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Method for determining relaxation factor for modified Newton-Raphson method

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Method for Determining Relaxation Factor for Modified Newton-Raphson Method

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Abstract **– In order to reduce the CPU time for the modified Newton-Raphson method which introduces a relaxation factor, the effect of the relaxation factor on the residual of the Galerkin method is examined in detail. It is shown that a relaxation factor, which always provides convergent solutions, can be easily searched for. Various methods for searching for the relaxation factor to be used are compared with each other.**

I. INTRODUCTION

When nonlinear magnetic fields are analyzed by using the conventional Newton-Raphson method, the iterative process often fails to provide convergent solutions [1,2], especially in the analysis using the magnetic scalar potential. If the optimum relaxation factor α_m [3], which minimizes the total square residual W for the Galerkin method, is introduced at each step of the nonlinear iteration, convergent solutions can be always obtained [3]. However, it takes a very long time to find α_m, because a large number of repeating calculations of W is required.

In this paper, in order to reduce the total CPU time for the modified Newton-Raphson method which introduces the relaxation factor α [3-5], the general tendency of W-α curves is examined. Various methods [3,4] for searching for the relaxation factor are compared quantitatively by analyzing the 3-D standard benchmark problem from the standpoints of the number of nonlinear iterations and the total CPU time.

II. EFFECT OF RELAXATION FACTOR ON RESIDUAL OF THE GALERKIN METHOD

The modified Newton-Raphson method is reviewed briefly. The magnetic scalar potential $\Omega_i^{(k+1)}$ of a node i at the (k+1)-th step of the nonlinear iteration can be represented by the following equation:

$$\Omega_i^{(k+1)} = \Omega_i^{(k)} + \alpha^{(k)} \delta\Omega_i^{(k)} \qquad (1)$$

where $\delta\Omega$ is an increment of Ω and α is the relaxation factor. The same α is used for all of the unknown potentials. The case of α = 1 corresponds to the conventional Newton-Raphson method. The optimum relaxation factor α_m, which minimizes the objective function W, defined as the total square residual of the Galerkin method as shown in (2), is searched for by using an iterative method, because W cannot be represented explicitly as a function of α [3].

$$W^{(k+1)} = \sum_{i=1}^{nu} \{G_i^{(k+1)}\}^2 \qquad (2)$$

where nu is the number of unknown variables. The residual $G_i^{(k+1)}$ is given by

$$G_i^{(k+1)} = -\iiint \text{grad}\, N_i \cdot \{\mu^{(k+1)} (\boldsymbol{T_o} - \text{grad}\, \Omega^{(k+1)})\} dV \qquad (3)$$

where N_i is the interpolation function and μ is the permeability. $\boldsymbol{T_o}$ is the current vector potential corresponding to the magnetizing current density [6,7].

In order to know the general tendency of W-α curves, a 2-D model shown in Fig.1 is analyzed for various exciting Ampere-turns and yoke width L. The B-H curve of the core is shown in Fig.2. The shape of the W-α curve is affected by the operating point on the B-H curve, namely the analyzed conditions such as the exciting Ampere-turns and the size and shape of the core as shown in Fig.3. The shape of the W-α curve is also changed by each step of the nonlinear iteration, because the flux density at each step is different. When the B-H curve is nonlinear, the W-α curve has some local peaks. If the W-α curve is calculated while keeping permeability constant, the W-α curve forms a parabola as shown in Fig.4, because W is represented by the quadratic equation of α from (1) to (3).

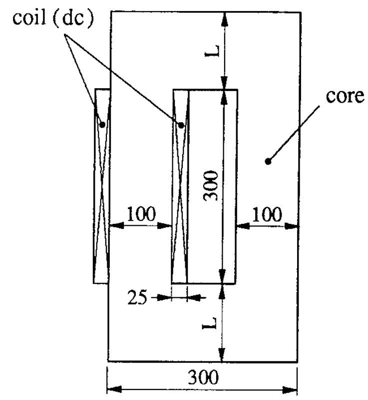

Fig.1 Analyzed model.

