# Proportion in school mathematics textbooks: A comparative study[1]

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ABSTRACT
This paper analyses how proportion is introduced and developed in selected mathematics textbooks for middle school students of Portugal, Spain, Brazil, and USA. The analysis focuses on the nature of the approach and on the cognitive demand, structure, and context of the tasks. The results show that the textbooks tend to present tasks at an intermediate level of cognitive demand and with a closed structure. Non-mathematical contexts predominate in three of the four textbooks. However, there are marked differences in the way textbooks approach the conceptual and procedural aspects of proportion. The way the students are addressed also varies, ranging from a questioning/problem solving style, to an explaining/practicing style, each of these styles supporting a rather different kind of activity.

Keywords**:** Textbooks, Task analysis, Proportion, Curriculum, Middle School Mathematics

RESUMO
Este artigo analisa como *proporção* é introduzida e desenvolvida nos livros didáticos matemática para alunos do ensino médio (Ensino Fundamental II) em Portugal, Espanha, Brasil e EUA. A análise incide sobre a natureza da abordagem e na demanda cognitiva, estrutura e contexto das tarefas. Os resultados mostram que os livros didáticos tendem a apresentar as tarefas em um nível intermediário de demanda cognitiva e com uma estrutura fechada. Contextos não-matemáticos predominam em três dos quatro livros didáticos analisados. No entanto, há diferenças marcantes no modo como os livros abordam os aspectos conceituais e procedimentais de proporção. A forma didática de tratar o assunto também varia, indo de um estilo de questionamento / resolução de problemas, a um modo explicativo/prático; cada um deles ampara-se em um tipo diferente de atividade.

Palavras-chave: Livros didáticos, análise de tarefas, Proporção, Curriculum, Matemática no Ensino Fundamental II.

Textbooks have a strong influence in mathematics teaching and learning. An aspect of concern for mathematics educators is how these educational instruments present key

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mathematical topics in relation to current curriculum orientations. This paper analyses how the topic of proportion – a central notion in the school mathematics curriculum – is presented in textbooks for 11-12 year old students of four different countries: Portugal, Brazil, Spain, and USA. We are particularly interested in how textbooks introduce and develop the notion of direct proportion, in the tasks that they present to support students in consolidating and systematizing their knowledge, and in identifying other similarities and differences among textbooks. The main focus of analysis is the nature of the tasks proposed to students, given their key role in structuring students' learning activities (Christiansen & Walter, 1986).

## Proportion in the mathematics curriculum and in mathematics education research

Proportion plays an important role in the curriculum of the four countries present in this study. In Portugal, the syllabus and the national curriculum documents (Ministério da Educação, 1991, 2001) recommend the development of the concept of direct proportion through real life situations from grade 6 on. In Brazil, the curriculum documents, *Parâmetros Curriculares Nacionais* (Ministério da Educação e Ciência, 1998), suggest that the preparation for the study of proportion must begin at grades 4 and 5, through rational numbers and percents, and, in grades 6 and 7, address the direct variation of proportional magnitudes. In Spain, the curriculum documents *Reais Decretos* (2003, 2004), suggest that direct proportional magnitudes is studied at grade 7 (the first year of ESO, *enseñanza secundaria obligatoria*); however, percent is studied in grade 6 (last year of EP, *enseñanza primaria*). In the USA there is no national curriculum but we can have an idea of the trends in this country by looking at the documents issued by the NCTM (1989, 2000, 2006). These documents present proportion as a central notion that relates numbers and other topics in algebra and geometry. They support a preparation for understanding this concept since the first years of school through the study of patterns and regularities, common fractions, decimal fractions, and percents. NCTM (2006) indicates "Connecting ratio and rate to multiplication and division" as a curriculum focal point for grade 6, and "Developing an understanding of and applying proportionality, including similarity" as a focal point for grade 8.

