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August 2005

CWPE 0537

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August 2005

Abstract

This paper shows that a power utility specification of preferences over total expenditure (ie. CRRA preferences) implies that intratemporal demands are in the PIGL/PIGLOG class. This class generates (at most) rank two demand systems and we can test the validity of power utility on cross-section data. Further, if we maintain the assumption of power utility, and within period preferences are not homothetic, then the intertemporal preference parameter is identified by the curvature of Engel curves. Under the power utility assumption, neither Euler equation estimation nor structural consumption function estimation is necessary to identify the power parameter. In our empirical work, we use demand data to estimate the power utility parameter and to test the assumption of the power utility representation. We find estimates of the power parameter larger than obtained from Euler equation estimation, but we reject the power specification of within period utility.

Keywords: elasticity of intertemporal substitution, Euler equation estimation, demand systems
JEL Classification: D91, E21, D12

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For useful discussions and helpful comments we are grateful to Sule Alan, Orazio Attanasio, Richard Blundell, Martin Browning, Costas Meghir, Ian Preston, Guglielmo Weber and participants in the session on Aggregate Implications of Microecnomic Consumption Behaviour at the 2005 NBER Summer Institute. For help with the data, we are very grateful to Andrew Leicester and Zoe Smith. The usual caveat applies. For financial support we thank the Social Science Humanities Research Council of Canada, and Trinity College, Cambridge. The first author was a visitor at the Institute for Fiscal Studies when this paper was written, and he thanks that institution for their hospitality.

1 Introduction

Optimizing models of the intertemporal allocation of consumption are the work-horses of modern macroeconomics and public finance. Almost always, such models assume the power form for within period utility (also called the constant relative risk aversion or isoelastic form). The reason is that in combination with additivity over time, this gives homothetic (intertemporal) preferences and this homotheticity allows for the analysis of steady states in growth models. However, these are not innocuous assumptions. For example, they imply that the elasticity of intertemporal substitution (EIS), and its inverse, fluctuation/risk aversion, are constant. Rich and poor agents are equally averse to proportional fluctuations in consumption, and this may have significant implications, for example when evaluating the costs of business cycles or evaluating a policy change in a dynamic general equilibrium model with heterogeneous agents. The underlying aim of this paper is to assess the power utility assumption. We do this by showing the implications of the assumption of power utility for within period demands and then to exploit these implications to estimate preference parameters and to test the validity of the power utility specification.

An intertemporal substitution elasticity can be defined for individual goods. Goods with high intertemporal substitution elasticities will also tend to have high income elasticities[1] ie., they will be luxuries (Deaton, 1992 and Browning and Crossley, 2000). Browning and Crossley (2000) also note that the aggregate overall EIS is the (budget) share, s_i, weighted average of the intertemporal substitution elasticities of individual goods

$$EIS = s_1 EIS_1 + s_2 EIS_2 \tag{1}$$

This quantity is unlikely to be constant since (i) individual goods have different EIS (eg., luxuries and necessities) and (ii) the budget shares of luxuries and necessities will change with the level of within period expenditure.

The starting point for this paper is to observe that we can be more precise. In particular, we specify the conditions under which the EIS is constant and show that the power utility form for preferences (over total expenditure in each period) requires simple (and testable) restrictions on the form of *intra*temporal preferences: within period preferences must be from the PIGL/PIGLOG

[1] The income elasticity is defined as the elasticity with respect to total within period expenditure.

class, and therefore be of rank 2.[2] This means that the power utility assumption can - and has (implicitly) - been tested on cross section data. This connection between intra- and inter-temporal allocations can be exploited further to estimate intertemporal preferences: if we (i) restrict inter-temporal preferences to be additive and have the power utility representation, and (ii) rule out homothetic intratemporal preferences, then the parameter of the power utility function is pinned down completely by the shape of Engel curves. Thus, if the power assumption is correct, the parameter governing the inter-temporal substitution elasticity can be estimated from cross-section demand data, obviating the need for Euler equation estimation.

A recent literature has discussed the difficulties in using Euler equations to estimate intertemporal preference parameters from panel data on consumption (see for example, Ludvigson and Paxson (2001), Carroll (2001), Attanasio and Low (2004) and Alan and Browning (2003)). The problem arises because our panel data on consumption is typically short and noisy, and we have limited variation in the intertemporal price (the interest rate). In contrast, we have large quantities of good quality cross-sectional demand data. Thus, the ability to identify intertemporal preference parameters and to test the utility specification from intratemporal allocations is very useful.

In our empirical work, we use demand data to estimate the power utility parameter at different points in time, for individuals of different characteristics (eg. age, education) and using different goods. Our estimates of the power parameter are typically greater than 1 and this is larger than those usually estimated in the Euler equation literature. We can use our estimates to test the power utility specification in two ways: first, the rank of the demand system places restrictions on the curvature of individual Engel curves that can be tested. Second, estimates derived from the demand equations for different goods should all imply the same power utility parameter. We reject the power utility form.

Section 2 shows the theoretical connections between intratemporal and intertemporal allocations under the power utility assumption. Section 3 extends these results to the HARA class of utility functions. Section 4 reports the results of our empirical work. Section 5 concludes.

[2] The rank of a demand system is the rank of the matrix of coefficients on income terms, or equivalently the dimension of the space spanned by Engel curves, Lewbel (1991).

