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Abstract **– The effect of the scalar potential ϕ in the *A*-ϕ method on the convergence characteristic of the Incomplete Cholesky Conjugate Gradient (ICCG) method using the edge element is investigated. Several 3-D eddy current models are analyzed both by taking into account ϕ and neglecting ϕ to compare the convergence characteristics. It is illustrated that the CPU time using ϕ is less than 1/2 of that neglecting ϕ, and there are some models in which the use of ϕ enables us to obtain convergent solutions.**

I. Introduction

In the $\boldsymbol{A}$-ϕ method using edge elements, the gauge condition for $\boldsymbol{A}$ is proposed, in which $\boldsymbol{A}$ should be solved on co-tree edges only [1],[2]. The number of unknown variables can be reduced by this gauge condition. However, we have strongly recommended not to impose the gauge condition for $\boldsymbol{A}$, because the gauge condition leads to substantially longer CPU time due to the large number of iterations for the ICCG method [3],[4]. Another gauge condition, namely ϕ=0, can be imposed [2]. It seems that the software could be simpler and the CPU time could be shorter than that taking into account ϕ, because the number of unknown variables is decreased. Recently, it has been found that the convergence characteristic of the ICCG iteration is fairly improved, if ϕ is added as unknown variables. The addition of ϕ can be regarded as an extension of our recommendation mentioned above. In [5], the improvement of the convergence characteristic was found independently in high frequency problems.

In this paper, the $\boldsymbol{A}$-ϕ method using edge elements is applied to various 3-D eddy current models in cases taking into account ϕ and neglecting ϕ, and the convergence characteristics of both cases are compared to illustrate the effectiveness of the addition of ϕ.

II. Formulation

When the electric scalar potential ϕ as well as the magnetic vector potential $\boldsymbol{A}$ are chosen as unknown variables, the following two residual equations for all unknowns i are discretized in the finite element method even in the case of the edge elements:

$$G_i = \iiint_V rot\,\boldsymbol{N}_i \bullet (\nu\, rot\,\boldsymbol{A})\,dV - \iiint_{V_c} \boldsymbol{N}_i \bullet \boldsymbol{J}_0\,dV + \iiint_{V_e} \boldsymbol{N}_i \bullet \left\{\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + grad\,\phi\right)\right\} dV - \iint_S \boldsymbol{N}_i \bullet \{(\nu\, rot\,\boldsymbol{A}) \times \boldsymbol{n}\}\,dS, \tag{1}$$

$$G_{di} = \iiint_{V_e} grad\,N_i \bullet \sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + grad\,\phi\right) dV + \iint_{S_e} N_i \left\{-\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + grad\,\phi\right)\right\} \bullet \boldsymbol{n}\,dS, \tag{2}$$

where $\boldsymbol{J}_0$ is the magnetizing current density. ν and σ are the reluctivity and the conductivity respectively. V, V_c and V_e are the whole region, the region of the winding and the eddy current region respectively. S and S_e are the boundaries surrounding V and V_e respectively. $\boldsymbol{n}$ is the unit normal vector. $\boldsymbol{N}_i$ and N_i are the edge and the nodal shape functions respectively [6],[7]. If only $\boldsymbol{A}$ is solved, the $grad\phi$ term in (1) can be omitted and (2) is not required.

The difference in discretization between the edge and the nodal elements is only in the shape functions. Nothing is special in the other procedures. If analysis codes using edge and nodal elements have already been finished separately, it is fairly easy to develop a new code which discretize (1) and (2) by combining those codes.

III. Effectiveness of ϕ

Since the convergence characteristic of the ICCG method depends on the models under analysis, several kinds of 3-D eddy current problems are selected to discuss its general tendency, such as ac steady-state and transient eddy current problems including both linear and nonlinear magnetic materials. The models are classified by data types from the standpoint of solving simultaneous equations.