There are many research studies dealing with teaching and learning of proportion. An important research program in this field was developed by Lesh, Post and Behr (1988). These mathematics educators regard proportion as the capstone of learning numbers and operations and consider it as an essential basis for learning algebra and other topics. They suggest that working with multiplicative comparisons in the first years of school leads the student to understand the equivalence of fractions, the conversion of measures, and how to deal with percents, ratio, and rate. They argue that proportional reasoning evolves through the development of local competencies based on contextualized knowledge. In their view, the student attains proportional reasoning when he or she is able to reason based on global relationships between rational expressions (fractions, quotients, ratios, and rates).

Spinillo (1992, 1996), from the standpoint of educational psychology, regards the development of proportional reasoning as progressing through the acquisition of flexible strategies by the student. She supports the idea that the development of proportional reasoning begins in the first years of school, as intuitive knowledge, giving particular attention to first order relations that are the basis for the establishment of

second order relations (involving two first order relations). This author underlines the importance of using the notion of "a half" as well as estimation activities to promote an effective differentiation between absolute and relative magnitudes and between first and second order relations.

Currently, most textbooks present a proportion as an equality between two ratios: $\frac{a}{b} = \frac{c}{d}$. However, in the past, a proportion was often presented like this:

If *a* is for *b* such as *c* is for *d*, we represent this as

a ---- b

X

c ---- d

and we say that *a*, *b*, *c* and *d* are in proportion and *ad* = *bc*.

This is often called the "rule of three simple" or "rule of three" – assuming that we know three of the values *a*, *b*, *c*, *d*, we can figure out the fourth. From the diagram above, the products *ad* and *bc* are named as "cross products".

The way textbooks present direct proportion is addressed in several studies (e.g., Cabrita, 1996; Ruggiero & Basso, 2003; Shield & Dole, 2002). These studies identify two main approaches to deal with problems involving proportions. One uses the rule of three and the other emphasizes the "fundamental property of proportions", stating that "if $\frac{a}{b} = \frac{c}{d}$ is a proportion, then *ad* = *bc*". The main difference is that the rule of three and the cross product are just relationships involving four numbers whereas the "fundamental property of proportions" involves the algebraic notion of equation.

Cabrita (1996) studies how seven Portuguese grade 7 textbooks revisit the notion of direct proportion previously taught at grade 6. She indicates that, to solve problems, some of the textbooks use the fundamental property of proportions and others use the rule of three. Shield and Dole (2002) analyse two chapters ("ratio and proportion" and "ratio and rates") of an Australian grade 7 textbook. The authors indicate that the most common method of solving a proportion task involves "cross-multiplication and algebraic methods" (p. 7), solving the equation in order to *x*. They contend that textbooks do not make a proper distinction between ratio (part-part) and fraction and percent (part-whole). In their view, textbooks fail to promote the development of proportional reasoning. Ruggiero and Basso (2003) analyze a Brazilian textbook that received a high rank in a national textbook assessment. They conclude that the distribution of topics is horizontal, but the textbook fails to promote learning with understanding because the new concepts are not presented based on relationships with other concepts or fields of knowledge.

## Mathematical tasks and connections

Mathematical tasks and connections play a central role in the present study. Tasks are regarded as the proposals and challenges set by the teacher or the textbook (and in some cases even by the student) that constitute a goal to attain (Christiansen & Walther, 1986; Ponte, 1995). The activity is what the students really do in terms of their thinking and actions. Therefore, tasks are external and the activity is internal to the student.

Skovsmose (2000) says that the teacher "invites" the student to get involved in the task. The way the student responds to such invitation becomes his or her activity. The NCTM (2000) also refers to the tasks as something constructed by the teacher, to be proposed to the student. In another document, the NCTM (1991) stresses that tasks must support students in developing their ability to formulate and solve problems, communicate ideas, and establish mathematical connections.

