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IMPROVEMENTS OF THE T-Ω METHOD FOR 3-D EDDY CURRENT ANALYSIS

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ABSTRACT

An improved **T**-Ω method which can analyze magnetic fields produced not only by eddy currents but also by magnetizing currents is proposed. The method is applied to the analysis of 3-D eddy current models with holes, and the usefulness of the method is investigated by comparing calculated results with measured ones .

1. INTRODUCTION

The "**T**-Ω method"[1] is especially effective when most of the analyzed region is the current free one. This method is investigated by C.J.Carpenter[1,2], T.W.Preston [3,4], D.Rodger[5] et al, and some applications to actual machinery are reported. In the conventional **T**-Ω method, however, the analysis of magnetic fields produced by magnetizing currents is not simple, because some techniques such as two scalar potentials[6] should be introduced. Moreover, the systematical analysis of eddy currents in conductors with holes has not been completely discussed[5,7].

In this paper, the **T**-Ω method is improved so that the magnetic field produced by the magnetizing current is calculated using a current vector potential **T**o[7] instead of introducing the total-reduced scalar formulation[6]. A technique for treating holes in conductors is investigated in detail. The validity of the method is examined experimentally, and the results calculated are compared with those obtained by the **A**-ϕ method.

2. METHOD OF ANALYSIS

2.1 Outline of the method

In the 3-D analysis of magnetic fields which uses a magnetic vector potential **A**, the number of unknown variables and the CPU time considerably increase compared with those of the 2-D analysis. If 3-D magnetic fields are analyzed using a magnetic scalar potential Ω, the number of unknown variables can be decreased. The **T**-Ω method is a simple and economical method for calculating 3-D eddy currents which is developed from the above-mentioned viewpoint.

The analyzed region R can be divided into the current free region Ro and the current carrying region Rj as shown in Fig.1. In the **T**-Ω method, the magnetic scalar potential Ω is defined in the current free region Ro, and Ω and a current vector potential T are defined in the current carrying region Rj (for example, in the conductor).

The method is especially effective when most of the analyzed region is the current free region, because the **T** vector is defined only in the current carrying region. If a plate, in which eddy currents flow, is assumed to be sufficiently thin, the method is also effective, because the vector **T** can be confined to one component.

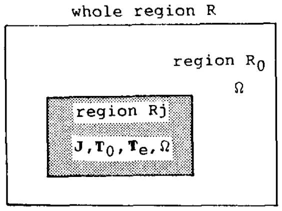

Fig.1 Eddy current problem.

2.2 Basic equations

The basic equations for the **T**-Ω method are given by[1]

$$\mathrm{div}\,\mu(\mathbf{T}-\mathrm{grad}\,\Omega)=0 \tag{1}$$

$$\mathrm{rot}\left(\frac{1}{\sigma}\mathrm{rot}\,\mathbf{T}\right)=-\frac{\partial}{\partial t}\{\mu(\mathbf{T}-\mathrm{grad}\,\Omega)\} \tag{2}$$

where μ is the permeability and σ is the conductivity. The current vector potential **T** is defined by

$$\mathbf{J}=\mathrm{rot}\,\mathbf{T} \tag{3}$$

The magnetic scalar potential Ω is defined by

$$\mathbf{H}-\mathbf{T}=-\mathrm{grad}\,\Omega \tag{4}$$

where **H** is the magnetic field intensity.

2.3 Consideration of magnetizing currents

(1) Introduction of a new current vector potential

In the conventional method, **T** in Eq.(3) is equal to the current vector potential **T**e corresponding to the eddy current density **J**e as follows:

$$\mathbf{J}_e=\mathrm{rot}\,\mathbf{T}_e \tag{5}$$

Similarly, a new current vector potential **T**o corresponding to the magnetizing current density Jo can be defined as follows:

$$\mathbf{J}_0=\mathrm{rot}\,\mathbf{T}_0 \tag{6}$$

The basic equations for the calculation of magnetic fields which are produced by the magnetizing currents and eddy currents are obtained by replacing **T** in Eqs.(1) and (2) with the following equation:

$$\mathbf{T}=\mathbf{T}_0+\mathbf{T}_e \tag{7}$$

If To is obtained beforehand from the magnetizing current, the magnetic field in the region having magnetizing currents and eddy currents can be directly calculated by Eqs.(1) and (2). Te and Ω are unknown variables in Eqs.(1) and (2).

