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Abstract

Central bank independence and transparency have become best practice in monetary policy. This paper cautions that transparency about economic information may not be beneficial in the absence of central bank independence. The reason is that it reduces monetary uncertainty, which could make the government less inhibited to interfere with monetary policy. In fact, a central bank could use monetary mystique to obtain greater insulation from political pressures, even if the government faces no direct cost of overriding. As a result, economic secrecy could be beneficial and provide the central bank greater political independence.

Keywords: Transparency, monetary policy, political pressures
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1 Introduction

Central bank independence and transparency have become best practice in monetary policy. But only 20 years ago, the most successful central banks, including the US Federal Reserve and the German Bundesbank, tended to be notorious for their secrecy. This paper shows that opacity may be desirable when a central bank could be subject to political interference. The reason is that greater monetary uncertainty makes the government more reluctant to intervene in monetary policy. In particular, opacity about the economic shocks to which the central bank responds makes it more difficult to assess the central bank's intentions from its monetary policy actions. This gives the central bank greater leeway to set monetary policy without government interference. As a result, a central bank could use monetary mystique to insulate itself from political pressures.

This paper helps to explain how central banks managed to gain independence through secrecy before the advent of the new paradigm of central bank independence-cum-transparency. For instance, the 'monetary veil' introduced by Chairman Paul Volcker in October 1979 helped to keep US Congress at bay while the Federal Reserve pursued its painful disinflation in the early 1980s. Furthermore, this paper cautions that in the absence of central bank independence, economic transparency may be detrimental as it could lead to greater political interference. Although central bank independence is prevalent in advanced economies, it is much less common in developing countries. In fact, in the survey of 94 central banks by Fry, Julius, Mahadeva, Roger and Sterne (2000, Table 4.4), 93% of central banks in industrialized countries report they enjoy independence without significant qualifications, whereas this holds for only 57% of central banks in developing countries. For those central banks that lack independence, economic secrecy could be an effective way to stave off unwanted political meddling with monetary policy.

This argument is formally developed using a stylized monetary policy game in the spirit of Kydland and Prescott (1977) and Barro and Gordon (1983). The government has a motive to stimulate output beyond the natural rate, whereas monetary policy is set by a conservative central banker (Rogoff 1985). The government can override the central bank's policy decision but only at a cost (Lohmann 1992). Uncertainty about the central bank's true intentions and the economic situation complicate the government's decision whether or not to interfere. There is rational updating of beliefs (Cukierman and Meltzer 1986, Backus and Driffill 1985, Barro 1986) and the modeling of transparency builds on Faust and Svensson (2001) and Geraats (2005).

Transparency of monetary policy could be defined as the extent to which monetary authorities disclose information that is relevant for the policymaking process; so, perfect transparency amounts to symmetric information. There is a growing literature on central bank transparency that covers many aspects.[1] This paper considers the effects of transparency about the central bank's preferences and about the economic information to which the central bank responds, in an institutional framework in which the central bank is subject to political interference.

The effect of preference transparency on government interference has been considered by Eijffinger and Hoeberichts (2002), who extend the Lohmann (1992) model by introducing uncertainty about the central bank's preferences. They find that greater preference transparency reduces the region of independence for the central bank. This result relies on the assumption that the central bank directly sets inflation and that the government has the same economic information as the central bank. However, in practice, inflation can only be influenced indirectly through a monetary policy instrument, such as the money supply. In addition, the monetary policy action also reflects economic disturbances, such as money market shocks, about which the government may not have the same information as the central bank. The contribution of the present paper is to incorporate these two realistic assumptions. The result is that the government can no longer perfectly infer the central bank's intentions and faces uncertainty about overriding. In fact, it is shown that economic opacity gives the central bank greater freedom from political interference, even if the government faces no direct cost of overriding. Greater economic transparency reduces the region of independence, whereas greater preference transparency actually increases it. So in contrast to Eijffinger and Hoeberichts (2002), this paper shows that in the presence of political pressures, greater preference uncertainty is detrimental but mystique about monetary disturbances is beneficial.

There are many other papers on preference and economic transparency, but none explicitly analyze government interference with monetary policy in the form of overriding. In general, an important benefit of transparency is that it reduces uncertainty. But, greater transparency could also be detrimental. For instance, it could cause financial markets to increase their reliance on public information to coordinate their actions, which could lead to greater volatility if the public information is sufficiently noisy (Morris and Shin 2002). In addition, opacity about central bank preferences could moderate wage demands by unions (Sørensen 1991) or give rise to beneficial reputation effects (e.g. Faust and Svensson 2001, Geraats 2005), thereby reducing inflation. Furthermore, transparency about economic information could hamper stabilization policy when the public incorporates supply shocks into inflation expectations and negatively affects the contemporaneous inflation-output trade-off (Cukierman 2001, Gersbach 2002, Jensen 2002). The present paper provides another argument against economic transparency, namely that it could make central banks prone to greater political pressures and thereby increase average inflation.

[1] For a comprehensive survey of the literature, see Geraats (2002).

The remainder of this paper is organized as follows. The basic model is presented in section 2 and the solution derived in section 3. Section 4 considers a few extensions to the basic model that feature more realistic objective functions and a richer economic structure, and it shows that the conclusions are robust. In addition, this section provides some empirical support for the theoretical prediction of this paper that central banks with lower independence are more likely to have low transparency to ward off political interference. Section 5 summarizes the results and concludes that economic opacity could be desirable when the central bank lacks institutional independence. This helps to explain the past practice of independence-through-secrecy. Furthermore, it suggests that countries that wish to adopt the new paradigm of central bank independence-cum-transparency should first grant the central bank political independence before insisting on economic transparency.

2 Model

The structure of the economy is described by the simple money market equation[2]

$$\pi = m + v \tag{1}$$

and the Lucas aggregate supply equation

$$y = \bar{y} + \theta\left(\pi - \pi^e\right) \tag{2}$$

where π is inflation, π^e private sector expectations of inflation, m money supply growth, y real aggregate output, $\bar{y}$ the natural rate of output, and θ the extent to which surprise inflation stimulates output ($\theta > 0$). There is a velocity shock v that is stochastic: $v \sim N\left(0, \sigma_v^2\right)$, with $\sigma_v^2 > 0$.

The government has the objective function[3]

$$W_G = -\frac{1}{2}\left(\pi - \bar{\tau}\right)^2 + \beta\left(y - \bar{y}\right) \tag{3}$$

where $\bar{\tau}$ is the government's inflation target and β the relative weight on output stimulation ($\beta > 0$). The government delegates monetary policy to a central bank, without granting it complete (instrument) independence. The central bank is conservative in the sense that it puts greater weight on inflation stabilization than the government (Rogoff 1985). For simplicity,

[2]Some extensions to this economic structure are discussed in section 4 and yield the same qualitative results.

[3]More plausible objective functions for the government and the central bank are discussed in section 4 and yield the same conclusions.

assume that the central bank only cares about inflation stabilization ($\beta = 0$) and that its objective function is

$$W_{CB} = -\frac{1}{2}(\pi - \tau)^2 \tag{4}$$

The central bank has an unknown, stochastic inflation target τ, where $\tau \sim N(\bar{\tau}, \sigma_\tau^2)$ with $\sigma_\tau^2 > 0$, and τ and v independent. There could be several reasons for this preference uncertainty. First, preferences of central bankers cannot be directly observed and are therefore subject to uncertainty. Also, central bank preferences could change because of new appointments to the central bank's governing body. In addition, the central bank may have goal independence. Even if there is an explicit inflation target, such targets often take the form of a range, leaving significant uncertainty about the central bank's intentions. The assumption that $\mathrm{E}[\tau] = \bar{\tau}$ implies that on average, the inflation target of the central bank and the government coincide.

The central bank does not enjoy complete independence and the government can decide to override the central bank's policy decision m, either explicitly (e.g. through an act of parliament) or implicitly through political pressure. Assume that the government suffers a direct cost of overriding $C > 0$. This could involve loss of reputation in the form of higher inflation expectations in the future, or electoral losses due to reduced voter confidence.

Although these assumptions no longer tend to apply to most monetary institutions today, they do effectively describe the era in which formal (instrument) independence was not the norm, as well as the situation that still prevails in many developing countries.

The government's decision to override the central bank is complicated by two information asymmetries. First, as already mentioned, the government is uncertain about the central bank's inflation target τ. Second, the velocity shocks v are observed by the central bank, but not by the government.[4] Instead, the government only observes a stochastic signal s such that

$$v = s + \eta \tag{5}$$

where $\eta \sim N(0, (1-\kappa)\sigma_v^2)$ with $0 \leq \kappa \leq 1$, and s, η and τ are independently distributed. The variable η could be interpreted as the government's forecast error of the velocity shock. In the special case of $\kappa = 1$ there is no asymmetric information about the velocity shock so that $v = s$, whereas for $\kappa = 0$ the signal provides no clues about the velocity shock and $s = 0$. The parameter κ is a measure of economic transparency, where $\kappa = 1$ amounts to perfect transparency.

Timing in the model is as follows. Initially, the central bank's inflation target τ is realized, but only known to the central bank, and the public forms its inflation expectations π^e. Subsequently, the government gets a (noisy) signal s of the velocity shock v, whereas the

[4] One could allow for imperfect central bank forecasts, but the conclusions would be the same.

central bank also observes the noise η and therefore knows the actual velocity shock v. The central bank sets the money supply m_{CB}, which is observed by the government. Then, the government decides whether to override the central bank and implement its policy m_G under transparency or m_O under opacity. After that, inflation π and output y are realized.