The NCTM (2000) indicates that in designing a task one needs to take into account the content, the level of difficulty, the routine or non-routine nature, the complexity, and the degree of openness. Gimeno (1998) indicates that tasks may promote different cognitive processes: memorization, comprehension, opinion, and discovery. In the PISA study (OCDE, 2004) there are three different kinds of tasks, according to the level of cognitive demand: reproduction, connection, and reflection. Also to differentiate the cognitive complexity of tasks, Smith and Stein (1998) and Stein and Smith (1998) speak of routine and non-routine tasks. They suggest that routine tasks include memorization tasks and tasks with no connections and that non-routine tasks include tasks with connections and "doing mathematics". Ponte (2005) suggests a model that differentiates tasks according to the degree of openness and the degree of challenge, comprising exercises, explorations, problems, and investigations.

The tasks may differ in their context. Skovsmose (2000) proposes that contexts may be of reality, semi-reality, or purely mathematical. He contends that, at first glance, semi-reality situations may look like reality, but, in fact, they often are meaningless for the students. In recent years, context and connections become an increasingly important idea in school mathematics. For example, Pepin and Haggarty (2004) underline the importance of textbooks establishing connections within and outside mathematics. The NCTM (1991, 2000) indicates that mathematical connections constitute a basic standard in mathematics learning through all grade levels, from primary to secondary school. This includes both connections within mathematics and connections of mathematics with other fields. Portuguese curriculum documents (ME, 1991, 2001) indicate the need of relating teaching units, developing transversal themes and using mathematics as a tool for understanding and acting upon the real world. The PISA study (OCDE, 2004) puts a particular emphasis in the nature of the context underlying each task.

The nature of mathematics tasks is also addressed by textbook research. For example, Li (2000) compares American and Chinese mathematics grade 7 textbooks, studying the kinds of tasks proposed in the topic of whole numbers addition and subtraction. This author analyses five American and four Chinese textbooks using three criteria of analysis: the mathematical dimension, the contextual dimension, and performance requirements. She concludes that both American and Chinese textbooks mostly propose tasks that only require a single operation and are framed in pure mathematical contexts. Also, Shield and Dole (2002) stress that tasks in Australian textbooks tend not to be contextualized in reality and to present few connections among mathematical topics as, for example, between proportion and geometry.

## Methodology

As we indicated earlier, this paper aims to indicate how textbooks introduce and develop the notion of direct proportion, to analyse the tasks that they use to support students in consolidating and systematizing their knowledge, and to identify similarities and differences among textbooks. To achieve these aims we sought to gather information about the following points: (i) How is the textbook regarding its physical features, language and illustrations? (ii) What is the organization of the textbook and the structure of the chapters? (iii) How does the textbook consider the use of technology, especially the calculator? (iv) At what point of the textbook is the concept of proportion introduced? (v) What concepts pave the way to the notion of proportion? (vi) How does the textbook approach the notion of proportion? (vii) What is the level of cognitive demand of the tasks proposed in the textbook? (viii) What is the structure of the tasks proposed? (ix) In what contexts are tasks presented in the textbook? (x) What are the common features among the textbooks? and (xi) What are the significant differences among the textbooks?

This study is based in documental analysis and uses a technique of content analysis. First, we begun by choosing the topic. We selected proportion because, in most countries, it is a central topic of the curriculum for the 11-12 year old age range. Second, we selected the countries to study. We were interested in comparing the Portuguese textbooks with the textbooks of countries with which Portugal has stronger cultural or educational connections. We chose: (i) Brazil, as it has the same language; (ii) Spain, as it has a similar language and has a common cultural root; and (iii) the USA, because it plays a leading role at international level in setting the mathematics curriculum. One must note that the NCTM (1989, 1991, 2000) documents have been quite influential in Portugal. On the other hand, curriculum documents and educational materials from Brazil and Spain are little known in our country and we are interested in knowing possible similarities and differences given the cultural affinities. Third, and finally, in these four countries we selected a textbook with a strong share of the market, thus representing a specific trend in mathematics education (Table 1).

