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Abstract

Despite the recent trend towards greater transparency of monetary policy, in many respects central bankers still prefer to speak with mystique. This paper shows that the resulting perception of ambiguity could be desirable. Under the plausible assumption that there is imperfect common knowledge about the degree of central bank transparency, economic outcomes are affected by both the actual and perceived degree of transparency. It is shown that actual transparency is beneficial but that it may be useful to create the perception of opacity. The optimal communication strategy for the central bank is to provide clarity about the inflation target and to communicate information about the output target and supply shocks with perceived ambiguity. In this respect, the central bank benefits from sustaining transparency misperceptions, which helps to explain the mystique of central bank speak.

Keywords: Transparency, monetary policy, communication, transparency misperceptions. JEL-classification: E52, E58, D82

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"Since I have become a central banker, I have learned to mumble with great incoherence. If I seem unduly clear to you, you must have misunderstood what I said."

Alan Greenspan, in testimony to US Congress, 1987.

1 Introduction

Central banks have long been associated with secrecy. Even the recent trend towards greater transparency of monetary policy has not dispelled the mystique with which central bankers often speak. This paper provides an economic explanation for the role of oblique communication. Under the plausible assumption that there is imperfect common knowledge about the degree of transparency, economic outcomes are determined by both actual and perceived transparency. It is shown that it may be beneficial to combine actual transparency with perceived opacity. The optimal communication strategy for the central bank is to provide clarity about the inflation target, but to provide information with perceived ambiguity about the output gap target and supply shocks. Thus, the central bank benefits from sustaining transparency misperceptions, which helps to explain why transparency of monetary policy has not eliminated the mystique of central bank speak.

Intuitively, transparency is beneficial as it reduces private sector uncertainty. However, transparency can only be achieved through central bank communications that could upset market expectations. Since markets respond strongest to signals that are perceived to be clear, market volatility could be muted by creating a perception of ambiguity.

For both the central bank's inflation and output target it is shown to be optimal to be transparent because it reduces erratic responses of market expectations. In addition, it is beneficial to be perceived to be transparent about the inflation target (e.g. by publishing an explicit numeric target) because it aligns private sector inflation expectations with the central bank's target. However, it is desirable to create the perception of ambiguity about the output gap target since it makes it easier to reach the target without upsetting inflation expectations. Similarly, for supply shocks it is useful to combine maximum actual with minimum perceived transparency.

In practice, many central banks have a quantitative inflation target but central bankers still tend to be notorious for their 'mumbling', as is illustrated by the introductory quote. Alan Greenspan has even used the term 'constructive ambiguity' to describe his style of communication. This paper establishes that the perception of ambiguity could indeed be a constructive way to achieve transparency because it reduces volatility of market expectations.

This paper builds on two different strands of the transparency literature. There are several papers that model monetary uncertainty faced by the public by making a parameter in the central bank's objective function stochastic, completely abstracting from any communication of information (e.g. Sørensen 1991, Eijffinger, Hoeberichts and Schaling 2000, Beetsma and Jensen 2003). Such monetary uncertainty directly increases the variability of economic outcomes, although it could also have indirect effects such as lower average inflation.¹ This 'monetary uncertainty' literature provides an important argument in favor of transparency, namely that it reduces private sector uncertainty and economic volatility.

A second strand of the transparency literature explicitly models information transmission and incorporates the static effect that the information has on the formation of private sector inflation expectations (e.g. Cukierman 2001, Hahn 2004).² In this 'information approach' transparency could be detrimental because it leads to greater fluctuations in private sector expectations and increases economic volatility. In a similar vein, Morris and Shin (2002) find that transparency could generate greater variability when agents disregard private information and rely on a sufficiently noisy public signal to coordinate their actions. A more comprehensive review of the transparency literature is provided in the survey by Geraats (2002).

Other interesting insights on central bank mystique are provided by Goodfriend (1986) who reviews the Federal Reserve's defense of secrecy in response to a Freedom of Information Act suit, including the argument that disclosure of information could be prone to misinterpretation and cause inappropriate market reaction. In addition, Winkler (2002) discusses central bank communication and proposes to view transparency in terms of openness, clarity, honesty and common understanding.

The present paper synthesizes the 'monetary uncertainty' and 'information' approaches. It allows for stochastic central bank preferences and it features public signals that convey information about those preferences but could also generate undesirable market reactions.

The main innovation of this paper is that it relaxes the ubiquitous assumption of perfect common knowledge about the degree of transparency. This assumption requires perceived and actual stochastic distributions to be identical, which precludes an analysis of the role of transparency (mis)perceptions. Furthermore, in practice it is very hard for the private sector to know how transparent the central bank actually is because the public cannot observe how much information the central bank withholds. Even if the private sector manages to perfectly predict monetary policy decisions, this need not imply complete transparency since the forecasts may have been accurate despite asymmetric information about variables relevant for (future) policy decisions. So, it seems more realistic to allow for transparency misperceptions.

This paper deviates from the perfect common knowledge assumption by introducing asymmetric information about the degree of transparency, which allows for a discrepancy between actual transparency and private sector perceptions of it. The result is that both the practice and perceptions of transparency matter for economic outcomes. It is shown that the drawbacks of

¹Sørensen (1991) provides an interesting example. However, it should be noted that many of the other indirect effects reported in this strand of the literature (including those in Eijffinger et al. (2000)) are spurious due to a biased specification of stochastic relative preferences (Geraats 2004).

²A third strand of the literature focuses on the dynamic effect of transparency on reputation (e.g. Faust and Svensson 2001, Jensen 2002, Geraats 2005). In this 'reputation approach', transparency about central bank preferences reduces beneficial reputation effects, whereas transparency about economic shocks strengthens them.

transparency emphasized by the 'information' approach stem not from the actual reduction of information asymmetries but from private sector responses induced by transparency perceptions. So, it may be beneficial for perceived transparency to be less than actual transparency. To be precise, although it is best to have perfect actual and perceived transparency about the inflation target, for the output target and supply shocks it is desirable for the central bank to combine actual transparency with perceived opacity.

The remainder of the paper is organized as follows. The model is presented in section 2. First, the case with perfect common knowledge about the degree of transparency about the central bank's inflation and output target is analyzed in section 2.1. Subsequently, imperfect common knowledge is introduced and the role of transparency perceptions is investigated in section 2.2. The robustness of the results is assessed in section 3, which analyzes four extensions to the model, including alternative social welfare functions and transparency about supply shocks (section 3.1). It also presents a new measure of transparency (section 3.2) and it discusses other arguments related to monetary mystique (section 3.3). Finally, section 4 concludes that there is an economic rationale for central bank communications that sustain transparency misperceptions.

2 Model

The central bank has the objective function

$$U = -\frac{1}{2}\alpha (\pi - \theta)^{2} - \frac{1}{2} (1 - \alpha) (y - \kappa)^{2}$$
(1)

where π denotes inflation, y the output gap, θ the central bank's inflation target, κ the central bank's output gap target, and α the relative weight on inflation stabilization ($0 < \alpha < 1$). The inflation target θ and output gap target κ are allowed to be stochastic with $\theta \sim N(\bar{\theta}, \sigma_{\theta}^2)$ and $\kappa \sim N(\bar{\kappa}, \sigma_{\kappa}^2)$, and θ and κ independent.

The economy is described by the expectations augmented Phillips curve

$$\pi = \pi^e + y + s \tag{2}$$

where π^e denotes the inflation expectations of the private sector and s is a supply shock, which is assumed to be i.i.d. white noise with variance σ_s^2 . For analytical convenience, the slope of the Phillips curve is normalized to one, but this does not affect any of the qualitative conclusions below. Furthermore, for simplicity it is assumed that the central bank directly controls the output gap y.³ It would be straightforward to extend the model with an aggregate demand equation that relates the output gap to an interest rate controlled by the central bank,

³Alternatively, one could assume a neo-monetarist transmission mechanism in which the central bank controls inflation π and faces the Lucas supply equation $y = \pi - \pi^e - s$, but this leads to exactly the same analytical results as for the Keynesian transmission mechanism in the model.

but this would merely clutter the analytical expressions without affecting any of the qualitative results.

There are two important information asymmetries between the central bank and the private sector. First, the private sector does not observe the central bank's inflation target θ and output gap target κ . Instead, it receives the public signals

$$\xi_{\theta} = \theta + \varepsilon \tag{3}$$

$$\xi_{\kappa} = \kappa + \eta \tag{4}$$

where ε and η are i.i.d. white noise, $\varepsilon \sim N(0, \sigma_{\varepsilon}^2)$ and $\eta \sim N(0, \sigma_{\eta}^2)$. The noise ε and η stems from the difficulty the private sector has interpreting the central bank's fuzzy communication. When $\sigma_{\varepsilon}^2 = \sigma_{\eta}^2 = 0$, the signals ξ_{θ} and ξ_{κ} communicate θ and κ without any noise, so the information asymmetry is eliminated and there is perfect transparency about the central bank's targets.

The accuracy of the signals ξ_{θ} and ξ_{κ} is described by

$$\tau_{\theta} = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2} \text{ and } \tau_{\kappa} = \frac{\sigma_{\kappa}^2}{\sigma_{\kappa}^2 + \sigma_n^2}$$
(5)

respectively, where $0 \le \tau_{\theta}, \tau_{\kappa} \le 1$. This measure of the *actual* degree of transparency follows Faust and Svensson (2002), who consider an announcement about a monetary control error. When the signals are completely accurate ($\sigma_{\varepsilon}^2 = \sigma_{\eta}^2 = 0$), there is perfect transparency ($\tau_{\theta} = \tau_{\kappa} = 1$) about the central bank's targets, which is defined as a situation of symmetric information between the central bank and the private sector. A shortcoming of the transparency measure in (5) is that a constant target ($\sigma_{\theta}^2 = 0, \sigma_{\kappa}^2 = 0$) implies minimum transparency ($\tau_{\theta} = 0, \tau_{\kappa} = 0$) regardless of the informativeness of the signal ($\xi_{\theta}, \xi_{\kappa}$). This drawback can be overcome if private sector perceptions are allowed to deviate from the actual stochastic distributions.⁴

The second information asymmetry is about the degrees of transparency τ_{θ} and τ_{κ} . The public is unsure how transparent the central bank really is. In particular, it does not know the actual stochastic distributions of θ , κ , ε and η . Instead, the public uses the perceived (or prior) distributions $\theta \sim N(\bar{\theta}, \tilde{\sigma}_{\theta}^2)$, $\kappa \sim N(\bar{\kappa}, \tilde{\sigma}_{\kappa}^2)$, $\varepsilon \sim N(0, \tilde{\sigma}_{\varepsilon}^2)$ and $\eta \sim N(0, \tilde{\sigma}_{\eta}^2)$. As a result, the *perceived* degrees of transparency are given by

$$\tilde{\tau}_{\theta} = \frac{\tilde{\sigma}_{\theta}^2}{\tilde{\sigma}_{\theta}^2 + \tilde{\sigma}_{\varepsilon}^2} \text{ and } \tilde{\tau}_{\kappa} = \frac{\tilde{\sigma}_{\kappa}^2}{\tilde{\sigma}_{\kappa}^2 + \tilde{\sigma}_{\eta}^2}$$
(6)

where $0 \leq \tilde{\tau}_{\theta}, \tilde{\tau}_{\kappa} \leq 1$. This (Bayesian) transparency measure does not depend on the actual variances σ_{θ}^2 and σ_{κ}^2 , so it also applies when the central bank's targets θ and κ are deterministic.