2 The Intratemporal Implications of Power Utility

We consider an agent that has an additive inter-temporal utility function of the form:

$$\sum \beta^t u(x_t)$$

where $u(x_t)$ is the "felicity" function that captures the utility derived from per period "consumption", x_t. Of course, households consume many goods. Therefore, we interpret $u(x_t)$ as an indirect utility function derived over total expenditure within the period and within period prices. This interpretation follows from two-stage budgeting which holds because intertemporal preferences are additive. We should therefore write $u(x_t; p_t)$, where p_t is a vector of within-period relative (normalized) prices. This interpretation has been adopted by a number of papers that simultaneously examine inter- and intra-period allocation (Blundell, Browning, and Meghir, 1994, and Attanasio and Weber, 1995, among others).

An alternative interpretation would treat $u(x_t)$ as a direct utility function defined over the composite consumption good, x_t. This relies on either (Hicks) composite commodity arguments (which require constant relative prices) or the assumption of within-period homotheticity. We do not consider either to be credible. The assumption of constant relative prices is contrary to everyday experience and is particularly difficult to defend for an open economy (movements in gasoline prices are a good counter example.) More formally, the fact that demand systems - including price responses - can be estimated on aggregated data (Deaton and Muellbauer, 1980a, is a classic example) is itself evidence of substantial variation in relative prices. The alternative assumption of within period homotheticity implies that there are neither luxuries nor necessities, which, as Deaton (1992) notes "contradicts both common sense and more than a hundred years of empirical research". It is perhaps worth noting that homotheticity over goods is rejected not just by micro data but also in aggregate data (see, for example, Deaton and Muellbauer, 1980a).

We define the elasticity of intertemporal substitution (EIS) as the derivative of log total expenditure with respect to the log of the intertemporal price (that is, the interest rate) holding within-period relative prices and the discounted marginal utility of expenditure constant. It is well known in this intertemporally additive setup this quantity is the inverse of the coefficient of relative risk (or fluctuation) aversion:

$$EIS = -\frac{\partial \log x_t}{\partial r_t} = -\frac{u_x}{u_{xx} x_t}$$

For the EIS to be constant, it must be independent of relative prices and of total expenditure.

Remark 1 *The indirect utility function defined over total expenditure within the period has a power utility representation, if and only if within period preferences take one of the following two forms:*

$$u = a(p)\frac{x^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + b(p) \qquad \theta \neq 1 \tag{2}$$

$$u = a(p)\log(x) + b(p) \qquad \theta = 1$$

These are of the PIGL and PIGLOG class, Muellbauer (1975, 1976).[3]

Sufficiency follows directly from repeated differentiation of these utility functions with respect to total expenditure. To see necessity, note that assuming that utility has the power representation implies

$$-\frac{u_{xx}}{u_x} = \frac{k}{x}$$

which we can use to solve for u_x by integrating to give

$$\log u_x = -k \log x + \log a$$

$$u_x = a x^{-k}$$

giving the general form for utility

$$u(x) = \frac{1}{1-k} a(p) x^{1-k} + b(p)$$

The point of this remark is to note that the indirect utility functions resulting from the assumption of the power utility form correspond to well known demand systems.[4] A number of

[3]Note also that PIGL/PIGLOG preferences are a subset of the Generalized Gorman Polar Form (Gorman, 1959): $u(x) = F[x/\alpha(p)] + \beta(p)$ where F is a monotone, increasing function.

[4]PIGL/PIGLOG preferences have the convenient property of allowing exact nonlinear aggregation (over agents). See Muellbauer (1975, 1976) or Deaton and Muellbauer (1980a, 1980b).

previous authors (for example, Blundell, Browning and Meghir, 1994) have noted that this form of PIGL/PIGLOG within period preferences were sufficient for intertemporal power utility, but the necessity of this condition and its implications have received little attention. An important implication is that the assumption of the power utility form can be tested by testing the rank of the demand system.

A question that this remark raises is whether it is possible to take any monotonic transform of $u(\cdot)$ to generate a different intertemporal parameter and yet maintain the power functional form and preserve within period demand. Proposition 1 proves that, in general, this is not possible. An implication of this is that the intertemporal parameter is pinned down by the shape of Engel curves.[5]

Proposition 1 *Suppose that intertemporal preferences have the power utility form, and within-period preferences are non-homothetic, so that the indirect utility function, $u(x;p)$ has the PIGL form, (2). There is no transformation of these preferences, $v = F(u(x;p))$, such that: (i) within-period demands are unchanged by $F(\cdot)$; (ii) v has the power utility form (a constant EIS) and (iii) the (constant) power parameter associated with v is different from the (constant) power parameter implied by u.*

Proof. The proof is by contradiction, and proceeds in three steps.

Suppose that such an $F(\cdot)$ does exist.

1. Demands are unchanged if and only if F is monotone in u and independent of prices: $F_u > 0$ and $F_p = 0$.

2. v has the power utility form and the power parameter is different from u if and only if:

$$\frac{F_{uu} u_x x}{F_u} = k \ , \qquad k \neq 0. \tag{3}$$

   This follows from using the chain rule in differentiating v to give

$$-\frac{v_{xx} x}{v_x} = -\frac{u_{xx} x}{u_x} - \frac{F_{uu} u_x x}{F_u} = \frac{1}{\theta} - k$$

[5] Our analysis is related to Muellbauer (1987). Muellbauer shows that our remark follows from results in Gorman (1959), but does not develop the implications for testing the power assumption. He also considers estimation of the intertemporal parameter from information on demands, but without stating or proving proposition 1.

where θ is the power parameter (EIS) defined by the function u. Note that linear transformations of course preserve both the power utility functional form and the value of the elasticity of substitution ($k = 0$), so $F(\cdot)$ must be nonlinear ($F_{uu} \neq 0$).

