Engineering

Industrial & Management Engineering fields

Okayama University *Year* 2006

Modeling and control of robotic yo-yo with visual feedback

Koichi Hashimoto
Okayama University

Toshiro Noritsugu
Okayama University

Proceedings of the 1996 IEEE
International Conference on Robotics and Automation
Minneapolis, Minnesota - April 1996

Modeling and Control of Robotic Yoyo with Visual Feedback

Koichi Hashimoto and Toshiro Noritsugu

Department of Mechanical Engineering
Okayama University
3-1-1 Tsushima-naka, Okayama 700 JAPAN

Abstract

Yoyo is a toy made of two thick circular pieces of wood, plastic, etc., connected with a short axle that can be made to run up and down by moving a string tied to it. Humans can play with a yoyo without difficulty. However, developing a robot system that can play with a yoyo presents a significant challenge to controller design, because the dynamics of the yoyo are difficult to model precisely. Moreover, since the dynamics are not continuous and there are integral-type constraints on the hand trajectory, conventional continuous time feedback control theory does not work well. This paper presents a model and a control scheme for robotic yoyo with visual feedback. Experiments on a PUMA 560 are carried out to evaluate the validity of the discrete time formulation and the controller design.

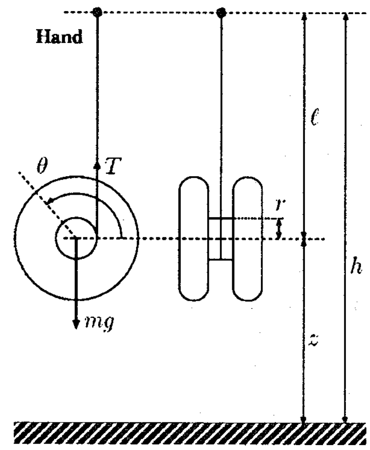

Figure 1: Schematic Illustration of a Yoyo

I Introduction

Humans can play with toys by using various kinds of sense organs. The outputs of the sensors are fused to present information for motor coordination of our body. However, it is not easy to do the same thing by robot manipulators. Developing a robot system that can play with toys presents a significant challenge to controller design, e.g., [1, 3, 4, 5, 6, 7]. An important reason is the lack of sensors to measure the motions of the toys. Another important reason is the lack of the mathematical models of the toys. In this paper, we adopt yoyo as an example of dynamic *easy for human but difficult for robot* task. We would like to show how the complicated model can be reduced to a simple model and how the controller works well based on the simple model to perform a dynamic task.

Figure 1 shows a schematic illustration of a yoyo. A string is tied to the axle and the operator moves it to control the yoyo. The yoyo runs up and down by winding and unwinding the string with the axle. The objective is to keep it running by moving the string.

Humans can play with yoyo without difficulty. However, once one closes the eyes, one may feel difficulty keep it running [2]. Thus the visual information is important for playing yoyo. The visual information contains the direction of the direction of the axis, the height, and the time at the bottom. One can know, by sensing the tension of the string, the time at the bottom but this is not sufficient. The direction of hand motion should be upward when the yoyo reaches the bottom. Thus, it is important to *predict* the bottom time by using vision. This is the reason why we use visual feedback instead of force feedback.

To model the equation of motion of the yoyo, it is useful to consider the energy balance. When the yoyo goes up, it converts the kinetic energy to the potential energy by winding the string. When it goes down, it converts the potential energy to the kinetic energy by unwinding the string. It dissipates the energy by the friction between the string and yoyo. Thus it will stop if the operator does not move the string. To keep it running the operator must supply energy by moving the hand. It seems that the yoyo gets energy only at the bottom, however, this observation is not true. The yoyo also gets energy during the up and down periods [2].