⁴The transparency measure in (5) also has the peculiar feature that it is increasing in 'monetary uncertainty' $(\sigma_{\theta}^2, \sigma_{\kappa}^2)$. This correctly reflects the relative accuracy of the signal $(\xi_{\theta}, \xi_{\kappa})$, but it is an odd implication for a transparency measure. A more general measure of transparency that does not suffer from this shortcoming is presented in section 3.2.

Furthermore, it describes transparency from the public's perspective, which makes it more relevant to understanding the behavior of the private sector.

The timing of events is as follows. First, the inflation target θ and output gap target κ are realized but only observed by the central bank. Subsequently, the private sector receives the public signals ξ_{θ} and ξ_{κ} , which are used to rationally form private sector inflation expectations π^{e} . Then, the supply shock *s* is realized and observed by the central bank. Finally, the central bank sets the output gap *y* and the level of inflation π is realized.

The central bank maximizes the expected value of its objective (1) with respect to y subject to (2) and given π^e . This yields the optimal output gap

$$y = \alpha \left(\theta - \pi^e - s\right) + (1 - \alpha) \kappa \tag{7}$$

The output gap is increasing in the central bank's inflation target θ and output gap target κ . In addition, higher private sector inflation expectations π^e cause the central bank to reduce the output gap to achieve price stability, and the same holds for a higher supply shock *s*. Substituting (7) into (2) produces the level of inflation

$$\pi = \alpha \theta + (1 - \alpha) \left(\pi^e + \kappa + s \right) \tag{8}$$

Inflation is increasing in the inflation target θ , the output gap target κ , the level of private sector inflation expectations π^e , and the supply shock s.

To fully understand the role of the two information asymmetries in the formation of the private sector's inflation expectations, subsection 2.1 assumes that the private sector only has asymmetric information about the central bank's inflation target θ and output gap target κ , but perfect common knowledge about the actual degrees of central bank transparency τ_{θ} and τ_{κ} . Then, in subsection 2.2 the assumption of asymmetric information about the degree of transparency is added and the role of transparency (mis)perceptions is analyzed.

2.1 Perfect Common Knowledge

The private sector has rational expectations so it uses all available information, including the public signals ξ_{θ} and ξ_{κ} , to form its inflation expectations π^{e} . Taking expectations of (8) and solving for π^{e} gives

$$\pi^{e} = \operatorname{E}\left[\pi|\xi_{\theta},\xi_{\kappa}\right] = \operatorname{E}\left[\theta|\xi_{\theta}\right] + \frac{1-\alpha}{\alpha} \operatorname{E}\left[\kappa|\xi_{\kappa}\right]$$
(9)

using the fact that ξ_{κ} is uninformative about θ and ξ_{θ} about κ . Private sector inflation expectations depend on the private sector's expectations of the central bank's inflation target θ and output gap target κ , which it attempts to infer from the public signals ξ_{θ} and ξ_{κ} . Using (3), (4)

and (5),⁵

$$\mathbf{E}\left[\theta|\xi_{\theta}\right] = \bar{\theta} + \frac{\sigma_{\theta}^{2}}{\sigma_{\theta}^{2} + \sigma_{\varepsilon}^{2}} \left(\xi_{\theta} - \bar{\theta}\right) = (1 - \tau_{\theta})\bar{\theta} + \tau_{\theta}\xi_{\theta}$$
(10)

$$\mathbf{E}\left[\kappa|\xi_{\kappa}\right] = \bar{\kappa} + \frac{\sigma_{\kappa}^{2}}{\sigma_{\kappa}^{2} + \sigma_{\eta}^{2}} \left(\xi_{\kappa} - \bar{\kappa}\right) = \left(1 - \tau_{\kappa}\right)\bar{\kappa} + \tau_{\kappa}\xi_{\kappa} \tag{11}$$

The private sector faces a signal extraction problem and its expectation of θ (κ) equals a weighted average of its prior belief $\overline{\theta}$ ($\overline{\kappa}$) and the public signal ξ_{θ} (ξ_{κ}). For a higher degree of transparency τ_{θ} (τ_{κ}), the public signal ξ_{θ} (ξ_{κ}) is relatively more informative, so the private sector attaches greater weight to it. In the case of perfect transparency, $\tau_{\theta} = \tau_{\kappa} = 1$ and $\sigma_{\varepsilon}^2 = \sigma_{\eta}^2 = 0$, so the inflation target and output gap target are perfectly inferred: $E[\theta|\xi_{\theta}] = \xi_{\theta} = \theta$ and $E[\kappa|\xi_{\kappa}] = \xi_{\kappa} = \kappa$. In the case of complete opacity ($\tau_{\theta} = \tau_{\kappa} = 0$), the private sector rationally ignores the signals so that $E[\theta|\xi_{\theta}] = \overline{\theta}$ and $E[\kappa|\xi_{\kappa}] = \overline{\kappa}$. Substituting (10) and (11) into (9) and using (3) and (4) gives

$$\pi^{e} = \bar{\theta} + \tau_{\theta} \left(\theta - \bar{\theta} \right) + \tau_{\theta} \varepsilon + \frac{1 - \alpha}{\alpha} \left[\bar{\kappa} + \tau_{\kappa} \left(\kappa - \bar{\kappa} \right) + \tau_{\kappa} \eta \right]$$
(12)

The private sector's inflation expectations are determined by its prior expectations $\bar{\theta}$ and $\bar{\kappa}$ of the central bank's targets, the deviations of the central bank's targets from the private sector's priors, and the noise ε and η in the public signals. The latter shows how misinterpretation of monetary policy communications causes inappropriate market reaction. The variability of private sector inflation expectations depends on the degrees of transparency. In particular,

$$\operatorname{Var}\left[\pi^{e}\right] = \tau_{\theta}^{2} \sigma_{\theta}^{2} + \tau_{\theta}^{2} \sigma_{\varepsilon}^{2} + \left(\frac{1-\alpha}{\alpha}\right)^{2} \left[\tau_{\kappa}^{2} \sigma_{\kappa}^{2} + \tau_{\kappa}^{2} \sigma_{\eta}^{2}\right]$$
$$= \tau_{\theta} \sigma_{\theta}^{2} + \left(\frac{1-\alpha}{\alpha}\right)^{2} \tau_{k} \sigma_{\kappa}^{2}$$

using the fact that (5) implies $\sigma_{\varepsilon}^2 = \frac{1-\tau_{\theta}}{\tau_{\theta}}\sigma_{\theta}^2$ and $\sigma_{\eta}^2 = \frac{1-\tau_{\kappa}}{\tau_{\kappa}}\sigma_{\kappa}^2$. This shows that inflation expectations π^e are most stable when the central bank is least transparent ($\tau_{\theta} = \tau_{\kappa} = 0$). Intuitively, the complete lack of transparency makes the public signal so noisy that the public no longer relies on it and only uses its prior expectations.⁶

Substituting (12) into (7) and using (2) gives the levels of the output gap y and inflation π :

$$y = \alpha \left[(1 - \tau_{\theta}) \left(\theta - \bar{\theta} \right) - \tau_{\theta} \varepsilon \right] + (1 - \alpha) \left[(1 - \tau_{\kappa}) \left(\kappa - \bar{\kappa} \right) - \tau_{\kappa} \eta \right] - \alpha s \quad (13)$$

$$\pi = \bar{\theta} + (\alpha + (1 - \alpha) \tau_{\theta}) \left(\theta - \bar{\theta} \right) + (1 - \alpha) \tau_{\theta} \varepsilon$$

$$+ \frac{1 - \alpha}{\alpha} \left[\bar{\kappa} + (\alpha + (1 - \alpha) \tau_{\kappa}) \left(\kappa - \bar{\kappa} \right) + (1 - \alpha) \tau_{\kappa} \eta \right] + (1 - \alpha) s \quad (14)$$

⁵This uses the fact that for two jointly normally distributed variables x and z, $E[x|z] = E[x] + \frac{Cov\{x,z\}}{Var[z]}(z - E[z]).$

⁶This case in which private sector expectations do not incorporate any communications resembles the 'monetary uncertainty' literature mentioned in section 1. It features deterministic private sector inflation expectations π^e and the degree of monetary uncertainty is described by σ_{θ}^2 and σ_{κ}^2 .

The output gap and inflation depend on the central bank's targets θ and κ , the private sector's priors $\overline{\theta}$ and $\overline{\kappa}$, the signal noise ε and η , and the supply shock s. Although the degrees of transparency τ_{θ} and τ_{κ} influence the output gap and inflation, they have no effect on the expected values E[y] and $E[\pi]$. In the case of perfect transparency ($\tau_{\theta} = \tau_{\kappa} = 1$, so $\varepsilon = \eta = 0$), the expressions simplify to $y = -\alpha s$ and $\pi = \theta + (1 - \alpha) (\kappa + \alpha s) / \alpha$, which gives the familiar rational expectations outcome that the targets θ and κ only affect inflation and do not influence output.

The variability of the output gap and inflation are given by

$$\begin{aligned} \operatorname{Var}\left[y\right] &= \alpha^{2} \left[\left(1 - \tau_{\theta}\right)^{2} \sigma_{\theta}^{2} + \tau_{\theta}^{2} \sigma_{\varepsilon}^{2} \right] + \left(1 - \alpha\right)^{2} \left[\left(1 - \tau_{\kappa}\right)^{2} \sigma_{\kappa}^{2} + \tau_{\kappa}^{2} \sigma_{\eta}^{2} \right] + \alpha^{2} \sigma_{s}^{2} \\ &= \alpha^{2} \left(1 - \tau_{\theta}\right) \sigma_{\theta}^{2} + \left(1 - \alpha\right)^{2} \left(1 - \tau_{\kappa}\right) \sigma_{\kappa}^{2} + \alpha^{2} \sigma_{s}^{2} \end{aligned}$$
$$\begin{aligned} \operatorname{Var}\left[\pi\right] &= \left(\alpha + \left(1 - \alpha\right) \tau_{\theta}\right)^{2} \sigma_{\theta}^{2} + \left(1 - \alpha\right)^{2} \tau_{\theta}^{2} \sigma_{\varepsilon}^{2} \\ &+ \frac{\left(1 - \alpha\right)^{2}}{\alpha^{2}} \left[\left(\alpha + \left(1 - \alpha\right) \tau_{\kappa}\right)^{2} \sigma_{\kappa}^{2} + \left(1 - \alpha\right)^{2} \tau_{\kappa}^{2} \sigma_{\eta}^{2} \right] + \left(1 - \alpha\right)^{2} \sigma_{s}^{2} \end{aligned}$$
$$\end{aligned}$$
$$\begin{aligned} &= \left(\alpha^{2} + \left(1 - \alpha^{2}\right) \tau_{\theta}\right) \sigma_{\theta}^{2} + \frac{\left(1 - \alpha\right)^{2}}{\alpha^{2}} \left(\alpha^{2} + \left(1 - \alpha^{2}\right) \tau_{\kappa}\right) \sigma_{\kappa}^{2} + \left(1 - \alpha\right)^{2} \sigma_{s}^{2} \end{aligned}$$

where (5) is used to substitute for σ_{ε}^2 and σ_{η}^2 . This shows that the output gap is most stable when the central bank is perfectly transparent ($\tau_{\theta} = \tau_{\kappa} = 1$). The reason is that greater transparency makes private sector inflation expectations more sensitive to the central bank's targets. For a change in the inflation target, the stronger response of private sector inflation expectations means that a smaller adjustment of the output gap is required to reach the inflation target. For a change in the output gap target, the output gap is adjusted by less because the larger shift in inflation expectations hampers inflation stabilization.⁷ However, inflation is most stable when the central bank is least transparent ($\tau_{\theta} = \tau_{\kappa} = 0$). This is due to the greater stability of private sector inflation expectations.

