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INVESTIGATION OF EFFECTIVENESS OF EDGE ELEMENTS

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Abstract - The effectiveness of the edge element is investigated by comparing systematically the number of unknown variables with that of the nodal element. It is shown that the edge element is superior to the nodal element from the standpoints of the computer storage and the CPU time.

The 3-D periodic boundary condition for the edge element is also derived in order to reduce the computer storage and the CPU time.

I. INTRODUCTION

Recently, the edge element has drawn the attention of many researchers[1-5]. The total number of unknown variables which affects the computer storage and the CPU time is changed by the kinds of the element (nodal or edge element) and the unknown variable (A-ϕ or T-Ω method) and the volume ratio of the conductor region.

Though the 3-D periodic boundary condition for the nodal element has been examined[6], the periodic boundary condition for the edge element has not previously been investigated.

In this paper, two kinds of element and two methods of analysis are compared in terms of the number of unknown variables and the number of non-zero entries in the coefficient matrix. The most suitable element and method of analysis for the respective problems are discussed. The periodic boundary condition for the edge element is also examined.

II. COMPARISON OF NUMBER OF UNKNOWN VARIABLES FOR OR VARIOUS ELEMENTS AND METHODS OF ANALYSIS

Only brick elements are used for the comparison. It is assumed that the number of elements is so large that the decrease of unknown variables by applying boundary conditions can be neglected. The gauge condition is ignored[3].

A. Number of Unknown Variables and Non-zero Entries

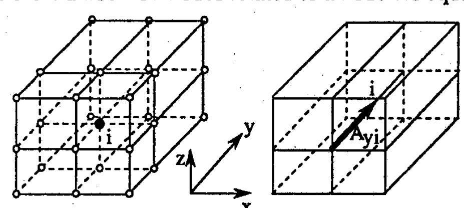
1) Nodal element: Since the number of nodes in one nodal element is equal to 8 and the number of elements which share one node is equal to 8 as shown in Fig.1(a), the average number of nodes per element is equal to unity (=8/8) when the number of elements is extremely large. Therefore, the relationship between the total number, ne, of the elements and the total number, nt, of the nodes can be represented as follows:

$$\mathrm{nt} = \mathrm{ne} \tag{1}$$

In the A-ϕ method using the nodal element, 4 kinds of unknown variables (ϕ and three components of A) are defined per node in the conductor region Rj and three kinds of unknown variables (three components of A) in the air region Ro as shown in Table I. Therefore, the total number, nu(nodal, A-ϕ), of unknown variables can be expressed as

$$\begin{aligned}\mathrm{nu(nodal,A\text{-}\phi)} &= 4\,\alpha \mathrm{nt} + 3\,(1-\alpha)\,\mathrm{nt} \\ &= (\alpha + 3)\,\mathrm{ne} \end{aligned} \tag{2}$$

where α is the ratio of the number of elements in the conductor region Rj to that of the whole region (Rj+Ro). Since the number of nodes related to a node i is equal to 27



(a) nodal element (b) edge element

Fig.1 Elements related to node i or edge i

Table I Numbers of unknown variables and non-zero entries

element	method	variables	number of unknown variables(nu)	number of non-zero entries(nz)
nodal	A-ϕ	R0 A(3) Rj A(3), ϕ(1)	(α+3) ne	27(7α+9) ne
	T-Ω	Ω(1) T(3), Ω(1)	(3α+1) ne	27(15α+1) ne
edge	A	A(1) A(1)	3 ne	9 9 ne
	T-Ω	Ω(1) T(1), Ω(1)	(3α+1) ne	9(23α+3) ne

( ) : number of variables defined on node or edge
ne : number of total elements
α : volume ratio of conductor region to whole region

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as shown in Fig.1(a), the number of unknown variables related to the node i is 27x4 in Rj and 27x3 in Ro. As a result, the total number, nz(nodal, **A**-ϕ), of non-zero entries can be given by

$$\begin{aligned} nz(nodal, A\text{-}\phi) &= 4\,\alpha\, n\, t \times 27 \times 4 \\ &\quad + 3\,(1 - a)nt \times 27 \times 3 \\ &= 27(7\,\alpha + 9)ne \end{aligned} \tag{3}$$

