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# INVESTIGATION OF EFFECTIVENESS OF VARIOUS METHODS WITH DIFFERENT UNKNOWN VARIABLES FOR 3-D EDDY CURRENT ANALYSIS

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## ABSTRACT

Computer codes using the $\mathbb{A}$-$\phi$, $\mathbb{A}$-$\phi$-$\Omega$, $\mathbb{A}^*$-$\Omega$, $\mathbb{T}$-$\Omega$ and $\mathbb{E}$-$\Omega$ methods were developed. The characteristics of these methods depend on the object to be analyzed.

In this paper, the effects of the volume ratio of the conductor region to the whole region, the shape of conductor and the ratio of the hole region to the conductor region on the computer storage, the CPU time and the accuracy are investigated systematically using a few simple models. The effect of the conductivity of conductor is also examined.

Features of various methods have been illustrated by this research. For example, the computer storage and the CPU time are affected by the number of nodes on the boundaries of the conductor, on which some components of unknown variables are given. The reasons having such features have also been discussed quantitatively.

## 1. INTRODUCTION

Various methods, such as the $\mathbb{A}$-$\phi$[1], $\mathbb{A}$-$\phi$-$\Omega$[2], $\mathbb{A}^*$-$\Omega$[3], $\mathbb{T}$-$\Omega$[4] and $\mathbb{E}$-$\Omega$[5] methods, have been proposed for 3-D eddy current analysis. In order to use the most suitable analyzing method for each problem, the advantages and disadvantages of respective methods should be examined systematically. Though we already compared the computer storage, the CPU time and the accuracy for various analyzing methods using a few examples[6], the comparison was not systematic. Therefore, we could not get enough information to decide preferred (accurate and fast) method.

In this paper, the effects of the following factors on the computer storage, the CPU time and the accuracy are investigated systematically, and the features of various methods have been discussed:

(1) the volume ratio of conductor region to whole region,
(2) the shape of conductor (width and thickness),
(3) the ratio of hole region to conductor region,
(4) the conductivity of conductor.

## 2. MODELS FOR COMPARING VARIOUS METHODS

Models for investigating the characteristics of various methods were chosen from the following viewpoints:

(a) The factors mentioned in Section 1 can be changed systematically.
(b) The geometry should be simple so that the mesh generation is easy, and the dimensions of the model can be easily changed.

Elementary models which satisfy these requirements mentioned above may be the brick conductors with and without hole as shown in Fig.1. Then, the distributions of eddy current and flux density of these models are analyzed. The relative permeability of the conductor is equal to 1 and the conductivity is $1.0\times10^7$(S/m) unless otherwise specified.

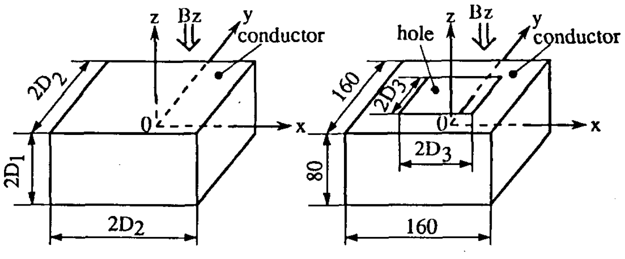
The applied magnetic field in the z-direction is uniform in space and changes with time as follows:

$$B_z = \begin{cases} 0 & (t<0) \\ 1000\times t\ (\mathrm{T}) & (t\geq 0) \end{cases} \qquad (1)$$

The effects of the volume ratio of the conductor region to the whole region, the shape of conductor and the ratio of the hole region to the conductor region on the computer storage, the CPU time and the accuracy are investigated by changing the dimensions $2D_1$, $2D_2$ and $2D_3$ of the models shown in Figs.1(a) and (b). The effect of the conductivity of conductor is also examined using the model shown in Fig.1(a).

1/8 region is analyzed as shown in Fig.2. The applied flux density in Eq.(1) is given by the boundary conditions.

Figure 3 shows the finite element subdivision. In every case except for examining the effects of the shape of conductor, the same subdivision shown in Fig.3 is used. Figure 4 shows the examples of subdivisions of long and flat conductors used in the investigation of the effects of the shape of conductor. nu and ns are the numbers of nodes on the upper surface and one side surface of conductor.



(a) without hole (b) with hole

Fig.1 Conductor models.