To determine the optimal degrees of transparency, substitute (8) and (7) into (1), use (12) and rearrange to get

$$U = -\frac{1}{2}\alpha (1-\alpha) (\pi^{e} - \theta + \kappa + s)^{2}$$

$$= -\frac{1}{2}\frac{1-\alpha}{\alpha} \left[\alpha (\tau_{\theta} - 1) (\theta - \bar{\theta}) + \alpha \tau_{\theta} \varepsilon + \bar{\kappa} + (\alpha + (1-\alpha) \tau_{\kappa}) (\kappa - \bar{\kappa}) + (1-\alpha) \tau_{\kappa} \eta + \alpha s\right]^{2}$$
(15)

When there is imperfect transparency about the inflation target ($\tau_{\theta} \neq 1$), the deviation between the actual target θ and the private sector's prior expectation $\bar{\theta}$ affects the level of U. The prior expectation $\bar{\kappa}$ also matters, unless there is perfect transparency about the output gap target ($\tau_{\kappa} = 1$). So, the outcome is distorted when there is incomplete transparency.

Taking unconditional expectations of (15) and substituting for σ_{ε}^2 and σ_{η}^2 using (5) gives

⁷For the neo-monetarist transmission mechanism in which the central bank directly controls inflation, the intuition is that greater transparency reduces inflation surprises, which makes the output gap more stable.

the ex ante expected central bank payoff

$$E[U] = -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\alpha^2 (\tau_{\theta} - 1)^2 \sigma_{\theta}^2 + \alpha^2 \tau_{\theta}^2 \sigma_{\varepsilon}^2 + \bar{\kappa}^2 + (\alpha + (1-\alpha)\tau_{\kappa})^2 \sigma_{\kappa}^2 + (1-\alpha)^2 \tau_{\kappa}^2 \sigma_{\eta}^2 + \alpha^2 \sigma_{s}^2 \right]$$

$$= -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\alpha^2 (1-\tau_{\theta}) \sigma_{\theta}^2 + \bar{\kappa}^2 + (\alpha^2 + (1-\alpha^2)\tau_{\kappa}) \sigma_{\kappa}^2 + \alpha^2 \sigma_{s}^2 \right]$$

As a result, it would be optimal to have maximum transparency about the inflation target $(\tau_{\theta} = 1)$ and minimal transparency about the output gap target $(\tau_{\kappa} = 0)$. Although transparency about the inflation target increases the variance of inflation, this drawback is dominated by the benefits that transparency makes the output gap more stable and brings inflation closer to the inflation target. In addition, opacity about the output gap target makes the output gap more volatile, but this disadvantage is more than offset by the greater stability of inflation and the smaller deviation between the output gap and its target. The optimality of opacity about the output gap target is similar in spirit to the result in the seminal paper by Cukierman and Meltzer (1986), where ambiguity about the output preference parameter allows the central bank to successfully stimulate output when it is most desirable. Cukierman and Meltzer (1986) assume that ambiguity is created through monetary control errors, whereas the present paper assumes perfect control over the monetary policy instrument but opacity caused by imperfect communications.

To summarize the key results:

Proposition 1 When there is asymmetric information about the central bank's inflation target θ and output gap target κ , and perfect common knowledge about the degree of central bank transparency τ_{θ} and τ_{κ} ,

(i) greater transparency (τ_{θ} and/or τ_{κ}) increases the variability of private sector inflation expectations π^{e} and inflation π , but reduces the volatility of the output gap y; (ii) it is optimal to have maximum transparency about the inflation target ($\tau_{\theta} = 1$) and minimal transparency about the output target ($\tau_{\kappa} = 0$).

In the next subsection, the assumption of perfect common knowledge about the degree of transparency is relaxed, allowing for a difference between actual and perceived transparency.

2.2 Transparency Misperceptions

The assumption of perfect common knowledge about transparency has the critical drawback that private sector perceptions are restricted to be determined by the actual volatilities σ_{θ}^2 , σ_{κ}^2 , σ_{ε}^2 and σ_{η}^2 . This is problematic because it is hard for the private sector to establish how transparent the central bank actually is. For instance, what is the noise σ_{η}^2 associated with a central banker's speech? It could easily vary, which means that the public is unlikely to know the level of transparency τ . So, it is realistic to allow for imperfect common knowledge about the degree of transparency. This has the virtue that it decouples private sector perceptions of uncertainty from actual stochastic volatility.⁸

In contrast to the previous subsection, assume now that the private sector does not know the actual stochastic distribution of the central bank's inflation target θ and output gap target κ , and the noise ε and η . Instead, it uses the perceived (or prior) distributions $\theta \sim N(\bar{\theta}, \tilde{\sigma}_{\theta}^2)$, $\kappa \sim N(\bar{\kappa}, \tilde{\sigma}_{\kappa}^2), \varepsilon \sim N(0, \tilde{\sigma}_{\varepsilon}^2)$ and $\eta \sim N(0, \tilde{\sigma}_{\eta}^2)$. This gives rise to the perceived degree of transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$ in (6).

Note that transparency perceptions do not affect the optimization by the central bank, so (7) and (8) continue to hold. In addition, the private sector still receives the public signals (3) and (4), which it uses rationally to form its inflation expectations $\pi^e = \tilde{E}[\pi|\xi]$, where $\tilde{E}[.]$ denotes the private sector expectation based on the perceived distributions of θ , κ , ε and η . But the signal-extraction process is affected by private sector perceptions. To be precise, (10) and.(11) are replaced by

$$\widetilde{\mathbf{E}}\left[\theta|\xi_{\theta}\right] = \bar{\theta} + \frac{\tilde{\sigma}_{\theta}^{2}}{\tilde{\sigma}_{\theta}^{2} + \tilde{\sigma}_{\varepsilon}^{2}} \left(\xi_{\theta} - \bar{\theta}\right) = (1 - \tilde{\tau}_{\theta})\bar{\theta} + \tilde{\tau}_{\theta}\xi_{\theta}$$
(16)

$$\widetilde{\mathbf{E}}\left[\kappa|\xi\right] = \bar{\kappa} + \frac{\tilde{\sigma}_{\kappa}^{2}}{\tilde{\sigma}_{\kappa}^{2} + \tilde{\sigma}_{\eta}^{2}}\left(\xi_{\kappa} - \bar{\kappa}\right) = \left(1 - \tilde{\tau}_{\kappa}\right)\bar{\kappa} + \tilde{\tau}_{\kappa}\xi_{\kappa}$$
(17)

So, with imperfect common knowledge about the degree of transparency, it is the perceived transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$ that matters for the updating of private sector expectations. As a result, private sector inflation expectations now equal

$$\pi^{e} = \bar{\theta} + \tilde{\tau}_{\theta} \left(\theta - \bar{\theta} \right) + \tilde{\tau}_{\theta} \varepsilon + \frac{1 - \alpha}{\alpha} \left[\bar{\kappa} + \tilde{\tau}_{\kappa} \left(\kappa - \bar{\kappa} \right) + \tilde{\tau}_{\kappa} \eta \right]$$
(18)

The variability of private sector inflation expectations depends on the perceived degrees of transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$. But now there are two measures of variability: $\widetilde{\text{Var}}[.]$ is based on the perceived stochastic distribution of θ , κ , ε and η , and measures private sector uncertainty (ex ante); and Var[.] is based on the actual stochastic distribution of θ , κ , ε and η , and measures average volatility (ex post).

The perceived variance of private sector inflation expectations equals

$$\widetilde{\operatorname{Var}} \begin{bmatrix} \pi^e \end{bmatrix} = \widetilde{\tau}_{\theta}^2 \widetilde{\sigma}_{\theta}^2 + \widetilde{\tau}_{\theta}^2 \widetilde{\sigma}_{\varepsilon}^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \left(\widetilde{\tau}_{\kappa}^2 \widetilde{\sigma}_{\kappa}^2 + \widetilde{\tau}_{\kappa}^2 \widetilde{\sigma}_{\eta}^2\right) \\ = \widetilde{\tau}_{\theta} \widetilde{\sigma}_{\theta}^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \widetilde{\tau}_{\kappa} \widetilde{\sigma}_{\kappa}^2$$

using the fact that (6) implies $\tilde{\sigma}_{\theta}^2 = \frac{1-\tilde{\tau}_{\theta}}{\tilde{\tau}_{\theta}} \tilde{\sigma}_{\theta}^2$ and $\tilde{\sigma}_{\eta}^2 = \frac{1-\tilde{\tau}_{\kappa}}{\tilde{\tau}_{\kappa}} \tilde{\sigma}_{\kappa}^2$. This shows that private sector uncertainty about inflation expectations is smallest when the central bank is perceived to be

⁸In a perceptive contribution, Hahn (2004) aims to analyze transparency about the central bank's relative preference weight α independently of the stochastic distribution of α . However, the private sector's ex ante distribution and the actual distribution of α are assumed to be the same, so there is no effective separation.

least transparent ($\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$). The reason is that the perceived lack of transparency makes the public signals ξ_{θ} and ξ_{κ} unreliable, so the private sector only uses its prior expectations $\bar{\theta}$ and $\bar{\kappa}$.

The actual variance of private sector inflation expectations equals

$$\operatorname{Var}\left[\pi^{e}\right] = \tilde{\tau}_{\theta}^{2} \sigma_{\theta}^{2} + \tilde{\tau}_{\theta}^{2} \sigma_{\varepsilon}^{2} + \left(\frac{1-\alpha}{\alpha}\right)^{2} \left(\tilde{\tau}_{\kappa}^{2} \sigma_{\kappa}^{2} + \tilde{\tau}_{\kappa}^{2} \sigma_{\eta}^{2}\right)$$
$$= \frac{\tilde{\tau}_{\theta}^{2}}{\tau_{\theta}} \sigma_{\theta}^{2} + \left(\frac{1-\alpha}{\alpha}\right)^{2} \frac{\tilde{\tau}_{\kappa}^{2}}{\tau_{\kappa}} \sigma_{\kappa}^{2}$$

using the fact that (5) implies $\sigma_{\varepsilon}^2 = \frac{1-\tau_{\theta}}{\tau_{\theta}}\sigma_{\theta}^2$ and $\sigma_{\eta}^2 = \frac{1-\tau_{\kappa}}{\tau_{\kappa}}\sigma_{\kappa}^2$. This shows that the volatility of private sector inflation expectations is increasing in perceived transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$ and decreasing in actual transparency τ_{θ} and τ_{κ} . Intuitively, lower perceived transparency causes the private sector to rely less on the noisy public signals (ξ_{θ} and ξ_{κ}), and greater actual transparency reduces the variance of the noise (σ_{ε}^2 and σ_{η}^2), both making inflation expectations π^e less volatile.