In the **T**-Ω method, four kinds of unknown variables (Ω and three components of **T**) are defined per node in Rj and one unknown variable (Ω) in Ro as shown in Table I. Therefore, the total number, nu(nodal, **T**-Ω), of unknown variables and the total number, nz(nodal, **T**- Ω) of non-zero entries for the **T**-Ω method can be given by

$$\begin{aligned} nu(nodal, T - \Omega) &= 4\,\alpha\, n\, t + 27 + (1 - \alpha)nt \times 27 \\ &= (3\,\alpha + 1)ne \end{aligned} \tag{4}$$

$$\begin{aligned} nz(nodal, T - \Omega) &= 4\,\alpha\, n\, t \times 27 \times 4 + (1 - \alpha)nt \times 27 \\ &= 27\,(15\,\alpha + 1)ne \end{aligned} \tag{5}$$

2) Edge element: Since the number of edges in one element is equal to 12 and the number of elements which share one edge is equal to 4 as shown in Fig.1(b), the average number of edges per element is equal to 3(=12/4). Therefore, the relationship between the total number, ne, of the elements and the total number, nh, of the edges can be represented as follows:

$$nh = 3ne \tag{6}$$

In the A-ϕ method using the edge element, ϕ can be set at zero[3]. The only one component of **A** is defined along each edge. For example, only the y-component Ayi is defined along the edge i as shown in Fig.1(b). Therefore, the total number, nu(edge, **A**), of unknown variables can be expressed as:

$$nu\,(edge\,, A) = nh = 3ne \tag{7}$$

Since the number of edges related to an edge i is equal to 33 as shown in Fig.1(b), the total number, nz(edge,**A**), of non-zero entries can be given by

$$nz\,(edge\,, A) = nu\,(edge\,, A) \times 33 = 99ne \tag{8}$$

In the **T**-Ω method, one component of **T** is defined along each edge in Rj, and Ω is defined on each node in (Rj+Ro). Therefore, the total number, nu(edge, **T**- Ω), of unknown variables is given by

$$\begin{aligned} nu\,(edge\,, T - \Omega) &= \alpha\, nh + \alpha\, nt + (1 - \alpha)\, nt \\ &= (3\,\alpha + 1)\, ne \end{aligned} \tag{9}$$

The numbers of edges and nodes related to the edge i in Rj in Fig.1(b) are 33 and 18 respectively. The numbers of edges and nodes related to the node i in Rj in Fig.1(a) are equal to 54 and 27 respectively. The number of nodes related to the node i in Ro in Fig.1(a) is equal to 27. Therefore, the total number, nz (edge, **T**-Ω), of non-zero entries for the **T**-Ω method can be given by

$$\begin{aligned} nz(edge\,, T - \Omega) &= \alpha\, nh \times (33 + 18) \\ &\quad + \alpha\, nt \times (54 + 27) + (1 - \alpha)nt \times 27 \\ &= 9\,(23\,\alpha + 3)\, ne \end{aligned} \tag{10}$$

B. Comparisons

Fig.2(a) shows the effects of the ratio α on the total number, nu/ne, of unknown variables per element. The figure can be obtained from (2), (4), (7) and (9). When α is nearly equal to zero (the conductor region is very small in the analyzed region), nu/ne for the **T**-Ω method is about 1/3 of nu/ne for the **A**-ϕ method. When α is greater than 0.7, the **A** method using the edge element has the smallest value of nu/ne. Fig.2(b) shows the effects of the ratio α on the total number, nz/ne, of non-zero entries per element. The figure can be obtained from (3), (5), (8) and (10). nz/ne of the **T**-Ω method is smaller than that of the **A**-ϕ method using the nodal element. nz/ne for the edge element is smaller than that for the nodal element. When α is greater than 0.4, nz/ne for the **A** method using the edge element is the smallest of all.

In order to verify the features illustrated in Fig.2, the numbers of unknown variables and non-zero entries are examined using a real model. Fig.3 shows a model proposed by the IEE of Japan. The features of the model are described in the reference[7]. An ac current of which the effective value is 1000AT (frequency: 50Hz) is applied. The ratio α for the model without hole is equal to 0.18 and that with hole is

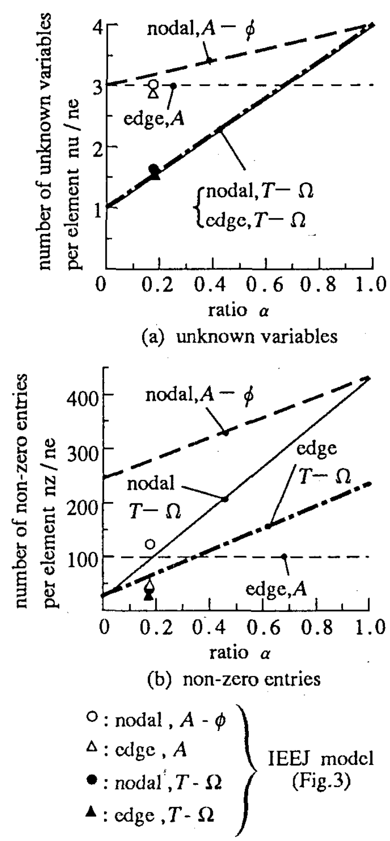

Fig.2 Effects of ratio α on number of unknown variables and number of non-zero entries.