The remaining assumption concerns the formation of expectations. The central bank, government and private sector all have rational expectations. The central bank has perfect information, whereas the government and private sector face imperfect information. To be precise, the information set available to the private sector when it forms its inflation expectations π^e equals $\Omega \equiv \{\beta, \theta, \bar{y}, \bar{\tau}, \kappa, \sigma_\tau^2, \sigma_v^2\}$; the government's information set when it makes the override decision is $\{m_{CB}, s, \Omega\}$. The solution of the model is described in the next section.

3 Solution

In the absence of political pressure, the conservative central bank would implement[5]

$$\tilde{m} = \tau - v \tag{6}$$

to achieve the economic outcome

$$\begin{aligned} \pi &= \tau \\ y &= \bar{y} + \theta\left(\tau - \pi^e\right) \end{aligned}$$

However, the government would prefer[6]

$$m_G = \bar{\tau} + \beta\theta - v \tag{7}$$

to obtain a higher expected level of output (given inflation expectations) but at the cost of higher inflation:

$$\begin{aligned} \pi &= \bar{\tau} + \beta\theta \\ y &= \bar{y} + \theta\left(\bar{\tau} + \beta\theta - \pi^e\right) \end{aligned}$$

The government's desire to stimulate output beyond the natural rate ($\beta > 0$) leads to the celebrated inflationary bias of discretionary monetary policy ($\pi > \bar{\tau}$) first advanced by Kydland and Prescott (1977).

[5]This follows from maximization of (4) with respect to m subject to (1) and (2), and given π^e.

[6]Maximize (3) with respect to m, subject to (1) and (2), and given π^e.

The discrepancy between (6) and (7) suggests that the government would like to override the central bank if τ is sufficiently different from $\bar{\tau}+\beta\theta$. However, its decision is complicated by the presence of asymmetric information about τ and v.

It is instructive to first consider the case of complete economic transparency ($\kappa = 1$). Then, the velocity shock v is known to the government, so it can use the central bank's policy decision m_{CB} to infer information about its inflation target τ. The government abstains from overriding m_{CB} and implementing its preferred policy m_G if[7]

$$W_G(m_G) - C \leq W_G(m_{CB})$$

Using the fact that in the absence of government interference $m_{CB} = \tilde{m}$, it is straightforward to show that this inequality reduces to[8]

$$\frac{1}{2}(\tau - \bar{\tau} - \beta\theta)^2 \leq C \tag{8}$$

So, the government decides not to override the central bank if $\bar{\tau} + \beta\theta - \sqrt{2C} \leq \tau \leq \bar{\tau} + \beta\theta + \sqrt{2C}$. This region of independence is increasing in the cost of overriding C. But if the central bank's inflation outcome $\pi = \tau$ is too far from the level preferred by the government $\pi = \bar{\tau}+\beta\theta$, (8) no longer holds and the government interferes with monetary policy. Since the central bank is worse off if the government overrides its policy decision, it adjusts its policy to prevent this. In particular, it optimally implements the monetary policy action that makes the government indifferent between interference and independence. So, for $\tau < \bar{\tau} + \beta\theta - \sqrt{2C}$ the central bank sets $m_{CB} = \bar{\tau} + \beta\theta - \sqrt{2C} - v$, and for $\tau > \bar{\tau} + \beta\theta + \sqrt{2C}$ it sets $m_{CB} = \bar{\tau} + \beta\theta + \sqrt{2C} - v$. As a result, the government never overrides, but the possibility of political interference does affect the monetary policy outcome.[9] In particular, it leads to higher average inflation: $\mathrm{E}[\pi] > \bar{\tau}$. Intuitively, without political pressures average inflation would be $\bar{\tau}$, but the threat of overriding brings average inflation closer to the government's preferred level of $\bar{\tau} + \beta\theta$. These results are all similar to Lohmann (1992).

When there is incomplete economic transparency ($0 \leq \kappa < 1$), the government can no longer infer the central bank's inflation target τ from its policy action m_{CB}. But there is an additional complication: The government is unable to implement its desired policy $m_G = \bar{\tau} + \beta\theta - v$ because it does not observe the velocity shock v. So, it tries to extract information about v from the central bank's actions m_{CB}.

The government's preferred policy action under opacity maximizes $\mathrm{E}[W_G(m_O)|m_{CB}]$ subject to (1) and (2), and given π^e. All expectations operators $\mathrm{E}[.]$ are implicitly conditional

[7]This assumes that the government does not override when it is indifferent; otherwise, there is no equilibrium.
[8]Substitute (1), (2), (7) and (6) into (3) and rearrange.
[9]If there would be uncertainty about the government's preferences, overriding could occur.

on the public information set $\{s, \Omega\}$. The first order condition implies

$$m_O = \bar{\tau} + \beta\theta - \mathrm{E}\left[v|m_{CB}\right] \tag{9}$$

This is the same as the government's preferred policy under economic transparency, m_G in (7), except that v has been replaced by $\mathrm{E}\left[v|m_{CB}\right]$.

The government abstains from overriding m_{CB} and implementing its policy m_O if

$$\mathrm{E}\left[W_G\left(m_O\right)|m_{CB}\right] - C \leq \mathrm{E}\left[W_G\left(m_{CB}\right)|m_{CB}\right] \tag{10}$$

It is shown in Appendix A.1 that this no-override condition reduces to

$$\frac{1}{2}\left(\bar{\tau} + \beta\theta - \mathrm{E}\left[v|m_{CB}\right] - m_{CB}\right)^2 \leq C \tag{11}$$

So, the central bank enjoys independence if

$$\bar{\tau} + \beta\theta - \mathrm{E}\left[v|m_{CB}\right] - \sqrt{2C} \leq m_{CB} \leq \bar{\tau} + \beta\theta - \mathrm{E}\left[v|m_{CB}\right] + \sqrt{2C} \tag{12}$$

As a result, the region of independence equals $m_{CB} \in [\underline{m}, \bar{m}]$, where the thresholds $\underline{m}$ and $\bar{m}$ only depend on publicly available information. The government overrides the central bank only if $m_{CB} < \underline{m}$ or $m_{CB} > \bar{m}$. But the central bank adjusts its policy to prevent the government from intervening. Since $\underline{m} < m_O < \bar{m}$, it directly follows from (4) that the central bank optimally sets

$$m_{CB} = \begin{cases} \underline{m} & \text{if} \quad \tilde{m} \leq \underline{m} \\ \tilde{m} & \text{if} \quad \underline{m} < \tilde{m} < \bar{m} \\ \bar{m} & \text{if} \quad \tilde{m} \geq \bar{m} \end{cases} \tag{13}$$

To compute the thresholds $\underline{m}$ and $\bar{m}$ it is necessary to obtain an expression for the conditional expectation $\mathrm{E}\left[v|m_{CB}\right]$, which involves a signal-extraction problem. For $\underline{m} < m_{CB} < \bar{m}$, it follows from (13) that $\mathrm{E}\left[v|m_{CB}\right] = \mathrm{E}\left[v|\tilde{m}\right]$. Note that (5) and (6) imply that v and $\tilde{m}$ are jointly normal because of their common dependence on η, so[10]

$$\begin{aligned} \mathrm{E}\left[v|\tilde{m}\right] &= s - \frac{\left(1-\kappa\right)\sigma_v^2}{\sigma_\tau^2 + \left(1-\kappa\right)\sigma_v^2}\left(\tilde{m} + s - \bar{\tau}\right) \\ &= \lambda s - \left(1-\lambda\right)\left(\tilde{m} - \bar{\tau}\right) \end{aligned} \tag{14}$$

where $\lambda \equiv \frac{\sigma_\tau^2}{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}$, so that $0 < \lambda \leq 1$. The magnitude of λ is increasing in the degree of economic transparency ($\partial\lambda/\partial\kappa > 0$), reflecting the fact that the signal s becomes more

[10]Use the fact that when x and z have a jointly normal distribution then $\mathrm{E}\left[x|z\right] = \mathrm{E}\left[x\right] + \frac{\mathrm{Cov}\{x,z\}}{\mathrm{Var}[z]}\left(z - \mathrm{E}\left[z\right]\right)$. Note that all moment operators are implicitly conditional on s.

reliable. In the limiting case of perfect transparency ($\kappa = 1$, so $s = v$), $\lambda = 1$ and $\mathrm{E}[v|\tilde{m}] = v$. In the case of economic opacity ($\kappa < 1$), both the signal s and the policy decision $\tilde{m}$ are used to infer information about the velocity shock v. A higher level of $\tilde{m}$ is partly attributed to a lower velocity shock and therefore reduces the expectation $\mathrm{E}[v|\tilde{m}]$.

For $m_{CB} = \underline{m}$, the signal-extraction problem is a bit more complicated since (13) implies $\mathrm{E}[v|m_{CB}] = \mathrm{E}[v|\tilde{m} \leq \underline{m}]$. It follows from (14), (6) and (5) that[11]

$$\begin{aligned} \mathrm{E}[v|\tilde{m} \leq \underline{m}] &= \lambda s + (1-\lambda)\bar{\tau} - (1-\lambda)\mathrm{E}[\tilde{m}|\tilde{m} \leq \underline{m}] \\ &= s + (1-\lambda)\sqrt{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}\frac{\phi(\underline{z})}{\Phi(\underline{z})} \end{aligned} \tag{15}$$

where $\phi(z)$ and $\Phi(z)$ denote the probability density function and the cumulative distribution function of the standard normal distribution, respectively, and $\underline{z} \equiv \frac{\underline{m}-(\bar{\tau}-s)}{\sqrt{\sigma_\tau^2+(1-\kappa)\sigma_v^2}}$ is the normalized lower threshold. The low level of $\tilde{m} \leq \underline{m}$ is partly attributed to high velocity shocks so that $\mathrm{E}[v|\tilde{m} \leq \underline{m}] \geq s$.

