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Numerical Study of Effects of Tsunami Wave Generated on Nankai Trough

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Numerical techniques to simulate tsunami waves are described, and numerical results are introduced. A finite difference method is applied to shallow water equations to analize the propagation of tsunami wave. Numerical results to simulate a tsunami wave generated on the Nankai Trough are introduced.

Key words: *tsunami, finite difference, staggered grid, leapfrog*

1 INTRODUCTION

The earthquake of magnitude 8.9 occurred in an offshore area of the north end of Sumatra around 1 a.m. GMT on December 26, 2004 led to the generation of the tsunami wave of the maximum height 10.5 m. The casualties over twenty thousand among eleven countries were reported Results introduced in this paper are motivated by previous papers on simulation of the tsunami wave of Indian Ocean.

A finite difference method is applined to equations of fluid dynamics to simulate tsunami waves generated on the Nankai Trough in the North Pacific Ocean. The sytem of partial differential equations analyzed in this paper consists of a continuity equation and equations of motion. In Section 2, it is shown how a leapfrog scheme with a staggered grid can be applied to the system of partial differential equations (土木学会水理委員会水理公式集改訂小委員会編, 1999). In Section 3, numerical techniques introduced in Section 2 is illustrated with an example. Grid data concerning the depth of the North Pacific Ocean (Japan Hydrographic Association) are converted with Gauss-Kruger projection method to generate depth data for a staggered grid. It was assumed that a tsunami wave was generated on the Nankai Trough where the Eurasian Plate overlapps Philippine Sea Plate. Then the propagation of the tsunami wave generated on Nankai Trough was simulated numerically, and numerical results are introduced in Section 3. The discussion of numerical results is given in Section 4.

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2 NUMERICAL MODEL

The propagation of tsunami waves is simulated by the following system of partial differential equations (土木学会水理委員会水理公式集改訂小委員会編, 1999).

$$\begin{aligned}
&\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}\left(\frac{M^2}{D}\right)+\frac{\partial}{\partial y}\left(\frac{MN}{D}\right)+gD\frac{\partial \eta}{\partial x} = -\frac{gn^2MQ}{D^{7/3}},\\
&\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}\left(\frac{MN}{D}\right)+\frac{\partial}{\partial y}\left(\frac{N^2}{D}\right)+gD\frac{\partial \eta}{\partial y} = -\frac{gn^2NQ}{D^{7/3}},\\
&\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0.
\end{aligned} \tag{1}$$

The surface of the ocean is represented by $z=\eta$ and the ocean floor is represented by $z=-h$. The variables M and N are defined by

$$M=\int_{-h}^{\eta} u\,dz, \quad N=\int_{-h}^{\eta} v\,dz,$$

where u and v represent x-component and y-component of the velocity, respectively. The constant n is the Manning‘s roughness coefficient, which is equal to $n=0.025$, and the total depth D is given by $D=h+\eta$ and Q is given by $Q=\sqrt{M^2+N^2}$.

Using the notation

$$\begin{aligned}
&\frac{\partial M}{\partial t}+gD\frac{\partial \eta}{\partial x}=-F_M\\
&\frac{\partial N}{\partial t}+gD\frac{\partial \eta}{\partial y}=-F_N
\end{aligned} \tag{2}$$

the system of equations (1) is written as

$$F_M = \frac{\partial}{\partial x}\left(\frac{M^2}{D}\right) + \frac{\partial}{\partial y}\left(\frac{MN}{D}\right) + \frac{gn^2 M\sqrt{M^2+N^2}}{D^{7/3}}$$
$$F_N = \frac{\partial}{\partial x}\left(\frac{MN}{D}\right) + \frac{\partial}{\partial y}\left(\frac{N^2}{D}\right) + \frac{gn^2 N\sqrt{M^2+N^2}}{D^{7/3}}$$
$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{3}$$