The energy supplied by the hand via string depends on the hand motion in a nonlinear way. Thus, it is difficult to derive the energy balance equation if the hand is controlled by pure continuous time feedback. Also there are other reasons why the continuous time feedback control is not adequate to control the yoyo:

1. the parameters of the motion changes the signs at the bottom (discontinuity),
2. it is difficult to measure the state (e.g., position, speed and time) at the bottom (observability),
3. the hand motion should be upward when the yoyo is at the bottom (motion constraint),
4. the hand position should be the same height before and after one stroke (integral-type constraint).

Thus, we derive a discrete time model which describes the top height of the yoyo.

First, a nominal motion of the hand is decided to analyze the yoyo's motion. The natural frequency and the shape are decided based on the motions of human operators. The amplitude is left undecided as a control parameter. Next, we compute the yoyo motion by integrating the equations of motion. Since our objective is to keep the top at the desired height, only the recurrence formula of the top height is derived. Note that the top height multiplied by the mass of yoyo and gravitational acceleration is considered as the potential energy of the yoyo. Thus the relation of the top heights is also the energy balance equation. The potential energy stored at a top position is the sum of the potential energy at the previous top position, the energy lost during the previous up-down and the energy supplied by the hand during the last stroke. It is interesting to note that the last term, the energy supplied by the hand, is linear with the amplitude of the hand stroke, though the coefficients are nonlinear and complicated functions of hand trajectory and friction. Thus, the energy balance equation can be considered as a linear discrete time equation of motion which describes the top height.

To design the controller, we consider the top height as the state, the amplitude of the hand trajectory as the input and the energy loss as the disturbance. Noting that the coefficients and the disturbance are time varying, we apply a robust servo controller based on the internal model principle.

Experiments with two yoyos are carried out to evaluate the validity of the formulation and the effectiveness of the controller design. To measure the top height and the top time, a free standing camera is used. The results show good recovery against the disturbance along the up-down direction. However, the controller can not recover other disturbances, e.g., the rotation around the string, the swings of back-forth and right-left.

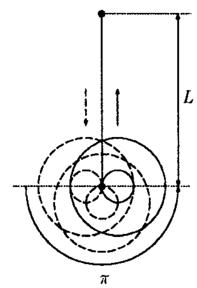

Figure 2: Rotation of π at the bottom

II Motion of Yoyo

A Equations of Motion

To simplify the analysis, the following assumptions are made:

Assumption 1 *The diameter and the mass of the string are neglected.*

Assumption 2 *The string is always vertical. The mass center moves only in vertical direction. The direction of the rotational axis is constant and orthogonal to the vertical axis.*

Assumption 3 *All frictions are viscous frictions which are proportional to the rotational velocity of the yoyo.*

Assumption 4 *The rotational velocity of yoyo does not change at the bottom. The rotation of π at the bottom (Figure 2) is neglected.*

As shown in Figure 1, let the diameter of the yoyo axle be r, the rotation angle of the yoyo be θ, the current (unwind) length of the string be ℓ, the heights of the yoyo and the hand be z and h, respectively. Suppose that $\ell = 0$ when $\theta = 0$, then we have

$$\ell = r\theta = h - z. \tag{1}$$

Let the friction coefficient be ϵ, the tension of the string be T, the mass and inertia of the yoyo be m and I, respectively. Then, when the yoyo goes down, the equations of translation of mass center and rotation about the mass center are given by

$$m\ddot{z} = -mg + T, \qquad I\ddot{\theta} = rT - r\epsilon\dot{\theta}. \tag{2}$$

On the other hand, after the yoyo turns its direction, we have

$$\ell = L - r\theta, \tag{3}$$

where L is the total length of the string. From assumption 4, the energy is not dissipated at the bottom. Thus, the equation of motion when the yoyo goes up becomes

$$I\ddot{\theta} = -rT - r\epsilon\dot{\theta}. \tag{4}$$

Summarizing equations (1)-(4), the equation of motion of yoyo is given by

$$(I + mr^2)\ddot{\theta} = \pm mr(\ddot{h} + g) - r\epsilon\dot{\theta}, \tag{5}$$

where the + sign denotes the down motion and the − sign denotes the up motion.