Similarly, for $m_{CB} = \bar{m}$ it holds that $\mathrm{E}[v|m_{CB}] = \mathrm{E}[v|\tilde{m} \geq \bar{m}]$, where[12]

$$\begin{aligned} \mathrm{E}[v|\tilde{m} \geq \bar{m}] &= \lambda s + (1-\lambda)\bar{\tau} - (1-\lambda)\mathrm{E}[\tilde{m}|\tilde{m} \geq \bar{m}] \\ &= s - (1-\lambda)\sqrt{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} \end{aligned} \tag{16}$$

where $\bar{z} \equiv \frac{\bar{m}-(\bar{\tau}-s)}{\sqrt{\sigma_\tau^2+(1-\kappa)\sigma_v^2}}$ is the normalized upper threshold. The high level of $\tilde{m} \leq \underline{m}$ is partly attributed to low velocity shocks so that $\mathrm{E}[v|\tilde{m} \geq \bar{m}] \leq s$.

The conditional expectations (14), (15) and (16) show how the government extracts information about the velocity shock v from the central bank's policy decision. For perfect economic transparency ($\kappa = \lambda = 1$), the expressions reduce to $\mathrm{E}[v|m_{CB}] = s = v$, so the no-override condition (11) amounts to (8).

Using (12), (13), (15) and (16), and substituting λ yields the following conditions for the thresholds $\underline{m}$ and $\bar{m}$:

$$\begin{aligned} \underline{m} &= \bar{\tau} + \beta\theta - \mathrm{E}[v|\tilde{m} \leq \underline{m}] - \sqrt{2C} \\ &= \bar{\tau} + \beta\theta - s - \frac{(1-\kappa)\sigma_v^2}{\sqrt{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}}\frac{\phi(\underline{z})}{\Phi(\underline{z})} - \sqrt{2C} \end{aligned} \tag{17}$$

$$\begin{aligned} \bar{m} &= \bar{\tau} + \beta\theta - \mathrm{E}[v|\tilde{m} \geq \bar{m}] + \sqrt{2C} \\ &= \bar{\tau} + \beta\theta - s + \frac{(1-\kappa)\sigma_v^2}{\sqrt{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} + \sqrt{2C} \end{aligned} \tag{18}$$

[11]Use the fact that for a normally distributed variable $x \sim N(\mu, \sigma^2)$, $\mathrm{E}[x|x \leq \underline{x}] = \mu - \sigma\phi\left(\frac{\underline{x}-\mu}{\sigma}\right)/\Phi\left(\frac{\underline{x}-\mu}{\sigma}\right)$.

[12]Now use the fact that for a normally distributed variable $x \sim N(\mu, \sigma^2)$, $\mathrm{E}[x|x \geq \bar{x}] = \mu + \sigma\phi\left(\frac{\bar{x}-\mu}{\sigma}\right)/\left[1-\Phi\left(\frac{\bar{x}-\mu}{\sigma}\right)\right]$.

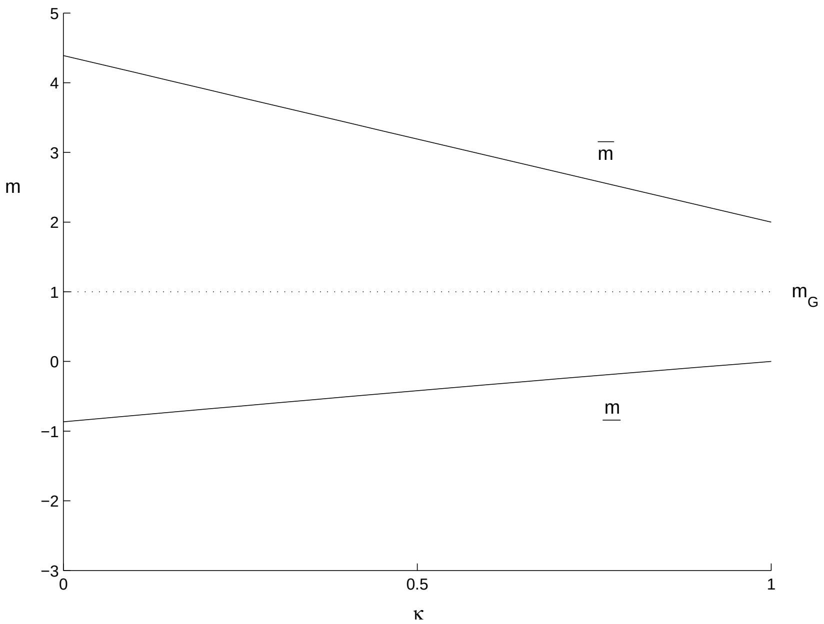

Figure 1: The effect of economic transparency on the region of independence.

The thresholds satisfy $\underline{m} < \bar{\tau} + \beta\theta - s < \bar{m}$. Note that (17) and (18) only provide an implicit expression for $\underline{m}$ and $\bar{m}$ that depends on $\underline{z}$ and $\bar{z}$, respectively. There is no analytical solution for $\underline{m}$ and $\bar{m}$, except for the special case in which there is perfect economic transparency ($\kappa = 1$, so $s = v$). Then, (17) and (18) reduce to $\underline{m} = \bar{\tau} + \beta\theta - v - \sqrt{2C}$ and $\bar{m} = \bar{\tau} + \beta\theta - v + \sqrt{2C}$, as before. For other values of κ, $\underline{m}$ and $\bar{m}$ need to be computed numerically.

Figure 1 illustrates the thresholds $\underline{m}$ and $\bar{m}$ over the range $\kappa \in [0, 1]$ for the parameter values $\bar{\tau} = s = 0$, $\beta = \theta = 1$, $\sigma_\tau^2 = \sigma_v^2 = 1$ and $C = 1/2$. This implies that with perfect economic transparency ($\kappa = 1$), the government's desired policy is $m_G = 1$ and the region of independence is $[0, 2]$. When there is economic opacity ($0 \leq \kappa < 1$), Figure 1 shows that the boundaries of the region of independence $\bar{m}$ and $\underline{m}$ are not symmetric around m_G. Intuitively, the government has expansionary preferences ($\beta > 0$), so it is willing to give the central bank more leeway to expand the money supply.[13] Furthermore, Figure 1 shows that $\bar{m}$ is decreasing and $\underline{m}$ is increasing in the degree of economic transparency κ, thereby shrinking the region of independence $[\underline{m}, \bar{m}]$. In fact, this result holds more generally:

[13] Formally, when $\beta > 0$ the government anticipates a larger surprise shock $|\eta|$ at $\bar{m}$ than at $\underline{m}$: $|\mathrm{E}[v|\tilde{m} \geq \bar{m}] - s| > |\mathrm{E}[v|\tilde{m} \leq \bar{m}] - s|$. So, the government tolerates greater deviations on the upside than on the downside. But for $\beta = 0$, $|\mathrm{E}[v|\tilde{m} \geq \bar{m}] - s| = |\mathrm{E}[v|\tilde{m} \leq \bar{m}] - s|$ and the region of independence is symmetric around $\bar{\tau} - s$.

Proposition 1 *The region of independence* $[\underline{m}, \bar{m}]$ *is decreasing in the degree of economic transparency* κ*.*

The proof is in Appendix A.2. It shows analytically that $\mathrm{d}\,\bar{m}/\,\mathrm{d}\,\kappa < 0$ and $\mathrm{d}\,\underline{m}/\,\mathrm{d}\,\kappa > 0$, so that $\mathrm{d}\left(\bar{m} - \underline{m}\right)/\,\mathrm{d}\,\kappa < 0$. Intuitively, when there is economic opacity, the government does not observe the velocity shock v, so is not sure whether it is appropriate to intervene and what level of the money supply to set. Greater economic opacity makes the government more cautious and less likely to interfere with monetary policy. As a result, less economic transparency κ increases the region of independence. Figure 1 shows that reducing transparency (from $\kappa = 1$ to $\kappa = 0$) could more than double the size of the region of independence (from 2 to over 5). Economic opacity also increases the probability that the central bank enjoys independence.[14] This in turn reduces average inflation, because there is less need to adjust monetary policy towards the higher inflation level $\bar{\tau} + \beta\theta$. So, greater economic transparency increases the probability of political pressures and lead to higher inflation on average.[15]

The effect of a higher variance of velocity shocks σ_v^2 is the same as a reduction in economic transparency κ.[16] However, greater uncertainty about the central bank's inflation target σ_τ^2 gives rise to different effects.