Table 1 – Textbooks selected for the study.

| Country | Textbook | Grade level |
|---|---|---|
| Portugal | Neves, M. A., Faria, L., & Azevedo, A. (2000). *Matemática* (6.º ano, 2.ª parte). Porto: Porto Editora. | 6 |
| Brazil | Lopes, A. J. (2000). *Matemática hoje é feita assim* (6.ª série). São Paulo: FTD. | 6 |
| Spain | Colera, J., & Gaztelu, I. (2005). *Matemáticas* (Educación Secundaria – 1). Madrid: Anaya. | 7 |
| USA | Maletsky, E. & Askey, R. (2007). *Math*. Orlando: Harcourt Brace & Company. | 6 |

The four textbooks (6 chapters altogether) were analysed using an instrument consisting on two frameworks: one for global analysis and another for the analysis of tasks. The first framework takes into consideration, on a general level, the physical appearance, accessibility of language, nature and role of illustrations, and structure of the chapters. On a level specifically related to the topic of proportions, this first framework addresses how the concept of proportion is presented, how its study is prepared, and what are the main notions and strategies for solving tasks that are emphasised.

The second framework is related to the tasks and includes three points: cognitive demand, structure, and context. Following closely the PISA framework (OCDE, 2004), cognitive demand is classified as reproduction, connection, and reflection tasks. *Reproduction tasks* are routine tasks that involve the use of knowledge previously acquired and practiced, have a low degree of mathematical complexity and do not ask for arguments in the response. Their interpretation is straightforward and they do not require the use of different kinds of representations. Furthermore, reproduction tasks tend to be quite structured and to be presented in a simple and familiar context. *Connection tasks* require the establishment of relationships or chains of reasoning, procedures, or computations and ask for a certain level of interpretation. They may include a request for justification or a simple explanation. They tend to have a closed structure and to be presented in a familiar or almost familiar context. *Reflection tasks* are more complex and require a high level of interpretation and reasoning, ask for a solution that involves the coordination of several steps, and often demand a response with some written communication and argumentation. The structure of these tasks is sometimes open or semi open and they may refer to less familiar situations.

The structure of the tasks was classified in three subcategories: closed, semi open and open (Ponte, 2005). In *closed* tasks the mathematical givens, goals, and conditions are clearly indicated. In *open* tasks the student needs to provide some further specification of givens, goals, and conditions. *Semi open* tasks lie in between these two.

Finally, we classified the context of tasks also according to a scheme similar to the PISA framework (OCDE, 2004). Such context may be (intra) mathematical or non mathematical. Non mathematical contexts include six subcategories: *daily life situations* involve personal situations or situations directly related to students' daily activities; *school situations* refer to activities and processes that occur in the school context; *professional situations* correspond to a professional activity that students may be involved with in the future; *life in society* concerns tasks related to life in community and in society*; other areas of knowledge* include tasks from subjects such as physics, geography, sports, language and so on; *imagination/fiction* concern tasks drawn in a fantasy world. Mathematical contexts include situations devoid of explicit non-mathematical elements; they are *between topics* if they refer explicitly to concepts taught in other chapters; otherwise, they are on the *same topic*.

## Results

### General aspects

First we consider the physical features of the textbooks, and the language and illustrations that they present. The analysis of the four textbooks shows that they address the notion of proportion at different grade levels – Portugal, Brazil and USA at

grade 6, and Spain at grade 7. The four textbooks have similar size formats (about A4 or letter) and come in one volume, except the Portuguese that is presented in four volumes. The American textbook, with 783 pages, is much thicker than the others (the Brazilian has 304 pages, the Spanish 215 pages, and the Portuguese 315 pages altogether). The covers of the American, Brazilian, and Portuguese textbooks do not have any apparent connection with mathematics, whereas the cover of the Spanish textbook shows a strong relationship with the subject.