A. Complex Data Type of Analysis

Fig.1 shows the convergence characteristic of the ICCG method for various 3-D linear ac steady-state

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eddy current problems. The ordinate denotes the ratio of Euclidean norm $\|\boldsymbol{R}^{(k+1)}\|_2$ of the residual vector $\boldsymbol{R}^{(k+1)}$ at the (k+1)-th iteration to that $\|\boldsymbol{G}\|_2$ of vector $\boldsymbol{G}$ on the right-hand side of the simultaneous equations to be solved. The convergence criterion for the ICCG method [8] is 10^{-7}. As all the models are linear, the time derivative can be replaced by $j\omega$ (ω : angular frequency). Model (a) is proposed by the Institute of Electrical Engineers of Japan (IEEJ) [9] as a verification model. Model (b) corresponds to an induction heater. Both models (a) and (b) have massive conductors of non-magnetic materials. The skin depth of the model (b) is about 10 times larger than that of the model (a). In the model (c), eddy currents in thin conducting plates (thickness: 1mm) of two layers with very high permeability (relative permeability: 10^5) are analyzed. All the models are solved both by taking into account ϕ (the $\boldsymbol{A}$-ϕ method) and by neglecting ϕ (the $\boldsymbol{A}$ method). The convergence characteristic of the $\boldsymbol{A}$-ϕ method is fairly improved in comparison with that of the $\boldsymbol{A}$ method.

Table I shows the discretization data, CPU time and memory requirement. In the model (a), the number of iterations for the ICCG method using the $\boldsymbol{A}$-ϕ method is much smaller than that using the $\boldsymbol{A}$ method, and the CPU time is reduced to about 1/2. In the model (b), the CPU time using the $\boldsymbol{A}$-ϕ method is less than 1/5 of that using the $\boldsymbol{A}$ method, although the number of unknown variables is

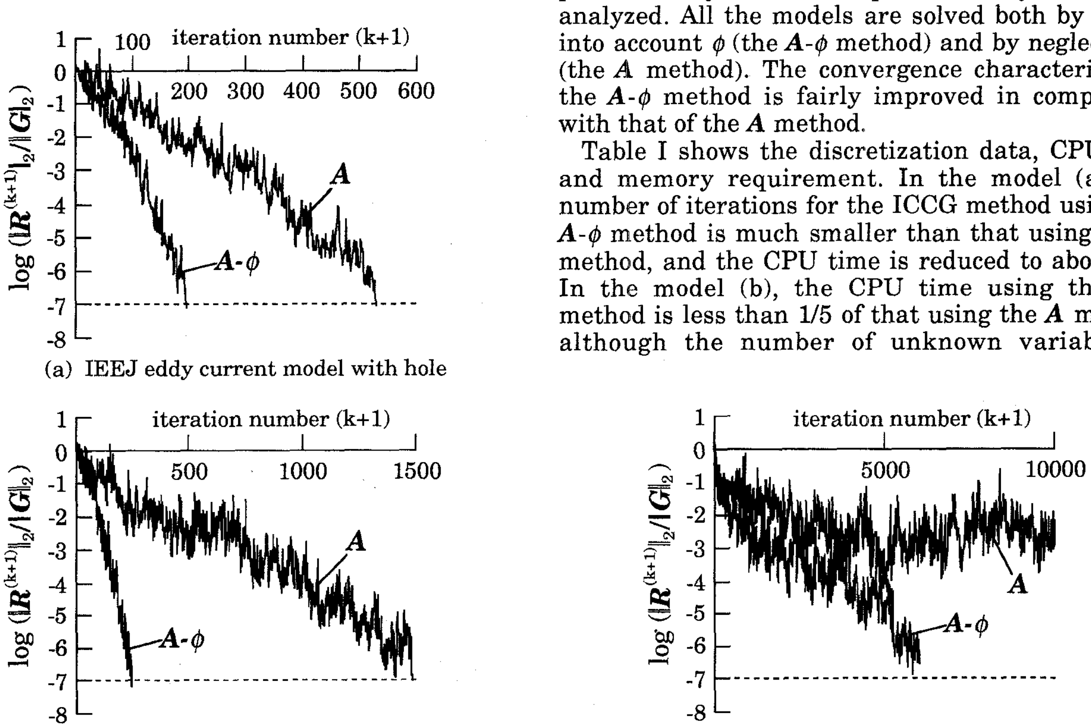

Fig. 1 Convergence characteristics of ICCG method (complex data type).