Proposition 2 *Under economic transparency (*$\kappa = 1$*), the region of independence* $[\underline{m}, \bar{m}]$ *is not affected by preference uncertainty* σ_τ^2*. Under economic opacity (*$0 \leq \kappa < 1$*), the region of independence* $[\underline{m}, \bar{m}]$ *is decreasing in the amount of preference uncertainty* σ_τ^2 *for* $\beta\theta \leq \sqrt{2C}$*.*

The proof is in Appendix A.2. Intuitively, when there is complete economic transparency ($\kappa = 1$) the government can perfectly infer from the central bank's policy decision m_{CB} whether or not it is appropriate to intervene. In addition, it also knows exactly what policy to implement. As a result, the amount of preference uncertainty σ_τ^2 is immaterial. But when there is some economic opacity ($0 \leq \kappa < 1$), greater preference uncertainty σ_τ^2 makes the policy action m_{CB} a more useful indicator of the central bank's intentions, so the government becomes more responsive to it and allows for less variation in m_{CB} before intervening. The proof shows that $\beta\theta \leq \sqrt{2C}$ is a sufficient condition for the negative relation between preference uncertainty and the region of independence. For $\beta\theta > \sqrt{2C}$, numerical simulations

[14]Formally, the probability of independence (i.e. no government interference) equals $p_I \equiv \Phi\left(\bar{z}\right) - \Phi\left(\underline{z}\right)$, so $\frac{\mathrm{d}\,p_I}{\mathrm{d}\,\kappa} = \frac{1}{\sqrt{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}}\left(\phi\left(\bar{z}\right)\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\kappa} - \phi\left(\underline{z}\right)\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\kappa}\right) < 0$.

[15]Interestingly, economic secrecy is not only desired by the central bank but it is also preferred by the government at the beginning of the game, because it gives rise to lower inflation without affecting average output due to rational private sector inflation expectations.

[16]To see this, note that $\underline{m}$ and $\bar{m}$ only depend on κ and σ_v^2 through $(1-\kappa)\,\sigma_v^2$, so a drop in κ has qualitatively the same effect as an increase in σ_v^2.

indicate that $\bar{m} - \underline{m}$ still tends to be decreasing in σ_τ^2, although it can be non-monotonic for small σ_τ^2.

In the limiting case of perfect preference transparency ($\sigma_\tau^2 \to 0$), no finite boundaries $\underline{m}$ and $\bar{m}$ exist.[17] With perfect preference transparency ($\sigma_\tau^2 \to 0$), the central bank's inflation target converges to the government's target $\bar{\tau}$ and the central bank enjoys complete independence for $\beta\theta \leq \sqrt{2C}$. Intuitively, the central bank's policy already gives an inflation rate of $\bar{\tau}$, so if the government's inflation bias $\beta\theta$ is sufficiently small, the benefit of overriding is less than the cost C. However, for $\beta\theta > \sqrt{2C}$ the government's expansionary preferences outweigh the overriding cost, so the government always interferes and the central bank has no independence under perfect preference transparency.

More generally, lower overriding costs reduce the independence of the central bank:

Proposition 3 *The region of independence $[\underline{m}, \bar{m}]$ is increasing in the overriding cost C.*

The proof is in Appendix A.2. This result is very intuitive. When the government faces a higher overriding cost it because more reluctant to interfere with monetary policy. So, the region of independence increases and the probability of independence rises as well.[18] As a result, average inflation declines when overriding costs increase.

The size of the region of independence is equal to

$$\bar{m} - \underline{m} = \frac{(1-\kappa)\,\sigma_v^2}{\sqrt{\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}} \left(\frac{\phi(\bar{z})}{1 - \Phi(\bar{z})} + \frac{\phi(\underline{z})}{\Phi(\underline{z})} \right) + 2\sqrt{2C}$$

This reveals that in the presence of economic opacity ($0 < \kappa < 1$), the size of the region of independence remains strictly positive even if the direct overriding cost C is zero. The reason is that the government cannot observe the velocity shock, so it faces uncertainty about the appropriate monetary policy stance. This makes the government reluctant to override the central bank, whose policy decision is based on superior economic information. Thus, economic opacity could serve as a substitute for direct overriding costs. In particular, a central bank that suffers from a government with low overriding costs C could envelop itself in economic secrecy to effectively make political interference more costly.

To summarize the (Bayesian Nash) equilibrium outcome of the model, the central bank's policy action is given by (6), (13), (17) and (18), and there is no overriding by the government. The properties of the central bank's region of independence are given by Propositions 1, 2 and 3.

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[17]To see this, note that $\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}$ has an asymptote of $\bar{z}$ as $\bar{m} \to \infty$, so for $\sigma_\tau^2 \to 0$ the right-hand side of (18) goes to $\beta\theta + \bar{m} + \sqrt{2C}$. This means that (18) yields no fixed point for $\bar{m}$. Similarly, the right-hand side of (17) goes to $\beta\theta + \underline{m} - \sqrt{2C}$ as $\sigma_\tau^2 \to 0$ so that there is no fixed point for $\underline{m}$.

[18]Formally, $\frac{\mathrm{d}\,p_I}{\mathrm{d}\,C} = \frac{1}{\sqrt{\sigma_\tau^2 + (1-\kappa)\sigma_v^2}} \left(\phi(\bar{z}) \frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,C} - \phi(\underline{z}) \frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,C} \right) > 0$.

4 Discussion

The model considered so far is based on several simplistic assumptions regarding the economic structure and the objective functions of the central bank and the government. It is now shown that the results in Propositions 1, 2 and 3 hold more generally. First, an extension of the model is considered with standard objective functions that exhibit a concern about the stabilization of both inflation and output. Second, a richer economic structure is discussed that also includes supply shocks.

Suppose that the government not only aims to stimulate output beyond the natural rate but also cares about output stabilization, so that

$$W_G = -\frac{1}{2}\alpha\left(\pi - \bar{\tau}\right)^2 - \frac{1}{2}\left(y - \bar{k}\bar{y}\right)^2 \tag{19}$$

where α denotes the concern for inflation stabilization ($\alpha > 0$) and $\bar{k}\bar{y}$ is the government's output target ($\bar{k} > 1$). Such a quadratic objective function is consistent with microfoundations and the assumption that the output target exceeds the natural rate ($\bar{k} > 1$) could be based on a plausible market imperfection such as imperfect competition. In addition, suppose that the central bank is no longer an 'inflation nutter' that puts no weight on output stabilization. Instead, the central bank cares as much about output stabilization as the government but it is 'responsible' in the sense that it does not attempt to stimulate output beyond the natural rate (Blinder 1997):

$$W_{CB} = -\frac{1}{2}\alpha\left(\pi - \tau\right)^2 - \frac{1}{2}\left(y - \bar{y}\right)^2 \tag{20}$$

Appendix A.3 derives the results for this model extension. It shows that the algebraic expressions become messier but Propositions 1 and 3 continue to hold. Proposition 2 also holds when the sufficient condition $\beta\theta \leq \sqrt{2C}$ is replaced by $\frac{\theta}{\sqrt{\alpha+\theta^2}}\left(\bar{k} - 1\right)\bar{y} \leq \sqrt{2C}$, which again means that the overriding cost C dominates the government's expansionary preferences ($\bar{k} > 1$).

Now consider a less simplistic economic structure. The simple money market equation (1) could be replaced by the quantity equation

$$\pi = m + v - y$$

It is straightforward to check that this only makes the expressions for the money supply m and the corresponding thresholds more complicated because of an additional intercept term, without affecting any of the qualitative economic results.

A more realistic economic structure would feature aggregate supply shocks ε, replacing (2) by

$$y = \bar{y} + \theta\left(\pi - \pi^e\right) + \varepsilon \tag{21}$$

The introduction of supply shocks ε has no effect on the conclusions of the model when there is symmetric information about the supply shocks. When the central bank has private information about the supply shocks ε, the results in the basic model of section 2 are not affected since ε does not affect the money supply m. In the extended model with the quadratic objectives (19) and (20), opacity about the supply shocks ε does influence the outcomes, but in a similar way to opacity about the velocity shocks v. In particular, when the degree of transparency κ is the same for the economic shocks ε and v, the results can simply be obtained by replacing v by $v_\varepsilon \equiv v + \frac{\theta}{\alpha+\theta^2}\varepsilon$ in all the algebraic expressions. So, Propositions 1, 2 and 3 continue to hold.

In addition, instead of the neo-monetarist framework in this paper, there could be an interest rate transmission mechanism. Then the monetary policy instrument is the interest rate and (1) would be replaced by an aggregate demand relation with demand shock d, while (21) could be interpreted as an expectations-augmented Phillips curve. In that case, aggregate demand shocks d and aggregate supply shocks ε matter for economic transparency, but otherwise the conclusions are similar.

It is useful to compare the results of this paper with Eijffinger and Hoeberichts (2002), who assume (19), (20) and (21). In contrast to Proposition 2, Eijffinger and Hoeberichts (2002) find that preference transparency decreases the expected region of independence. They model greater preference transparency as a reduction in uncertainty about the central bank's preference parameter for inflation stabilization α, which essentially makes the central bank more conservative. But this critically depends on how relative preference uncertainty is modelled.[19] Less uncertainty about the parameter for output stabilization would make the central bank less conservative and reverse the results. Using an 'unbiased' specification that does not distort average conservativeness, greater preference transparency would have no effect on average economic outcomes in the Eijffinger and Hoeberichts (2002) model. Since they (implicitly) assume economic transparency, this is consistent with the result in Proposition 2 that preference uncertainty does not affect the region of independence for $\kappa = 1$. So, the contribution of the present paper is that it establishes that economic (rather than preference) transparency reduces the region of independence for the central bank. Furthermore, it derives the novel result that economic opacity gives the central bank greater freedom from political pressures even if there is no direct overriding cost ($C = 0$).

Thus, this paper provides a theoretical argument for the observation that central banks could adopt secrecy to obtain greater independence.[20] An interesting example is the way the

[19]This was first pointed out by Beetsma and Jensen (2003). Geraats (2004) provides further details on the pitfalls of modeling relative preference uncertainty.