(2) Determination of To from Jo

In order to obtain **T**o from the magnetizing current **J**o, the relationship between **T**o and **J**o is examined using an example shown in Fig.2. The Figure shows one-eighth of the analyzed model. It is assumed

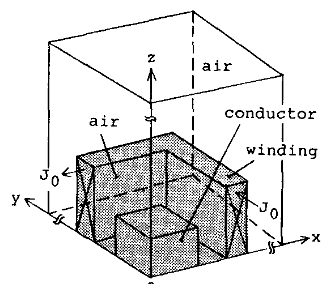

Fig.2 Model with winding and conductor.

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that the eddy current flows only in the conductor. Then, **Te** is defined only in the conductor[8]. **To** is defined in the dotted part which is surrounded by the winding. If it is assumed that the magnetizing current flows two-dimensionally, the x- and y-components Tox and Toy of **To** become zero. Then, the x- and y-components Jox and Joy of **Jo** can be represented from Eq.(6) as follows:

$$J_{0x}=\frac{\partial T_{0z}}{\partial y}, \quad J_{0y}=-\frac{\partial T_{0z}}{\partial x} \tag{8}$$

where Toz is the z-component of **To**. Eq.(8) denotes that Toz changes with x or y linearly, if the magnetizing current is distributed uniformly in the winding. Therefore, Toz is distributed linearly in the winding as shown in Fig.3. Toz is constant and equal to $|J_0|\cdot L$ in the region surrounded by the winding, because there is no magnetizing current in that region. L is the width of the winding. The value of $|J_0|\cdot L$ is obtained from Eq.(8).

When the shape of the winding is complicated and the distribution of **Jo** is not known beforehand, the following Laplace's equation of electric potential ϕ should be solved[9].

$$\mathrm{div}\ \mathrm{grad}\,\phi = 0 \tag{9}$$

Jo can be calculated from the obtained ϕ.

As the magnetic fields produced by both magnetizing currents and eddy currents can be calculated using **T** and Ω without introducing two scalar potentials[6], the calculation is simplified.

2.4 Problems in treating holes in conductors

The difficulty in treating holes in a conductor is that $1/\sigma$ in Eq.(2) becomes infinite, because the conductivity of the hole (air) is equal to zero. In order to overcome the difficulty, the holes are treated as conductors with very low conductivity. The effect of the conductivity σa of a hole in Fig.4 on the accuracy of eddy currents is investigated. Figure 4 may be thought as a typical model with a hole. The following sinusoidal flux is applied in the z-direction.

$$Bz=0.1\cdot\sin(2\pi\cdot 50t) \tag{10}$$

The conductivity σc of the conductor is 0.25×10^{8} (S/m). The analysis has been done by using the double-precision (72 bits) computer.

Figure 5 shows the distribution of eddy current

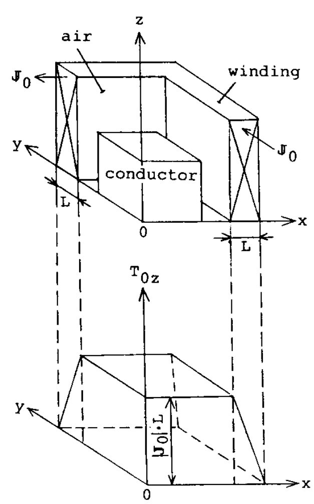

Fig.3 T_{0z} corresponding to J_0.

vectors on the top surface of the conductor. In this case, σa is equal to 1(S/m). Very small eddy current flows in the hole (for example, Je = 6.23×10^{-2} (A/m^2) at a point A). As it is negligibly small compared with the eddy current in the conductor (for example, Je= 1.92×10^{6} (A/m^2) at a point B), this does not affect the accuracy of eddy current.