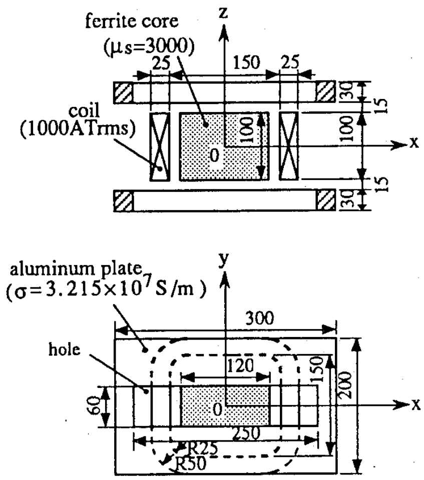

Fig.3 Analyzed model
(with hole).

0.14. The **A**-ϕ and **T**-Ω methods with nodal and edge elements are applied. 1/8 region is analyzed.

The computer storage, the CPU time, etc. are shown in Table II. The number of non-zero entries for the **T**-Ω method is extremely decreased compared with that for the **A**-ϕ method. The CPU times for the **A**-ϕ and **T**-Ω methods using the edge element are about 1/6 and 1/2 of those using the nodal element in the case without hole. As an example, the number nu/ne, of unknown variables per element and the number, nz/ne, of non-zero entries per element for α=0.18 (without hole) are illustrated in Fig.2. Numbers nu/ne and nz/ne for this model are not the same as the values described by (2)-(5) and (7)-(10), because the boundary condition is applied, and the number of elements is not extremely large. However, the tendency of them is similar to Fig.2. From the standpoint of the CPU time, the **T**-Ω method with the edge element is favorable for this model (α=0.14, 0.18).

Table II Discretization data and CPU time

item	without hole				with hole			
	$A-\phi$		$T-\Omega$		$A-\phi$		$T-\Omega$	
	nodal	edge	nodal	edge	nodal	edge	nodal	edge
number of elements	14400							
number of nodes	16275							
number of unknowns	43417	41060	22844	22412	42885	41060	22844	22412
number of non-zero entries	1781644	653718	632859	423056	1734684	653718	632859	423056
computer storage (MB)	72.2	28.4	30.7	19.4	70.5	28.4	30.7	19.4
number of iterations of ICCG method	1306	513	172	192	1264	582	1141	327
CPU time (s)	6242	947	533	290	5870	1069	2001	442

Computer used : NEC supercomputer SX-1E
(maximum speed : 285 MFLOPS)
convergence criterion of ICCG method : 10^{-7}

III. PERIODIC BOUNDARY CONDITION

The periodic boundary condition for the edge element is investigated by examining the distributions of flux and magnetic vector potential of the model shown in Fig.4 which is the problem 13 of the TEAM workshop[8]. The coil is excited by dc current of 1000AT. For simplicity, the magnetic characteristic of the steel is assumed to be linear (μs=1000). The model is subdivided into 847 first order brick edge elements or brick nodal elements.

Fig.5 shows the flux density vectors $\mathbf{B}_P$ and $\mathbf{B}_Q$ and magnetic vector potentials $\mathbf{A}_P$ and $\mathbf{A}_Q$ at the corresponding points P and Q on the planes o-p' and o-q' (y-z plane) on which the periodic condition is satisfied. $\mathbf{B}_P$ and $\mathbf{B}_Q$ satisfy the following conditions [5]:

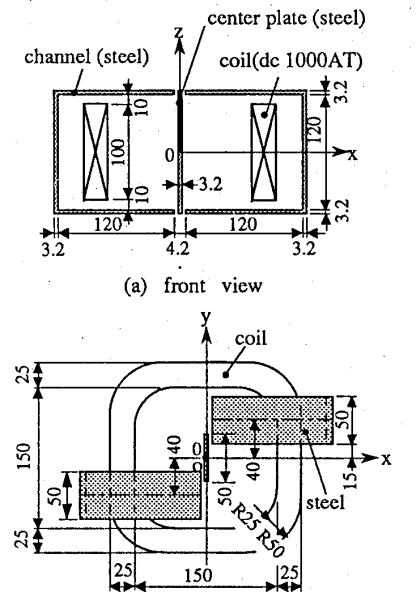

Fig. 4 3-D non-linear magnetostatic model.