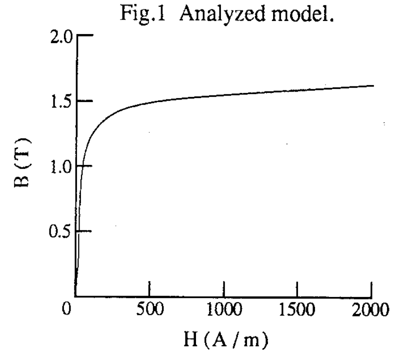

Fig.2 B-H curve.

From Figs.3 and 4, the following tendency is obtained: The W-α curve decreases monotonically near α = 0 as shown in Fig.5, because $W^{(k+1)}$ should be smaller than the previous objective function $W^{(k)}$ which corresponds to $W^{(k+1)}$ for α = 0. Therefore, if the relaxation factor which is nearly equal to zero is used, the nonlinear iterations can always provide convergent solutions. However, if the relaxation factor which is near zero is used, the number of iterations is increased. The relaxation factors $1/2^i$ and $1/2^{i+1}$ represented by symbols □ and △ in Fig.5 are described later.

The optimum relaxation factor α_m, which minimizes W, is represented by the symbol ● in Figs.3-5. In the previous paper [3], we proposed to search for α_m by using the linear search method. However, it takes a very long time to find α_m.

III. DETERMINATION OF RELAXATION FACTOR

A. Various Methods for Determining Relaxation Factor

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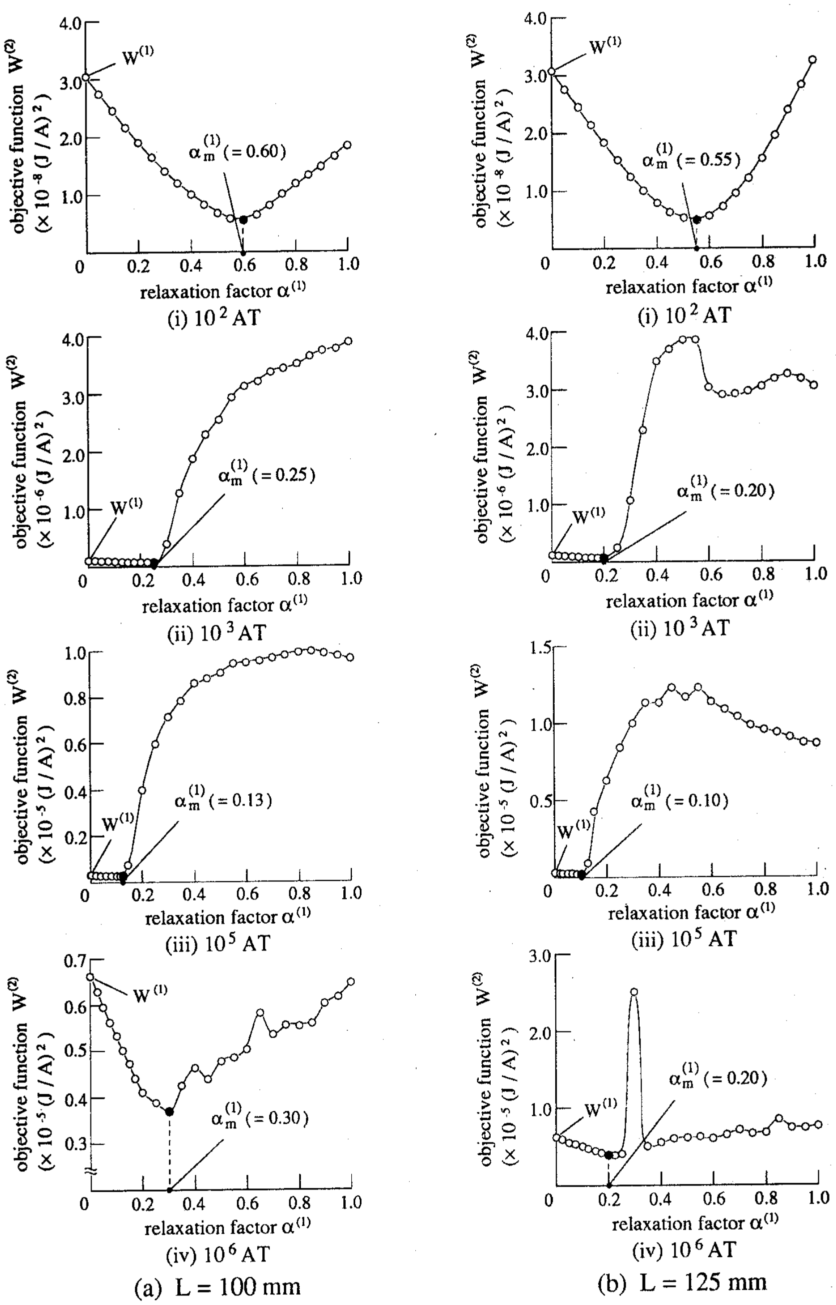