Table 1 Discretization data and CPU time (complex data type)

model	(a) IEEJ		(b) induction heater		(c) magnetically shielded room	
unknown variables	$\boldsymbol{A}$-ϕ	$\boldsymbol{A}$	$\boldsymbol{A}$-ϕ	$\boldsymbol{A}$	$\boldsymbol{A}$-ϕ	$\boldsymbol{A}$
element type (1st order edge element)	brick		brick		hexahedron	
number of elements	14,400		58,725		90,720	
number of nodes	16,275		63,480		97,356	
number of unknowns	43,552	41,060	171,884	170,984	270,503	259,319
number of non-zeros	816,037	653,718	2,844,986	2,790,192	4,952,579	4,246,435
memory requirement (MB)	35.3	29.2	126.7	123.6	215.3	188.3
number of iterations for ICCG method *1	195	528	246	1,483	6,027	10,000 *2
CPU time for ICCG method (s)	444 *3	843 *3	813 *4	4,584 *4	64,027 *3	104,046 *3

*1 convergence criterion for ICCG method : 10^{-7}
*2 Calculation is terminated forcibly at 10^4 iterations.
*3 computer used : HP workstation 735 (40 MFLOPS)
*4 computer used : IBM workstation 3AT (49.7 MFLOPS)

nearly the same for both. In the model (c), the calculation is stopped forcibly at 10^4 iterations when the $\boldsymbol{A}$ method is applied, because the accuracy of calculation could not reach an allowable range. The $\boldsymbol{A}$-ϕ method, however, can give the convergent solution.

The memory requirement for the $\boldsymbol{A}$-ϕ method is always larger than that for the $\boldsymbol{A}$ method, because ϕ is solved additionally. Its increase depends on the mesh used, and on the ratio of the number of elements in eddy current regions to the total number of elements. If eddy currents flow in all elements, the memory requirement for the $\boldsymbol{A}$-ϕ method is about twice as large as that for the $\boldsymbol{A}$ method [10]. It means that if the CPU time is more important than the memory requirement, ϕ should be added. On the contrary, if the memory constraints are more severe (for example, if a small computer is used), the $\boldsymbol{A}$ method should be used.

B. Real Data Type of Analysis

Fig.2 shows the convergence characteristic of the ICCG method. The three models of Problems 4 [11], 10 [12] and 21 [13] proposed by the FELIX and TEAM Workshops, which are linear transient, nonlinear transient and nonlinear ac steady-state problems respectively. Problem 21 is solved by the time-periodic finite element method [14]. In the nonlinear problems, the convergence characteristic at the first step of nonlinear iterations is illustrated. The tendency of the convergence characteristic is similar to that of the complex data type of analysis. Namely, the convergence can be accelerated by adding ϕ. Problem 4, however, shows that even the $\boldsymbol{A}$ method can give a fairly fast convergence.

Table II shows the discretization data, CPU time and memory requirement. In the nonlinear problems, the total number of iterations for the ICCG method is described, which is summed up in the whole nonlinear iterations. In Problem 4, the number of iterations for the ICCG method is not decreased by adding ϕ, and the CPU time becomes slightly longer because of the increase of the number of unknowns. In Problems 10 and 21, the addition of ϕ enables us to reduce the CPU times to 1/6 and 1/2 respectively. Such a reduction in the CPU time is especially effective in the nonlinear analysis, because the simultaneous equations should be solved repeatedly until the nonlinear iteration can give the convergent result and its repetition requires a substantially longer CPU time.