Using the staggered leapfrog scheme (土木学会水理委員会水理公式集改訂小委員会編, 1999)

$$M^*_{i+\frac{1}{2},j} = M^n_{i+\frac{1}{2},j} - gDC_x\left[\eta^n_{i+1,j} - \eta^n_{i,j}\right]$$
$$N^*_{i,j+\frac{1}{2}} = N^n_{i,j+\frac{1}{2}} - gDC_y\left[\eta^n_{i,j+1} - \eta^n_{i,j}\right] \tag{4}$$

where C_x and C_y are given as

$$C_x = \frac{\Delta t}{\Delta x}, \quad C_y = \frac{\Delta t}{\Delta y}$$

the system of equations (1) leads to the following system of difference equations

$$M^{n+1}_{i+\frac{1}{2},j} = M^*_{i+\frac{1}{2},j} - \Delta t\left(\frac{F^*_{M_{i,j}} + F^*_{M_{i+1,j}}}{2}\right)$$
$$N^{n+1}_{i,j+\frac{1}{2}} = N^*_{i,j+\frac{1}{2}} - \Delta t\left(\frac{F^*_{N_{i,j}} + F^*_{N_{i,j+1}}}{2}\right)$$
$$\eta^{n+1}_{i,j} = \eta^n_{i,j} - C_x\left[M^{n+1}_{i+\frac{1}{2},j} - M^{n+1}_{i-\frac{1}{2},j}\right] - C_y\left[N^{n+1}_{i,j+\frac{1}{2}} - N^{n+1}_{i,j-\frac{1}{2}}\right] \tag{5}$$

where F^*_M and F^*_N are the upwind differences of F_M and F_N, respectively. This system of equations was solved numerically to simulate a tsunami wave generated on the Nankai Trough in the North Pacific Ocean (Liu Ying, Shinya Sumida, Majda Ceric, Kazuhiro Yamamoto, Masaji Watanabe, Submitted)..

3 NUMERICAL RESULTS FOR TSUNAMI SIMULATION

3.1 Gauss-Kruger projection

The coodinates according to a reference ellipsoid can be converted approximately to those of a rectangular coordinate system by the Gauss-Kruger projection. Let a and b be the major and the minor axis of a reference ellipsoid. According to the World Geodetic System 1984 (WGS-84), $a = 6378137.0$ m and $b = 6356752.31425$ m) (B. ホフマン/H. リヒテネガー/J. コリンズ, 2005). Let λ_0 and ϕ_0 be correspond to the origin of the rectangular coordinate system. Let

$$\Delta\varphi = \varphi - \varphi_0, \quad \Delta\lambda = \lambda - \lambda_0.$$

The first and second eccentricities are given by

$$e^2 = \frac{a^2-b^2}{a^2} = \frac{(e')^2}{1+(e')^2} = 2f - f^2$$
$$(e')^2 = \frac{a^2-b^2}{b^2} = \frac{(e^2}{1-e^2} = \frac{f(2-f)}{1-f^2} \tag{6}$$

where

$$f = \frac{a-b}{a}$$

denotes the flattening of the ellipsoid. The meridian arc length from equator to latitude φ is approximated by

$$B = a\left(1-e^2\right)\left(A'\varphi - \frac{B'}{2}\sin 2\varphi + \frac{C'}{4}\sin 4\varphi - \frac{D'}{6}\sin 6\varphi + \frac{E'}{8}\sin 8\varphi - \frac{F'}{10}\sin 10\varphi\right) \tag{7}$$

where

$$A' = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{175}{256}e^6 + \frac{11025}{16384}e^8 + \frac{43659}{65536}e^{10}$$
$$B' = \frac{3}{4}e^2 + \frac{15}{16}e^4 + \frac{525}{512}e^6 + \frac{2205}{2048}e^8 + \frac{72765}{65536}e^{10}$$
$$C' = \frac{15}{64}e^4 + \frac{105}{256}e^6 + \frac{2205}{4096}e^8 + \frac{10395}{16384}e^{10}$$
$$D' = \frac{35}{512}e^6 + \frac{315}{2048}e^8 + \frac{31185}{131072}e^{10}$$
$$E' = \frac{315}{16384}e^8 + \frac{3465}{65536}e^{10}$$
$$F' = \frac{693}{131072}e^{10} \tag{8}$$