All textbooks have a language accessible to the students of these grade levels. Furthermore, all textbooks are widely illustrated by pictures, drawings, and photographs that usually illustrate the question but often do not present any relevant information to solve it. However, there are noteworthy differences in the discourse. The most deviant case is the textbook from Brazil that is based in solving problematic situations that allows the introduction and systematization of concepts based in discussions about the issues presented. In contrast, the discourse adopted in the Portuguese, Spanish and American textbooks assumes that the role of the student is, first, to understand the explanations and examples solved, and, second, to solve a battery of tasks, most of reproduction and connection. The way the students are addressed ranges from a questioning/problem solving style (in the Brazilian textbook), to an explaining/practicing style (in the Portuguese, Spanish, and American textbooks).

The organization of the chapters follows a pattern that does not vary much from textbook to textbook. All the textbooks include introductory tasks, application tasks, and consolidation tasks in different quantities and levels of complexity. All textbooks, except the Brazilian, present revision tasks in the beginning of a new chapter. In introducing new concepts, the Portuguese and Spanish textbooks begin by presenting a problematic situation and its solution, explain the concepts that follow from that example, present a synthesis, and, finally, propose a battery of tasks for practice. The American textbook follows the same pattern, but it stresses even more than the others the need for revision, introducing at the beginning of each new section a small set of routine tasks to review concepts formerly studied (figure 1). The Brazilian textbook is noteworthy because it begins with problematic situations that are solved through a "conversation" among cartoon characters and only afterwards it proposes some tasks for the student to do. Furthermore, this is the only textbook that does not present a very quick formalization of the concepts.

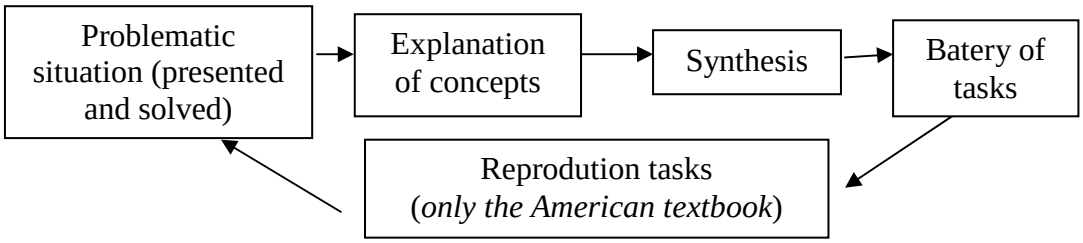


Figure 1 – Pattern of presenting new concepts in textbooks.

All textbooks make few references to technology or to the use of the calculator. The Spanish and American textbooks ask the pupil to use the calculator just a few times. The Portuguese and Brazilian textbooks do not include tasks that explicitly suggest the pupil to use the calculator.

## Previous work and introduction to proportion

Next we consider how the topic of direct proportion is introduced in the four textbooks and also how its study is prepared by previous chapters. For example, the Portuguese grade 6 textbook includes only one chapter on direct proportion. Before direct proportion, this textbook presents chapters about rational numbers and its operations and also on construction of triangles. In the Spanish textbook of grade 7, direct proportion is the object of one chapter. The previous chapters study natural, integer and decimal numbers, powers and square roots, divisibility, metric system. There is also a review of notions about rational numbers already taught in grade 6. The Brazilian textbook for grade 6 presents proportion in two chapters, the second of which focuses on the study of geometric situations. Before introducing proportion, the previous chapters study the arithmetic mean, angles, operations with rational numbers, polygons and regularities, square root, negative numbers and equations. The grade 6 American textbook deals with direct proportion in two chapters, the second of which focuses on percents. Before addressing proportion, it includes the study of numbers and operations, statistics and graphing, fraction concepts and operations, algebraic expressions, equations and patterns, geometry and plane figures, measurement in one and two dimensions and solid figures and measurement.

The textbooks introduce the concept of proportion in different ways. In the Portuguese textbook the study of the concept is based on the previous study of rational numbers. The same happens with Spanish textbook that also builds on the previous work on percent. Meanwhile, in the Brazilian and American textbooks, the study of proportion is based in the work on rational numbers, equations, patterns, and regularities, in line with the NCTM's (2000) recommendations. It is clear that the four textbooks organize the mathematical content in rather different ways, preparing the study of this concept and carrying it out differently, thus proposing different learning trajectories. The pros and cons of each choice are a matter for empirical investigation.

