DEVELOPING ALGEBRAIC AND DIDACTICAL KNOWLEDGE IN PRE-SERVICE PRIMARY TEACHER EDUCATION

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This study analyzes the contribution of a teaching experiment for the development of prospective primary teachers regarding knowledge of algebra and of algebra teaching as well as their professional identity. The case study of a prospective teachersuggests that an exploratory approach combining content and pedagogy supports this development, especially in the need to propose challenging tasks, to provide opportunity for students' autonomous work and collective discussions and to be attentive to children's representations and strategies in order to promote algebraic thinking.

INTRODUCTION

In addition to consistent mathematical knowledge, prospective teachers need to have an appropriate knowledge of curriculum and didactics. Such knowledge is essential to select tasks and prepare and manage students' work, providing a classroom dynamics that promotes seeking generalizations, sharing strategies, and establishing connections among mathematical ideas. A major challenge in the prospective teachers' future teaching practice is supporting the development of students' algebraic thinking. The goal of this paper is to analyse the contribution of a teaching experiment in an algebra course, in preservice primary and kindergarten teacher education, for the development of prospective teachers' algebraic thinking and knowledge of key aspects for teaching this subject, so that, in the future, they may use them in their teaching practice. In addition, we seek to know the influence of this teaching experiment in the development of the participants' professional identity.

ALGEBRAIC THINKING AND TEACHER EDUCATION

Recent curriculum guidelines (NCTM, 2000) and researchers (Carraher & Schliemann, 2007; Kieran, 2004) point the importance of promoting algebraic thinking from an early age. This does not mean that the topics usually taught in algebra in later school years now arise in primary school (Carraher & Schliemann, 2007), but rather that algebraic ideas are tacked in an informal way. Cai and Knuth (2011) indicate that the development of algebraic thinking requires analyzing relations between quantities, paying attention to structures, studying changes, generalizing, solving problems, modeling, justifying, proving and predicting. Generalization is central to algebraic thinking as well as expressingit symbolically (Kaput, 2008; Mason, Graham, & Johnston-Wilder, 2005). A generalization may be expressed in different ways and, at primary school, students may do this in their own words, based on what they observe

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and learning gradually to express it symbolically (Blanton, 2008). Algebra also involves "syntactically guided actions on reasoning and generalizations expressed in conventional symbol systems" (Kaput, 2008, p. 11).

In preservice teacher education, prospective teachers must have learning experiences aimed at different aspects of algebraic thinking so that, in their teaching practice, they can promote it in their students (Magiera, van den Kieboom, & Moyer, 2011). Ponte and Chapman (2008) address three aspects to consider in preservice teacher education: (i) knowledge of mathematics for teaching, (ii) knowledge of mathematics teaching or didactics, and (iii) professional identity. Both knowledge of mathematics and mathematics teaching are included in the development of identity. Knowledge of mathematics involves knowing how to use mathematics and also understanding its meanings and foundations (Albuquerque et al., 2006). The teacher must know and use procedures and why these procedures work. NCTM (2000) states that "teachers must know and understand deeply the mathematics they are teaching" (p. 17). Prospective teachers need to know also about mathematics teaching, namely about tasks to propose, classroom work, students' learning processes and curriculum guidelines. Ponte and Chapman (2008) suggest that preservice teacher education faces the challenge of combining content and pedagogy, as well as "teaching preservice teachers the same way that they are expected to teach their students" (p. 256). Therefore, they must knowledge algebra and what its teaching involves in primary school to be able to mobilize it later in their practice, creating learning situations to develop their students'algebraic thinking.

Teacher education must also foster the development of prospective teachers' professional identity. This includes the appropriation of the values and standards of the profession, the notion of what is teaching in the envisaged school level, an image of the teacher he/she wants to be, as well as an understanding of his/her own learning and of the role of reflecting on experience (Ponte & Chapman, 2008). Prospective teachers' past experience as school students influences their identity, bringing up these memories in shaping their role as teachers (Brady, 2007). With regard to algebraic thinking in primary school, future teachers will face challenges and demands, most of which they did not experience as students.

METODOLOGY

This research is carried out in the context of a teaching experiment that takes into account current guidelines for preservice teacher education and for kindergarten and basic education (grades 1-6). It aims at two intertwined aspects, the development of participants' algebraic thinking and their learning how to promote the development of students' algebraic thinking. The teaching experiment follows an exploratory approach (most tasks are exploratory and investigative) and the classroom dynamics aims at involving participants in discussing algebraic concepts and analyzingissues on algebra teaching and learning. The experiment involves 7 tasks on topics such as relationships, patterns, sequences, functions, and modeling, their mutual relationship and with other themes. Each task aims at deepen aspects of algebraic knowledge and provides

opportunities to discuss learning situations seeking to develop participants' didactical knowledge. Some situations refer to primary classroom episodes involving students' work, teaching practice or to students' solutions of algebraictasks. Therefore the teaching experiment addresses mathematics and didactic knowledge, providing participants with learning experiences regarding aspects that they will meet in their future practice (Albuquerque et al. 2006; Ponte & Chapman, 2008).