B Hand and Yoyo Trajectories

The equation of motion (5) depends on the acceleration of the hand $\ddot{h}$. Thus we decide a shape of the hand motion h except the amplitude. The nominal motion should satisfy the following boundary conditions:

1. when the yoyo is at the top, say $t = t_0$, the hand is at the origin, i.e., $h(t_0) = h_0, \dot{h}(t_0) = 0, \ddot{h}(t_0) = 0$.
2. when the yoyo is at the bottom, say $t = t_c$, the direction of the hand motion is up, i.e., $\dot{h}(t_c) > 0$.
3. when the yoyo goes up to the top again, say $t = t_f$, the hand should return to the origin, i.e., $h(t_f) = h_0, \dot{h}(t_f) = 0, \ddot{h}(t_f) = 0$;

An example of the trajectory is shown in Figure 3. To simplify the notation, we assume $t_0 = 0$. Then, on the basis of the data of human operators, a nominal motion is decided as follows:

$$\begin{aligned} h(t) &= h_0 + \alpha\eta(t) && (6) \\ \ddot{\eta}(t) &= \sum_{k=1}^{n} a_k \sin(\pi_k t) \\ & \qquad (\bar{T} \le t < \bar{T} + 2T) && (7) \\ \ddot{\eta}(t) &= \dot{\eta}(t) = \eta(t)0 \\ & \qquad (0 \le t < \bar{T}, \bar{T} + 2T \le t \le t_f) && (8) \end{aligned}$$

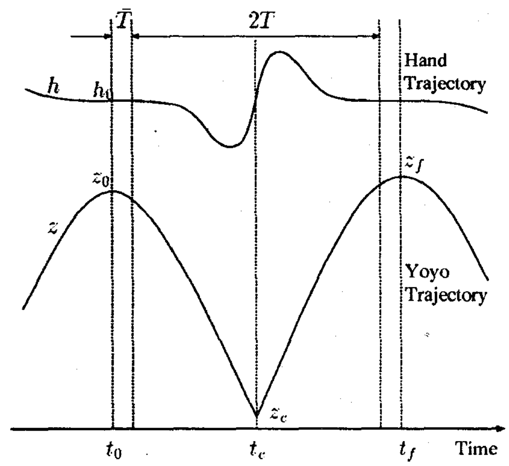

Figure 3: Trajectories of Hand and Yoyo

where $\pi_k = \frac{k\pi}{T}$, α is the amplitude, $2T$ is the period of one stroke and $\bar{T}$ is the static period. From the implementational point of view, $\bar{T}$ is required to judge whether yoyo is at the top or not. Note that one period t_f depends on z_0 and z_f. Since the hand stroke must be completed before t_f, it is necessary to satisfy $2T + \bar{T} < t_f$. However, for simplicity, we ignore $\bar{T}$ to derive the yoyo's equation of motion.

Once the hand trajectory and boundary conditions are obtained, it is straightforward to derive the motion of yoyo. Substituting (6) and (7) into (5) yields

$$\theta(t) = C_1 + C_2 e^{-\beta t} \pm \frac{mg}{\epsilon} t \pm \alpha\gamma f(t) \tag{9}$$

where C_1 and C_2 are integration constants, $\beta = \frac{r\epsilon}{I+mr^2}, \gamma = \frac{mr}{I+mr^2}$ and

$$f(t) = -\sum_{k=1}^{n} \frac{a_k\{\pi_k \sin(\pi_k t) + \beta\cos(\pi_k t)\}}{\pi_k(\pi_k^2 + \beta^2)}. \tag{10}$$

The + sign is for down and the − sign is for up.