3. Given $u = a(p) \frac{x^{1-1/\theta}}{1-1/\theta} + b(p)$, we know $u_x = a(p) x^{-\frac{1}{\theta}}$.Substituting this into equation (3) gives,

$$\frac{F_{uu}}{F_u} a(p) x^{1-\frac{1}{\theta}} = k \tag{4}$$

Note that $x = c(u,p)$ where $c(u,p)$ is the cost function corresponding to the within period indirect utility function $u(x,p)$,

$$c(u,p) = \left(\frac{u - b(p)}{a(p)} \left(1 - \frac{1}{\theta}\right) \right)^{\frac{\theta}{\theta-1}}.$$

Substituting back into equation (4), gives

$$\frac{F_{uu}}{F_u} (u - b(p)) \left(1 - \frac{1}{\theta}\right) = k$$

Defining Φ and Γ to be constants of integration, this differential equation has the general solution:

$$F(u,p) = \Phi \frac{1}{k\frac{1}{1-\frac{1}{\theta}} + 1} (u - b(p))^{k\frac{1}{1-\frac{1}{\theta}}+1} + \Gamma$$

However, this $F(\cdot)$ is a function of p: $F_p \neq 0$, which is a contradiction (of 1).

■

To understand this proposition, consider taking a power transform of within period utility:

$$v = \frac{1}{1-\gamma} \left(a(p) \frac{x^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + b(p) \right)^{1-\gamma}$$

This gives the general expression for the EIS as

$$\frac{1}{EIS} = \gamma \left[\frac{a(p) x^{1-\frac{1}{\theta}}}{a(p) x^{1-\frac{1}{\theta}} + b(p)} \right] + \frac{1}{\theta}.$$

The EIS will in general depend on x and p. There are two special cases:

1. If $b(p) = 0$ (the case with within period homotheticity), the EIS does not depend on x or p and can vary without changing intratemporal demands.

2. If within period preferences are PIGLOG (so that all shares are linear in log expenditure), we can write

$$v = \frac{1}{1-\gamma}\left(Exp\left[a(p)\ln x + b(p)\right]\right)^{1-\gamma}$$

In this case the EIS is given by

$$\frac{1}{EIS} = \gamma a\left(p\right) - a\left(p\right) + 1$$

and is independent of x but dependent on p. This is the functional form used in Attanasio and Weber (1995).

If neither of these special cases holds, any nonlinear transformation results in an EIS that depends on x (and p). Thus if we want to have a constant EIS, we must have $\gamma = 0$ and a single curvature parameter, θ, controls both intertemporal and intratemporal allocation. The proposition shows that in general there is no nonlinear transformation that maintains a constant EIS.

Corollary 1 *If intertemporal preferences are additive and represented by the power utility form (constant EIS), and within-period preferences are not homothetic, then the intertemporal preference parameter is identified by the curvature of Engel curves.*

To take these ideas to data, note that we can use Roy's identify to derive the budget share equation for good k from the utility function (2):

$$w_k = -\frac{a_k}{a(p)(1-\frac{1}{\theta})} - \frac{b_k}{a(p)}\ x^{\frac{1}{\theta}-1} \qquad \theta \neq 1 \tag{5}$$

$$w_k = -\frac{a_k}{a\left(p\right)}\log\left(x\right) - \frac{b_k}{a\left(p\right)} \qquad \theta = 1 \tag{6}$$

where a_k and b_k are the derivatives of $a(p)$ and $b(p)$ with respect to the price of good k, p_k.

As shown in equation (5), each member of the PIGL/PIGLOG family has a different curvature for Engel curves. Choosing between members of the PIGL/PIGLOG family amounts to choosing the box-cox transformation of total expenditure that best fits the data - a very manageable nonlinear regression problem. Proposition 1 says that if within-period preferences are non-homothetic, then under the power utility assumption there is a one-to-one relationship between the curvature of Engel Curves and the power utility parameter. One Engel curve is sufficient to identify the power utility parameter. The fact that this restriction does not hold in the homothetic case makes intuitive sense: with homothetic preferences, $b(p) = 0$, Engel curves are linear, and hence are not informative about curvature.

As noted above there is a substantial recent literature (Ludvigson and Paxson, 2001, Carroll, 2001, Attanasio and Low, 2004) documenting the difficulties associated with estimating the EIS from data on consumption growth and the intertemporal price (Euler equation estimation). These problems arise because consumption growth data contains considerable measurement error, because expectation errors are only mean zero in the "T" direction while our panels are typically short (the Chamberlain,1984, critique), and because there is limited variation in the intertemporal price.[6] One response to these problems has been to move to structural estimation, as in Gourinchas and Parker (2002). However, structural estimation of consumption functions requires that the environment in which agents operate is completely (and correctly) specified. The empirical characterization of features of the economic environment (such as the income process) is itself a difficult task. And because structural estimation of the consumption function is computationally intensive, it is not yet possible to investigate how sensitive the estimates are to minor mis-specifications of the economic environment.