The first author is also the teacher in this experiment. This option establishes a close link to the classroom, allowing the results to inform her practice. We present the case of Diana, a prospective primary school teacher that was a successful mathematics student up to grade 12. Data was collected by two questionnaires with mathematical and didactical tasks, administered before (Qi) and after (Qf) the teaching experiment and three interviews (E1, E2, E3) madebefore, during, and after the teaching experiment. Data is also collected by participant observation, recording field notes (FN) and collecting documents produced by the prospective teacher. Data analysis is descriptive and interpretive, seeking to highlight the contribution of the teaching experiment for the participant's developmentof knowledge of algebra and algebra teaching and professional identity.

DEVELOPMENT OF DIANA'S ALGEBRAIC THINKING

Since the beginning of the study, Diana demonstrates significant algebraic thinking, making generalizations, using algebraic representations and procedures, and relating natural language, algebraic and graphical representations. For example, regarding modeling situations, in the initial questionnaire, she represents in algebraic language a problem with two unknown quantitiesstated in pictures and natural language. She writes a system of two 1st degree equations with two unknowns and solves it by the substitution method. She displays ease in using and manipulating algebraic symbols, showing to know formal procedures to solve systems of equations (Qi).

In another problem involving three unknown quantities (Task 2 of the teaching experiment), Diana identifies relationships between known and unknown quantities and performs basic operations. She writes a system of three 1st degree equations that she uses to find the value of one unknown, showing some difficulty in solving it. After the collective discussion of the solution of the system, she improves her understanding of the procedures (FN). Then, she analyzes solutions of grade 6 students, identifying strategies and representations, and a new problem is proposed (figure 1):

> Three friends walk in different routes. We know thatJoão and Tiago together walk 19 km, Tiago and Diogo together walk 24 km, and João and Diogo together 29 km.What distance does each friend walk?

Figure 1: Problem from Task 2 of the teaching experiment

Diana writes the system of three 1st degree equations that she solves correctly by the substitution method. Based on the solution of a student to the former problem (FN), she learns a new strategy that she also uses to solve this problem (figure 2):

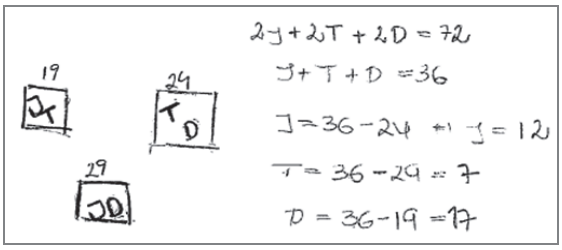

Figure 2: Diana's solution to the problem of the friends walking in different routes

Diana adds the three totals for each pair getting the double of the combined walking of the three friends. She considers this an efficient strategy for this context and uses algebraic language to present relationships in a formal way. She continues to do generalizations and to use different representations, notably pictorialand algebraic and torelate different representations. She improves her comprehension of the algebraic language and procedures that she uses, indicating that she now understands "why".

In the final questionnaire, Diana represents the problem proposed in figure 3 by a system of equations and solves it correctly by the substitution method (Qf).

Maria and Raquel went shopping. Maria bought glasses and two equal bags by 64 euros. Raquel spent 101 euros buying similar objectsbut in different quantity, as she brought two glasses and three bags. Find the price of the glasses and the bag. Explain what you did.

Figure 3: Problem from final questionary

In the interview, she thinks in another strategy to solve the problem that may be closer of the strategiesof primary students, without using a system of equations: "Maybe multiply this [picture 2] by 2 would yield 6 more 4 [6 bagsand 4 glasses] and then take this2 from here [picture 1]… Or 3, that is. Exactly" (E3).

Figure 4: Drawing made by Diana

As the interviewer asks her for clarification, she goes on:

> This is what is here [drawspicture 2]. If I multiply this by 2 I get… [draws 4 glasses and 6 bags]. And here, if I multiply by 3 I would get 3 glasses… [draws 3 glassesand 6 bags]. And then, if I go to this one [points towards4 glasses and 6 bags] and took out this [points to3 glasses and 6 bags], this is eliminated [3 glasses] this also [6 bags] and I would get just the glasses. That is, 64 had to multiply by 3, 64×3 [writes in the picture] and here 202 [writes in the picture]… This less that yields the costof the glasses. (E3)

The interviewer asks what the final result is and she indicates:

> $202 - 192$ is 10. Exactly, the glasses cost 10 euros. (E3)

Diana relies on the pictorial representation that she considers to promote students' understanding of the situation. However, this strategy involves the method of

subtraction. She multiplies each equation by the values that she chooses, subtracts the two equations and obtains an equation with one unknown. She shows, once more, a good command of the algebraic language and procedures and her ability to make generalizations and to interpret and use different representations.