Figure 6 shows the effects of the conductivity σa of the hole on the distributions of Tez along the x-axis. Figure 7 shows the y-components Jey of Je calculated from Eq.(8) using Tez in Fig.6. Figures 8 and 9 show the effects of $\sigma a/\sigma c$ on JeB, Jec and JeA/JeB, where JeA, JeB and Jec are the y-components of eddy current densities at the points A, B and C in Fig.5. Figures 7 and 8 denote that the distribution of Jey deviates from the proper one, when $\sigma a/\sigma c$ is less than 10^{-12}. Figure 9 denotes that $\sigma a/\sigma c$ should be less than 10^{-2} under the condition that the eddy current density in the hole is within 1(%) of the minimum value of that in the conductor. Therefore, the conductivity of the hole should be in the region of $10^{-12} < \sigma a/\sigma c < 10^{-2}$.

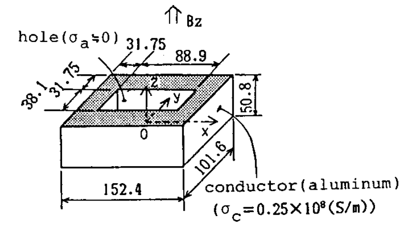

Fig.4 Model with a hole.

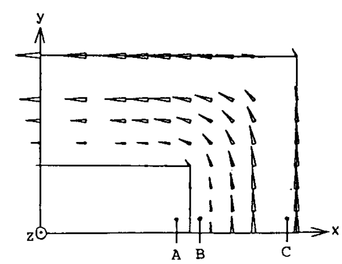

Fig.5 Distribution of eddy current vectors (z=25.4(mm)).

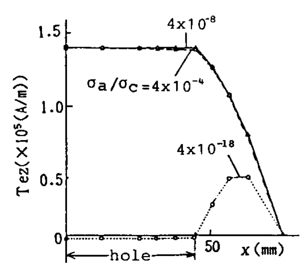

Fig.6 Effects of σ_a on T_{ez} (y=0(mm),z=25.4(mm)).

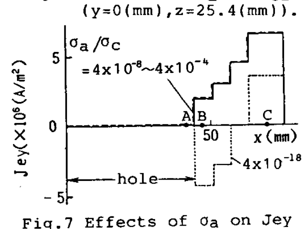

Fig.7 Effects of σ_a on Jey (y=0(mm),z=25.4(mm)).

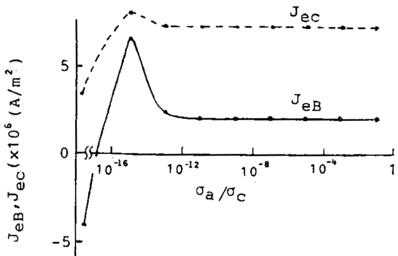

Fig.8 Effects of σ_a/σ_c on J_{eB} and J_{ec} (y=0(mm),z=25.4(mm)).

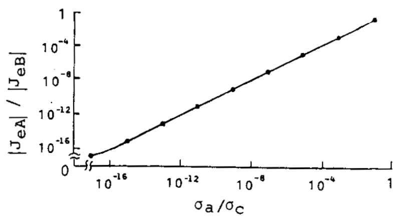

Fig.9 Effects of σ_a/σ_c on $|J_{eA}|/|J_{eB}|$ (y=0(mm),z=25.4(mm)).

3. EXAMPLES OF APPLICATION

3.1 Three times model of Bath plate

In order to verify the new method, it is applied to a model shown in Fig.10. The size is three times as large as the Bath plate [10,11]. Therefore, the experimental results are more reliable and the skin effect is more remarkable than the Bath plate. The conductivity of the plate is 0.27×10^8(S/m), the frequencies of the exciting current are 50 and 200(Hz) respectively, and the ampere-turns are 3780(AT). As the

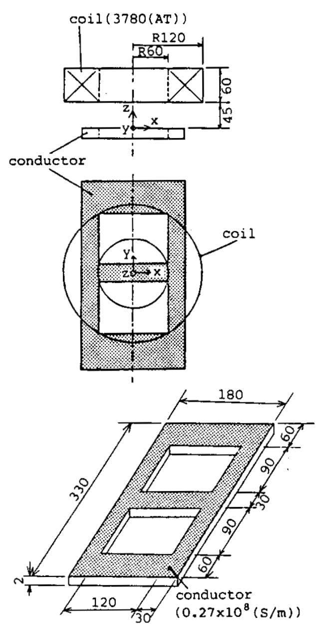

Fig.10 Three times model of Bath plate with two holes.

plate is very thin, the x- and y-components Tex and Tey of the current vector potentials Te can be neglected. This means that the eddy currents flow only in the x-y plane.