Fig.3 Various W-α curves (2nd step of Newton-Raphson iteration).

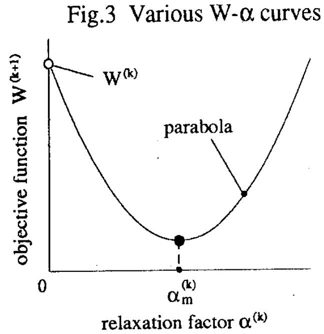


Fig.4 W-α curve with constant permeability.

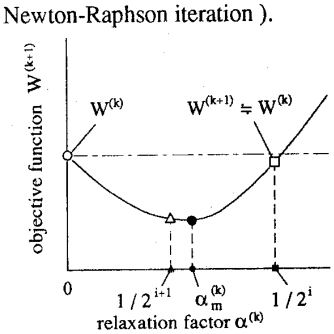


Fig.5 Characteristic of W-α curve near $\alpha = 0$.

Three kinds of methods for determining the relaxation factor are investigated.

(1) Method 1 [3]: The optimum relaxation factor α_m, which minimizes the objective function W, is introduced, and α_m is searched for by using the golden section method [8].

(2) Method 2 [4]: The relaxation factor is determined so that the objective function $W^{(k+1)}$ at the (k+1)-th step of the nonlinear iteration is less than $W^{(k)}$ at the previous step [5] as follows :

$$W^{(k+1)} < W^{(k)} \tag{4}$$

The relaxation factor which satisfies (4) is searched for by using the following equation * :

$$\alpha^{(k)} = 1 / 2^n \quad (n = 0, 1, \cdots, i) \tag{5}$$

When (4) is satisfied, the calculation of (5) changing n is terminated at n = i.

(3) Method 3: This method is a revised version of Method 2. After (4) is satisfied for $\alpha^{(k)} = 1/2^i$, the objective function $W^{(k+1)}$ is calculated once more by setting $\alpha^{(k)} = 1/2^{i+1}$ as shown in Fig.5. The relaxation factor which corresponds to the smaller objective function is adopted. In the case of Fig.5, $\alpha^{(k)} = 1/2^{i+1}$ is adopted instead of $1/2^i$.

B. Comparison and Discussion

In order to compare the number of nonlinear iterations, the change in the objective function W and the total CPU time for the various methods, the TEAM Workshop Problem 13 [1,2], which is a 3-D nonlinear magnetostatic model shown in Fig.6, is analyzed. The exciting Ampere-turns are changed, because the convergence characteristic is affected by the analyzed condition.