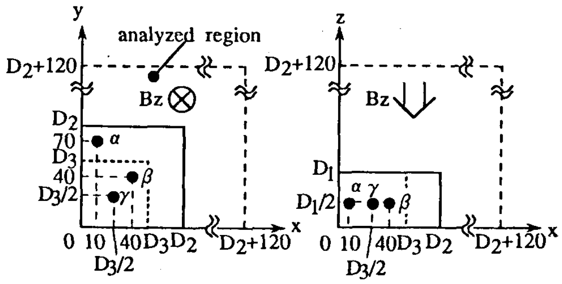
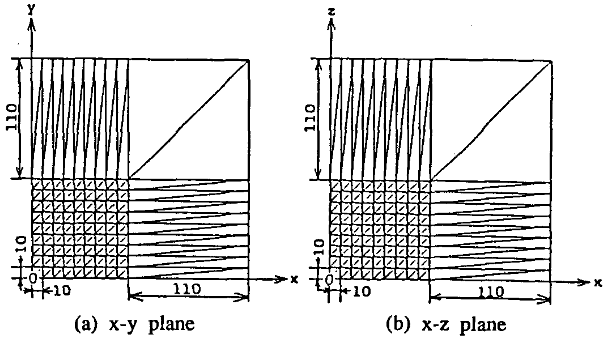
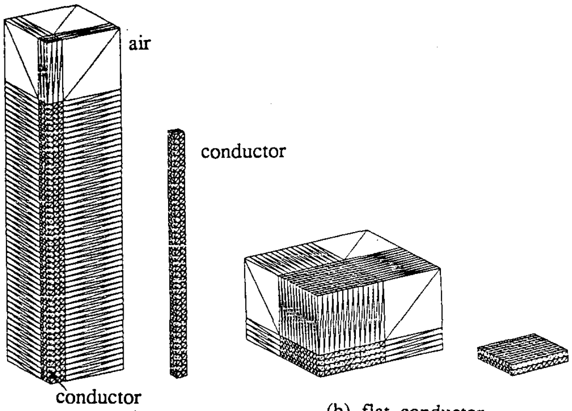


Fig.2 Analyzed region.



(a) x-y plane (b) x-z plane

Fig.3 Finite element subdivision.



(a) long conductor (nu=9, ns=135)

(b) flat conductor (nu=144, ns=36)

Fig.4 Finite element subdivisions.

## 3. COMPARISONS AND DISCUSSIONS

### <3.1> Effects of volume ratio of conductor region to whole region

The effects of the ratio of the number of nodes nc in the conductor region to the number of nodes nt in the whole region on the computer storage, the CPU time and the accuracy are investigated by changing the thickness $2D_1$ of the conductor shown in Fig.1(a). In this case, $D_2$ is 80mm.

Figure 5 shows the comparisons of the computer storage and the CPU time. The supercomputer SX-1E which was manufactured by NEC is used. The matrix of FEM is solved by the ICCG method. The computer storage (=3.4MB) and the CPU time (=231s) for the A-$\phi$ method at nc/nt=0 are normalized to unity.

The number of variables of the A-$\phi$-$\Omega$ method[2] should be ideally equal to that of the T-$\Omega$ method[4], if the variables on the boundary are neglected. In Fig.5, the computer storage and the CPU time of the T-$\Omega$ method, however, are smaller than those of the A-$\phi$-$\Omega$ method. This is because the number of unknown variables of the T-$\Omega$ method on the boundary of the conductor is smaller than that of the A-$\phi$-$\Omega$ method as shown in Figs.6(b) and (d).

The computer storage and the CPU time of the T-$\Omega$ method are smaller than those of the $A^*$-$\Omega$(E-$\Omega$) method, if nc/nt is less than 0.25(flat conductor). This is explained as follows:
In the case of the flat conductor, the number of unknown variables is mainly affected by that on the upper or lower surface of the conductor. Therefore, the above-mentioned results can be obtained, because the number of unknown variables of the T-$\Omega$ method on the upper or lower surface is smaller than that of the $A^*$-$\Omega$ method as shown in Figs.6(c) and (d).

The accuracy is examined using an analytical solution[7]. Table 1 shows the errors $\varepsilon_{Jx^*}$ of the x-components Jx of eddy currents in the conductor ($D_1$=20, $D_2$=80mm) at t=100ms. The compared points $\alpha$ and $\beta$ are shown in Fig.2. The error $\varepsilon_{Jx^*}$ is defined by

$$\varepsilon_{Jx^*}=\frac{Jx-Jxo}{Jx}\times 100\,(\%) \qquad (2)$$

where Jxo is the analytical solution of eddy current density. Table 1 shows that the error of the $A^*$-$\Omega$ (E-$\Omega$) method increases at the point $\beta$.