The quantities η^2 and t are given by

$$\eta^2 = (e')^2\cos^2\varphi, \quad t = \tan\varphi,$$

and the radius of the prime vertical is given by

$$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}. \tag{9}$$

Gauss-Kruger two-dimensional coordinates are given by

$$x = B + \frac{N(\Delta\lambda)^2}{2}\sin\varphi\cos\varphi + \frac{N(\Delta\lambda)^4}{24}(5 - t^2 + 9\eta^2 + 4\eta^4)\sin\varphi\cos^3\varphi + \frac{N(\Delta\lambda)^6}{720}(61 - 58t^2 + t^4 + 270\eta^2 - 330t^2\eta^2) \times \sin\varphi\cos^5\varphi + \frac{N(\Delta\lambda)^8}{40320}(1385 - 3111t^2 + 543t^4 - t^6)\sin\varphi\cos^7\varphi \tag{10}$$

$$y = N(\Delta\lambda)\cos\varphi + \frac{N(\Delta\lambda)^3}{6}(1 - t^2 + \eta^2)\cos^3\varphi + \frac{N(\Delta\lambda)^5}{120}(5 - 18t^2 + t^4 + 14\eta^2 - 58t^2\eta^2)\cos^5\varphi + \frac{N(\Delta\lambda)^7}{5040}(61 - 479t^2 + 179t^4 - t^6)\cos^7\varphi \quad (11)$$

(原田健久, 2004). Two-dimensional rectangular coordinates are given by

$$X = m_0 k(x - B_0) + X_0, \quad Y = m_0 k y + Y_0 \quad (12)$$

where

$$k = 1 + h_0/r_0. \quad (13)$$

On the surface of the ellipsoid, $h_0 = 0$.

The radius of curvature along the meridian is given by

$$M = a\left(1 - e^2\right)\left(1 - e^2\sin^2\varphi\right)^{3/2}. \quad (14)$$

The mean radius of curvature r_0 is given by

$$r_0 = \sqrt{M_0 N_0}, \quad (15)$$

where M_0 and N_0 are obtained from Equations (14) and (9) for $\varphi = \varphi_0$.

The following values of the parameters were taken.

$$m_0 = 0.9999, \quad X_0 = 0, \quad Y_0 = 0, \quad k = 1, \quad (16)$$

where m_0 stands for the zero meridian scale factor, k represents the plane elevation coefficient, and the point B_0 corresponds to the east longitude 133°30′ and the north latitude 33° (原田健久 著, 2004).

3.2 Initial conditions

A recangular coordinate system with the origin corresponding to $\lambda_0 = 133°30'$ and $\varphi_0 = 33°0'$ was set. The transition of tsunami waves was simulated in the rectangular region

$$x_{min} \le x \le x_{max}, \quad y_{min} \le y \le y_{max},$$

where

$$x_{min} = 250000, \quad x_{max} = 550000,$$

$$y_{min} = -300000, \quad y_{max} = 300000.$$

The interval $[x_{min}, x_{max}]$ was divided into 200 intervals. Similarly, the interval $[y_{min}, y_{max}]$ was divided into 150 intervals, and a rectangular grid was set.

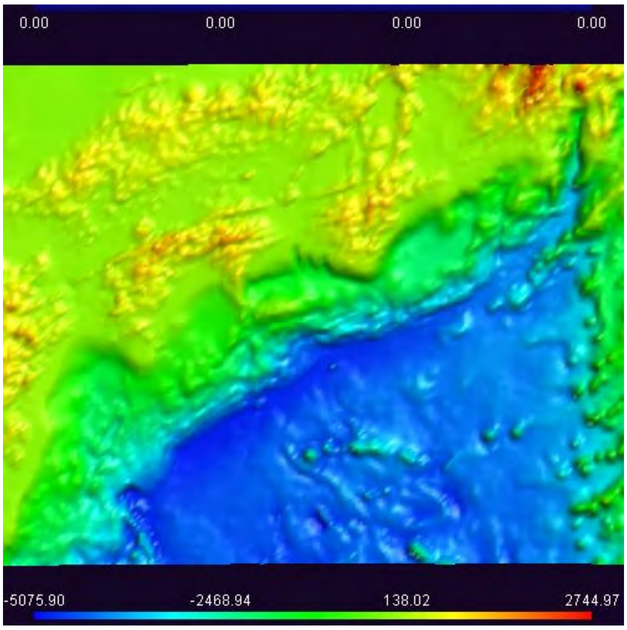

Fig. 1: Topography of the ocean floor.