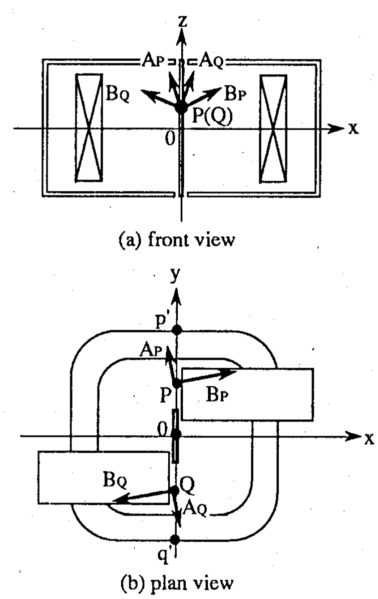

Fig.5 Relationship between flux density and potential vectors for periodic condition.

$$B_{Px} = -B_{Qx} \quad (11)$$
$$B_{Py} = -B_{Qy} \quad (12)$$
$$B_{Pz} = -B_{Qy} \quad (13)$$

The vector potential **A** is perpendicular to the flux density **B** on the periodic boundaries[5] as shown in Fig.5. Then, the relationship between $\mathbf{A}_P$ and $\mathbf{A}_Q$ is represented by the following equations:

$$A_{Px} = -A_{Qx} \quad (14)$$
$$A_{Py} = -A_{Qy} \quad (15)$$
$$A_{Pz} = -A_{Qy} \quad (16)$$

1) Edge element: Since only components parallel to the edges are defined in the edge element[1], (14) is ignored on the periodic boundaries in the case of Fig.4.

Fig.6 shows the spatial distribution of average flux density in the steel plate. The result which is obtained by analyzing the 1/4 region with the periodic boundary condition coincides with the result which is obtained by analyzing the 1/2 region without the periodic boundary condition. Therefore, we can conclude that one condition of (14)-(16) can be ignored for the edge element.

2) Nodal element: In the nodal element, all of three conditions ((14)-(16)) mentioned above can be satisfied on the periodic boundaries[5]. Even in the nodal element, however, it can be proved that only the components parallel to the periodic boundaries are sufficient to satisfy the above-mentioned conditions. The comparison between the flux distribution which is obtained using all of three conditions and that which is obtained ignoring one condition is also shown in Fig.6. As both results using the nodal element are the same, we can conclude that one condition of (14)-(16) can be ignored even for the nodal element. It is better, however,

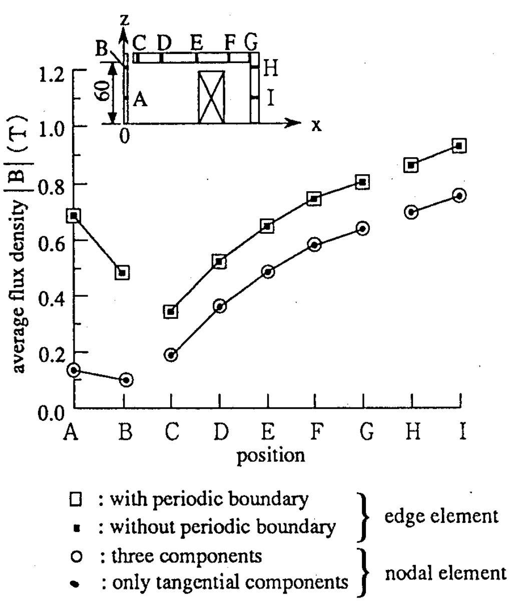

Fig.6 Distributions of flux densities along steel plates.

to impose three conditions in order to reduce the number of unknown variables.

The results of edge element are different from those of nodal element due to the difference of the accuracy of both types of element[9].

IV. CONCLUSIONS

The results obtained can be summarized as follows:

(1) When the ratio α of the number of elements in the conductor region to that of the whole region is small, the number, nz, of non-zero entries, which affects the CPU time, for the T-Ω method using the edge element is the smallest of all. When α is large, nz for the **A** method using the edge element is the smallest.

(2) The periodic boundary condition can also be used for the edge element in order to reduce the analyzed region.

The reason why only two components of **A** parallel to the periodic boundary are sufficient for both nodal and edge elements should be examined in future.

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