Evaluating (9) and (10) with the boundary conditions yields a complete motion. Since the derivation is straightforward, we omit the derivation and the result is as follows:

$$z_f = z_0 + \Phi\alpha + \Psi \tag{11}$$

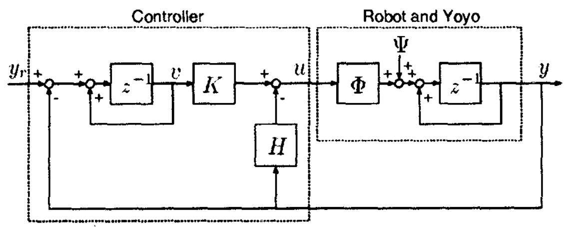

Figure 4: Block Diagram of Controller

where

$$
\begin{aligned}
\Phi &= r\gamma\{f(0) - f(t_f) + \frac{1}{\beta}\dot{f}(0)(1 + e^{\beta t_f}) \\
&\quad - \frac{2}{\beta}\dot{f}(t_c)(1 + e^{-\beta(t_f - t_c)})\} \\
\Psi &= \frac{mgr}{\epsilon\beta}(e^{-\beta t_f} - 2e^{-\beta(t_f - t_c)} - 1) - \frac{mgr}{\epsilon}t_f
\end{aligned} \quad (12)
$$

Multiplying mg for both sides of (11) yields an energy balance equation. The terms $mg\Psi$ and $mg\Phi\alpha$ are the energy dissipation by the friction and the energy supply from the hand. The energy supply can be controlled by using α, the amplitude of the hand motion.

III Control

Equation (11) can be written as follows:

$$z_f = z_0 + (\bar{\Phi} + \Delta\Phi)\alpha + (\bar{\Psi} + \Delta\Psi) \quad (13)$$

where $\bar{\Phi}$ and $\bar{\Psi}$ are nominal values of Φ and Ψ at the steady state. Thus we have a discrete time state equation with the state being the yoyo height as follows:

$$z_{i+1} = z_i + \bar{\Phi}u_i + w, \qquad y_i = z_i \quad (14)$$

where $u_i = \alpha$ is the input (amplitude of the hand motion) and $w = \bar{\Psi}$ is the disturbance (energy consumption due to friction). The deviations $\Delta\Phi$ and $\Delta\Psi$ are considered as the parameter variation of the plant. A robust servo controller for this plant is easily obtained by using the internal model principle. The control law is as follows

$$u_i = -Hz_i + Kv_i, \qquad v_{i+1} = v_i + (y_r - y_i) \quad (15)$$

where y_r is the reference height. The block diagram of the controller is depicted in Figure 4. The feedback gain $[H\ K]$ are decided to minimize the performance index

$$J = \sum_{i=0}^{\infty}\{Q_1 z_i^2 + Q_2 v_i^2 + Ru_i^2\}. \quad (16)$$

It is straightforward to see that the servo system is robust, i.e.,

1. It is internally stable.
2. The controlled output tracks the reference against constant disturbances.
3. It satisfies 1 and 2 even if there exist parameter deviations in the controlled plant.

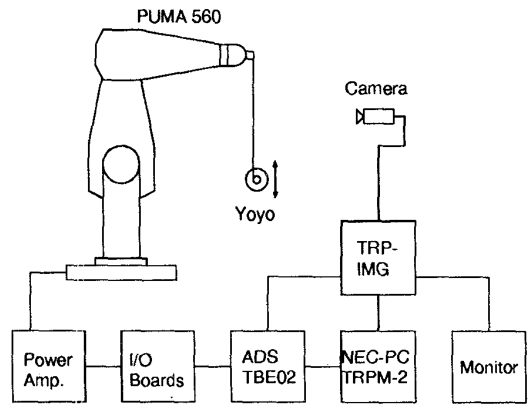

Figure 5: Experimental Set Up

IV Experiments

A Set Up

A PUMA 560 robot is used to perform yoyo. A CCD camera is used to measure the height. We have developed a transputer network for robot control. The block diagram of the system is depicted in Figure 5. Eight transputers on the TBE02 board are used to control the PUMA and two transputers on the TRP-IMG board are used for image processing. The sampling periods of the visual feedback and the joint servo are 33 ms and 1 ms, respectively.

We use two yoyos, namely 'Y1' and 'Y2', whose parameters are listed in Table 1. The parameters m, r, I are measured and $2T, \bar{T}, \epsilon$ are decided based on the motions of human operators.