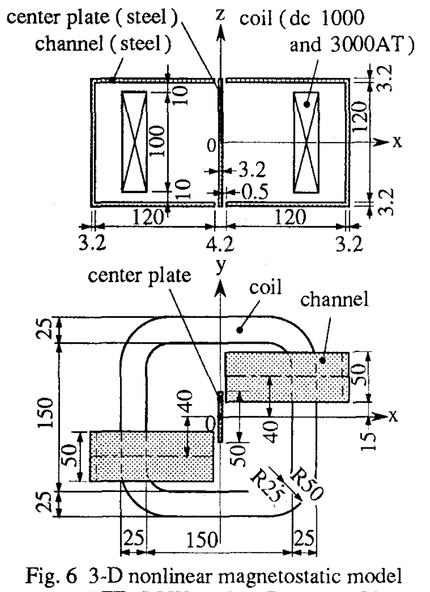


Fig. 6 3-D nonlinear magnetostatic model (TEAM Workshop Problem 13).

Table I Discretization data

method	Ω		*A*	
mesh	coarse	fine	coarse	fine
number of elements	3,564	6,930	3,564	6,930
number of nodes	4,370	8,184	4,370	8,184
number of unknowns	2,944	5,950	10,080	20,009
number of non-zero entries	36,012	74,949	150,074	314,599

* : Private communications with R.Albanese and G.Miano at the Sorrento Compumag, July 1991.

Table I shows the discretization data for the Ω method and the magnetic vector potential *A* method. Roughly and finely divided meshes are examined. The number of non-zero entries shown in Table I means the size of coefficient matrix.

Fig.7 shows the number of nonlinear iterations and the change in W. The convergence criterion for the Newton-Raphson iteration is chosen as 0.01T. All the methods show nearly the same number of nonlinear iterations. Method 1 does not always give the smallest value of W as shown in Fig.7(a)(ii). This is because the W-α curves for the respective methods are different at each step of the nonlinear iterations, and W corresponding to α_m for Method 1 is not always the minimum compared with those for the other methods. Methods 2 and 3 converge stably, because these methods satisfy (4) which guarantees that the iterative process always provides convergent solutions. It is expected that Method 3 shows the faster convergence than Method 2, because Method 2 may choose the relaxation factor for which $W^{(k+1)}$ is nearly equal to $W^{(k)}$ as shown in Fig.5. However, these methods have similar speeds of the convergence, because of the same reason as for α_m mentioned above.

Table II shows the convergence characteristics. N_W is the total number of repeating calculations of W. Method 1 requires many iterations of N_W for determining α_m. In Methods 2 and 3, α satisfying (4) can be easily searched for even if the W-α curve has many local peaks as shown in Fig.3 (a)(iv), because α approaches zero with increasing n by (5). Therefore, the CPU times for calculating W, and consequently the total CPU times for these methods are much less than that for Method 1.

If the magnetic vector potential *A* is applied, the same problem can be solved within the comparable number of nonlinear iterations without any special technique, that is, using the conventional Newton-Raphson method (α = 1) as shown in Table II. The total CPU times for the *A* method are about twice as long as those for the Ω method, because the number of unknowns of *A* is larger than that of Ω as shown in Table I.