The initial displacement of the ocean is given by Gaussian hump (Lye 2005, Watanabe *et al.* 2006).

$$\eta = \begin{cases} 5.0e^{-\delta}, & \begin{cases} \delta = \frac{(x-x_0)^2}{80000^2} + \frac{(y-y_0)^2}{40000^2} < 6 & (y > y_0) \\ \delta = \frac{(x-x_0)^2}{80000^2} + \frac{(y-y_0)^2}{20000^2} < 6 & (y \le y_0) \end{cases} \\ 0, & \delta \ge 6 \end{cases}$$

where $(x_0, y_0) = (50000, -120000)$ which corresponds to the north latitude 33°26′ and to the east longitude 132°12′. Initial values of η_i were generated according to Gaussian hump, and the initial values of M_i and N_i were given as

$$M = 0, \quad N = 0$$

when $D^n = h_i + \eta_i^n \le 0$, and

$$\sqrt{M^2 + N^2} = \pm\eta\sqrt{gh}$$

when $D^n > 0$.

3.3 Results of the computation

Fig. 1 shows the topography of the ocean floor generated by the Gauss-Kruger projection (Liu Ying, Shinya Sumida, Majda Ceric, Kazuhiro Yamamoto, Masaji Watanabe, Submitted). The depth of the ocean in the area ranges approximately from 0 m to 5000 m.

Fig. 2 shows the surface of the ocean at two minutes after the tsunami wave is generated.

Fig. 3-Fig. 8 show the surface of the ocean at every five minutes for thirty minutes. Numbers on the upper scale represent height of the wave.

4 CONCLUSION

The results of Section 3 indicate that a tsunami wave generated in the north pacific ocean near the Nankai

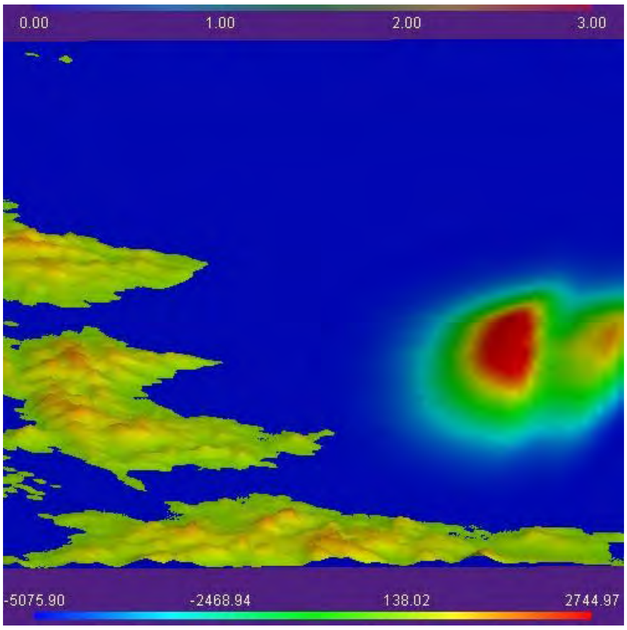

Fig. 2: Surface of the sea at $t = 120$sec.

