and K. Sugimoto, "A design method of multivariable model reference adaptive control system using coprime factorization approach", *Proc. 32nd IEEE Conf. Dec. Contr.*, pp.3092-3097, 1993.

[13] A. Inoue, S. Masuda, V. T. Kroumov and K. Sugimoto, " Multivariable MRACS for Systems with Rectangular Transfer Matrix Using Coprime Factorization Approach,", *Proc. 1994 American Contr. Conf.*, Vol.1, pp.460-pp.464, 1994.

[14] H. Elliott and W.A. Wolovich, "A Parameter Adaptive Control Structure for Linear Multivariable Systems", *IEEE Trans. on Automat. Contr.*, Vol.27, No.2, pp.340-352, 1982.

[15] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, 1989.

[16] S. Sastry and M. Bodson, *Adaptive Control-Stability, Convergence, and Robustness*, Prentice-Hall, 1989.

[17] P. Ioannou and A. Datta, " Robust Adaptive Control: Design, Analysis and Robustness Bounds", *Foundations of Adaptive Control*, P. V. Kokotovic(ED) Springer-Verlag, 1989.

A. The proof of theorem 4.3

In order to prove Theorem 4.3, the following lemma and corollary are necessary. These results are the extension of the single-input single-output case shown in [17] to the multi-input multi-output case. Since the proof of these results can be done in a similar way as in the single-input single-output case, it is omitted.

Lemma A.1 *Let* $\Phi : \mathbf{R}^+ \to \mathbf{R}^{m\times n}$ *be defferentiable, and let* $\omega : \mathbf{R}^+ \to \mathbf{R}^n$. *Let* $W(s) \in \mathbf{RH}_\infty$ *with a minimal realization* (A, b, c, d), *i.e.*

$$W(s) = c^T(sI - A)^{-1}b + d. \tag{A.1}$$

Then the following equation is satisfied.

$$W(s)\left[\Phi(t)\omega(t)\right] = \Phi(t)W(s)\left[\omega(t)\right] + W_c(s)\left[\dot{\Phi}(t)W_b(s)\left[\omega(t)\right]\right] \tag{A.2}$$

where

$$W_c(s) = -c^T(sI - A)^{-1}, \quad W_b(s) = (sI - A)^{-1}b. \tag{A.3}$$

Corollary A.1 *In Lemma A.1 if* $\dot{\phi}_i(t) \in L^2$, $i = 1, \cdots, m$, *then*

$$\sup_{\tau\le t} \|e(\tau)\| = o(\sup_{\tau\le t} \|\omega(\tau)\|) \tag{A.4}$$

where $e(t) \in \mathbf{R}^m$ *is defined by*

$$e(t) = W(s)\left[\Phi(t)\omega(t)\right] - \Phi(t)W(s)\left[\omega(t)\right]. \tag{A.5}$$

Proof of Theorem 4.3:

Step 1 From eq.(4.21) eq.(4.22) can be rewritten into

$$\dot{\Phi}(t) = -\gamma\frac{\epsilon(t)\zeta(t)^T}{\kappa + \zeta(t)^T\zeta(t)}. \tag{A.6}$$

If a Lyapunov function candidate $V(t)$ is chosen as

$$V(t) = \mathrm{Tr}\left(\frac{1}{2}\gamma^{-1}\Phi(t)^T\Phi(t)\right) + \frac{1}{4(\kappa + \zeta(t)^T\zeta(t))}\int_t^\infty \|\eta_f(\tau)\|^2\, d\tau \tag{A.7}$$

where $\mathrm{Tr}(\cdot)$ represents the trace of matrix. Here noting that the second term of the right hand side in eq.(A.7) can be assured to be finite because $\eta_f(t)$ belongs to $L^2 \cap L^\infty$ owing to $d_1, d_2 \in L^2 \cap L^\infty$ in the assumptions of Theorem 4.3

The time derivative $\dot{V}$ along the trajectories of eq.(A.6) is given by

$$\begin{aligned}\dot{V}(t) &= \mathrm{Tr}\left(\gamma^{-1}\Phi(t)^T\dot{\Phi}(t)\right) - \frac{\|\eta_f(t)\|^2}{4(\kappa+\zeta(t)^T\zeta(t))} \\ &= -\frac{1}{\kappa+\zeta(t)^T\zeta(t)}\left\|\epsilon(t)+\frac{3}{2}\eta_f(t)\right\|^2. \qquad (A.8)\end{aligned}$$

Hence $\dot{V}(t) \le 0$ and $\Phi(t)$ is uniformly bounded for all $t \ge 0$. Therefore we can get

$$\theta_i(t), \phi_i(t) \in L^\infty, \quad i = 1, \cdots, m \qquad (A.9)$$

where $\theta_i(t)$ and $\phi_i(t)$, $i = 1, \cdots, m$ represent row vectors of $\hat{\Theta}(t)$ and $\Phi(t)$ respectively.