Substituting (18) into (7) and using (2) gives the levels of the output gap y and inflation π for transparency perceptions $\tilde{\tau}$:

$$y = \alpha \left[(1 - \tilde{\tau}_{\theta}) \left(\theta - \bar{\theta} \right) - \tilde{\tau}_{\theta} \varepsilon \right] + (1 - \alpha) \left[(1 - \tilde{\tau}_{\kappa}) \left(\kappa - \bar{\kappa} \right) - \tilde{\tau}_{\kappa} \eta \right] - \alpha s$$
(19)
$$\pi = \bar{\theta} + (\alpha + (1 - \alpha) \tilde{\tau}_{\theta}) \left(\theta - \bar{\theta} \right) + (1 - \alpha) \tilde{\tau}_{\theta} \varepsilon$$
$$+ \frac{1 - \alpha}{\alpha} \left[\bar{\kappa} + (\alpha + (1 - \alpha) \tilde{\tau}_{\kappa}) \left(\kappa - \bar{\kappa} \right) + (1 - \alpha) \tilde{\tau}_{\kappa} \eta \right] + (1 - \alpha) s$$
(20)

These expressions are identical to their counterparts under common knowledge, (13) and (14), except that the actual degrees of transparency τ_{θ} and τ_{κ} are replaced by the perceived degrees of transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$. The same holds for $\widetilde{\text{Var}}[y]$ and $\widetilde{\text{Var}}[\pi]$ when σ_{θ}^2 and σ_{κ}^2 are also replaced by $\tilde{\sigma}_{\theta}^2$ and $\tilde{\sigma}_{\kappa}^2$, so the perceived variances only depend on private sector perceptions. The actual variance is equal to

$$\begin{aligned} \operatorname{Var}\left[y\right] &= \alpha^{2}\left[\left(1-\tilde{\tau}_{\theta}\right)^{2}\sigma_{\theta}^{2}+\tilde{\tau}_{\theta}^{2}\sigma_{\varepsilon}^{2}\right]+\left(1-\alpha\right)^{2}\left[\left(1-\tilde{\tau}_{\kappa}\right)^{2}\sigma_{\kappa}^{2}+\tilde{\tau}_{\kappa}^{2}\sigma_{\eta}^{2}\right]+\alpha^{2}\sigma_{s}^{2}\right] \\ &= \alpha^{2}\left(1-2\tilde{\tau}_{\theta}+\frac{\tilde{\tau}_{\theta}^{2}}{\tau_{\theta}}\right)\sigma_{\theta}^{2}+\left(1-\alpha\right)^{2}\left(1-2\tilde{\tau}_{\kappa}+\frac{\tilde{\tau}_{\kappa}^{2}}{\tau_{\kappa}}\right)\sigma_{\kappa}^{2}+\alpha^{2}\sigma_{s}^{2}\right] \\ \operatorname{Var}\left[\pi\right] &= \left(\alpha+\left(1-\alpha\right)\tilde{\tau}_{\theta}\right)^{2}\sigma_{\theta}^{2}+\left(1-\alpha\right)^{2}\tilde{\tau}_{\theta}^{2}\sigma_{\varepsilon}^{2} \\ &\quad +\frac{\left(1-\alpha\right)^{2}}{\alpha^{2}}\left[\left(\alpha+\left(1-\alpha\right)\tilde{\tau}_{\kappa}\right)^{2}\sigma_{\kappa}^{2}+\left(1-\alpha\right)^{2}\tilde{\tau}_{\kappa}^{2}\sigma_{\eta}^{2}\right]+\left(1-\alpha\right)^{2}\sigma_{s}^{2}\right] \\ &= \left[\alpha^{2}+2\alpha\left(1-\alpha\right)\tilde{\tau}_{\theta}+\left(1-\alpha\right)^{2}\frac{\tilde{\tau}_{\theta}^{2}}{\tau_{\theta}}\right]\sigma_{\theta}^{2} \\ &\quad +\frac{\left(1-\alpha\right)^{2}}{\alpha^{2}}\left[\alpha^{2}+2\alpha\left(1-\alpha\right)\tilde{\tau}_{\kappa}+\left(1-\alpha\right)^{2}\frac{\tilde{\tau}_{\kappa}^{2}}{\tau_{\kappa}}\right]\sigma_{\kappa}^{2}+\left(1-\alpha\right)^{2}\sigma_{s}^{2}\end{aligned}$$

where (5) is used to substitute for σ_{ε}^2 and σ_{η}^2 . The variability of the output gap and inflation depends on both the perceived and actual degrees of transparency. In the special case in which

 $\tilde{\tau}_{\theta} = \tau_{\theta}$ and $\tilde{\tau}_{\kappa} = \tau_{\kappa}$, the common knowledge results in section 2.1 are obtained. With imperfect common knowledge, the volatility of the output gap is decreasing in actual transparency τ_{θ} and τ_{κ} , and is minimized for $\tilde{\tau}_{\theta} = \tau_{\theta} = 1$ and $\tilde{\tau}_{\kappa} = \tau_{\kappa} = 1.9$ The variability of inflation is also decreasing in actual transparency τ_{θ} and τ_{κ} , but increasing in perceived transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$. Intuitively, greater transparency corresponds to fewer inflation surprises and therefore more output gap stability, whereas lower perceived and higher actual transparency reduces the volatility of private sector expectations and thereby the variance of inflation.

To derive the optimal degrees of actual and perceived transparency, substitute (18) into (15) and rearrange to get:

$$U = -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\alpha \left(\tilde{\tau}_{\theta} - 1 \right) \left(\theta - \bar{\theta} \right) + \alpha \tilde{\tau}_{\theta} \varepsilon + \bar{\kappa} + \left(\alpha + \left(1 - \alpha \right) \tilde{\tau}_{\kappa} \right) \left(\kappa - \bar{\kappa} \right) + \left(1 - \alpha \right) \tilde{\tau}_{\kappa} \eta + \alpha s \right]^2 \right]^2$$

This is identical to the expression under common knowledge, except that τ_{θ} and τ_{κ} are replaced by $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$, respectively. It shows that in the presence of transparency misperceptions, it is the lack of perceived transparency that causes the prior expectations $\bar{\theta}$ and $\bar{\kappa}$ to exert their influence on the outcome, regardless of the stochastic distribution of the central bank targets.

Taking expectations using the distributions perceived by the private sector yields

$$\widetilde{\mathbf{E}}\left[U\right] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\alpha^2\left(1-\tilde{\tau}_{\theta}\right)\tilde{\sigma}_{\theta}^2 + \bar{\kappa}^2 + \left(\alpha^2 + \left(1-\alpha^2\right)\tilde{\tau}_{\kappa}\right)\tilde{\sigma}_{\kappa}^2 + \alpha^2\tilde{\sigma}_s^2\right]$$

This reflects the ex ante expectation based on private sector perceptions. It is the same as the expression for E[U] under common knowledge after replacing τ by $\tilde{\tau}$ and σ^2 by $\tilde{\sigma}^2$.

Taking unconditional expectations based on the actual distributions and substituting for σ_{ε}^2 and σ_{η}^2 using (5) yields

$$E[U] = -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\alpha^2 \left(\tilde{\tau}_{\theta} - 1 \right)^2 \sigma_{\theta}^2 + \alpha^2 \tilde{\tau}_{\theta}^2 \sigma_{\varepsilon}^2 + \bar{\kappa}^2 + \left(\alpha + (1-\alpha) \tilde{\tau}_{\kappa} \right)^2 \sigma_{\kappa}^2 + (1-\alpha)^2 \tilde{\tau}_{\kappa}^2 \sigma_{\eta}^2 + \alpha^2 \sigma_{s}^2 \right]$$

$$= -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\alpha^2 \left(1 - 2\tilde{\tau}_{\theta} + \frac{\tilde{\tau}_{\theta}^2}{\tau_{\theta}} \right) \sigma_{\theta}^2 + \bar{\kappa}^2 + \left(\alpha^2 + 2\alpha \left(1 - \alpha \right) \tilde{\tau}_{\kappa} + (1-\alpha)^2 \frac{\tilde{\tau}_{\kappa}^2}{\tau_{\kappa}} \right) \sigma_{\kappa}^2 + \alpha^2 \sigma_{s}^2 \right]$$

This reflects the central bank's ex ante expectation and it corresponds to the average ex post experience. It shows that E[U] is increasing in the actual degrees of transparency τ_{θ} and τ_{κ} , so that perfect transparency is optimal ($\tau_{\theta} = \tau_{\kappa} = 1$). In addition, E[U] is maximized for $\tilde{\tau}_{\theta} = \tau_{\theta}$ and $\tilde{\tau}_{\kappa} = 0$.¹⁰ So, it is best to have complete perceived and actual transparency about the inflation target ($\tilde{\tau}_{\theta} = \tau_{\theta} = 1$), but maximum actual transparency ($\tau_{\kappa} = 1$) and minimal perceived transparency ($\tilde{\tau}_{\kappa} = 0$) about the output gap target. Intuitively, it is desirable to have actual transparency about the central bank's targets because it avoids erratic reactions of private sector expectations. Furthermore, it is beneficial to have perceived transparency about

⁹Formally, these results follow from differentiating $\operatorname{Var}[y]$ with respect to $\tau_{\theta}, \tau_{\kappa}, \tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$.

¹⁰Formally, $\partial E[U] / \partial \tilde{\tau}_{\theta} = -\alpha (1-\alpha) \frac{\tilde{\tau}_{\theta} - \tau_{\theta}}{\tau_{\theta}} \sigma_{\theta}^2$ and $\partial^2 E[U] / \partial \tilde{\tau}_{\theta}^2 < 0$ implies that $\tilde{\tau}_{\theta} = \tau_{\theta}$ is optimal, and $\partial E[U] / \partial \tilde{\tau}_{\kappa} = -\frac{(1-\alpha)^2}{\alpha} \left(\alpha + (1-\alpha) \frac{\tilde{\tau}_{\kappa}}{\tau_{\kappa}} \right) \sigma_{\kappa}^2 < 0$ implies the corner solution $\tilde{\tau}_{\kappa} = 0$.

the inflation target so that private sector inflation expectations are more responsive and become more closely aligned with the inflation target. However, perceived transparency about the output gap target is detrimental because the response of private sector inflation expectations hampers the stabilization of inflation.

This shows that the optimal communication strategy is different for the central bank's inflation and output gap target. It is best to be transparent and unambiguously clear about the inflation target. But for the output gap target it is desirable to provide information with perceived ambiguity.

To summarize the results:

Proposition 2 When there is asymmetric information about the central bank's inflation target θ and output gap target κ , and about the degree of central bank transparency τ_{θ} and τ_{κ} (i) greater actual transparency (τ_{θ} and/or τ_{κ}) reduces the variability of private sector inflation expectations π^{e} , inflation π and the output gap y.