The exploratory approach of the teaching experiment promotes Diana's involvement in different learning situations in algebra that contribute towards the development of her knowledge regarding generalization, using and understanding different representations and learning the justification of procedures. In addition to algebraic and graphical representations, she uses also pictorial representations.

KNOWLEDGE OF ALGEBRA TEACHING

Before the teaching experiment, Diana shows to know the main topics of school algebra, functions and equations. However, she indicates that these topics will not be addressed in primary school, at least in the formal way she learned them. She considers that the problems involving unknown quantities may be complex for primary students and therefore the unknown values must be numbers that students can easily find by trial and error. The strategy she suggests does not show the relationships between given and unknown values, verifying that it is necessary to satisfy each condition.

During the teaching experiment, Diana recognizes the possibility of working with situations concerning unknown quantities in primary school although these involve equations and unknowns that are not formally addressed by students and she suggests that this work may takes place supported in pictorial representation and in the establishment of relations based on this representation:

> More through images... I think it's much better, at least for children from grades1-4, because if we put this on paper with no pictures I think it would be much harder for them to understand the exercise. In this way they have something tangible. With images in the exercises it is easier for them to work. (E2)

As a school student, Diana learned in a very different way: "we got to some point and it was just mathematics, mathematics... Everything with computation and we did not ever think of simpler ways" (E2). Thus, she recognizes that some tasks may contribute to the development of students' algebraic thinking and knows how to propose them. She says that, if students just practice exercises, they memorize the procedures without understanding: "if the exercises are similar, just with different numbers, they end up memorizing, they just copy from above just changing the numbers, and often do not understand the exercise" (E2). The teaching experiment led her to solve problems using strategies and representations tailored to the skills and knowledge of her future students, establishing relationships and meeting conditions. She adds that this work in primary school may improve students' understanding, particularly, of equations. She appreciates the practical work of analysis of students' solutions because she considers important to understand what students do and how they think and the discussions about their understanding in different situations.

For Diana, it was important to examine different strategies and representations to identify the work that may be developed with primary students in identifying regularities and establishing generalizations in order to promote their algebraic thinking. Furthermore, the analysis of teaching practice and the dynamics created in the teaching experiment contributed for her recognition of the importance of classroom working modes and the roles of students and teacher, highlighting moments of group work and collective discussion and the teacher's questioning. After the teaching experiment, she relates algebra teaching and learning, in general, to algebraic thinking. She also refers specific aspects related to sequences and functions. She recognizes now that students may solve problems involving unknown quantities based in the exploration of relationships and not just by trial and error as she formerly thought. Concerning the tasks to propose, she indicates: "in the second task they may get some lessons they gained in the first, [tasks may] form a sequence… They may learn in the second task something else, using what they learned in the previous example" (E3). That is, Diana emphasizes the sequences of tasks that gradually increase the cognitive level.

PROFESSIONAL IDENTITY

Diana indicates a clear intention of becoming a teacher for grades 5-6. At the beginning of the teaching experiment, influenced by her former experience as a secondary school student, she views work on algebra as very formal, and does not regard that as appropriate for these grade levels. However, the work on the teaching experiment allows her to verify that working on algebra may be a rather different activity, exploring relationships and patterns aimed at developing students' algebraic thinking. The proposed activities provided her more confidence to work with her future students, especially in grades 1-4, a level that she originally did not intended to teach.

During and after the teaching experiment she identifies important features of the professional knowledge of the teacher of this subject, with which she identifies herself. She considers that the teacher must be able to solve a task in different ways, analyzedifferent answers from students and support them learning from their mistakes:

> [The teacher] has to know how to solve [the task] in a variety of ways, because a child can get there with a different solution and the teacher cannot say that is wrong, because something may be right. And the teacher must know, must understand what the child did... And use what the child knows (…). The child may know something, he/she may be wrong, but not totally wrong, one may use something... (E3)

Diana stresses that teachers must hold a formal knowledge in algebra, knowing the algebraic language and procedures. In her view, the teacher must use this knowledge to "getting the simplest ways to do and to explain" (E2) and to prepare tasksfor her students. The teacher must understand grade 1-6 students' thinking, the strategies that they use and adapt her language to the knowledge and understanding of students. She shows capacity to reflect about her experience and about her development and recognizes the importance of analyzing the students' answers and reasoning:

> I think it is very good that we analyzed how children solved the exercise, because, on one hand, we must know to solve the exercises, and, on the other hand, we must understand what kids do. Because sometimes they think in a way that we do not thought of, and it may be correct. And I think it's good that we do not practice just how the exercises may be done, but also understand how they did them. Because, in our future practice we will need, we have to understand. (E2)

The analysis of teaching situations and their relation to experience contributes for her understanding of the work to be done on these gradesand her recognition of some of the challenges that the teacher faces and of the specificity of professional knowledge.