Further, since

$$-\int_0^\infty \dot{V}(t)\,dt = -V(\infty)+V(0) < \infty \qquad (A.10)$$

it follows that $\epsilon(t)/(\kappa+\zeta(t)^T\zeta(t))^{1/2} \in L^2$ or equivalently

$$\epsilon(t) = m(t)\sqrt{\kappa+\zeta(t)^T\zeta(t)}, \quad m(t) \in L^2. \qquad (A.11)$$

From eqs.(A.8),(A.10),(A.11) it can be concluded that

$$\dot{\theta}_i(t), \dot{\phi}_i(t) \in L^2, \quad i = 1, \cdots, m. \qquad (A.12)$$

Step 2 Since the controller parameter matrix $\hat{\Theta}(t)$ is bounded, the signals of the system can grow at most exponentially. Hence they can be assumed to belong to the class $PC_{[0,\infty)}$[15], and we can compare the growth rate of unbounded signals. Let the signals of the system grow in an unbounded fashion. Now comparing the growth rate in the signals of the system, we can consequently conclude that

$$\sup_{\tau\le t}\|\omega(\tau)\| \sim \sup_{\tau\le t}\|y_p(\tau)\| \sim \sup_{\tau\le t}\|\omega_2(\tau)\| \sim \sup_{\tau\le t}\|\zeta(\tau)\|, \qquad (A.13)$$

$$\sup_{\tau\le t}\|u(\tau)\|,\ \sup_{\tau\le t}\|\nu(\tau)\|,\ \sup_{\tau\le t}\|\zeta(\tau)\| = O\left[\sup_{\tau\le t}\|y_p(\tau)\|\right]. \qquad (A.14)$$

The derivation of eqs.(A.13),(A.14) is omitted because it can be done in a similar way as one in the SISO system.

Step 3 From eqs.(4.18),(4.24) $\nu(t)$ becomes

$$\nu(t) = -\frac{1}{f[s]}\hat{\Theta}(t)\omega(t) + \frac{1}{f[s]}K(s)r(t) - Q_1(s)\frac{1}{f[s]}X(t) \qquad (A.15)$$

where

$$X(t) = y_p(t) - L[s]^{-1}\left(\hat{\Theta}(t)+u(t)\right). \qquad (A.16)$$

Substituting eq.(A.15) into eq.(4.19) we can get

$$\hat{e}_f(t) = \left[\hat{\Theta}(t)\zeta(t) - \frac{1}{f[s]}\hat{\Theta}(t)\omega(t)\right] - Q_1(s)\frac{1}{f[s]}X(t). \qquad (A.17)$$

Here noting that $\hat{\Theta}(t)$ is differentiable and $\frac{1}{f[s]} \in \mathbf{RH}_\infty$, we can apply Lemma A.1 to get

$$\begin{aligned}\hat{e}_f(t) &= -c_f^T\left(sI-A_f\right)^{-1}\left[\dot{\hat{\Theta}}(t)\left(sI-A_f\right)^{-1}b_f[\omega(t)]\right] \\ &\quad -Q_1(s)\frac{1}{f[s]}X(t). \qquad (A.18)\end{aligned}$$

where (c_f^T, A_f, b_f) is a minimal realization of $\frac{1}{f[s]}$.

Since $\dot{\theta}_i(t) \in L^2$, $i = 1, \cdots, m$, we can get from Corollary A.1

$$\|\hat{e}_f(t)\| \le o\left(\sup_{\tau\le t}\|\omega(\tau)\|\right) + \left\|Q_1(s)\frac{1}{f[s]}X(t)\right\|. \qquad (A.19)$$