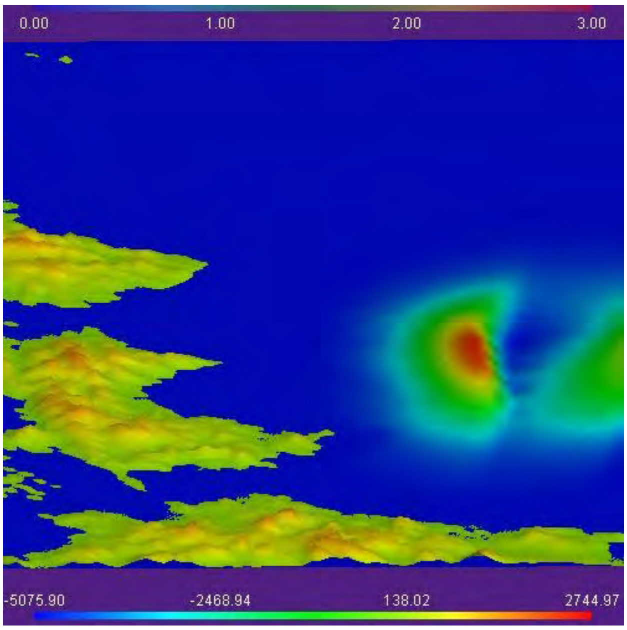

Fig. 3: Surface of the sea at $t = 300$sec.

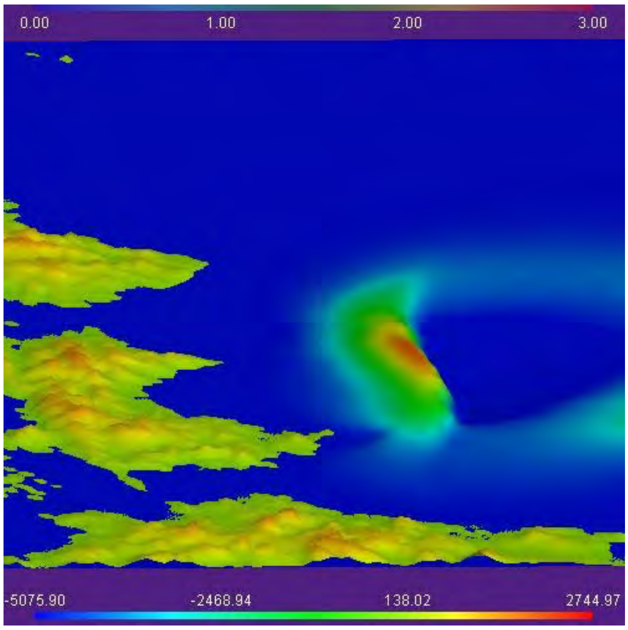

Fig. 4: Surface of the sea at $t = 600$sec.

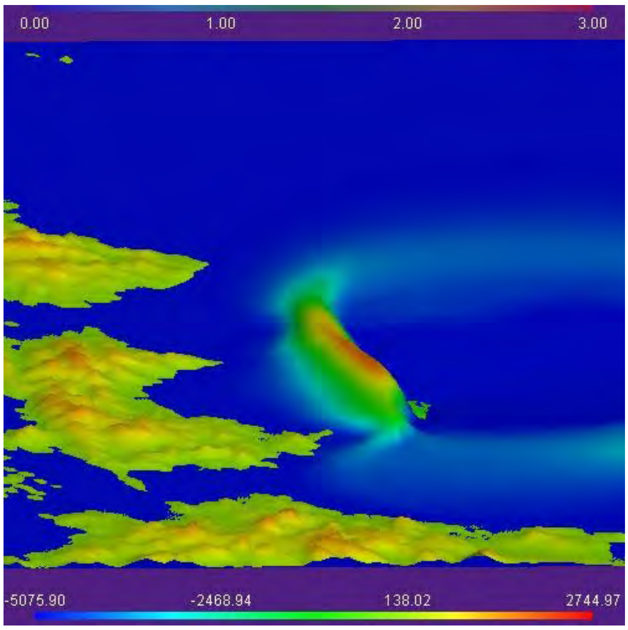

Fig. 5: Surface of the sea at $t = 900$sec.