However, all of the above papers *assume* that intertemporal preferences have the power utility representation. Under this assumption, Corollary 1 says that the power parameter can be estimated directly from cross-section data by examining the curvature of Engel curves, and all of the above problems can be avoided. Of course cross-section expenditure data contains measurement error too. But the signal-to-noise ratio is almost surely much higher in levels data than in differences, and moreover, good methods for dealing with measurement error in the estimation of Engel curves or demands are available (for example, Hausman, Newey and Powell, 1995, or Lewbel, 1996).[7]

Turning to testing, PIGL/PIGLOG implies testable restrictions on the shapes of Engel curves.

[6] A further issue is that many of our panel surveys (for example the Panel Study of Income Dynamics) collect information on food expenditure but not total expenditure. Estimation of the power utility parameter from an Euler equation in a single good (such as food) requires within period additive separability. and within period homotheticity.

[7] Against this, Engel curves are not informative about second intertemporal parameter in the standard model - the subjective discount factor. The two-step structural approach of Gourinchas and Parker (2002), the simulated residual estimation approach of Alan and Browning (2003) and exact Euler equation estimation all provide an estimate of this parameter in addition to an estimate of the EIS (although approximate Euler equation estimation does not.) Of course, data on intra-temporal and inter-temporal allocations could be combined to give more precise estimates of both parameters.

It has not been common, in the literature, to test PIGL/PIGLOG preferences against more general alternatives.[8] Note however, that PIGL/PIGLOG preferences are at most rank 2 (the homothetic form being rank 1). The literature contains numerous tests of demand system rank. For example, Lewbel (1991), Banks, Blundell and Lewbel (1997) and Donald (1997) provide nonparametric tests of rank, and Lewbel (2004) describes a parametric approach using a rational rank four demand system that nests rational rank three polynomial demands. The typical finding of this literature is that demands are at least rank 3, and possibly rank 4. In other words, there is a strong consensus that rank 2 demand systems (including PIGL/PIGLOG) provide an inadequate representation of intra-period allocation, which implies the power utility functional form plus additivity is an inadequate representation of intertemporal utility.

In terms of equation (5), we can test PIGL/PIGLOG preferences in two ways: first, we can test whether the curvature of individual Engel curves is adequately captured by equation (5), (testing against more general specifications) . Second, estimates of equation (5) for different goods should all give the same power utility parameter.

This approach to testing power utility has considerable advantages over tests using consumption growth. The balance of evidence from testing using consumption growth is against power utility (Blundell, Browning and Meghir, 1994, Atkeson and Ogaki, 1996, and Attanasio and Browning, 1996). However, if such tests include a proper treatment of uncertainty, then they necessarily face the same difficulties that were described for Euler equation estimation above. These tests are using limited variation in the intertemporal price and noisy consumption data to assess not just how consumption growth responds to the intertemporal price (as in the usual Euler equation estimation exercise) but also how that relationship varies with the level of consumption. It is perhaps not surprising then that the results of these tests are often suggestive but not strongly statistically significant (as in Blundell, Browning and Meghir, 1994).[9] In contrast, tests of power utility based on the curvature of Engel curves should be powerful simply because we have so much cross-section demand data, and so much variation in total consumption.

[8] More commonly, a particular member of this class - usually a parameterization of PIGLOG - is tested against a non-PIGL/PIGLOG parametric demand system which nests only that particular member of the class. See for example, Banks, Blundell and Lewbel (1997).

[9] Another approach to testing CRRA is based on asset prices: see Ait-Sahalia and Lo (2000).

3 HARA Utility

HARA is the general class of utility functions which includes CRRA. It is defined by risk tolerance being an affine transform of total expenditure. Other members of the HARA class are quadratic, negative exponential, and translated power. Within the HARA class, both increasing and decreasing relative risk (or fluctuation) aversion is possible. Many important results in finance rest on the assumption of HARA preferences, such as the two-fund separation theorems (See for example Eeckhoudt, Gollier and Schlesigner, 2005).[10]

In this section we show the restrictions on within period utility implied by this more general specification of preferences. In particular, analogously to remark 1 and proposition 1, we show that assuming intertemporal preferences are additive with utility functions of the HARA class implies demand functions which are of at most rank 3.

HARA implies

$$-\frac{u_x}{u_{xx}} = \gamma + \theta x.$$

Solving for u_x

$$u_x = A(p)(\gamma + \theta x)^{-\frac{1}{\theta}}$$

$$u = \frac{1}{\theta - 1} A(p)(\gamma + \theta x)^{1-\frac{1}{\theta}} + B(p)$$

Proposition 1 (above) holds for HARA demands, and hence its corollaries do too. To see this, define

$$Q = \frac{\gamma}{\theta} + x$$

Then the definition of HARA preferences and corresponding indirect utility function can be written:

$$-\frac{u_x}{u_{xx}} = \theta Q.$$

$$u = \frac{1}{\theta - 1} A(p)(\theta Q)^{1-\frac{1}{\theta}} + B(p)$$

[10] Just as additivity plus the CRRA functional form gives homothetic intertemporal preferences, additivity plus HARA preferences gives quasi-homothetic preferences over time periods or states (Pollack, 1971). Inter-temporal expansion paths (and Engel curves for periods or states) are linear but not through the origin. This linearity is crucial to aggregation results (just as in the intratemporal case).

and the proof to proposition 1 given above can be applied directly (noting that $u_Q = u_x$ because $\frac{\partial Q}{\partial x} = 1$).