Figure 7 shows the error $\varepsilon_{Jx}$ of eddy current density in the conductor at t=100ms. The error $\varepsilon_{Jx}$ is defined by

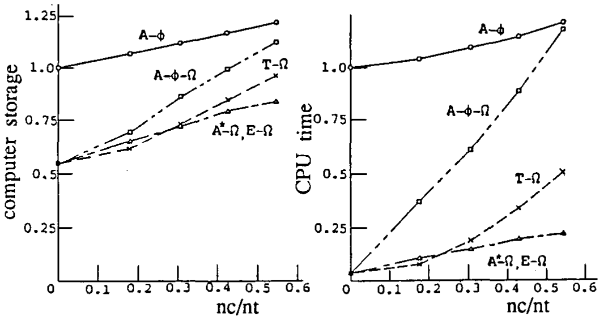
$$\varepsilon_{Jx}=\frac{Jx-Jx(A-\phi)}{Jx(A-\phi)}\times 100\,(\%) \qquad (3)$$

where Jx(A-$\phi$) is the x-component of the eddy current density obtained by the A-$\phi$ method which is assumed to be a true value.

Figure 8 shows the error $\varepsilon_{Bz}$ of the z-component Bz of the flux density at t=100ms. The error $\varepsilon_{Bz}$ is defined by

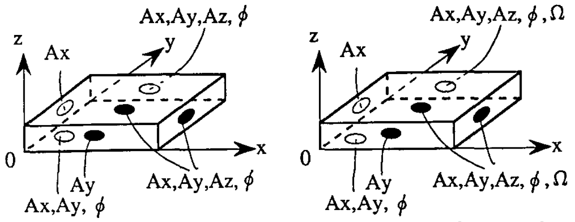
$$\varepsilon_{Bz}=\frac{Bz-Bz(A-\phi)}{Bz(A-\phi)}\times 100\,(\%) \qquad (4)$$

where Bz(A-$\phi$) is the flux density calculated by using the A-$\phi$ method. It can be seen that $\varepsilon_{Bz}$ is within 3%.

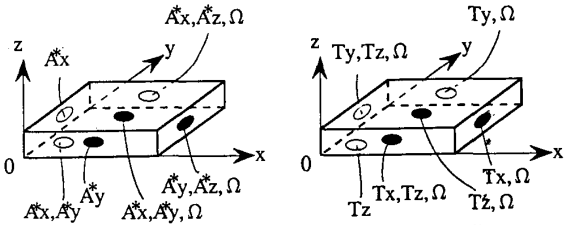


(a) computer storage (b) CPU time

Fig.5 Effects of ratio of conductor region on computer storage and CPU time.



(a) A-$\phi$ method (b) A-$\phi$-$\Omega$ method



(c) $A^*$-$\Omega$(E-$\Omega$) method (d) T-$\Omega$ method

Fig.6 Unknown variables on the boundary (in the case when the magnetic field is applied in the z-direction).

Table 1 Errors of various methods compared with an analytical solution

| method | error $\varepsilon_{Jx^*}$ (%) | |
|---|---|---|
| | point $\alpha$ | point $\beta$ |
| A-$\phi$ | 0.17 | 0.56 |
| A-$\phi$-$\Omega$ | 0.53 | -0.90 |
| $A^*$-$\Omega$(E-$\Omega$) | -0.08 | 15.00 |
| T-$\Omega$ | 0.08 | 0.52 |

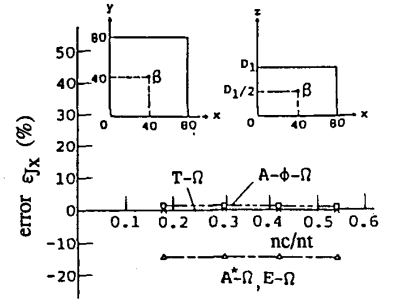


Fig.7 Effects of ratio of conductor region on error of eddy current density (point $\beta$).