CONCLUSION

This study aims to contribute for understanding how to integrate mathematics and didactical knowledge in prospective teachers' educational programs, in particular, in an algebra course and to identify its contribution for the development of these two aspects as well as in the development of professional identity. Diana intends to become a primary school teacher (grades 1-6) expressing preference for teaching mathematics and science. Being a successful secondary school mathematics student, before the teaching experiment she already makes an effectively use of the algebraic language, solving most tasks with no difficulty but in a formal way, using algebraic procedures, but she does not know what work may be developed in primary school.

With the teaching experiment, Diana recognizes that many algebraic tasks may be addressed in a different way. She strives to find different ways to solve them, and values the solution of a problemusing different strategies and representations, feeling much more prepared to interpret the diversity of students'solutions. The focus on relationships and seeking generalizations provides her a deeper understanding of the procedures she already knew and often used in a mechanized way, showing evolution of her syntactically guided reasoning. Diana considers that primary school students' algebraic thinking may be developed by the exploration of relationships, contributing to a better understanding of formal aspects of algebra later on (Blanton, 2008). In this experiment, she develops an understanding of the knowledge that the teacher need to promote algebra learning. She recognizes that the teacher must have mathematical knowledge to use in his/her teaching practice to prepare suitable tasks for students and to solve correctly different kinds of situations, and also have a deep knowledge of students, their prior knowledge and the way how they learn. She is also aware of the ways she can communicate with students in an effective way. The teaching experiment also influenced the way she regards the work with her future students, highlighting moments of autonomous work and the moments of collective discussion.

Besides changing her view regarding the role of teaching and learning of algebra in primary school, Diana also developed a much better image of the teacher that she wants to be, based on the reflection that she makes about her experience and the development provided by the teaching experiment. As Brady (2007) indicates, initially, her past experiences influence her identity. The memory of how she learned algebra makes she think it will be difficult to address this subject with primary students. This

view changes with the teaching experiment. Contrarily to the focus on calculations that she experienced as student, she now underlines the role of non-routine tasks aimed at the students' understanding and the activities that promote algebraic thinking in primary school. The integration of content and didactic knowledge (Ponte & Chapman, 2008) and the exploratory approach used in the teaching experiment contributed for development ofher knowledge of algebra for teaching, her knowledge of mathematics teaching, and her professional identity. In particular, the emphasis on prospective teachers working on algebraic tasks and analyzing learning situations, combining autonomous work and collective discussions, helped Diana to deepen her mathematical knowledge, understanding the rationale for certain procedures, and to develop her understanding of learning processes and knowledge of teaching practice.

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References

Albuquerque, C., Veloso, E., Rocha, I., Santos, L., Serrazina, L., & Nápoles, S. (2006). *A Matemática na formação inicial de professores*. Lisboa: APM and SPCE.

Blanton, M. (2008). *Algebra and the elementary classroom*. Portsmouth, NA: Heinemann.

Brady, K. (2007). Imagined classrooms: Prospective primary teachers visualise their ideal mathematics classroom. In J. Watson & K. Beswick (Eds.), *Proceedings of the 30th Annual conference of MERGA* (vol 1, pp. 143-152). Tasmania, Australia: MERGA.

Cai, J., & Knuth, E. (2011). Preface to part I. In J. Cai & E. Knuth (Eds.), *Early algebraization* (pp. 3-4). Berlin: Springer.

Carraher, D., & Schliemann, A. (2007). Early algebra and algebraic reasoning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (vol. 2, pp. 669-705). Charlotte, FN: Information Age.

Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 5-17). Mahwah, NJ: Erlbaum.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it*? The Mathematics Educator, 8*(1), 139-151.

Mason, J., Graham A., & Johnston-Wilder, S. (2005). *Developing thinking in algebra*. London: Sage.

Magiera, M., van den Kieboom, L., & Moyer, J. (2011). Relationships among features of pre-service teachers' algebraic thinking. In B. Ubuz (Ed.), *Proceedings of the 35thIGPME Conference* (vol. 3, pp. 169-176): Ankara, Turkey.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Ponte, J. P., & Chapman, O. (2008). Preservice mathematics teachers' knowledge and development. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 225-263). New York, NY: Routledge.