Thus, under the HARA functional form, the EIS is identified by the curvature of Engel curves as long as within period preferences are not homothetic, and the assumption of HARA preferences can be tested with cross-sectional data.

Using Roy's identity,

$$x_k = -\frac{A_{p_k}\left(\frac{1}{\theta-1}\right)(\gamma+\theta x)^{1-\frac{1}{\theta}} + B_{p_k}}{A(p)(\gamma+\theta x)^{-\frac{1}{\theta}}}$$

Thus share equations generated from an indirect utility function from the HARA class have the form:

$$w_k = \frac{x_k}{x} = \frac{\theta}{1-\theta}\frac{A_{p_k}}{A(p)} + \frac{1}{1-\theta}\gamma\frac{A_{p_k}}{A(p)}x^{-1} + \frac{B_{p_k}}{A(p)}\frac{(\gamma+\theta x)^{\frac{1}{\theta}}}{x} \tag{7}$$

Note that these demands are at most rank 3.[11]

Relative to power utility, this extra degree of rank means that there may be more scope to rationalize HARA intertemporal preferences with within period demand patterns. In practice, the two members of the HARA class which generate rank 3 demands are translated power utility and quadratic utility. Thus, if we wish to assume HARA preferences, but rule out quadratic utility (because of the evidence of precautionary behaviour), then evidence of demand being rank 3 can only be accommodated by translated power utility. Translated power utility exhibits decreasing relative risk aversion.

Note also that the term in equation (7) that gives the extra degree of flexibility goes to zero as total within period expenditure grows. As total expenditure grows, translated power utility converges to power utility, implying that the demands of the rich should be very close to rank 2. One way out of this might be to assume that the translation (γ) is determined by an external reference point (as in models of "external habits").[12] Note, however, that such a model would imply (see equation (7)) that budget shares depend - in potentially testable ways - on both total expenditure and the excess

[11] Lewbel and Perraudin (1995) show that there is a correspondence between the rank of demands and the degree of fund separation: for example, two -fund separation is consistent with rank 2 demands. Note that this is a very different result from ours: Lewbel and Perraudin are refering to the rank of (general) preferences over states and treating total consumption in each state as a single commodity.

[12] An internal reference point would break the intertemporal additivity and so we would be unable to work with the per period indirect utility function.

of total expenditure over the reference point.

4 Empirical Work

Our empirical work illustrates the estimation and testing strategy derived in Section 2. First, we use demand data to estimate the parameter of the power utility function, maintaining the assumption that the power specification is appropriate. Second, we test the assumption of power utility in two ways: by testing whether the same power parameter is estimated using different Engel curves and by testing the implied restrictions on the curvature of a given Engel curve.

We use micro data on expenditures from the UK Family Expenditure Survey. We have this data in suitable form, annually from 1974 to 2000. To avoid (unobserved) within-period price variation, we focus on households in London and the South East (as in Banks, Blundell and Lewbel, 1997). We also focus on a restricted set of family types: couples with and without children. We group data into two year bands to increase cell size.

As shown in equation (5), we can write the budget share equation for good k:

$$w_k = -\frac{a_k}{a(p)(1-\frac{1}{\theta})} - \frac{b_k}{a(p)}\ x^{\frac{1}{\theta}-1} \qquad \theta \neq 1 \tag{8}$$

If the assumption of power utility is valid, the curvature from θ should capture all the curvature in Engel curves and the estimate of the parameter θ should be the same for any Engel curve.

We estimate equation (8) by nonlinear least squares, using the standard iterative Gauss-Newton method.[13] It is worth stressing that although we impose that θ must be equal across households, we do not restrict the degree of heterogeneity entering via the slope or intercept coefficients. We then use artificial regressions (Gauss-Newton Regressions, see MacKinnon, 1992) for testing. The specific

[13] One problem with the estimation of equation (8) is that it is badly specified at $\theta = 1$ where the budget share becomes

$$w_k = -\frac{a_k}{a\,(p)}\log{(x)} - \frac{b_k}{a\,(p)}. \tag{9}$$

We address this by:

1. Estimating equation (8), trying starting values for θ above and below 1.
2. Checking convergence conditions carefully (via another GNR) at our best estimate of $\theta \neq 1$.
3. Estimating equation (9) and comparing the minimized sum of squared residuals to those from our best estimate from equation (5).

parametric alternative that we test against is the HARA share equation (7).[14]

We begin by showing estimates using data on food budget shares. The left hand graph in Figure 1 shows estimates of θ across time from food budget shares. The figure reports 95% confidence intervals around each estimate. It is not possible to reject a constant value of $\theta = 1.5$ although point estimates do move over time. The right hand graph reports tests of the implied curvature restriction. In most years, equation (8) is not rejected against more flexible alternatives.

Our estimates of θ are higher than those obtained from linearized Euler equation estimation. For example, Attanasio and Weber (1995) estimate the elasticity of intertemporal substitution to be 0.67, compared to our estimate of $\theta = 1.5$. Gourinchas and Parker (2002) implement a fully structural estimation strategy to recover θ (and the discount rate.) Their estimates range from $\theta = 0.7$ to $\theta = 2.0$.