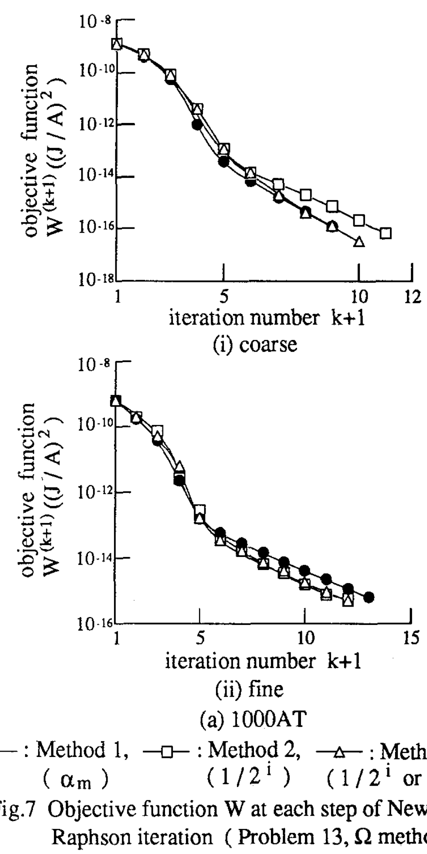
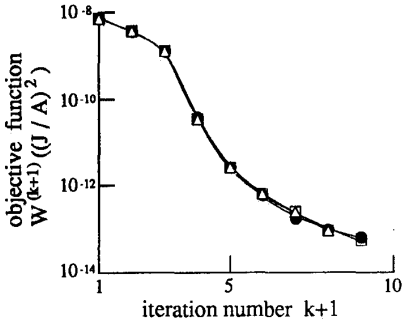
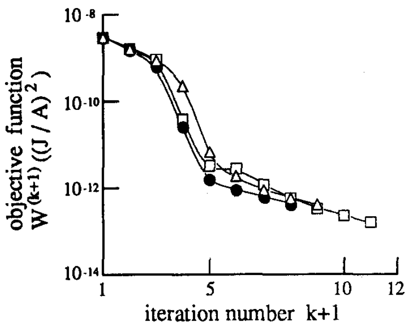


Fig.7 Objective function W at each step of Newton-Raphson iteration (Problem 13, Ω method).



(i) coarse



(ii) fine

(b) 3000AT

—●—: Method 1, —□—: Method 2, —△—: Method 3
(α_m) ($1/2^i$) ($1/2^i$ or $1/2^{i+1}$)

Fig.7 Objective function W at each step of Newton-Raphson iteration (continued).

Table II Convergence characteristics (Problem 13)

(a) 1000AT

unknown	method	number of iterations N_R*		CPU time (s) for calculations of W (total number of calculations N_W**)		total CPU time (s)	
		coarse	fine	coarse	fine	coarse	fine
Ω	1 (α_m)	9	13	113 (86)	239 (91)	241	626
	2 ($1/2^i$)	11	12	14 (11)	40 (15)	158	367
	3 ($1/2^i$ or $1/2^{i+1}$)	10	12	25 (19)	63 (24)	166	419
A	conventional ($\alpha=1$)	9	11	—	—	375	1043

(b) 3000AT

unknown	method	number of iterations N_R*		CPU time (s) for calculations of W (total number of calculations N_W**)		total CPU time (s)	
		coarse	fine	coarse	fine	coarse	fine
Ω	1 (α_m)	9	8	107 (82)	185 (70)	251	504
	2 ($1/2^i$)	9	11	17 (13)	63 (24)	155	481
	3 ($1/2^i$ or $1/2^{i+1}$)	8	9	22 (17)	57 (22)	152	419
A	conventional ($\alpha=1$)	9	11	—	—	351	924

computer used : IBM workstation 320H (11.7 MFLOPS)
convergence criterion for Newton-Raphson method : 0.01T
convergence criterion for ICCG method : 10^{-5}
* N_R : number of iterations for Newton-Raphson method
** N_W : total number of calculations of objective function

IV. CONCLUSIONS

In order to reduce the total CPU time for the modified Newton-Raphson method, the general tendency of the relationship between the relaxation factor α and the total square residual W of the Galerkin method is investigated. Various methods for searching for the relaxation factor are compared with each other.

The results obtained can be summarized as follows :

(1) The shape of the W-α curve depends on the analyzed conditions such as the B-H curve, the exciting Ampere-turns and the size and shape of the core.
(2) The W-α curve decreases monotonically near $\alpha = 0$.
(3) The relaxation factor, which always provides convergent solutions, can be easily searched for by using the simple method.
(4) It is recommended that the relaxation factor searched for by using the simple method should be adopted instead of the optimum relaxation factor.

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