Table 1: Parameters of Yoyo

	m (g)	r (cm)	$2T$ (sec)
Y1	100	0.50	1.90
Y2	180	0.50	1.80
	$\bar{T}$ (sec)	ϵ (gcm/s)	I (gcm^2)
Y1	0.160	195	306
Y2	0.0500	195	407

Table 2: Coefficients of Hand Trajectory ($n = 5$)

k	1	2	3	4	5
a_k	136.9	−366.5	174.8	−49.06	1.883

The trajectory of the hand is given by (6) and the coefficients a_k are decided based on the humans motions. These coefficients are the same for both yoyos and their values are listed in Table 2. The nominal ($\alpha = 1$) trajectories are shown in Figure 6.

B Experiments

An experimental result of Y1 is shown in Figure 7. The horizontal axis is the time and the vertical axis is the height of yoyo relative to the camera. Each circle denotes the measured yoyo position. The top positions are marked by crosses. Only the data marked by crosses are used by the feedback controller. Once the top is detected, the amplitude of the hand is computed and the hand starts to move after $\bar{T}$ seconds from the top time. The reference height is 0 mm. The commanded input (amplitude) is shown in Figure 8. The controller tries to recover the height and the height is almost recovered at $t = 11.7$. However, it can not keep the reference height because the yoyo rotates around the string (Assumption 2 is violated).

Another experimental result with Y2 is shown in Figure 9. The reference height is 50 mm. The commanded input is shown in Figure 10. The reference height is kept for 20 sec.

V Conclusion

This paper presented a model of yoyo suitable for controller design. A control scheme for robot hand by using visual feedback is also proposed. Since the dynamical equation of motion depends on the hand motion, we have decided the nominal hand motion a priori and used the amplitude of the motion as the control input. To avoid the difficulty due to the discontinuity of the motion at the bottom, we have derived a considerably simple discrete time equation, which can be considered as a energy balance equation. Since the state variable is the top height, we can use state feedback by observing the height of yoyo around the top. Fast motion around the bottom are not needed to measure, which is a big advantage of this scheme. Experiments with a PUMA 560 and a free-standing camera are carried out to evaluate the validity of the discrete time formulation. The approach is useful to control the height but the rotation around the string and the swings of back-forth and right-left should be regulated. Thus, to keep the yoyo running, one must add control action that suppresses these motions, which is our next subject.

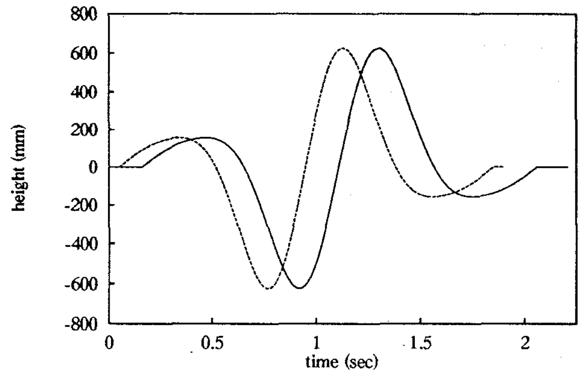

Figure 6: Nominal Trajectory of Hand
–: Y1, - -: Y2

References

[1] R. L. Andersson. *A Robot Ping-Pong Player: Experiment in Real Time Intelligent Control.* MIT Press, Cambridge, Massachusetts, 1988.

[2] S. Kurosu. Dynamics of a human-operated dynamical systems (Yoyo control system). *Trans. SICE (in Japanese)*, 14(3):277–282, 1978.

[3] A. A. Rizzi and D. E. Koditchek. Further progress in robot juggling: the spatial two-juggle. In *IEEE Int. Conf. Robotics and Automation*, pages 919–924, Atlanta, Georgia, 1993.