Figures 2,3 and 4 report similar estimates and tests for clothing, for fuel and light and for leisure services. For clothing and for fuel and light, we obtain similar estimates of θ to those from food Engel curves, but the estimates are less precise. For leisure services, we estimate a θ which is substantially lower, around 0.5, and very precisely estimated.

For none of the goods that we consider do we reject the functional form of equation (8) against more flexible alternatives. However, the restriction that θ must be common across goods is strongly rejected and so the power form for the felicity function is not consistent with this data.

5 Conclusions

In this paper we have demonstrated that within period preferences must be in the PIGL/PIGLOG class if they are to generate power utility. This means that the assumption of constant relative risk aversion requires that within period demands be at most Rank 2, a restriction which is typically rejected by demand data. On the other hand, if intertemporal preferences are additive and power and within period preferences are non-homothetic, then the inter-temporal substitution elasticity is pinned down completely by the shape of (within period) Engel curves. Finally, we showed that the broader class of HARA (intertemporal) preferences are consistent with a particular form of intratemporal demands which are at most rank 3 and that the plausible member of HARA generating

[14] We considered a number of other parametric alternatives. These all gave similar results.

this more general structure has decreasing relative risk aversion.

Our results follow from the fact that behavioral responses are all governed by the curvature of the (indirect) utility function. This has previously been noted by Deaton (1974) who showed that additivity over goods implied a connection between (intratemporal) price and income elasticities, and by Deaton (1992) and Browning and Crossley (2000) who note that additivity over time *and* goods implies that the intertemporal substitution elasticities of particular goods are related to their income elasticities. In this paper, we have moved significantly beyond those results. We do not assume within-period additivity, rather we have shown the necessary and sufficient conditions on within period demand implied by a power function over total expenditure. Moreover, in the Deaton and Browning and Crossley results, the relationship between the income elasticity of a good and its intertemporal substitution elasticity is mediated by the (unknown) overall EIS. In contrast, we have shown how under power utility, the curvature parameter can be directly estimated from cross-section data, or alternatively, how the power utility assumption can be tested on cross section data.

Our analysis shows that the assumption of the power (or CRRA) form for the felicity function is inconsistent with well documented features of the micro-data on intra-temporal allocation. An obvious question is: how wrong is the power form? In a statistical sense, our empirical results amount to a large rejection of the overidentifying restrictions implied by this functional form. It is more difficult to give an economic answer. The difficulty arises because outside of the power form, we must use data on consumption growth and variation in the intertemporal price to characterize the EIS (and how the EIS varies with the level of consumption). As elaborated above, this is a difficult task.[15] We do know that relaxing the CRRA assumption can significantly change our answers to substantive questions. For example, Ogaki and Zhang (2001) show that it is important to allow for declining relative risk aversion when testing the full risk-sharing hypothesis.

In the 1990s, it became apparent that quadratic utility, while analytically very convenient, was inconsistent with micro evidence on precautionary behavior. Consequently, quadratic utility was largely abandoned in intertemporal models of consumption and savings. Our analysis suggests a

[15] There is analogy here to the literature on equivalence scales. Equivalence scales are not, in general, identified by demand data (Pollak and Wales, 1979). They can, however, be estimated under the assumption that the equivalence scale does not depend on the level of utility. This restriction is referred to as base independence or equivalence scale exactness. Base independence does imply testable restrictions on the data, which are in some cases statistically rejected (see Pendakur, 1999). However, because equivalence scales are not in general identified, it is not clear how to determine how far (in an economic sense) true equivalence scales are from being base independent.

similar abandonment of power utility, despite the considerable conveniences it offers (for example in analysing steady states).

An alternative reaction to our results would be to interpret them as evidence (or perhaps further evidence) against intertemporal additivity. If homothetic intertemporal preferences are very desirable, and homotheticity plus additivity implies power utility, which in turn implies intratemporal demands that are rejected by the data, then the most promising direction may be to abandon intertemporal additivity. However, the need to reconcile inter- and intra-period allocation would remain.