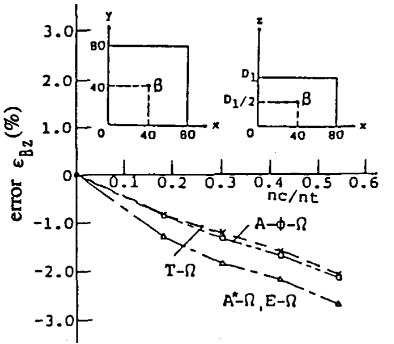
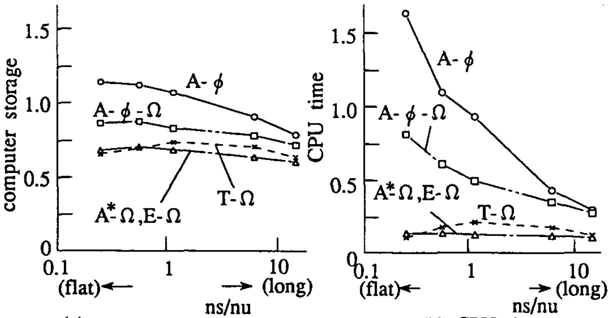


Fig.8 Effects of ratio of conductor region on error of flux density (point $\beta$).

### <3.2> Effects of shape of conductor

In the case of the model shown in Fig.1(a), the unknown variables on the surface to which the magnetic field is parallel are different from those to which the magnetic field is perpendicular. In such a model, the computer storage and the CPU time can be affected by the shape (width and thickness) of the conductor. Then, the effects of the number of nodes ns on one side of the conductor to the number of nodes nu on the upper surface of the conductor are investigated by changing the dimensions ($2D_1$ and $2D_2$ in Fig.1(a)) as shown in Fig.4. nc/nt for every shape of conductor which is examined here, is all nearly equal to 0.3.

Figure 9 shows the comparison of the computer storage and the CPU time. The large ns/nu means long conductor, and small ns/nu means flat conductor. The rate of decrease of computer storage of the A-φ method is larger than those of the other methods when the conductor becomes long. This is because the number of unknown variables nv per node of the A-φ method on the boundary of symmetry(x-z or y-z plane) to which the magnetic field is parallel is considerably decreased (nv=1) than other surfaces (nv=3,4) as shown in Fig.6(a). The CPU time of the A-φ or A-φ-Ω method is decreased with the increase of ns/nu, because the number of unknown variables is decreased, and moreover, the number of iterations of the ICCG method is also decreased. In the case of the T-Ω method, the computer storage and the CPU time of ns/nu=1 are larger than those of other values of ns/nu. This is because the number of nodes in the conductor having 4 unknown variables(Tx, Ty, Tz and Ω) becomes the maximum when the shape of the conductor approach a cube(ns/nu=1).



(a) computer storage (b) CPU time

Fig.9 Effects of shape of conductor on computer storage and CPU time (nc/nt≒0.3).

### <3.3> Effects of ratio of hole region to conductor region

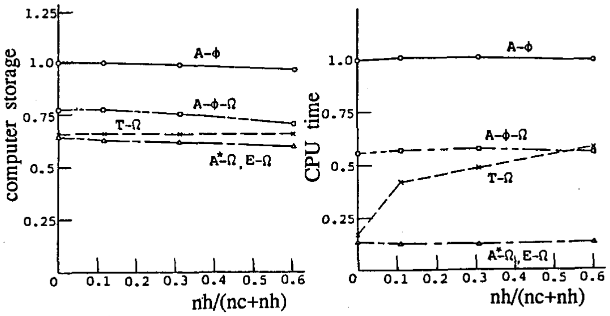
The effects of the ratio of the number of nodes nh in the hole region to (nc+nh) in the conductor and hole regions are investigated by changing the dimension $2D_3$ of the hole shown in Fig.1(b).

Figure 10 shows the comparisons of the computer storage and the CPU time. The ratio of the hole region to the conductor region is denoted by the ratio of the number of nodes in the hole region to the nodes in both the conductor and hole regions. The computer storage (=3.8MB) and the CPU time(=252s) for the A-φ method in the conductor without hole are normalized to unity.

Though the computer storage of the T-Ω method is not varied by nh/(nc+nh), the CPU time is changed. This is because the matrix becomes ill-conditioned with the increase of nh/(nc+nh), and as a result, the number of iterations of the ICCG method is increased. The computer storage and the CPU time of the other methods are not so greatly affected if the hole region is increased.

Figure 11 shows the errors of the x-component of eddy current density in the conductor with hole. The error of the A*-Ω(E-Ω) method is more considerable than the other methods.