[20]For instance, Goodfriend (1986, p. 82) argues that "secrecy makes it more difficult for particular political

Federal Reserve under Chairman Paul Volcker managed to implement a painful disinflation policy during the early 1980s. The introduction of monetary targeting in October 1979 made it more difficult for Congress to assess whether high interest rates where due to restrictive monetary policy or market forces. The change in monetary operating procedures effectively made the monetary policy instrument a less reliable signal of the policy stance due to imperfect information about money market disturbances. So, Congress felt more reluctant to challenge the monetary policy actions of the Federal Reserve. As a result, the 'monetary veil' provided cover to pursue the disinflation without political interference.

The present paper suggests that central banks with lower independence benefit more from secrecy to fend off government intervention, so they are less likely to be transparent. Thus, it predicts a positive relation between central bank independence and transparency. To investigate this empirically, the comprehensive survey of central banks by Fry, Julius, Mahadeva, Roger and Sterne (2000) is used. Fry et al. (2000, Table 4.6) construct an index for 'policy explanations' based on twelve items covering explanations of policy decisions, forecasts and forward looking analysis, and policy assessments and research. This measure is used as a proxy for economic transparency.[21] In addition, Fry et al. (2000, Table 4.4) provide an index for central bank independence that captures statutory objectives of price stability, goal and instrument independence, limits on monetary financing of budget deficits, and the length of central bankers' term of office. It also comprises a separate measure for instrument independence. Data is available for 92 countries.

Table 1: Relation between central bank transparency and independence.

Correlation with transparency [p-value]	Full sample	Excl. fixed FX	Fixed FX
Independence	0.430 [<0.001]	0.450 [<0.001]	0.261 [0.157]
Instrument independence	0.339 [0.001]	0.392 [0.002]	0.186 [0.316]
Sample size	92	61	31

Table 1 shows that there is a statistically significant positive correlation between transparency and central bank independence (with p-values in brackets). Using the more specific measure of instrument independence gives the same finding. This is consistent with the theoretical prediction of this paper that central banks with lower independence are likely to display lower transparency.

However, there is an alternative, public policy argument that also generates a positive relation between central bank independence and transparency. Institutional independence requires public accountability to safeguard democratic legitimacy, and accountability requires

groups to pressure the Federal Reserve regarding current policy actions".

[21] Three out of twelve items do not pertain to economic transparency and have a weight of 15.5%. Reconstructing the index to get a more accurate measure of economic transparency yields similar conclusions.

transparency. Fortunately, it is possible to distinguish between this public policy motive and the economic explanation advanced in this paper. The former should always apply regardless of the monetary policy framework, whereas the latter relies on the presence of discretionary monetary policy. In particular, the economic argument does not apply to countries that commit to a fixed exchange rate.

Table 1 shows that there is indeed a marked difference between countries with and without a fixed exchange rate regime. The correlation between transparency and (instrument) independence remains positive and highly significant for countries without a fixed exchange rate, but it is much weaker for countries that have abandoned discretion over monetary policy by the adoption of a fixed exchange rate regime.[22] These findings provide some tentative empirical support for the economic argument formalized in this paper that the positive relationship between central bank independence and transparency is caused by the use of secrecy to limit political interference.

Finally, it should be emphasized that the present paper analyzes the optimal degree of transparency for a given institutional framework. The override mechanism captures the lack of complete instrument independence that used to be prevalent and still applies to many developing countries. The seminal contributions by Walsh (1995) and Svensson (1997) suggest better institutional frameworks through contracting and inflation targeting. An interesting topic is the joint optimality of disclosure policy and institutional settings, but this is left for future research.

5 Conclusion

The new paradigm in monetary policy of central bank independence and transparency has rapidly gained ground. This paper cautions that transparency may not be beneficial without central bank independence. In particular, uncertainty about the economic information to which the central bank responds makes politicians more cautious about intervening in monetary policy because it is harder to interpret the central bank's actions. As a result, economic secrecy effectively gives the central bank greater political independence.

This paper has formalized this argument using a monetary policy game in which a conservative or responsible central bank without complete independence sets monetary policy. The government, which aims to stimulate output beyond the natural rate, can override the monetary policy decision, but this involves a direct override cost. The government's decision

[22]Rank correlations of transparency with independence and instrument independence give similar results: 0.504 [$<$0.001] and 0.373 [$<$0.001] for the full sample; 0.483 [$<$0.001] and 0.381 [0.003] excluding fixed exchange rates; and 0.360 [0.047] and 0.323 [0.073] for countries with a fixed exchange rate regime.

to override the central bank is complicated by the presence of uncertainty about the central bank's intentions and imperfect information about the economic situation. It is shown that the region of independence enjoyed by the central bank is declining in the degree of economic transparency and in the amount of preference uncertainty. Intuitively, economic transparency reduces the government's uncertainty about whether to override and how to set the policy instrument, so it makes the government less inhibited to interfere with monetary policy. Greater preference uncertainty makes the central bank's policy action a more useful signal of its intentions, so the government becomes more sensitive to it and leaves the central bank less leeway before overriding. The region of independence is increasing in the overriding cost for the government. More interestingly, this paper obtains the new result that even in the absence of a direct overriding cost, the size of the region of independence is strictly positive when there is economic opacity. Intuitively, if the government feels uninhibited to interfere with monetary policy, the central bank could effectively make overriding costly by depriving the government of important economic information. Thus, the central bank could insulate itself from political pressures by enveloping itself in economic secrecy.

The model generates the theoretical prediction that central banks with lower independence are more likely to display less transparency. Empirically, there is indeed a strong positive correlation between central bank independence and transparency. But this could also be for public policy reasons as central bank independence requires accountability and therefore transparency. Interestingly, the positive relation between independence and transparency does not hold for countries that maintain a fixed exchange rate regime. This seems at odds with the public policy argument, but it supports the economic explanation advanced in this paper, which relies on discretionary monetary policy.

The main conclusion of the paper is that economic opacity could be beneficial if the central bank lacks instrument independence because it makes it more difficult for the government to interfere with monetary policy. This helps to explain the past practice of independence-through-secrecy. The paper also has policy implications for countries that wish to adopt the new paradigm of central bank independence-cum-transparency. It is important to ensure that the central bank has political independence before insisting on economic transparency, since monetary mystique is an effective way to prevent political pressures.

A Appendix

This appendix derives the no-override condition (11) in the basic model of section A.1 with objective functions (3) and (4). The proofs to Propositions 1, 2 and 3 are in section A.2. The derivation of the results for the extended model with objective functions (19) and (20) is in section A.3.

A.1 No-override condition

The condition for no government interference is given by (10):

$$\mathrm{E}\left[W_G\left(m_O\right)|m_{CB}\right] - C \leq \mathrm{E}\left[W_G\left(m_{CB}\right)|m_{CB}\right]$$

This is equivalent to $\mathrm{E}\left[D|m_{CB}\right] \leq C$, where $D \equiv W_G\left(m_O\right) - W_G\left(m_{CB}\right)$. Substitute (2) and (1) into (3) to get

$$W_G = -\frac{1}{2}\left(m + v - \bar{\tau}\right)^2 + \beta\theta\left(m + v - \pi^e\right)$$

So,

$$D = -\frac{1}{2}\left(\left(m_O\right)^2 - \left(m_{CB}\right)^2\right) + \left(m_O - m_{CB}\right)\left(\bar{\tau} + \beta\theta\right) - \left(m_O - m_{CB}\right)v$$

Substituting (9) and rearranging,

$$D = \frac{1}{2}\left(\bar{\tau} + \beta\theta - m_{CB}\right)^2 - \frac{1}{2}\left(\mathrm{E}\left[v|m_{CB}\right]\right)^2 - \left(\bar{\tau} + \beta\theta - \mathrm{E}\left[v|m_{CB}\right] - m_{CB}\right)v$$

Taking expectations and simplifying gives

$$\begin{aligned} \mathrm{E}\left[D|m_{CB}\right] &= \frac{1}{2}\left(\bar{\tau} + \beta\theta - m_{CB}\right)^2 + \frac{1}{2}\left(\mathrm{E}\left[v|m_{CB}\right]\right)^2 - \left(\bar{\tau} + \beta\theta - m_{CB}\right)\mathrm{E}\left[v|m_{CB}\right] \\ &= \frac{1}{2}\left(\bar{\tau} + \beta\theta - \mathrm{E}\left[v|m_{CB}\right] - m_{CB}\right)^2 \end{aligned}$$

Hence, (10) if and only if (11).

A.2 Proof of Proposition 1, 2 and 3

To facilitate the derivation of results for the extended model of section 4, this section proves Propositions 1, 2 and 3 for a general model in which the no-override condition is

$$\frac{1}{2}b\left(B - \mathrm{E}\left[v|m_{CB}\right] - m_{CB}\right)^2 \leq C \tag{22}$$

So, the thresholds of the region of independence are determined by

$$\bar{m} = B - \mathrm{E}\left[v|\tilde{m} \geq \bar{m}\right] + \sqrt{2C/b} \tag{23}$$

$$\underline{m} = B - \mathrm{E}\left[v|\tilde{m} \leq \underline{m}\right] - \sqrt{2C/b} \tag{24}$$

The central bank's money supply without political pressures is assumed to satisfy $\tilde{m}|s \sim N\left(A - s, a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2\right)$. The corresponding expected velocity shock equals

$$\mathrm{E}\left[v|\tilde{m} \geq \bar{m}\right] = s - \frac{(1-\kappa)\,\sigma_v^2}{\sqrt{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}} \frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)} \tag{25}$$

$$\mathrm{E}\left[v|\tilde{m} \leq \underline{m}\right] = s + \frac{(1-\kappa)\,\sigma_v^2}{\sqrt{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}} \frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)} \tag{26}$$

and the normalized thresholds are

$$\bar{z} \equiv \frac{\bar{m} - (A-s)}{\sqrt{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}} \text{ and } \underline{z} \equiv \frac{\underline{m} - (A-s)}{\sqrt{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}} \tag{27}$$

The coefficients are assumed to satisfy $B > A$, $b > 0$ and $a > 0$. For the basic model of section 2, $B = \bar{\tau} + \beta\theta$, $A = \bar{\tau}$ and $b = a = 1$.