(ii) greater perceived transparency ($\tilde{\tau}_{\theta}$ and/or $\tilde{\tau}_{\kappa}$) increases the volatility of private sector inflation expectations π^{e} and inflation π , whereas the output gap is most stable in the absence of transparency misperceptions ($\tilde{\tau}_{\theta} = \tau_{\theta}$ and $\tilde{\tau}_{\kappa} = \tau_{\kappa}$).

(iii) it is optimal to have maximum actual and perceived transparency about the inflation target $(\tau_{\theta} = \tilde{\tau}_{\theta} = 1)$, and maximum actual transparency but minimal perceived transparency about the output gap target $(\tau_{\kappa} = 1, \tilde{\tau}_{\kappa} = 0)$.

A comparison with Proposition 1 reveals that the main drawback of transparency under common knowledge, namely the greater variability of inflation, is not due to the actual degree of transparency but the private sector's perceptions of it. The fact that the public is better informed is beneficial, but the correspondingly stronger response of private sector expectations leads to undesirable inflation volatility.

3 Discussion

It is important to assess the robustness of the results above, so several extensions are analyzed in section 3.1. Subsequently, section 3.2 addresses the limitation of τ as a measure of transparency and presents a more comprehensive alternative. In addition, alternative explanations for central bank mystique are discussed in section 3.3.

3.1 Extensions

Propositions 1(i) and 2(ii) show that transparency could have different effects on inflation and output gap variability, which may give the impression that the desirability of transparency depends on the weight attached to inflation versus output gap stabilization. To explore this

issue, suppose that the central bank's objective remains (1) but that social welfare is given by

$$W = -\frac{1}{2}\beta (\pi - \theta)^2 - \frac{1}{2} (1 - \beta) (y - \kappa)^2$$
(21)

where $0 < \beta < 1$. So, monetary policy has been delegated to a central bank with a different relative preference weight. For instance, $\alpha > \beta$ would amount to a 'conservative' central bank that is more concerned about inflation stabilization than society (Rogoff 1985). Interestingly, the degrees of transparency given in Propositions 1(ii) and 2(iii) that are optimal for the central bank are also socially optimal, regardless of the weight β . More precisely, both E[U] and E[W] are maximized for $\tau_{\theta} = 1$ and $\tau_{\kappa} = 0$ under common knowledge, and for $\tilde{\tau}_{\theta} = \tau_{\theta} =$ $\tau_{\kappa} = 1$ and $\tilde{\tau}_{\kappa} = 0$ with transparency misperceptions.¹¹ The reason that β is immaterial is that social welfare is not determined by Var[y] and $Var[\pi]$ but by $E[(\pi - \theta)^2]$ and $E[(y - \kappa)^2]$. The latter are always proportional when the central bank behaves optimally according to (7) and (8), so transparency affects them in the same way.

Suppose now that monetary policy is still delegated to a central bank that maximizes (1) but that the social welfare function equals

$$W = -\frac{1}{2}\beta \left(\pi - \bar{\theta}\right)^2 - \frac{1}{2}\left(1 - \beta\right) \left(y - \bar{\kappa}\right)^2$$
(22)

So, again the central bank attaches a different weight to inflation stabilization. In addition, although the targets of the central bank (θ and κ) and society ($\overline{\theta}$ and $\overline{\kappa}$) are the same on average, they typically differ due to idiosyncratic shocks ($\theta \neq \overline{\theta}$ and $\kappa \neq \overline{\kappa}$). This variation on the basic model is analyzed in appendix A.1. With perfect common knowledge, the degree of transparency that is socially optimal now depends on β . To be precise, $\tau_{\theta} = \tau_{\kappa} = 1$ is socially optimal for $\alpha^2 > \beta$, and $\tau_{\theta} = \tau_{\kappa} = 0$ for $\alpha^2 < \beta$. In other words, if the central bank is sufficiently conservative, the social optimum is transparency. Intuitively, if society cares a lot about output gap stabilization, the benefit of greater output gap stability under transparency outweighs the drawback of more inflation variability. This result is similar to Hahn (2004) who considers transparency about the central bank's relative preference weight α .

With imperfect common knowledge, perfect actual transparency about the central bank's targets ($\tau_{\theta} = \tau_{\kappa} = 1$) is socially optimal regardless of the value of β . The reason is that transparency avoids erratic movements of market expectations. Regarding perceived transparency, if the central bank is not conservative ($\alpha \leq \beta$), society benefits from complete perceived opacity ($\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$). Furthermore, for any other β the degree of perceived transparency in the social optimum is strictly positive but remains less than the degree of actual transparency ($0 < \tilde{\tau}_{\theta} < \tau_{\theta}$ and $0 < \tilde{\tau}_{\kappa} < \tau_{\kappa}$). Intuitively, the perception of opacity reduces the response of market expectations to noise in the signal and therefore limits volatility.

¹¹To see this, substitute (7) and (8) into (21) and rearrange to get $W = -\frac{1}{2} \left(\beta (1-\alpha)^2 + (1-\beta) \alpha^2 \right) (\pi^e - \theta + \kappa + s)^2$. This is directly proportional to (15) so that E[W] is maximized for the same degrees of transparency as E[U].

Another issue is whether the conclusions depend on the assumption that the central bank's inflation and output gap targets follow a normal distribution. In particular, the expressions for E[U] in section 2 give the impression that the degrees of actual and perceived transparency τ and $\tilde{\tau}$ are immaterial when the targets θ and κ are deterministic ($\sigma_{\theta}^2 = \sigma_{\kappa}^2 = 0$). The case of constant central bank targets is more closely examined in appendix A.2. This reveals that it is optimal to have complete perceived opacity about both targets ($\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$), but maximum actual transparency in the sense of minimally noisy signals ($\sigma_{\varepsilon}^2 = \sigma_{\eta}^2 = 0$). Intuitively, noisy signals lead to inflation and output gap variability, but this effect is muted when the signals are perceived to be opaque so that the private sector pays less attention to them. So again, it is desirable to have maximum actual transparency but to sustain transparency misperceptions such that perceived opacity exceeds actual opacity.

Another interesting extension is to consider transparency about the supply shock s. In particular, suppose that the private sector receives a public signal of the supply shock before it forms its inflation expectations π^e . This is analyzed in appendix A.3. In the case of perfect common knowledge, greater transparency τ_s about the supply shock s increases the volatility of both the output gap and inflation. Intuitively, greater transparency about the supply shock makes private sector inflation expectations π^e more sensitive to the supply shock s, so the central bank increases the output gap response to partially offset the increased volatility of inflation. Not surprisingly, minimum transparency about supply shocks ($\tau_s = 0$) is optimal. This result is consistent with Cukierman (2001), who compares limited ($\tau_s = 0$) and full ($\tau_s = 1$) transparency about the supply shock s in a model with a neo-monetarist transmission mechanism.

With imperfect common knowledge about the degree of transparency τ_s , the variance of the output gap y and inflation π are both minimized for minimum perceived transparency ($\tilde{\tau}_s = 0$) and maximum actual transparency ($\tau_s = 1$). The intuition behind this result is familiar. Minimum perceived transparency mutes the response of private sector expectations π^e to the supply shock s, which contributes to greater stability of the output gap and inflation. In addition, maximum actual transparency reduces the noise of the public signal, which makes inflation expectations more stable and thereby generates less volatility in the output gap and inflation. Not surprisingly, it is (socially) optimal to have minimum perceived and maximum actual transparency about supply shocks ($\tilde{\tau}_s = 0$ and $\tau_s = 1$).

So, the most effective communication strategy for supply shocks is to provide all the relevant information but to downplay its relevance. Perhaps, this could explain why some central banks (e.g. the European Central Bank) stress that the quarterly macroeconomic forecasts they publish are staff forecasts that come without any endorsement by the monetary policymakers.

These extensions of the model show that the key findings of section 2 are robust: When the assumption of perfect common knowledge is relaxed, actual transparency is beneficial and it is desirable to have a perceived degree of transparency that is no greater than the actual degree

of transparency ($\tilde{\tau} \leq \tau$).¹²

3.2 Transparency Measures

Since the transparency measure in (5) suffers from some drawbacks, it is useful to reconsider it. Although τ describes the relative accuracy of the signal ξ , it is less suitable as a measure of central bank transparency because it is increasing in 'monetary uncertainty' (σ_{θ}^2 , σ_{κ}^2). In the literature, transparency typically refers to the absence of an information asymmetry (e.g. Geraats 2002). So, transparency is decreasing in the extent to which the private sector faces asymmetric information. However, an increase in opacity due to greater variability of the central bank's targets has the awkward implication that it leads to a higher value of τ . This shows that (5) is not a good indicator of the degree of transparency.

Instead, it is useful to construct a more fundamental measure that is directly based on the definition of transparency. Focusing on the inflation target θ , the private sector has the prior $\bar{\theta}$ and symmetric information amounts to $\theta = \bar{\theta}$. The difference between θ and $\bar{\theta}$ gives an indication of the degree of asymmetric information. So, ex ante opacity can be described by $E\left[\left(\theta - \bar{\theta}\right)^2\right] = \sigma_{\theta}^2$, which is the 'monetary uncertainty' measure used in one strand of the literature.

However, the private sector is able to use the public signal ξ_{θ} to update its prior of θ , which leads to the posterior $E[\theta|\xi_{\theta}]$ in (10). Taking into account the information conveyed by the signal, the appropriate measure of opacity becomes

$$E\left[\left(\theta - E\left[\theta|\xi_{\theta}\right]\right)^{2}\right] = E\left[\left(\theta - (1 - \tau_{\theta})\bar{\theta} - \tau_{\theta}\left(\theta + \varepsilon\right)\right)^{2}\right]$$
$$= (1 - \tau_{\theta})^{2}\sigma_{\theta}^{2} + \tau_{\theta}^{2}\sigma_{\varepsilon}^{2}$$
$$= (1 - \tau_{\theta})\sigma_{\theta}^{2}$$

after substituting (10), (3), and using (5) to substitute for σ_{ε}^2 . This shows that opacity about θ is increasing in the amount of initial monetary uncertainty σ_{θ}^2 and decreasing in the relative accuracy τ_{θ} of the signal ξ_{θ} .

Taking the inverse of opacity and substituting (5) leads to the transparency measure

$$\gamma_{\theta} = \frac{1}{\left(1 - \tau_{\theta}\right)\sigma_{\theta}^{2}} = \frac{\sigma_{\theta}^{2} + \sigma_{\varepsilon}^{2}}{\sigma_{\theta}^{2}\sigma_{\varepsilon}^{2}} = \frac{1}{\sigma_{\theta}^{2}} + \frac{1}{\sigma_{\varepsilon}^{2}}$$

This measure of (actual) transparency depends positively on the relative accuracy of the signal τ_{θ} and negatively on monetary uncertainty σ_{θ}^2 . It has the intuitive property that transparency about θ could be enhanced in two, independent ways: (i) reduce the initial uncertainty (σ_{θ}^2), or (ii) reduce the noisiness of the signal (σ_{ε}^2). So, γ_{θ} has the desirable property that greater monetary uncertainty decreases transparency, which is in contrast to τ_{θ} .

¹²Another extension would be to incorporate the reputation approach. Since reputation effects are based on the updating of private sector inflation expectations, they would depend only on perceived transparency. So, actual transparency would remain desirable and transparency perceptions would again play a key role.