Figure 12 shows the error $\varepsilon_{Bz}$. Though the error $\varepsilon_{Jx}$ of eddy current density of the A*-Ω method is large, the error $\varepsilon_{Bz}$ of flux density is not so remarkable.



(a) computer storage (b) CPU time

Fig.10 Effects of ratio of hole region on computer storage and CPU time.

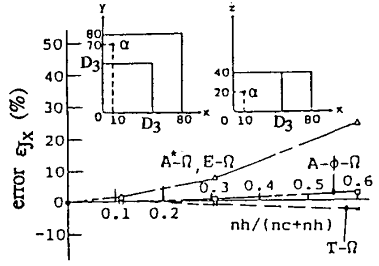


Fig.11 Effects of ratio of hole region on error of eddy current density (point $\alpha$).

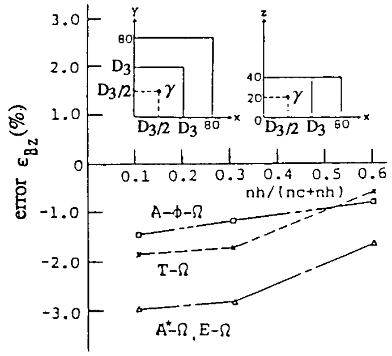


Fig.12 Effects of ratio of hole region on error of flux density (point γ ).

## <3.4> Effects of conductivity

The effects of the conductivity of conductor on the CPU time and the accuracy are investigated by using the conductor shown in Fig.1(a). In this case, $D_1$ and $D_2$ of the conductor are equal to 40 and 80mm respectively(nc/nt=0.3).

Figure 13 shows the comparison of the CPU time. The CPU time is decreased with the increase of the conductivity, because the number of iterations of the ICCG method is decreased.

Figures 14 and 15 show the effects of the conductivity on errors of eddy current density and flux density. The error of the eddy current density of the T-Ω or A-ϕ-Ω method is increased with the increase of the conductivity. This may be due to the insufficient number of elements near the surface of the conductor.

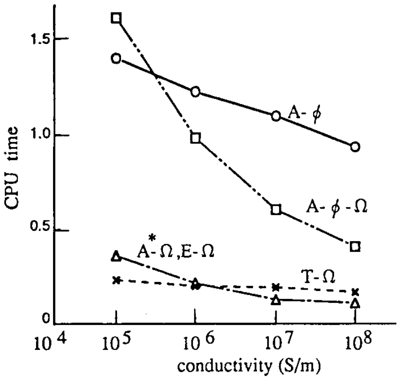


Fig.13 Effects of conductivity on CPU time (nc/nt=0.3).

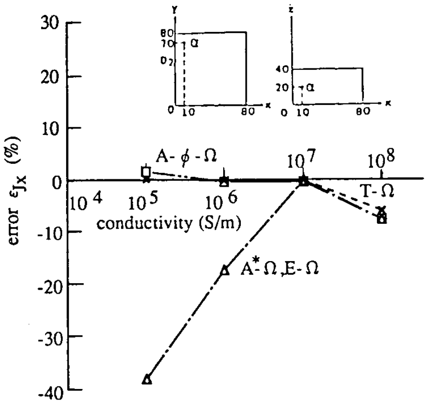


Fig.14 Effects of conductivity on error on eddy current density (point α ).

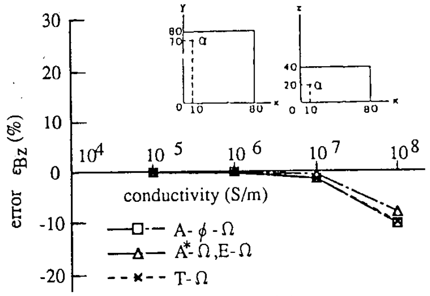


Fig.15 Effects of conductivity on error of flux density (point α ).

## 4. CONCLUSIONS

Features of various methods have been investigated by comparing the computer storage, the CPU time and the accuracy. The results obtained can be summarized as follows:

(1) The computer storage, the CPU time and the error are increased with the increase of the volume ratio of the conductor region to the whole region.

(2) The computer storage and the CPU time are affected by the shape of conductor in some methods of analysis.

(3) The error of the A*-Ω(E-Ω) method is more considerable than the other methods.

(4) As the effects of various factors on the computer storage, the CPU time and the accuracy are illustrated, it became possible to decide the preferred method for the given problem.

The effects of the permeability on the CPU time and the accuracy of each method will be reported later in another paper.

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