The proofs of Propositions 1, 2 and 3 make use of the following two results:

Lemma 1 *The function* $\frac{\phi(z)}{1-\Phi(z)}$ *is convex and has the property that* $0 < \frac{\mathrm{d}}{\mathrm{d}\,z}\frac{\phi(z)}{1-\Phi(z)} < 1$ *for* $z \in \mathbb{R}$.

Proof. See Sampford (1953). ■
Note that $\frac{\phi(z)}{1-\Phi(z)}$ is increasing, with a horizontal asymptote of 0 as $z \to -\infty$ and an asymptote of z as $z \to \infty$.

Lemma 2 *The function* $\frac{\phi(z)}{\Phi(z)}$ *is convex and has the property that* $-1 < \frac{\mathrm{d}}{\mathrm{d}\,z}\frac{\phi(z)}{\Phi(z)} < 0$ *for* $z \in \mathbb{R}$.

Proof. Using the fact that $\phi(z) = \phi(-z)$ and $\Phi(z) = 1 - \Phi(-z)$, $\frac{\phi(z)}{\Phi(z)} = \frac{\phi(-z)}{1-\Phi(-z)}$. So, the result is a corollary of Lemma 1. ■
Note that $\frac{\phi(z)}{\Phi(z)}$ is decreasing, with an asymptote of $-z$ as $z \to -\infty$ and a horizontal asymptote of 0 as $z \to \infty$.

Proposition 3 is derived first since it generates a result that is used to prove Proposition 1.

Proof of Proposition 3:
Differentiate (23) with respect to C and use (27) to get

$$\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,C} = \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)} \frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,C} + \frac{1}{\sqrt{2bC}}$$

Rearranging gives

$$\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,C} = \frac{1}{1 - \frac{(1-\kappa)\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}} \frac{1}{\sqrt{2bC}} > 0 \tag{28}$$

using Lemma 1.

Similarly, differentiate (24) with respect to C and use (27) to get

$$\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,C} = -\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,C} - \frac{1}{\sqrt{2bC}}$$

Rearranging gives

$$\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,C} = -\frac{1}{1+\frac{(1-\kappa)\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}}\frac{1}{\sqrt{2bC}} < 0 \tag{29}$$

using Lemma 2.

As a result, $\frac{\mathrm{d}(\bar{m}-\underline{m})}{\mathrm{d}\,C} > 0$ so that the region of independence $[\underline{m}, \bar{m}]$ is increasing in the override cost C. ■

Proof of Proposition 1:

The proof proceeds in two parts. First it is shown that $\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\kappa} < 0$, and then that $\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\kappa} > 0$.

(I) Differentiate (23) with respect to κ using (25) and (27) to get

$$\begin{aligned}\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\kappa} \;=\; & \frac{-\sigma_v^2\left(a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2\right)-\frac{1}{2}(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} \\ & +\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\left(\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\kappa}+\frac{1}{2}\sigma_v^2\frac{\bar{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)\end{aligned}$$

This already gives $\left.\frac{d\bar{m}}{d\kappa}\right|_{\kappa=1} = -\frac{b\sigma_v^2}{a\sigma_\tau}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} < 0$. Rearranging yields

$$\begin{aligned}\left(1-\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\kappa} = & -\frac{1}{2}\frac{\sigma_v^2\left(2a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2\right)}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} \\ & +\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\frac{\bar{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2} \equiv \bar{R}\end{aligned}$$

Note that the left-hand-side factor is strictly positive by Lemma 1 so that $\operatorname{sgn}(\mathrm{d}\,\bar{m}/\mathrm{d}\,\kappa) = \operatorname{sgn}\bar{R}$. The first term on the right-hand side is strictly negative, whereas the second term is positive using Lemma 1 and the fact that (23) and (25) imply $\bar{m} > B-s > A-s$. Substituting (23) and (25) and rearranging gives

$$\begin{aligned}\bar{R} \;=\; & -\frac{1}{2}\frac{\sigma_v^2\left(2a^2\sigma_\tau^2+\left(1-\frac{(1-\kappa)\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)(1-\kappa)\,\sigma_v^2\right)}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} \\ & +\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\frac{B-A+\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\end{aligned}$$

Using Lemma 1 and $B > A$, the first term is strictly negative, whereas the second term is positive. To determine $\operatorname{sgn} \bar{R}$ it is useful to consider a more tractable upper bound on $\bar{R}$ that can be obtained using Lemma 1:

$$\begin{aligned} \bar{R} &< -\frac{\sigma_v^2 a^2 \sigma_\tau^2}{\left(\sqrt{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}\right)^3} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})} + \frac{1}{2}\sigma_v^2 \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right) \frac{B - A + \sqrt{2C/b}}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \\ &< -\frac{\sigma_v^2 a^2 \sigma_\tau^2}{\left(\sqrt{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}\right)^3} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})} + \frac{1}{2}\sigma_v^2 \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \frac{B - A + \sqrt{2C/b}}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \equiv R_{\max} \qquad (30) \end{aligned}$$

It is now shown that $R_{\max} < 0$. Note that $\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}$ is increasing in $\bar{z}$ (by Lemma 1), and $\bar{z}$ is increasing in C (using (28)), so the first term of $R_{\max}$ is decreasing in C, whereas the second term is increasing in C. Since $\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}$ has an asymptote of $\bar{z}$ as $\bar{z} \to \infty$, and $\bar{m} \to \infty$ as $C \to \infty$ by (23), the first term of $R_{\max}$ dominates the second term as $C \to \infty$. So, $R_{\max} < 0$ for sufficiently large C. Since $R_{\max}$ may be non-monotonic, critical point(s) at which $\mathrm{d}\, R_{\max} / \mathrm{d}\, C = 0$ (if any) need to be checked to assess whether $R_{\max} < 0$ for all C. Substituting (28), the first order condition equals

$$\frac{a^2\sigma_v^2\sigma_\tau^2}{\left(a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2\right)^2} \left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right) \frac{1}{1 - \frac{(1-\kappa)\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\sigma_v^2} \frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})}} \frac{1}{\sqrt{2bC}} = \frac{1}{2}\sigma_v^2 \frac{(1-\kappa)\,\sigma_v^2}{\left(a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2\right)^2} \frac{1}{\sqrt{2bC}}$$

Rearranging and simplifying yields

$$\begin{aligned} 2a^2\sigma_\tau^2 \frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})} &= (1-\kappa)\,\sigma_v^2 \left(1 - \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right) \\ \left(\frac{2a^4\sigma_\tau^4 + 2a^2\sigma_\tau^2\,(1-\kappa)\,\sigma_v^2 + (1-\kappa)^2\,\sigma_v^4}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}\right) \frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})} &= (1-\kappa)\,\sigma_v^2 \\ \frac{\mathrm{d}}{\mathrm{d}\,\bar{z}} \frac{\phi(\bar{z})}{1-\Phi(\bar{z})} &= \frac{(1-\kappa)\,\sigma_v^2}{\left(\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2 + (1-\kappa)\sigma_v^2} + 1\right) a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2} \qquad (31) \end{aligned}$$

The left-hand side has a range of $(0,1)$ and it is monotonic in $\bar{z}$ as $\frac{\phi(z)}{1-\Phi(z)}$ is convex (Lemma 1), so (31) holds for exactly one $\bar{z}$ for $0 \le \kappa < 1$. If this implies a complex number for C, no (real) critical point exists and $R_{\max} < 0$. Otherwise, there is one critical point C^c and it is straightforward to check that $\mathrm{d}\, R_{\max} / \mathrm{d}\, C$ goes from positive to negative at C^c so that it represents a maximum. To evaluate $R_{\max}$ at C^c the corresponding expression for $\frac{\phi(z)}{1-\Phi(z)}$ is required. To this end, differentiate $\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}$ with respect to $\bar{z}$ and substitute (27), (23) and (25) to get[23]

$$\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)} = \left(-\bar{z}+\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)}\right)\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)}$$
$$= \left(-\frac{B-A+\sqrt{2C/b}}{\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}}-\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)}+\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)}\right)\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)}$$

Substituting this into (31) and rearranging gives

$$\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\phi\left(\bar{z}\right)}{1-\Phi\left(\bar{z}\right)}=\frac{(1-\kappa)\,\sigma_v^2}{\left(\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}+1\right)a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{1-\Phi\left(\bar{z}\right)}{\phi\left(\bar{z}\right)}+\frac{B-A+\sqrt{2C/b}}{\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}}$$

Substituting this into (30) gives

$$R^c_{\max} = -\frac{\sigma_v^2}{\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}}\frac{(1-\kappa)\,\sigma_v^2}{\left(\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}+1\right)a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{1-\Phi\left(\bar{z}\right)}{\phi\left(\bar{z}\right)}$$
$$-\sigma_v^2\left(1-\frac{1}{2}\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)\frac{B-A+\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}$$

Recall that $B > A$, so, $R^c_{\max} < 0$ at the critical point (if any). As a result, $\bar{R} < 0$ for all C. Therefore, $\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\kappa} < 0$.