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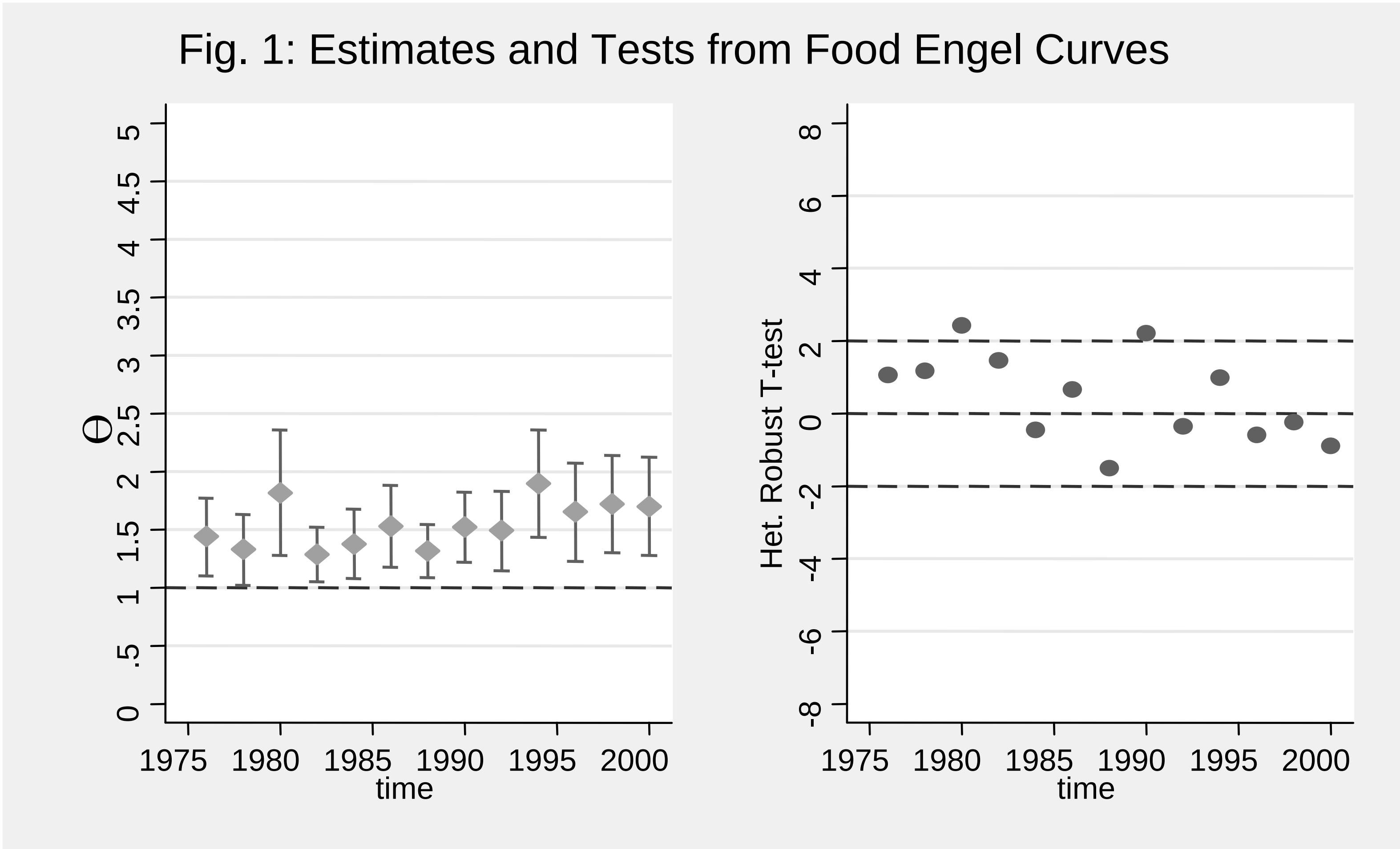

Fig. 1: Estimates and Tests from Food Engel Curves
Θ
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
1975
1980
1985
1990
1995
2000
time
Het. Robust T-test
-8
-6
-4
-2
0
2
4
6
8
1975
1980
1985
1990
1995
2000
time

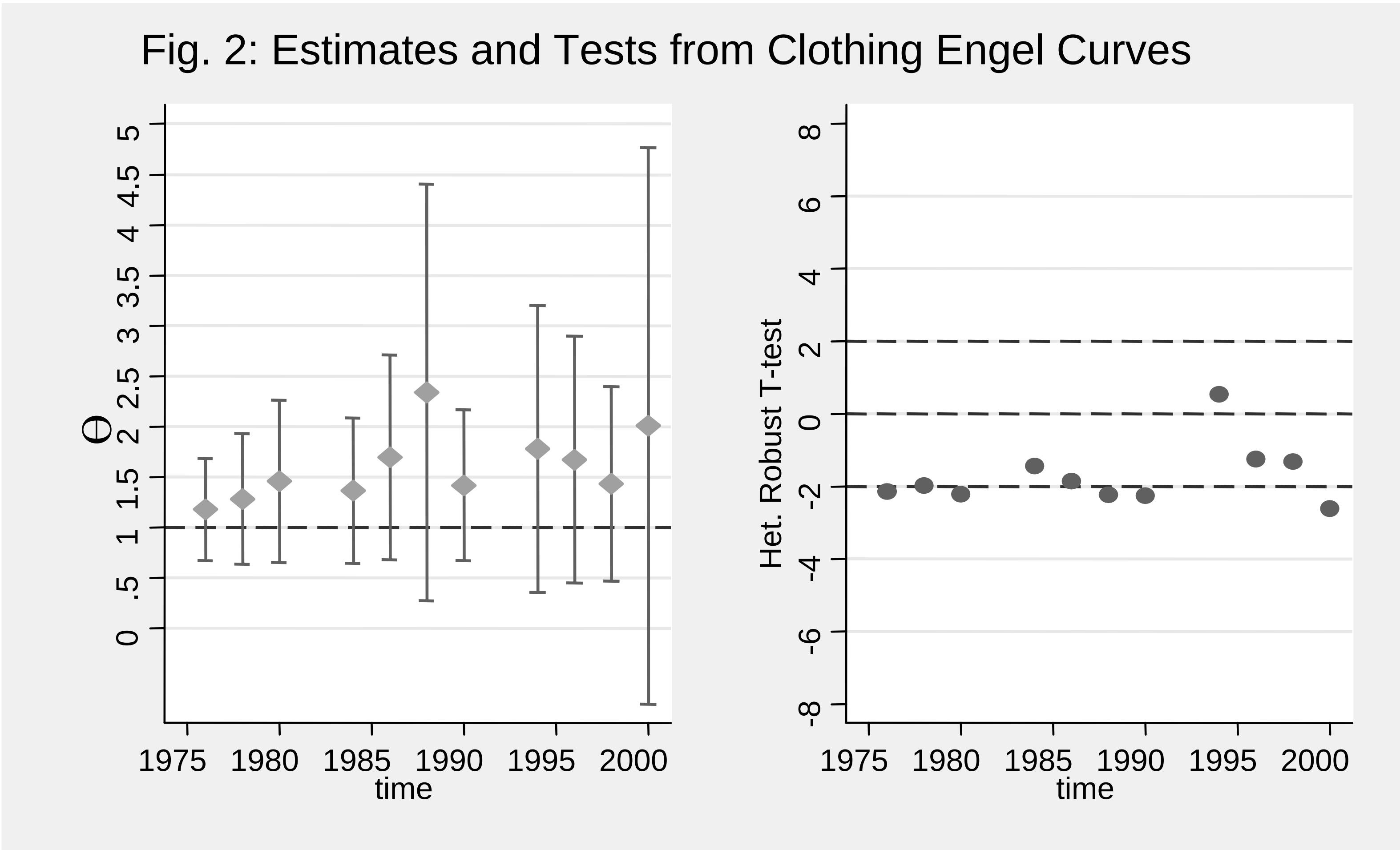
Fig. 2: Estimates and Tests from Clothing Engel Curves
Θ
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
1975
1980
1985
1990
1995
2000
time
Het. Robust T-test
-8
-6
-4
-2
0
2
4
6
8
1975
1980
1985
1990
1995
2000
time