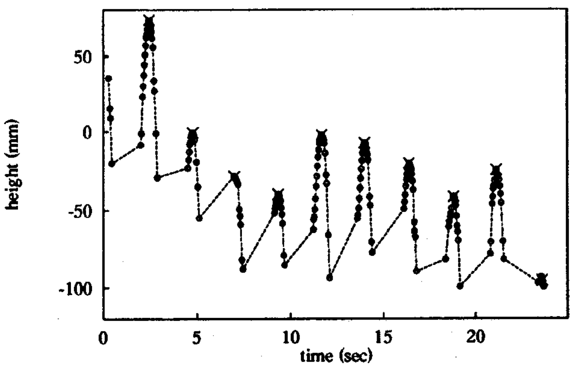

Figure 7: Trajectory of Yoyo
o: Yoyo height, x: Highest

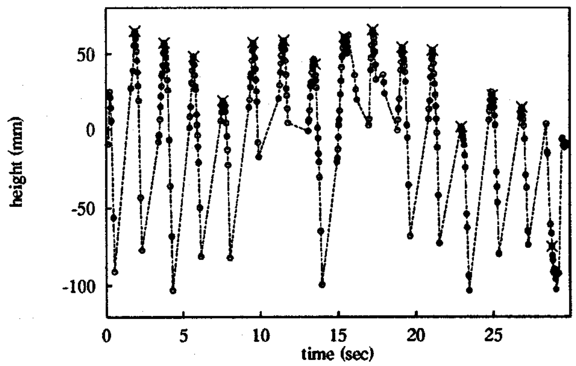

Figure 9: Trajectory of Yoyo
o: Yoyo height, x: Highest

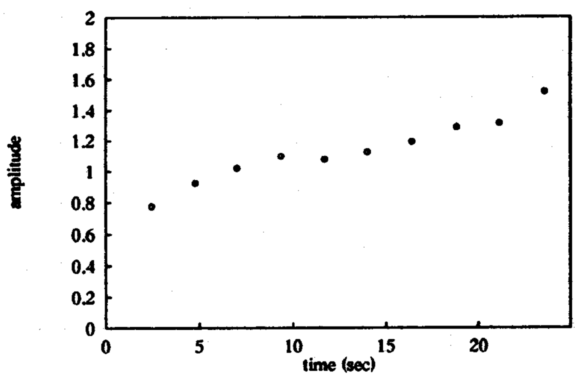

Figure 8: Control Input

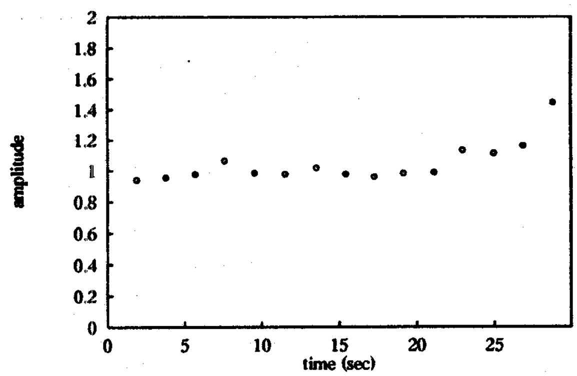

Figure 10: Control Input

[4] A. A. Rizzi and D. E. Koditchek. Further progress in robot juggling: solvable mirror laws. In *IEEE Int. Conf. Robotics and Automation*, pages 2935–2940, San Diego, Calif., 1994.

[5] T. Sakaguchi, M. Fujita, H. Watanabe, and F. Miyazaki. Motion planning and control for a robot performer. In *IEEE Int. Conf. Robotics and Automation*, pages 925–930, Atlanta, Georgia, 1993.

[6] T. Sakaguchi and F. Miyazaki. Dynamic manipulation of ball-in-cup game. In *IEEE Int. Conf. Robotics and Automation*, pages 2941–2940, San Diego, Calif., 1994.

[7] N. B. Zumel and M. A. Erdmann. Balancing of a planner bouncing object. In *IEEE Int. Conf. Robotics and Automation*, pages 2949–2954, San Diego, Calif., 1994.