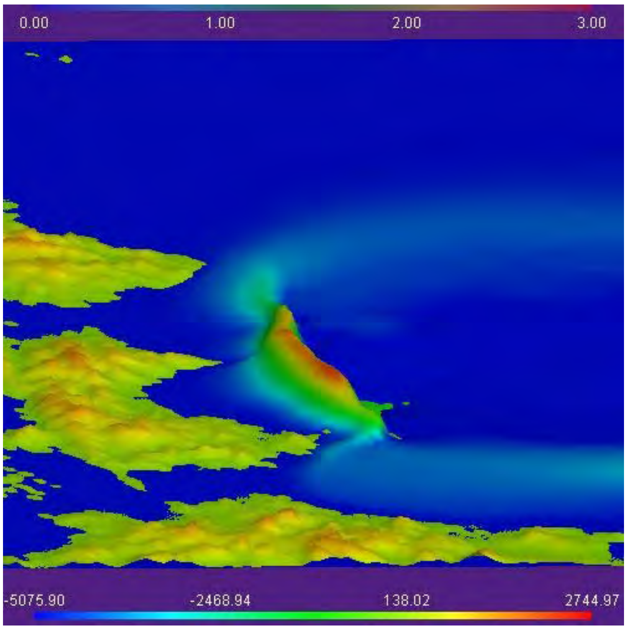

Fig. 6: Surface of the sea at $t = 1200$sec.

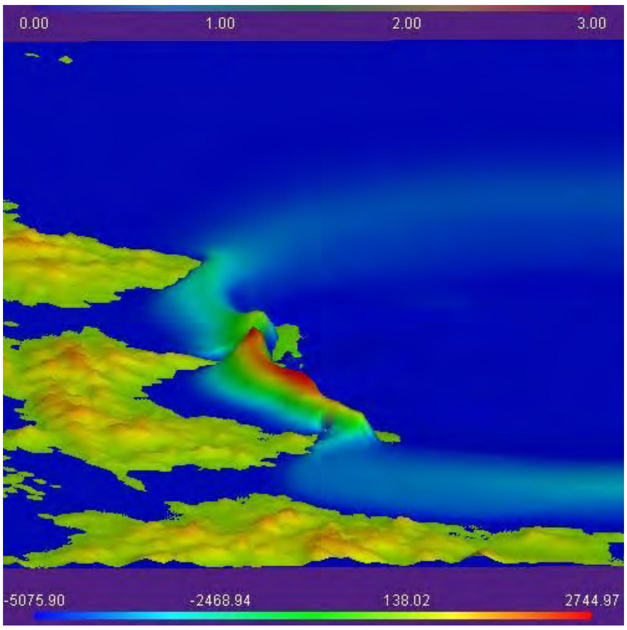

Fig. 7: Surface of the sea at $t = 1500$sec.

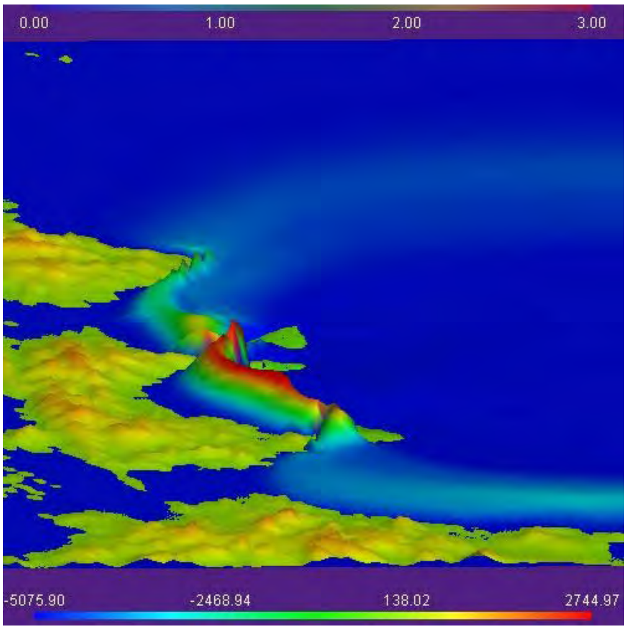

Fig. 8: Surface of the sea at $t = 1800$sec.

Trough reaches the Shikoku Island in approximately twenty minutes. It also indicates that the wave height can reach as much as three meters.

ACKNOWLEDGMENTS

Data concerning the depth of the ocean are based on the charts produced by the Japan Hydrographic Association, Marine Information Research Center, MIRC-JTOPO30, M1406, 2006/09/11, Ver. 1.0.1.

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