Fig. 3: Estimates and Tests from Fuel and Light Engel Curves

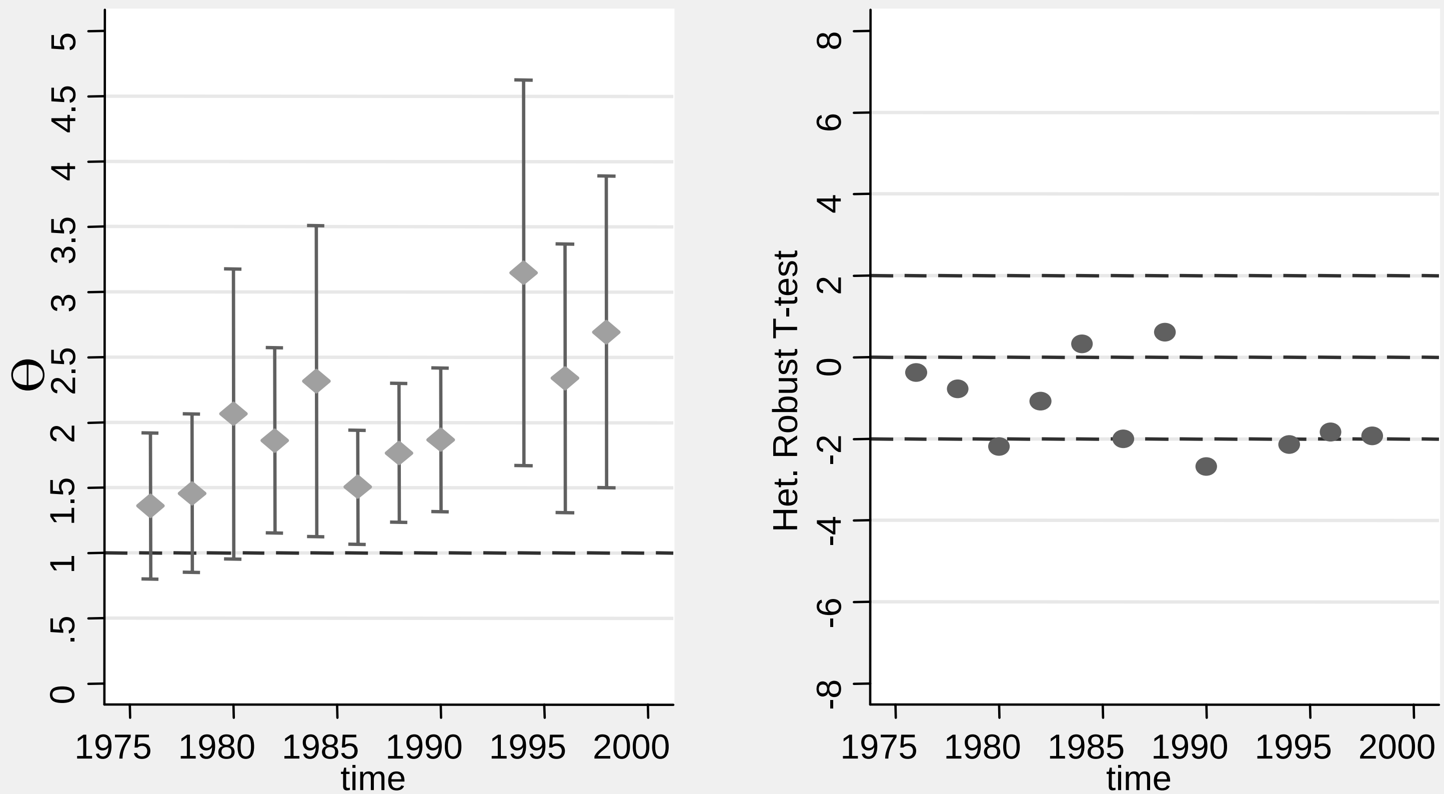

Fig. 4: Estimates and Tests from Leisure Services Engel Curves