Nevertheless, γ_{θ} still has the drawbacks that it depends on the actual stochastic distributions and implies infinite transparency if θ is deterministic ($\sigma_{\theta}^2 = 0$). These problems can be overcome by the following analogous measure of perceived transparency:

$$\tilde{\gamma}_{\theta} = \frac{1}{\left(1 - \tilde{\tau}_{\theta}\right)\tilde{\sigma}_{\theta}^2} = \frac{1}{\tilde{\sigma}_{\theta}^2} + \frac{1}{\tilde{\sigma}_{\varepsilon}^2}$$

If the private sector believes there is symmetric information about the inflation target θ , then the prior variance equals $\tilde{\sigma}_{\theta}^2 = 0$, so that perceived transparency $\tilde{\gamma}_{\theta}$ is infinite. On the other hand, an infinitely diffuse prior ($\sigma_{\theta}^2 \to \infty$) does not imply complete opacity ($\tilde{\gamma}_{\theta} = 0$) when the signal is informative (so $\tilde{\sigma}_{\varepsilon}^2$ is finite). Similarly, the transparency measures γ_{κ} , $\tilde{\gamma}_{\kappa}$, γ_s , and $\tilde{\gamma}_s$ can defined.

Although γ and $\tilde{\gamma}$ are better measures of the degree of asymmetric information, the economic effects are more easily understood in terms of the relative accuracy of the signal $(\tau, \tilde{\tau})$ and the extent of monetary uncertainty $(\sigma_{\theta}^2, \sigma_{\kappa}^2, \tilde{\sigma}_{\theta}^2, \tilde{\sigma}_{\kappa}^2)$. The reason is that the relative signal accuracy need not have the same effect as initial monetary uncertainty. In particular, when there is common knowledge about all the variance parameters σ^2 and thereby about τ , greater opacity through higher monetary uncertainty $\sigma_{\theta}^2, \sigma_{\kappa}^2$ and σ_s^2 is always detrimental because it increases the variance of output and inflation, $\operatorname{Var}[y]$ and $\operatorname{Var}[\pi]$, and reduces $\operatorname{E}[U]$.¹³ In contrast, greater opacity through a lower relative signal accuracy τ_{κ} is beneficial and actually increases $\operatorname{E}[U]$.

Nevertheless, one of the main findings of the paper, namely that actual transparency is beneficial in the presence of private sector misperceptions, not only holds for the measure τ but also for the more general measure γ . To be precise, a decrease in initial monetary uncertainty $(\sigma_{\theta}^2, \sigma_{\kappa}^2, \sigma_s^2)$ and in signal noise $(\sigma_{\varepsilon}^2, \sigma_{\eta}^2, \sigma_{\upsilon}^2)$ are both beneficial because of a reduction in Var [y] and Var $[\pi]$, and an increase in E[U].¹⁴ As a result, this conclusion remains robust even when a more comprehensive transparency measure is used.

3.3 Central Bank Mystique

Despite all the emphasis on transparency of monetary policy nowadays, central bankers still often speak with a remarkable lack of clarity. Although it is difficult to characterize 'central bank speak', according to an insider:

"[Fed speak] is a language in which it is possible to speak, without ever saying anything." (Mike Moskow, President of the Federal Reserve Bank of Chicago, December 7, 2002)

¹³This holds not only ceteris paribus (i.e. for a constant τ_{θ} , τ_{κ} and τ_{s}), but also for the total effects of σ_{θ}^{2} , σ_{κ}^{2} and σ_{s}^{2} .

¹⁴This refers to the total effect, which is straightforward (though tedious) to compute by differentiating Var [y], Var $[\pi]$ and E[U] after substituting for τ .

This paper shows that a central bank may try to give this impression and create the perception of opacity. This could be achieved by avoiding the publication of precise, quantitative information and instead resorting to qualitative statements. For example, a numeric inflation target is likely to contribute to a high degree of perceived (and actual) transparency, whereas speeches that provide ambiguous perspectives could lower transparency perceptions.

It is worthwhile to note that the conclusions of this paper regarding the desirability of perceived opacity are independent of the public's prior expectation of the central bank's output gap target, $\bar{\kappa}$. In particular, the results also hold for $\bar{\kappa} = 0$, in which case there is no average inflation bias, so the central bank has no systematic incentive to misrepresent its information. In that case, commitment to a truthful communication technology is perfectly credible. To the extent that this is not possible, there may be central bank 'cheap talk' such that communication of central bank private information is only credible when it is imprecise (Stein 1989).

In addition, there may be institutional reasons for central banks to be vague. For example, a central bank without an explicit legal primary objective of price stability, such as the US. Federal Reserve, may be more reluctant to adopt a numeric inflation target because it could give the impression that it is neglecting its other objectives.

There could also be other reasons for oblique communications by central bankers. For instance, evasiveness could be used to limit accountability or hide incompetence. In addition, secretive central bankers receive more media attention as their every word is scrutinized. Last but not least, vague communications could reflect the tremendous uncertainty faced by central bankers, which is often difficult to explicate.

The paper shows that under certain circumstances maximum perceived opacity is optimal. In principle, there are two ways to achieve this. The central bank could give the impression that the public signal ξ is infinitely noisy so that $\tilde{\tau} = 0$. Alternatively, the central bank could remain silent and not communicate at all so that $\xi \in \{\emptyset\}$ and $\pi^e = E[\pi]$. In the latter case, the actual and perceived degree of transparency always coincide: $\tau = \tilde{\tau} = 0$. In practice, few central bankers prefer to remain silent, but rather engage in oblique speak. This still gives them the benefits of perceived opacity, while allowing them to communicate relevant information to the private sector and achieve greater actual transparency.¹⁵

In practice, there are likely to be some constraints on the degree of transparency. In particular, it may not be feasible to achieve complete opacity or perfect transparency. Suppose that there are binding constraints on the degree of (perceived and actual) transparency such that $\tilde{\tau}_{MIN} \leq \tilde{\tau} \leq \tilde{\tau}_{MAX}$ and $\tau_{MIN} \leq \tau \leq \tau_{MAX}$. Then, an optimum of maximum actual transparency ($\tau = 1$) and minimum perceived transparency ($\tilde{\tau} = 0$) would not be achievable. In that case, the constrained optimum is maximum possible perceived opacity, $\tilde{\tau} = \tilde{\tau}_{MIN}$, and maximum attainable actual transparency, $\tau = \tau_{MAX}$.

¹⁵Another reason for not remaining completely silent is that most central banks face accountability requirements, such as testimony before parliament or the publication of inflation reports.

A key finding of the paper is that it tends to be desirable to have less perceived than actual transparency ($\tilde{\tau} < \tau$). The only exception is the inflation target θ , for which $\tilde{\tau}_{\theta} = \tau_{\theta}$ is preferred by the central bank but not necessarily by society. An important practical consideration is the extent to which it is possible to sustain systematic deviations between actual and perceived transparency. In particular, if all the parameters of the model were stable it would be possible for the private sector to learn the degree of transparency τ over time.¹⁶ For instance, inflation reports with consistently detailed information are likely to facilitate learning about the central bank's inflation target transparency τ_{θ} . However, if the accuracy of communications is variable so that σ_{ε}^2 , σ_{η}^2 and σ_v^2 are unstable, τ_{θ} , τ_{κ} and τ_s can never be learned. This is especially relevant for verbal communications such as speeches and testimonies, because their informativeness could easily vary from one occasion to another. Moreover, whenever $\tilde{\tau} = \tau$ is not optimal, it is actually desirable to inhibit private sector learning and maintain transparency misperceptions.

It could be quite challenging for central bankers to communicate with a sustained discrepancy between actual and perceived transparency. Perhaps, this is where part of the 'art' of central banking comes in. A 'maestro' like Alan Greenspan manages to effectively guide financial markets by means of statements that appear to be open to multiple interpretations. Although financial markets definitely take cues from speeches and congressional testimony by Greenspan, the fact that his statements are perceived to be rife with ambiguity is constructive and prevents financial markets from reacting too strongly.

4 Conclusion

Central banks are transparent in many respects nowadays, but there is still a lot of ambiguity in their communication. This paper shows that arcane statements by central bankers may serve an important purpose. They create the perception of opacity and make the market more cautious in its response to central bank communications, which reduces the volatility of private sector expectations.

The paper models this mechanism by relaxing the strong assumption of perfect common knowledge about the degree of central bank transparency. In practice, there is considerable disagreement among researchers and market participants how transparent central banks are. In addition, it would be difficult to verify the degree of transparency. So, it appears realistic to allow the actual and perceived degrees of transparency to differ from each other. This has the virtue that asymmetric information can be modeled regardless of the actual variability of parameters, thereby decoupling ex ante uncertainty and ex post volatility.

¹⁶To see this, note that s and v follow (ex post) from (2) and (23), so that σ_s^2 , σ_v^2 and τ_s could be learned over time. In addition, y, ξ_{θ} and ξ_{κ} could be used to estimate Var $[\xi_{\theta}]$, Cov $\{y, \xi_{\theta}\}$, Var $[\xi_{\kappa}]$ and Cov $\{y, \xi_{\kappa}\}$, from which $\sigma_{\theta}^2, \sigma_{\varepsilon}^2, \sigma_{\kappa}^2$ and σ_{η}^2 can be deduced. So, τ_{θ} and τ_{κ} would also be learnable.

Moreover, the analysis of transparency perceptions of the private sector gives a better understanding of some of the disadvantages of transparency suggested in the literature. Although transparency is likely to reduce private sector uncertainty, information disclosed by the central bank could alter private sector expectations and give rise to greater economic volatility. However, this drawback appears to be entirely due to transparency perceptions. In particular, the paper shows that actual transparency is beneficial because it reduces the noisiness of communication, but perceived transparency could be more problematic as it makes markets more sensitive to (noisy) information. This provides an economic rationale for transparent central bank communications that sustain transparency misperceptions. As a result, central banks may find it desirable to disclose information under a veil of perceived ambiguity.

The paper shows that the central bank's optimal communication strategy is to be crystal clear about the inflation target, but to be informative about the output gap target and supply shocks through statements that are perceived to be opaque. In that respect, central bankers should speak, but with mystique.

A Appendix

This appendix analyzes two extensions to the basic model that are discussed in section 3.1.