Figure 11 shows the distribution of eddy current vectors. Figure 12 shows the distribution of Bz along the y-axis (z=15mm). The total flux passing through a hole is shown in Table 1.

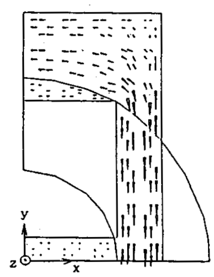

Fig.11 Distribution of eddy current vectors(z=0(mm)).

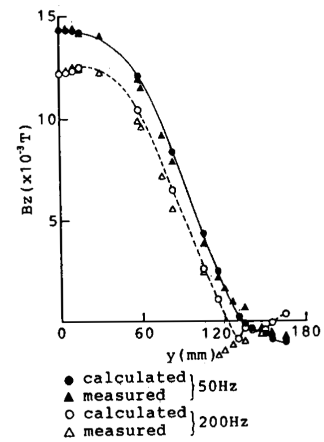

Fig.12 Flux distribution along y-axis(z=15(mm)).

Table 1 Comparison of total flux in a hole.

frequency (Hz)	total flux ($\times10^{-5}$Wb)	
	calculated	measured
50	7.37	7.47
200	4.39	4.78

3.2 The FELIX Brick

3-D eddy currents in a rectangular brick with a hole shown in Fig.4 are analyzed[12]. The applied magnetic field is uniform and decays exponentially with time as follows :

$$Bz=0.1e^{-t/0.0119} \tag{11}$$

Figures 13 and 14 show the total circulating current (eddy current) and the total magnetic field. Results obtained by the A-ϕ method[13] are also shown in the same Figures. ne is the number of tetrahedral elements. The Figures suggest that results obtained by the T-Ω method and the A-ϕ method approach to the same value, when ne is increased.

Table 2 shows the comparison of the computer storage and the CPU time. As the coefficient matrix of the T-Ω method is unsymmetrical[7], the computer storage of the T-Ω method is larger than that of the A-φ method. The CPU time of the T-Ω method can be considerably reduced than those of the A-φ method.

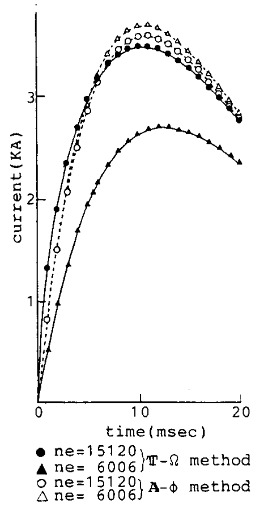

Fig.13 Total circulating current.

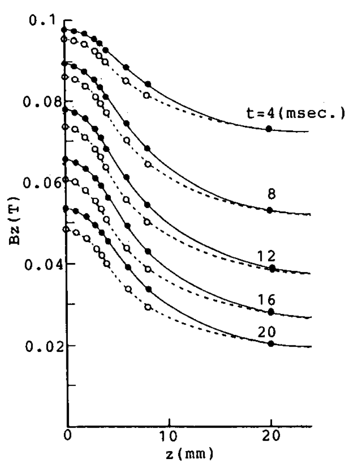

Fig.14 Total magnetic field along z-axis (x=y=0(mm)).

Table 2 Comparison of memory storage and CPU time (ne=15120).

method	number of unknown variables	memory storage (MByte)	CPU time (sec.)
T-Ω	3267	8.0	1266
A-φ	8141	3.4	3123

computer : SX-1E (NEC super computer)

4. CONCLUSIONS

The concept of the current vector potential T is expanded so that not only eddy currents but also magnetizing currents can be treated by T. It has been shown that suitable conductivities of holes should be chosen in the analysis of eddy currents in conductors with holes.

The T-Ω method has an advantage that the CPU time can be considerably reduced, when most of the analyzed region is the current free one and the eddy current flows two-dimensionally. The method is especially effective, when the shape of the winding is simple and the distribution of magnetizing current is known beforehand.

As T-Ω method is not effective when most of the analyzed region is the current carrying one, it is hoped that a more excellent method which is suitable for such a problem will be developed in future.

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