(II) Differentiate (24) with respect to κ using (26) and (27) to get

$$\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\kappa} = -\frac{-\sigma_v^2\left(a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2\right)-\frac{1}{2}(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}$$
$$-\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\left(\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\kappa}+\frac{1}{2}\sigma_v^2\frac{\underline{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)$$

This already gives $\left.\frac{d\underline{m}}{d\kappa}\right|_{\kappa=1}=\frac{b\sigma_v^2}{a\sigma_\tau}\frac{\phi(\underline{z})}{\Phi(\underline{z})} > 0$. Rearranging yields

$$\left(1+\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\kappa}=\frac{1}{2}\frac{\sigma_v^2\left(2a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2\right)}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}$$
$$-\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\frac{\underline{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\equiv\underline{R}$$

Note that the left-hand-side factor is strictly positive by Lemma 2 so that $\operatorname{sgn}\left(\mathrm{d}\,\underline{m}/\,\mathrm{d}\,\kappa\right) = \operatorname{sgn}\underline{R}$. The first term on the right-hand side is strictly positive, whereas the second term is

[23]Use the fact that $\phi'(z) = -z\phi(z)$ and $\Phi'(z) = \phi(z)$.

ambiguous. Substituting (24) and (26) and rearranging gives

$$\begin{aligned}\underline{R} &= \frac{1}{2}\frac{\sigma_v^2\left(2a^2\sigma_\tau^2+\left(1+\frac{(1-\kappa)\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)(1-\kappa)\,\sigma_v^2\right)}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\\ &\quad -\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\frac{B-A-\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\end{aligned}$$

Using Lemma 2, the first term is strictly positive and the second term is also positive if $B-A>\sqrt{2C/b}$. So, $B-A\geq\sqrt{2C/b}$ is a sufficient condition for $\underline{R}>0$. To determine $\operatorname{sgn}\underline{R}$ for $B-A<\sqrt{2C/b}$ it is useful to consider a more tractable lower bound on $\underline{R}$ that can be obtained using Lemma 2:

$$\begin{aligned}\underline{R} &> \frac{\sigma_v^2a^2\sigma_\tau^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}-\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\frac{B-A-\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\\ &> \frac{\sigma_v^2a^2\sigma_\tau^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}+\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{B-A-\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\equiv R_{\min}\end{aligned} \quad (32)$$

It is now shown that $R_{\min}>0$ for $\sqrt{2C/b}>B-A$. First, note that $R_{\min}$ is strictly positive for $C=0$ as $B>A$. In addition, $\frac{\phi(\underline{z})}{\Phi(\underline{z})}$ is decreasing in $\underline{z}$ (by Lemma 2), and $\underline{z}$ is decreasing in C (using (29)), so the first term of $R_{\min}$ is increasing in C, whereas the second term is decreasing in C. Since $\frac{\phi(\underline{z})}{\Phi(\underline{z})}$ has an asymptote of $-\underline{z}$ as $\underline{z}\to-\infty$, and $\underline{m}\to-\infty$ as $C\to\infty$ by (24), the first term of $R_{\min}$ dominates the second term as $C\to\infty$. So, $R_{\min}>0$ for sufficiently large C. Since $R_{\min}$ may be non-monotonic, critical point(s) at which $\mathrm{d}\,R_{\min}/\,\mathrm{d}\,C=0$ (if any) need to be checked to assess whether $R_{\min}>0$ for all C. Substituting (29), the first order condition equals

$$-\frac{\sigma_v^2a^2\sigma_\tau^2}{\left(a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2\right)^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\frac{1}{1+\frac{(1-\kappa)\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}}\frac{1}{\sqrt{2bC}}=\frac{1}{2}\sigma_v^2\frac{(1-\kappa)\,\sigma_v^2}{\left(a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2\right)^2}\frac{1}{\sqrt{2bC}}$$

Rearranging and simplifying yields

$$\begin{aligned}-2a^2\sigma_\tau^2\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)} &= (1-\kappa)\,\sigma_v^2\left(1+\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)}\right)\\ -\left(\frac{2a^4\sigma_\tau^4+2a^2\sigma_\tau^2\,(1-\kappa)\,\sigma_v^2+(1-\kappa)^2\,\sigma_v^4}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)} &= (1-\kappa)\,\sigma_v^2\\ \frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi\left(\underline{z}\right)}{\Phi\left(\underline{z}\right)} &= -\frac{(1-\kappa)\,\sigma_v^2}{\left(\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}+1\right)a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\end{aligned} \quad (33)$$

The left-hand side has a range of $(-1, 0)$ and it is monotonic in $\underline{z}$ as $\frac{\phi(\underline{z})}{\Phi(\underline{z})}$ is convex (Lemma 2), so (33) holds for exactly one $\underline{z}$ for $0 \leq \kappa < 1$. If this implies a complex number for C, no (real) critical point exists and $R_{\min} > 0$. Otherwise, there is one critical point C^c and it is straightforward to check that $\mathrm{d}\, R_{\min}/\,\mathrm{d}\, C$ goes from negative to positive at C^c so that it represents a minimum. To evaluate $R_{\min}$ at C^c the corresponding expression for $\frac{\phi(\underline{z})}{\Phi(\underline{z})}$ is required . To this end, differentiate $\frac{\phi(\underline{z})}{\Phi(\underline{z})}$ with respect to $\underline{z}$ and substitute (27), (24) and (26) to get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})} &= -\left(\underline{z}+\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\frac{\phi(\underline{z})}{\Phi(\underline{z})} \\ &= -\left(\frac{B-A-\sqrt{2C/b}}{\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}}-\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\phi(\underline{z})}{\Phi(\underline{z})}+\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\frac{\phi(\underline{z})}{\Phi(\underline{z})}\end{aligned}$$

Substituting this into (33) and rearranging gives

$$\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\phi(\underline{z})}{\Phi(\underline{z})}=\frac{(1-\kappa)\,\sigma_v^2}{\left(\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}+1\right)a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\Phi(\underline{z})}{\phi(\underline{z})}-\frac{B-A-\sqrt{2C/b}}{\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}}$$

Substituting this into (32) gives

$$\begin{aligned}R_{\min}^c &= \frac{\sigma_v^2}{\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}}\frac{(1-\kappa)\,\sigma_v^2}{\left(\frac{a^2\sigma_\tau^2}{a^2\sigma_\tau^2+(1-\kappa)\sigma_v^2}+1\right)a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\Phi(\underline{z})}{\phi(\underline{z})} \\ &\quad -\sigma_v^2\left(1-\frac{1}{2}\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)\frac{B-A-\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\end{aligned}$$

So, for $\sqrt{2C/b} > B-A$, $R_{\min}^c > 0$ at the critical point (if any). This implies that $R_{\min} > 0$ and thereby $\underline{R} > 0$ for $B-A < \sqrt{2C/b}$. As a result, $\underline{R} > 0$ for all $C \geq 0$. Therefore, $\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\kappa} > 0$.

Finally, combining the results under (I) and (II) yields $\frac{\mathrm{d}(\bar{m}-\underline{m})}{\mathrm{d}\,\kappa} < 0$, so that the region of independence is decreasing in the degree of economic transparency. ■

Proof of Proposition 2:

The proof proceeds in two parts. First it is shown that $\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\sigma_\tau^2} < 0$, and then that $\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\sigma_\tau^2} > 0$.

(I) Differentiate (23) with respect to σ_τ^2 using (25) and (27) to get

$$\begin{aligned}\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\sigma_\tau^2} &= -\frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})} \\ &\quad +\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\left(\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\sigma_\tau^2}-\frac{1}{2}a^2\frac{\bar{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)\end{aligned}$$

This gives $\left.\frac{d\bar{m}}{d\sigma_\tau^2}\right|_{\kappa=1} = 0$. Rearranging yields

$$\left(1 - \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\sigma_\tau^2} = -\frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}$$
$$-\frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\bar{z}}\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\right)\frac{\bar{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}$$

Note that the left-hand-side factor is strictly positive by Lemma 1. In addition, the right-hand side is (strictly) negative (for $\kappa \neq 1$) using Lemma 1 and the fact that (23) and (25) imply $\bar{m} > B - s > A - s$. As a result, $\frac{\mathrm{d}\,\bar{m}}{\mathrm{d}\,\sigma_\tau^2} < 0$ for $\kappa \neq 1$.