IV. Conclusions

The convergence characteristic of the ICCG method is fairly improved by adding the electric scalar potential ϕ as unknown variables in the $\boldsymbol{A}$-ϕ method using the edge elements. Even in the case when the $\boldsymbol{A}$ method fails to converge, the $\boldsymbol{A}$-ϕ method can give the convergent solution. However, the memory requirement is increased, if ϕ is taken into account. Therefore, the method should be selected according to the computer environment. If a sufficiently large memory is installed, the A-ϕ method is strongly recommended.

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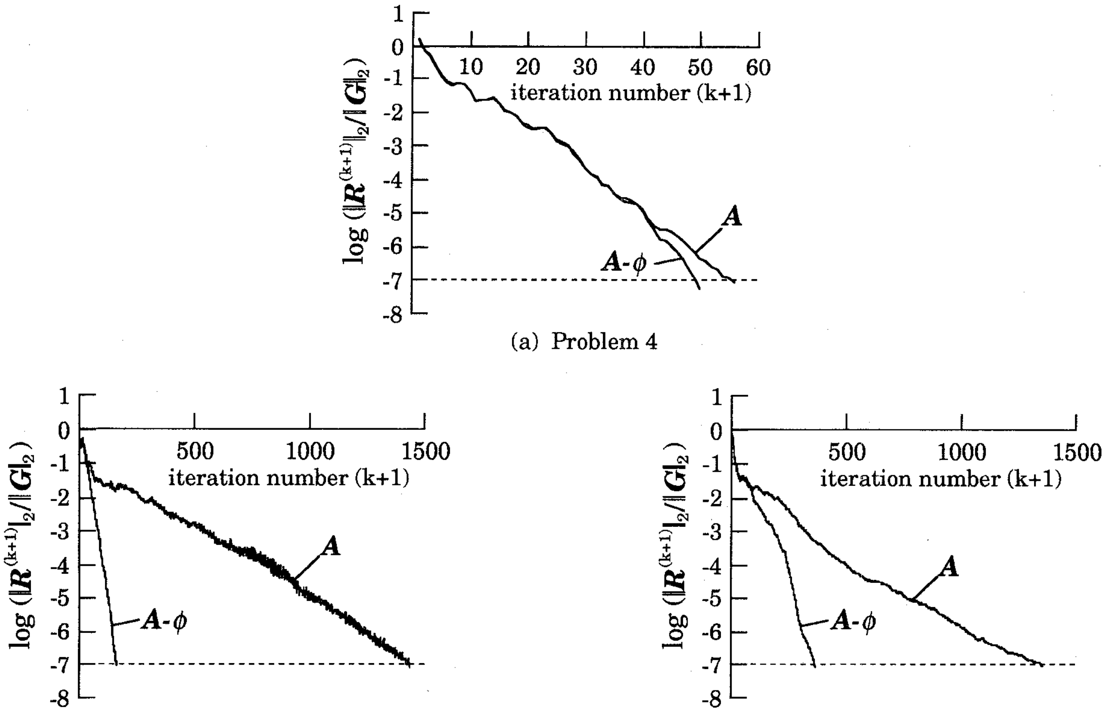

Fig. 2 Convergence characteristics of ICCG method (real data type).

Table II Discretization data and CPU time (real data type)

model	(a) Problem 4		(b) Problem 10		(c) Problem 21 model A	
unknown variables	***A*-*ϕ***	***A***	***A*-*ϕ***	***A***	***A*-*ϕ***	***A***
element type	1st order brick edge element					
number of elements	2,520		14,742		52,338	
number of nodes	3,135		16,720		56,916	
number of unknowns	8,032	7,402	44,581	41,853	158,880	148,220
number of non-zeros	149,510	111,766	835,405	664,760	3,111,548	2,409,558
memory requirement (MB)	3.7	2.9	20.6	17.0	85.8	68.2
number of iterations for ICCG method *1	1,000	1,115	48,332	473,678	17,932	62,354
CPU time for ICCG method (s) *2	75.3	65.3	26,151	166,880	33,500	79,525

*1 convergence criterion for ICCG method : 10^{-7} *2 computer used : IBM workstation 3AT (49.7 MFLOPS)