Further since

$$X(t) = -L[s]^{-1}\Phi(t)\omega(t) + \eta(t), \qquad (A.20)$$

it follows from eq.(4.20) and Lemma A.1 that

$$\begin{aligned}&\frac{1}{f[s]}X(t) - \frac{1}{f[s]}\eta(t) \\ &= -L[s]^{-1}\epsilon(t) - L[s]^{-1}\eta_f(t) \\ &\quad +L[s]^{-1}\left\{c_f^T\left(sI-A_f\right)^{-1}\left[\dot{\Phi}(t)\left(sI-A_f\right)^{-1}b_f[\omega(t)]\right]\right\}\end{aligned}$$

From (A.11), (A.13), (A.14). Corollary A.1, Lemma 2.9 in [15] it follows that

$$\left\|\frac{1}{f[s]}X(t)\right\| = o\left(\sup_{\tau\le t}\|\omega(\tau)\|\right). \qquad (A.21)$$

Finally from eqs.(A.17), (A.19), (A.21), it follows that

$$\|\hat{e}_f(t)\| = o\left(\sup_{\tau\le t}\|\omega(\tau)\|\right). \qquad (A.22)$$

Let $y_{pf}(t)$ be defined by $y_{pf}(t) = \frac{1}{f[s]}L[s]y_p(t)$. From the filtered output error (4.15) and estimated error (4.20) it follows that

$$\begin{aligned}\|y_{pf}(t)\| &= \left\|\hat{e}_f(t) - \epsilon(t) + y_{mf}(t)\right\| \\ &\le o\left(\sup_{\tau\le t}\|\omega(\tau)\|\right) \qquad (A.23) \\ &\quad + \left\|m(t)\sqrt{\kappa+\zeta(t)^T\zeta(t)}\right\| + \|y_{mf}(t)\|.\end{aligned}$$

Noting that $\dot{y}_m(t)$ is bounded, and $L[s]^{-1}$ is strictly proper, $\dot{\eta}(t)$ is bounded due to that $d_1(t)$ and $d_2(t)$ belong to $L^2 \cap L^\infty$ and $\dot{d}_2(t)$ belong to L^∞ in eq.(4.14), we can derive that the next relation.

$$\sup_{\tau\le t}\|\dot{y}_p(\tau)\| = O(\sup_{\tau\le t}\|y_p(\tau)\|). \qquad (A.24)$$

Therefore since $f[s]^{-1}L[s]$ has no zeros in $\mathbf{C}^+$, it follows that from Lemma 3.6.2 in [16]

$$\sup_{\tau\le t}\|y_p(\tau)\| = O(\sup_{\tau\le t}\|y_{pf}(\tau)\|). \qquad (A.25)$$

And since it is assumed that all signals in the system grow in unbounded fashion and $\|y_{mf}(t)\|$ is bounded, the following relation is satisfied.

$$\sup_{\tau\le t}\|y_{mf}(\tau)\| = o(\sup_{\tau\le t}\|y_{pf}(\tau)\|). \qquad (A.26)$$

Finally from (A.13), (A.14), (A.23), (A.25), (A.26). $m(t) \in L^2$, Lemma 2.9 in [15] it follows that

$$\sup_{\tau\le t}\|\omega_2(\tau)\| = o\left(\sup_{\tau\le t}\|y_p(\tau)\|\right). \qquad (A.27)$$

This contradicts eq.(A.13) according to which $y_p(t)$ and $\omega_2(t)$ grow at the same rate if they grow in an unbounded fashion. Hence, it is concluded that all signals in the feedback loop are bounded.

Step 4 Since all signals in the feedback loop are bounded from Step 3,

$$\lim_{t\to\infty}\|\hat{e}_f(t)\| = 0 \qquad (A.28)$$

is derived from (A.22). The derivative of $m(t)$ in eq.(A.11) is bounded noting that $\dot{\zeta}(t)$ and $\dot{\epsilon}(t)$ is bounded from the boundedness of all the signal in the system. Therefore from $m(t) \in L^2$ and Corollary 2.9 in [15] it follows that $\lim_{t\to\infty}\|m(t)\| = 0$. Since $\zeta(t)$ is bounded, from eq.(A.11) it follows that

$$\lim_{t\to\infty}\|\epsilon(t)\| = 0. \qquad (A.29)$$

Hence from eqs.(4.20),(A.28),(A.29) the following equation is obtained.

$$\lim_{t\to\infty}\|e_f(t)\| = 0. \qquad (A.30)$$

Since $\dot{e}_f(t)$ is bounded and $\frac{1}{f[s]}L[s]$ has no zeros in $\mathbf{C}^+$, it follows that from Lemma 3.6.2 in [16] eq.(2.5) is satisfied. ∎