(II) Differentiate (24) with respect to σ_τ^2 using (26) and (27) to get

$$\begin{aligned}\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\sigma_\tau^2} &= \frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\underline{z})}{\Phi(\underline{z})} \\ &\quad -\frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\left(\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\sigma_\tau^2} - \frac{1}{2}a^2\frac{\underline{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)\end{aligned}$$

This gives $\left.\frac{d\underline{m}}{d\sigma_\tau^2}\right|_{\kappa=1} = 0$. Rearranging yields

$$\left(1 + \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2 + (1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\sigma_\tau^2} = \frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\underline{z})}{\Phi(\underline{z})}$$
$$+\frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\frac{\bar{m}-(A-s)}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2} \equiv \underline{R}$$

Note that the left-hand-side factor is strictly positive by Lemma 2. So, $\operatorname{sgn}(\mathrm{d}\,\underline{m}/\,\mathrm{d}\,\sigma_\tau^2) = \operatorname{sgn}\underline{R}$. Substituting (24) and (26) and rearranging gives

$$\begin{aligned}\underline{R} &= \frac{1}{2}\left(1 - \frac{(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\frac{a^2(1-\kappa)\,\sigma_v^2}{\left(\sqrt{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\right)^3}\frac{\phi(\underline{z})}{\Phi(\underline{z})} \\ &\quad +\frac{1}{2}\frac{a^2(1-\kappa)\,\sigma_v^2}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,\underline{z}}\frac{\phi(\underline{z})}{\Phi(\underline{z})}\right)\frac{B-A-\sqrt{2C/b}}{a^2\sigma_\tau^2+(1-\kappa)\,\sigma_v^2}\end{aligned}$$

Using Lemma 2, the first term is strictly positive and the second term is also positive if $B - A < \sqrt{2C/b}$. So, $B - A \leq \sqrt{2C/b}$ is a sufficient condition for $\underline{R} > 0$ ($\kappa \neq 1$). However, for $B - A > \sqrt{2C/b}$, $\underline{R} < 0$ is possible. Therefore, $\frac{\mathrm{d}\,\underline{m}}{\mathrm{d}\,\sigma_\tau^2} > 0$ for $B - A \leq \sqrt{2C/b}$ and $\kappa \neq 1$.

Finally, combining the results under (I) and (II) yields that $\frac{\mathrm{d}(\bar{m}-\underline{m})}{\mathrm{d}\,\sigma_\tau^2} = 0$ for $\kappa = 1$, so the amount of preference uncertainty is immaterial for the region of independence with perfect economic transparency. For $0 \leq \kappa < 1$, $\frac{\mathrm{d}(\bar{m}-\underline{m})}{\mathrm{d}\,\sigma_\tau^2} < 0$ for $B - A \leq \sqrt{2C/b}$, so the region of independence is decreasing in the amount of preference uncertainty when the overriding cost is not too small. Note that for the basic model in section 2 the sufficient condition reduces to $\beta\theta \leq \sqrt{2C}$. ■

A.3 Derivations for extended model

This appendix derives the results for the extended model of section 4 with objective functions (19) and (20). The condition for no government interference is still equal to (10), which is equivalent to $\mathrm{E}[D|m_{CB}] \leq C$, where $D \equiv W_G(m_O) - W_G(m_{CB})$. Substitute (2) and (1) into (19) to get

$$W_G = -\frac{1}{2}\alpha(m + v - \bar{\tau})^2 - \frac{1}{2}\left(\theta(m + v - \pi^e) - \left(\bar{k} - 1\right)\bar{y}\right)^2 \tag{34}$$

So,

$$\begin{aligned} D &= -\frac{1}{2}\left(\alpha + \theta^2\right)\left((m_O)^2 - (m_{CB})^2\right) + (m_O - m_{CB})\left(\alpha\bar{\tau} + \theta^2\pi^e + \theta\left(\bar{k} - 1\right)\bar{y}\right) \\ &\quad - (m_O - m_{CB})\left(\alpha + \theta^2\right)v \end{aligned} \tag{35}$$

The policy action desired by the government follows from maximization of $\mathrm{E}[W_G|m_{CB}]$ using (34), subject to (2) and (1) and given π^e:

$$m_O = \frac{\alpha}{\alpha + \theta^2}\bar{\tau} + \frac{\theta^2}{\alpha + \theta^2}\pi^e + \frac{\theta}{\alpha + \theta^2}\left(\bar{k} - 1\right)\bar{y} - \mathrm{E}[v|m_{CB}] \tag{36}$$

Substituting (36) into (35) and rearranging,

$$\begin{aligned} D &= \frac{1}{2}\left(\alpha + \theta^2\right)\left(\frac{\alpha}{\alpha + \theta^2}\bar{\tau} + \frac{\theta^2}{\alpha + \theta^2}\pi^e + \frac{\theta}{\alpha + \theta^2}\left(\bar{k} - 1\right)\bar{y} - m_{CB}\right)^2 - \frac{1}{2}\left(\alpha + \theta^2\right)\left(\mathrm{E}[v|m_{CB}]\right)^2 \\ &\quad - \left(\alpha + \theta^2\right)\left(\frac{\alpha}{\alpha + \theta^2}\bar{\tau} + \frac{\theta^2}{\alpha + \theta^2}\pi^e + \frac{\theta}{\alpha + \theta^2}\left(\bar{k} - 1\right)\bar{y} - \mathrm{E}[v|m_{CB}] - m_{CB}\right)v \end{aligned}$$

Taking expectations and simplifying gives

$$\mathrm{E}[D|m_{CB}] = \frac{1}{2}\left(\alpha + \theta^2\right)\left(\frac{\alpha}{\alpha + \theta^2}\bar{\tau} + \frac{\theta^2}{\alpha + \theta^2}\pi^e + \frac{\theta}{\alpha + \theta^2}\left(\bar{k} - 1\right)\bar{y} - \mathrm{E}[v|m_{CB}] - m_{CB}\right)^2$$

Hence, the no-override condition equals (22) with $B = \frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e + \frac{\theta}{\alpha+\theta^2}\left(\bar{k} - 1\right)\bar{y}$ and $b = \alpha + \theta^2$.

The central bank now maximizes (20) subject to (2) and (1) and given π^e, so in the absence of political pressure it would implement

$$\tilde{m} = \frac{\alpha}{\alpha + \theta^2}\tau + \frac{\theta^2}{\alpha + \theta^2}\pi^e - v$$

This means that the expressions for $\mathrm{E}[v|m_{CB}]$ are affected. Using joint normality of $\tilde{m}$ and v,

$$\begin{aligned}\mathrm{E}[v|\tilde{m}] &= s - \frac{(1-\kappa)\sigma_v^2}{\frac{\alpha^2}{(\alpha+\theta^2)^2}\sigma_\tau^2 + (1-\kappa)\sigma_v^2}\left(\tilde{m} + s - \frac{\alpha}{\alpha+\theta^2}\bar{\tau} - \frac{\theta^2}{\alpha+\theta^2}\pi^e\right) \\ &= \lambda_2 s - (1-\lambda_2)\left(\tilde{m} - \frac{\alpha}{\alpha+\theta^2}\bar{\tau} - \frac{\theta^2}{\alpha+\theta^2}\pi^e\right) \qquad (37)\end{aligned}$$

where $1-\lambda_2 \equiv \frac{(1-\kappa)\sigma_v^2}{\frac{\alpha^2}{(\alpha+\theta^2)^2}\sigma_\tau^2+(1-\kappa)\sigma_v^2}$. Similarly,

$$\begin{aligned}\mathrm{E}[v|\tilde{m} \geq \bar{m}] &= \lambda_2 s + (1-\lambda_2)\left(\frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e\right) - (1-\lambda_2)\,\mathrm{E}[\tilde{m}|\tilde{m} \geq \bar{m}] \\ &= s - (1-\lambda_2)\sqrt{\frac{\alpha^2}{(\alpha+\theta^2)^2}\sigma_\tau^2 + (1-\kappa)\sigma_v^2}\,\frac{\phi(\bar{z})}{1-\Phi(\bar{z})}\end{aligned}$$

$$\begin{aligned}\mathrm{E}[v|\tilde{m} \leq \underline{m}] &= \lambda_2 s + (1-\lambda_2)\left(\frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e\right) - (1-\lambda_2)\,\mathrm{E}[\tilde{m}|\tilde{m} \leq \underline{m}] \\ &= s + (1-\lambda_2)\sqrt{\frac{\alpha^2}{(\alpha+\theta^2)^2}\sigma_\tau^2 + (1-\kappa)\sigma_v^2}\,\frac{\phi(\underline{z})}{\Phi(\underline{z})}\end{aligned}$$

where the normalized thresholds now equal

$$\bar{z} \equiv \frac{\bar{m} - \left(\frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e - s\right)}{\sqrt{\frac{\alpha^2}{(\alpha+\theta^2)^2}\sigma_\tau^2 + (1-\kappa)\sigma_v^2}} \text{ and } \underline{z} \equiv \frac{\underline{m} - \left(\frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e - s\right)}{\sqrt{\frac{\alpha^2}{(\alpha+\theta^2)^2}\sigma_\tau^2 + (1-\kappa)\sigma_v^2}}$$

Hence, for $a = \frac{\alpha}{\alpha+\theta^2}$ and $A = \frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e$ the expected velocity shock satisfies (25) and (26) and the normalized thresholds equal (27).

As a result, the extension of the model in section 4 is identical to the general model of appendix A.2 for $B = \frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e + \frac{\theta}{\alpha+\theta^2}\left(\bar{k}-1\right)\bar{y}$, $A = \frac{\alpha}{\alpha+\theta^2}\bar{\tau} + \frac{\theta^2}{\alpha+\theta^2}\pi^e$, $b = \alpha+\theta^2$ and $a = \frac{\alpha}{\alpha+\theta^2}$, and satisfies the conditions $B > A$, $b > 0$ and $a > 0$. Therefore, Propositions 1 and 3 continue to hold for the model extension. Proposition 3 also holds when the sufficient condition $\beta\theta \leq \sqrt{2C}$ is replaced by $B - A \leq \sqrt{2C/b}$, which reduces to $\frac{\theta}{\sqrt{\alpha+\theta^2}}\left(\bar{k}-1\right)\bar{y} \leq \sqrt{2C}$.

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