A.1 Alternative Social Welfare Function

This section computes the optimal degrees of transparency when the social welfare function equals (22). Substituting (7), (8) and (18) into (22) gives

$$W = -\frac{1}{2}\beta \left\{ \alpha\theta + (1-\alpha)\left(\pi^{e} + \kappa + s\right) - \bar{\theta} \right\}^{2} - \frac{1}{2}\left(1-\beta\right) \left\{ \alpha\left(\theta - \pi^{e} - s\right) + (1-\alpha)\kappa - \bar{\kappa} \right\}^{2} \\ = -\frac{1}{2}\beta \left\{ \left(\alpha + (1-\alpha)\tilde{\tau}_{\theta}\right)\left(\theta - \bar{\theta}\right) + (1-\alpha)\tilde{\tau}_{\theta}\varepsilon \\ + \frac{1-\alpha}{\alpha} \left[\bar{\kappa} + \left(\alpha + (1-\alpha)\tilde{\tau}_{\kappa}\right)\left(\kappa - \bar{\kappa}\right) + (1-\alpha)\tilde{\tau}_{\kappa}\eta + \alpha s \right] \right\}^{2} \\ - \frac{1}{2}\left(1-\beta\right) \left\{ \alpha\left(1-\tilde{\tau}_{\theta}\right)\left(\theta - \bar{\theta}\right) - \alpha\tilde{\tau}_{\theta}\varepsilon + (1-\alpha)\left(1-\tilde{\tau}_{\kappa}\right)\left(\kappa - \bar{\kappa}\right) - (1-\alpha)\tilde{\tau}_{\kappa}\eta - \bar{\kappa} - \alpha s \right\}^{2}$$

Taking expectations and substituting for σ_{ε}^2 and σ_{η}^2 using (5) gives

$$\begin{split} \mathbf{E}\left[W\right] &= -\frac{1}{2}\beta\left\{\left(\alpha + (1-\alpha)\,\tilde{\tau}_{\theta}\right)^{2}\sigma_{\theta}^{2} + (1-\alpha)^{2}\,\tilde{\tau}_{\theta}^{2}\sigma_{\varepsilon}^{2} \\ &+ \frac{(1-\alpha)^{2}}{\alpha^{2}}\left[\bar{\kappa}^{2} + (\alpha + (1-\alpha)\,\tilde{\tau}_{\kappa})^{2}\,\sigma_{\kappa}^{2} + (1-\alpha)^{2}\,\tilde{\tau}_{\kappa}^{2}\sigma_{\eta}^{2} + \alpha^{2}\sigma_{s}^{2}\right]\right\} \\ &- \frac{1}{2}\left(1-\beta\right)\left\{\alpha^{2}\left(1-\tilde{\tau}_{\theta}\right)^{2}\sigma_{\theta}^{2} + \alpha^{2}\tilde{\tau}_{\theta}^{2}\sigma_{\varepsilon}^{2} + (1-\alpha)^{2}\left(1-\tilde{\tau}_{\kappa}\right)^{2}\sigma_{\kappa}^{2} + (1-\alpha)^{2}\,\tilde{\tau}_{\kappa}^{2}\sigma_{\eta}^{2} + \bar{\kappa}^{2} + \alpha^{2}\sigma_{s}^{2}\right\} \\ &= -\frac{1}{2}\beta\left\{\left(\alpha^{2} + 2\alpha\left(1-\alpha\right)\tilde{\tau}_{\theta} + (1-\alpha)^{2}\frac{\tilde{\tau}_{\theta}^{2}}{\tau_{\theta}}\right)\sigma_{\theta}^{2} \\ &+ \frac{(1-\alpha)^{2}}{\alpha^{2}}\left[\bar{\kappa}^{2} + \left(\alpha^{2} + 2\alpha\left(1-\alpha\right)\tilde{\tau}_{\kappa} + (1-\alpha)^{2}\frac{\tilde{\tau}_{\kappa}^{2}}{\tau_{\kappa}}\right)\sigma_{\kappa}^{2} + \alpha^{2}\sigma_{s}^{2}\right]\right\} \\ &- \frac{1}{2}\left(1-\beta\right)\left\{\alpha^{2}\left(1-2\tilde{\tau}_{\theta} + \frac{\tilde{\tau}_{\theta}^{2}}{\tau_{\theta}}\right)\sigma_{\theta}^{2} + (1-\alpha)^{2}\left(1-2\tilde{\tau}_{\kappa} + \frac{\tilde{\tau}_{\kappa}^{2}}{\tau_{\kappa}}\right)\sigma_{\kappa}^{2} + \bar{\kappa}^{2} + \alpha^{2}\sigma_{s}^{2}\right\} \end{split}$$

Differentiating with respect to the degrees of actual transparency yields:

$$\frac{d \operatorname{E}[W]}{d\tau_{\theta}} = \frac{1}{2} \left(\beta \left(1 - \alpha \right)^{2} + \left(1 - \beta \right) \alpha^{2} \right) \frac{\tilde{\tau}_{\theta}^{2}}{\tau_{\theta}^{2}} \sigma_{\theta}^{2} > 0$$
$$\frac{d \operatorname{E}[W]}{d\tau_{\kappa}} = \frac{1}{2} \left(\beta \frac{\left(1 - \alpha \right)^{2}}{\alpha^{2}} + \left(1 - \beta \right) \right) \left(1 - \alpha \right)^{2} \frac{\tilde{\tau}_{\kappa}^{2}}{\tau_{\kappa}^{2}} \sigma_{\kappa}^{2} > 0$$

This implies that it is socially optimal to have perfect actual transparency about the central bank's targets ($\tau_{\theta} = \tau_{\kappa} = 1$).

Concerning perceived transparency, the first order conditions $d \ge [W] / d\tilde{\tau}_{\theta} = 0$ and $d \ge [W] / d\tilde{\tau}_{\kappa} = 0$

0 yield

$$\tilde{\tau}_{\theta} = \frac{\alpha (\alpha - \beta)}{\beta (1 - \alpha)^{2} + (1 - \beta) \alpha^{2}} \tau_{\theta} = \frac{\alpha (\alpha - \beta)}{\beta (1 - \alpha) + \alpha (\alpha - \beta)} \tau_{\theta}$$
$$\tilde{\tau}_{\kappa} = \frac{\alpha (\alpha - \beta)}{\beta (1 - \alpha)^{2} + (1 - \beta) \alpha^{2}} \tau_{\kappa} = \frac{\alpha (\alpha - \beta)}{\beta (1 - \alpha) + \alpha (\alpha - \beta)} \tau_{\kappa}$$

respectively. For $\alpha \geq \beta$, these are the socially optimal degrees of perceived transparency, since $d^2 \operatorname{E}[W]/d\tilde{\tau}_{\theta}^2 < 0$ and $d^2 \operatorname{E}[W]/d\tilde{\tau}_{\kappa}^2 < 0$. But for $\alpha < \beta$, the social optimum is the corner solution $\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$. So, if the central bank is not conservative, society benefits from complete perceived opacity. Regardless of the value of β , in the social optimum the degree of perceived transparency is strictly less than the degree of actual transparency ($\tilde{\tau}_{\theta} < \tau_{\theta}$ and $\tilde{\tau}_{\kappa} < \tau_{\kappa}$).

In the case of common knowledge about the degree of transparency ($\tilde{\tau}_{\theta} = \tau_{\theta}$ and $\tilde{\tau}_{\kappa} = \tau_{\kappa}$),

$$E[W] = -\frac{1}{2}\beta \left\{ \left(\alpha^{2} + (1 - \alpha^{2})\tau_{\theta}\right)\sigma_{\theta}^{2} + \frac{(1 - \alpha)^{2}}{\alpha^{2}}\left[\bar{\kappa}^{2} + (\alpha^{2} + (1 - \alpha^{2})\tau_{\kappa})\sigma_{\kappa}^{2} + \alpha^{2}\sigma_{s}^{2}\right] \right\}$$
$$-\frac{1}{2}(1 - \beta) \left\{\alpha^{2}(1 - \tau_{\theta})\sigma_{\theta}^{2} + \bar{\kappa}^{2} + (1 - \alpha)^{2}(1 - \tau_{\kappa})\sigma_{\kappa}^{2} + \alpha^{2}\sigma_{s}^{2}\right\}$$

Differentiating yields

$$\frac{d \operatorname{E} [W]}{d\tau_{\theta}} = -\frac{1}{2} \left[\beta \left(1 - \alpha^{2} \right) - (1 - \beta) \alpha^{2} \right] \sigma_{\theta}^{2} = -\frac{1}{2} \left(\beta - \alpha^{2} \right) \sigma_{\theta}^{2}$$

$$\frac{d \operatorname{E} [W]}{d\tau_{\kappa}} = -\frac{1}{2} \left[\beta \frac{\left(1 - \alpha \right)^{2}}{\alpha^{2}} \left(1 - \alpha^{2} \right) - (1 - \beta) \left(1 - \alpha \right)^{2} \right] \sigma_{\kappa}^{2} = -\frac{1}{2} \left[\frac{\beta}{\alpha^{2}} - 1 \right] (1 - \alpha)^{2} \sigma_{\kappa}^{2}$$

Note that $d \in [W] / d\tau_{\theta} = d \in [W] / d\tau_{\kappa} = 0$ for $\beta = \alpha^2$, and sgn $(d \in [W] / d\tau_{\theta}) = \text{sgn} (d \in [W] / d\tau_{\kappa}) = \text{sgn} (\alpha^2 - \beta)$. Hence, $\tau_{\theta} = \tau_{\kappa} = 1$ is socially optimal for $\alpha^2 > \beta$, and $\tau_{\theta} = \tau_{\kappa} = 0$ is socially optimal for $\alpha^2 < \beta$. So, if society attaches a sufficiently low weight to inflation stabilization or the central bank is sufficiently conservative, the social optimum is to have transparency about the central bank's targets.

To summarize the results for the social welfare function (22):

- With perfect common knowledge about the degrees of transparency τ_{θ} and τ_{κ} , it is socially optimal to have maximum transparency about the central bank targets ($\tau_{\theta} = \tau_{\kappa} = 1$) for $\alpha^2 > \beta$, and minimum transparency ($\tau_{\theta} = \tau_{\kappa} = 0$) for $\alpha^2 < \beta$.

- With transparency misperceptions, it is socially optimal to have maximum actual transparency about the central bank's targets ($\tau_{\theta} = \tau_{\kappa} = 1$) regardless of α and β , some perceived opacity ($0 < \tilde{\tau}_{\theta}, \tilde{\tau}_{\kappa} < 1$) for $\alpha > \beta$, and maximum perceived opacity ($\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$) for $\alpha \leq \beta$.

A.2 Constant Central Bank Targets

This section examines optimal transparency (mis)perceptions when the central bank's inflation target θ and output gap target κ are constant. More precisely, the actual distributions of θ and κ are degenerate, but the private sector still faces asymmetric information about these targets and has the perceived (or prior) distributions $\theta \sim N(\bar{\theta}, \tilde{\sigma}_{\theta}^2)$, $\kappa \sim N(\bar{\kappa}, \tilde{\sigma}_{\kappa}^2)$. The optimal output gap and inflation still satisfy (7) and (8). In addition, private sector expectations are again given by (16), (17) and (18).¹⁷ The difference with the model in section 2.2 is that the actual values of θ and κ are now deterministic so that $\sigma_{\theta}^2 = \sigma_{\kappa}^2 = 0$. As a result, the actual variance of inflation expectations equals

$$\operatorname{Var}\left[\pi^{e}\right] = \tilde{\tau}_{\theta}^{2} \sigma_{\varepsilon}^{2} + \left(\frac{1-\alpha}{\alpha}\right)^{2} \tilde{\tau}_{\kappa}^{2} \sigma_{\eta}^{2}$$

This shows that the volatility of inflation expectations is increasing in perceived transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$, and in the noise variances σ_{ε}^2 and σ_{η}^2 , so that it is essentially decreasing in actual transparency about θ and κ .

