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IMPROVEMENTS OF MODIFIED NEWTON-RAPHSON METHOD

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1. Introduction

When nonlinear magnetic fields are analyzed using the conventional Newton-Raphson method, the iterative processes often fail to converge, especially, in the analyses using the magnetic scalar potential. If the optimum value α_m of the relaxation factor which minimizes the total square residual W for Galerkin method is introduced at each iteration in the modified Newton-Raphson method, the convergence characteristic is fairly improved[1]. However, when the relationship between W and α is not unimodal[1], often α_m cannot be found within an allowable CPU time.

Two improvements are proposed in this paper in order to reduce the CPU time. One of them is to introduce a new relaxation factor α_n instead of α_m. Another is to reduce the range for searching α_n. The effectiveness of the proposed methods is illustrated quantitatively by analyzing some models.

2. Methods of reducing CPU time

In the modified Newton-Raphson method, the magnetic scalar potential $\Omega_i^{(k+1)}$ of a node i at the (k+1)-th iteration can be represented by the following equation :

$$\Omega_i^{(k+1)} = \Omega_i^{(k)} + \alpha^{(k)} \cdot \delta\Omega_i^{(k)} \qquad (1)$$

where $\delta\Omega$ is the increment of Ω. α is the relaxation factor. The same α is used for all unknown potentials. The case of $\alpha = 1$ corresponds to the conventional Newton-Raphson method.

As the objective function W shown in Eq.(2) which is the total square residual for Galerkin method cannot be represented explicitly as a function of α, the golden section method which is an iterative method is used to search α_m in the modified Newton-Raphson method[1].

$$W^{(k+1)} = f(\alpha^{(k)}) \qquad (2)$$

The CPU time for searching α_m using the iterative method cannot be neglected[1], especially when the W-α curve is not unimodal. Therefore, two improvements are proposed to reduce the CPU time.

2.1 Introduction of New Relaxation Factor

A new relaxation factor α_n which satisfies the following condition is introduced in order to reduce the number of calculations of W.

$$\left| \left(W_{(j)}^{(k+1)} - W_{(j+1)}^{(k+1)} \right) / W_{(j+1)}^{(k+1)} \right| \le \varepsilon_{GS} \qquad (3)$$

where $W_{(j+1)}^{(k+1)}$ is the objective function for the (j+1)-th iteration for the golden section method, which is calculated in the (k+1)-th nonlinear iteration. ε_{GS} is the convergence criterion for the golden section method. As the speed of convergence is affected by the ratio of $W^{(k+1)}$ to $W^{(k)}$, Eq.(3) is added to the conventional condition[2,3] which satisfies Eq.(4).

$$W^{(k+1)} < W^{(k)} \qquad (4)$$

If a very small value is chosen as ε_{GS} (for example, $\varepsilon_{GS} = 0.01$), α is nearly the same as α_m. It is found that even in the case of the large ε_{GS} (= 0.5), almost the same convergence characteristic as that using α_m can be obtained without waste of much CPU time for the golden section method.

2.2 Reduction of Range of Search

When α_n is determined using the golden section method, α is searched between α_{upper} and zero. If the range is small, the number of iterations for the golden section method can be decreased. α_{upper} is determined utilizing the fact that the direction of vector $\boldsymbol{H}$ of the magnetic field strength in each element for nonlinear analysis is nearly the same as that for linear analysis : that is, the angle between the vector for nonlinear analysis and that for linear one should be less than 90°.

When the magnetic scalar potential method is applied, the magnetic field strength $\boldsymbol{H}^{(k+1)}$ in a element for the (k+1)-th nonlinear iteration is calculated using $\boldsymbol{H}^{(k)}$ for the k-th iteration, its increment $\delta\boldsymbol{H}^{(k)}$ and the relaxation factor $\alpha^{(k)}$ as follows :

$$\boldsymbol{H}^{(k+1)} = \boldsymbol{H}^{(k)} + \alpha^{(k)} \cdot \delta\boldsymbol{H}^{(k)} \qquad (5)$$

If $\alpha^{(k)}$ is considerably large and the direction of $\delta\boldsymbol{H}^{(k)}$ is deviated from that of $\boldsymbol{H}^{(k)}$, the direction of $\boldsymbol{H}^{(k+1)}$ becomes considerably different from that of $\boldsymbol{H}^{(k)}$. When the angle $\theta^{(k)}$ between $\boldsymbol{H}^{(k+1)}$ and $\boldsymbol{H}^{(k)}$ is less than 90°, the following condition is obtained.

$$\cos\theta^{(k)} = \frac{\boldsymbol{H}^{(k+1)} \cdot \boldsymbol{H}^{(k)}}{\left|\boldsymbol{H}^{(k+1)}\right| \left|\boldsymbol{H}^{(k)}\right|} > 0 \qquad (6)$$

The upper bound α_{upper} for searching α_n is derived from Eqs.(5) and (6) so that the direction of $\boldsymbol{H}^{(k+1)}$ renewed is nearly the same as that of $\boldsymbol{H}^{(k)}$ as follows :

$$\alpha_{upper}^{(k)} = \min\left(\frac{-\boldsymbol{H}^{(k)} \cdot \boldsymbol{H}^{(k)}}{\boldsymbol{H}^{(k)} \cdot \delta\boldsymbol{H}^{(k)}} \right) \qquad (7)$$

where the minimization of Eq.(7) is performed for the elements in which $\boldsymbol{H}^{(k)} \cdot \delta\boldsymbol{H}^{(k)} < 0$.

3. Applications

The TEAM Workshop Problem 13[4] which is a 3-D nonlinear magnetostatic model is analyzed using magnetic scalar potential Ω in order to illustrate the effectiveness of the proposed methods. Table 1 shows the number of iterations N_R for Newton-Raphson method and the total CPU time. N_{GS} is the total number of iterations for the golden section method which is required for all nonlinear iterations. When α_m ($\varepsilon_{GS} = 0.01$) is adopted, N_{GS} is extremely increased. When only Eq.(4) is taken into account, many nonlinear iterations are required in order to get a convergence. If the magnetic vector potential $\boldsymbol{A}$ is applied, the same problem (using the same B-H curve) can be solved without special technique, namely using the conventional Newton-Raphson method (α= 1) as shown in Table 1.

Table 1 Convergence characteristics

unknown	condition	number of iterations N_R	total number of iterations N_{GS}	total CPU time (s)
Ω	Eq.(3), $\varepsilon_{GS} = 0.5$	8	15	251
	Eq.(3), $\varepsilon_{GS} = 0.01$	8	40	291
	only Eq.(4)	19	20	548
$\boldsymbol{A}$	$\alpha = 1$ (const.)	12	—	753

computer used : NEC supercomputer SX-1E (maximum speed : 285MFLOPS)
convergence criterion for Newton-Raphson method : 0.01T
convergence criterion for ICCG method : 10^{-5}

The other applications of the proposed methods for the models which have worse conditions will be reported in the full paper.

References

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