The level of the output gap and inflation are still given by (19) and (20), but their actual variances now equal

$$\operatorname{Var}\left[y\right] = \alpha^{2} \tilde{\tau}_{\theta}^{2} \sigma_{\varepsilon}^{2} + (1-\alpha)^{2} \tilde{\tau}_{\kappa}^{2} \sigma_{\eta}^{2} + \alpha^{2} \sigma_{s}^{2}$$
$$\operatorname{Var}\left[\pi\right] = (1-\alpha)^{2} \tilde{\tau}_{\theta}^{2} \sigma_{\varepsilon}^{2} + \frac{(1-\alpha)^{2}}{\alpha^{2}} (1-\alpha)^{2} \tilde{\tau}_{\kappa}^{2} \sigma_{\eta}^{2} + (1-\alpha)^{2} \sigma_{s}^{2}$$

So, the variability of the output gap and inflation are both increasing in perceived transparency $\tilde{\tau}_{\theta}$ and $\tilde{\tau}_{\kappa}$, and in the noise variances σ_{ε}^2 and σ_{η}^2 . As a result, the output gap and inflation are more stable when there is greater perceived opacity about the inflation and output gap targets, and greater transparency in the communications ξ_{θ} and ξ_{κ} .

Regarding welfare effects, taking unconditional expectations based on actual distributions,

$$\mathbf{E}\left[U\right] = -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\alpha^2 \tilde{\tau}_{\theta}^2 \sigma_{\varepsilon}^2 + \bar{\kappa}^2 + (1-\alpha)^2 \tilde{\tau}_{\kappa}^2 \sigma_{\eta}^2 + \alpha^2 \sigma_s^2\right]$$

Clearly, the best outcome is obtained for maximum perceived opacity ($\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$) and maximum actual transparency ($\sigma_{\varepsilon}^2 = \sigma_{\eta}^2 = 0$). So, again it is optimal to have transparency misperceptions.

The same conclusion holds for the social welfare functions in (21) and (22). Concerning the latter, expected social welfare now equals

$$\mathbf{E}\left[W\right] = -\frac{1}{2}\left[\left(\beta\left(1-\alpha\right)^2 + \left(1-\beta\right)\alpha^2\right)\right]\left\{\tilde{\tau}_{\theta}^2\sigma_{\varepsilon}^2 + \frac{1}{\alpha^2}\bar{\kappa}^2 + \frac{\left(1-\alpha\right)^2}{\alpha^2}\tilde{\tau}_{\kappa}^2\sigma_{\eta}^2 + \sigma_s^2\right\}\right]$$

So again, minimum perceived transparency ($\tilde{\tau}_{\theta} = \tilde{\tau}_{\kappa} = 0$) and maximum actual transparency ($\sigma_{\varepsilon}^2 = \sigma_{\eta}^2 = 0$) is optimal.

 $^{^{17}}$ Note that if the perceived distributions were not normal, (16) and (17) would still be the best linear predictors.

As a result, the conclusion that it is desirable to have transparency misperceptions does not depend on the assumption that the central bank targets θ and κ are stochastic, but it even holds when these targets are actually deterministic.

A.3 Transparency about Supply Shocks

This section analyzes the effect of transparency about the supply shock s, where $s \sim N(0, \sigma_s^2)$. In the model of section 2, transparency about the supply shock s is immaterial because s is only realized after the private sector has formed its inflation expectations π^e . Now suppose that the private sector receives a public signal ξ_s of the supply shock before it forms its inflation expectations π^e :

$$\xi_s = s + \upsilon \tag{23}$$

where $v \sim N(0, \sigma_v^2)$, independent of ε and η . Then, the actual degree of transparency about supply shocks is given by

$$\tau_s = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_v^2} \tag{24}$$

Similarly, the perceived degree of transparency about supply shocks is given by

$$\tilde{\tau}_s = \frac{\tilde{\sigma}_s^2}{\tilde{\sigma}_s^2 + \tilde{\sigma}_v^2} \tag{25}$$

where $\tilde{\sigma}_s^2$ and $\tilde{\sigma}_v^2$ are the private sector perceptions of the (prior) variance of s and v, respectively.

Note that the optimal degree of transparency about the inflation target θ and output gap target κ in section 2 is independent of the variability of the supply shock s. The reason is that σ_{θ}^2 , σ_{κ}^2 and σ_s^2 enter separably in E[U], and θ , κ and s are independent. Similarly, the optimal degree of transparency about the supply shock is independent of the variability of the inflation and output gap target. For simplicity, assume that the inflation target and output gap target are deterministic and known to the private sector so that $\theta = \bar{\theta}$, $\kappa = \bar{\kappa}$ with $\sigma_{\theta}^2 = \tilde{\sigma}_{\theta}^2 = \sigma_{\kappa}^2 = \tilde{\sigma}_{\kappa}^2 = 0$. Instead there is asymmetric information about the supply shock s. The central bank still maximizes (1) subject to (2) given π^e , which yields (7) and (8).

The results for imperfect common knowledge about the degree of transparency of supply shocks are derived first. Perfect common knowledge amounts to the special case in which there are no transparency misperceptions ($\tilde{\tau}_s = \tau_s$). Taking expectations of (8) and solving for π^e gives

$$\pi^{e} = \widetilde{\mathbf{E}}\left[\pi|\xi_{s}\right] = \bar{\theta} + \frac{1-\alpha}{\alpha}\left(\bar{\kappa} + \widetilde{\mathbf{E}}\left[s|\xi_{s}\right]\right)$$

Using (23) and (25),

$$\widetilde{\mathbf{E}}\left[s|\boldsymbol{\xi}_{s}\right] = \frac{\widetilde{\sigma}_{s}^{2}}{\widetilde{\sigma}_{s}^{2} + \widetilde{\sigma}_{v}^{2}}\boldsymbol{\xi}_{s} = \widetilde{\tau}_{s}\boldsymbol{\xi}_{s}$$

Substituting into π^e and using (23) gives

$$\pi^{e} = \bar{\theta} + \frac{1 - \alpha}{\alpha} \left(\bar{\kappa} + \tilde{\tau}_{s} s + \tilde{\tau}_{s} \upsilon \right)$$
(26)

Substituting this into (7) and (8) yields

$$y = -(\alpha + (1 - \alpha)\tilde{\tau}_s)s - (1 - \alpha)\tilde{\tau}_s v$$

$$\pi = \bar{\theta} + \frac{1 - \alpha}{\alpha} [\bar{\kappa} + (\alpha + (1 - \alpha)\tilde{\tau}_s)s + (1 - \alpha)\tilde{\tau}_s v]$$

The variance of the output gap and inflation depend on the degree of transparency:

$$\begin{aligned} \text{Var}\left[y\right] &= \left(\alpha + (1-\alpha)\,\tilde{\tau}_{s}\right)^{2}\sigma_{s}^{2} + (1-\alpha)^{2}\,\tilde{\tau}_{s}^{2}\sigma_{v}^{2} \\ &= \left[\alpha^{2} + 2\alpha\,(1-\alpha)\,\tilde{\tau}_{s} + (1-\alpha)^{2}\,\frac{\tilde{\tau}_{s}^{2}}{\tau_{s}}\right]\sigma_{s}^{2} \\ \text{Var}\left[\pi\right] &= \frac{(1-\alpha)^{2}}{\alpha^{2}}\left[\left(\alpha + (1-\alpha)\,\tilde{\tau}_{s}\right)^{2}\sigma_{s}^{2} + (1-\alpha)^{2}\,\tilde{\tau}_{s}^{2}\sigma_{v}^{2}\right] \\ &= \frac{(1-\alpha)^{2}}{\alpha^{2}}\left[\alpha^{2} + 2\alpha\,(1-\alpha)\,\tilde{\tau}_{s} + (1-\alpha)^{2}\,\frac{\tilde{\tau}_{s}^{2}}{\tau_{s}}\right]\sigma_{s}^{2} \end{aligned}$$

using the fact that (24) implies $\sigma_v^2 = \frac{1-\tau_s}{\tau_s} \sigma_s^2$. This shows that the variance of the output gap and inflation are decreasing in actual transparency τ_s and increasing in perceived transparency $\tilde{\tau}_s$.

Not surprisingly, perceived transparency about supply shocks is harmful, whereas actual transparency is beneficial. Formally, substitute (26) into (15) to get

$$U = -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\bar{\kappa} + (\alpha + (1-\alpha)\tilde{\tau}_s)s + (1-\alpha)\tilde{\tau}_s v \right]^2$$

Taking unconditional expectations and substituting $\sigma_{\varepsilon}^2 = \frac{1-\tau_s}{\tau_s}\sigma_s^2$ gives the ex ante expected central bank payoff

$$\mathbf{E}\left[U\right] = -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\bar{\kappa}^2 + \left(\alpha + (1-\alpha)\,\tilde{\tau}_s\right)^2 \sigma_s^2 + (1-\alpha)^2 \,\tilde{\tau}_s^2 \sigma_v^2\right]$$

$$= -\frac{1}{2} \frac{1-\alpha}{\alpha} \left[\bar{\kappa}^2 + \left(\alpha^2 + 2\alpha \left(1-\alpha\right)\tilde{\tau}_s + (1-\alpha)^2 \frac{\tilde{\tau}_s^2}{\tau_s}\right) \sigma_s^2\right]$$

As a result, for supply shocks, maximum actual transparency ($\tau_s = 1$) and minimum perceived transparency ($\tilde{\tau}_s = 0$) is optimal for the central bank. Formally, this follows from $\partial \operatorname{E}[U] / \partial \tau_s = \frac{1}{2} \frac{(1-\alpha)^3}{\alpha} \frac{\tilde{\tau}_s^2}{\tau_s^2} \sigma_s^2 > 0$ and $\partial \operatorname{E}[U] / \partial \tilde{\tau}_s = -\frac{(1-\alpha)^2}{\alpha} \frac{(1-\alpha)\tilde{\tau}_s + \alpha\tau_s}{\tau_s} \sigma_s^2 < 0$.

The results under common knowledge follow from imposing the restriction that $\tilde{\tau}_s = \tau_s$. The variance of the output gap and inflation are equal to

$$\operatorname{Var}\left[y\right] = \left[\alpha^{2} + \left(1 - \alpha^{2}\right)\tau_{s}\right]\sigma_{s}^{2}$$
$$\operatorname{Var}\left[\pi\right] = \frac{\left(1 - \alpha\right)^{2}}{\alpha^{2}}\left[\alpha^{2} + \left(1 - \alpha^{2}\right)\tau_{s}\right]\sigma_{s}^{2}$$

This shows that greater transparency about the supply shock s increases the volatility of both the output gap and inflation.

Not surprisingly, transparency about supply shocks is detrimental. Formally,

$$\mathbf{E}\left[U\right] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\bar{\kappa}^2 + \left(\alpha^2 + \left(1-\alpha^2\right)\tau_s\right)\sigma_s^2\right]$$

Clearly, minimum transparency about supply shocks $\tau_s = 0$ is optimal for the central bank. It is also socially optimal for the social welfare functions (21) and (22).

To summarize the results concerning supply shocks s:

- With perfect common knowledge about the degree of transparency τ_s , greater transparency τ_s increases the variability of inflation π and the output gap y, and minimum transparency ($\tau_s = 0$) is optimal for the central bank and society.

- With transparency misperceptions, greater actual transparency τ_s and smaller perceived transparency $\tilde{\tau}_s$ reduce the variability of inflation π and the output gap y, and it is optimal for the central bank and society to have maximum actual transparency ($\tau_s = 1$) but minimal perceived transparency ($\tilde{\tau}_s = 0$).

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