## Approach to proportion

There are also differences in the approach and content of the sections that made up the chapters of each of the textbooks analysed. As a starting point for the study of proportion, the Portuguese textbook presents the concept of ratio. Then, it addresses the notion of proportion and the fundamental propriety of proportions, and, afterwards, presents percents, scales, and direct proportion. The Spanish textbook for grade 7 approaches proportion using proportional series to discuss the relationship of proportional magnitudes, then presents direct and inverse proportion, and, finally, revisits the concept of percent in a deeper way. The Brazilian textbook grade 6, in a first chapter, presents ratio, percent, and change in proportional magnitudes, alluding to inverse proportional magnitudes. In a subsequent chapter, the textbook relates proportion with geometry, presenting the topics of scales, proportional rectangles, the fundamental property of proportions, and similar triangles. Finally, the American textbook introduces in chapter 24, the concepts of ratio, rate, proportion of similar figures and scales, and shows how to construct circular graphs and compute interest rates. Next, in chapter 25, it revisits the concept of percent and enlarges it.

The approaches used by textbooks show three interesting contrasts. First, it is noteworthy that in all countries except Spain the textbooks begin the chapter on direct

proportion with the concept of ratio. The Spanish textbook for grade 7 begins this chapter with "proportional series", addressing relationships between magnitudes. This is a rather unusual approach that has the advantage of leading to a close connection to the notion of function and to the use of tables to represent data but has the disadvantage of leaving aside the notion of ratio and the work with fractions. Second, in the Brazilian and American textbooks, we note a spiral approach as they present the notion of direct proportion in one chapter and come back to it at least a second time. This does not happen in the Portuguese and Spanish textbooks that address this notion only once. Third, the fundamental property of proportions is stated in the Brazilian and Portuguese textbooks but not in the others. In contrast, the American textbook uses the cross product and the Spanish the rule of three. As in the textbooks analysed by Cabrita (1996), these four textbooks suggest the rule of three or the cross product as the main strategy to solve proportion tasks. However, some textbooks present more than one strategy.

For example, the Portuguese textbook presents the fundamental property of proportions as the first method to solve direct proportion tasks but it also presents the rule of three and the reduction to the unit as possible forms for solving problems involving proportions (Annex 1). The Spanish textbook presents the rule of three followed by the cross product (Annex 2). This textbook presents the schema of the rule without explaining it, then constructs a pair of fractions (representing ratios) that are intended as equivalent and, finally, encourages the students to use the cross product. In another example it just moves from the rule of three to the cross product that is solved as an equation. The Brazilian textbook presents, by the end of the chapter, the fundamental property of proportions, justifying it in a deductive style (Annex 3). The American textbook begins by presenting two ways to solve proportion problems involving comparison of ratios. However, in the solved examples it places stronger emphasis in the cross product (Annex 4).

Tasks

We now address the way the tasks proposed in the textbooks vary in level of cognitive demand, structure and context.

*Cognitive demand*

The cognitive demand of the tasks proposed varies between a minimum level of reproduction, an intermediate level of connection, and a maximum level of reflection. In all textbooks analysed connection tasks (Annex 5) predominate – about 2/3 of the total (Table 1). In the Spanish and Brazilian textbooks such tasks represent 68% of the total, in the Portuguese textbook about 62% and in the American textbook only 47%. In a second place, textbooks present reproduction tasks. The American textbook stands up by the large percent of this kind of tasks (Annex 6). In contrast, reflection tasks are little represented in all textbooks. They have a stronger presence in the American (12%) textbook (Annex 7), closely followed by the Brazilian (10%), leaving at great distance the Portuguese (4%) and Spanish (3%) textbooks. It is interesting to note that the American textbook has the higher level of reflection tasks and has also the higher level of reproduction tasks.

Table 1 – Cognitive demand, structure and context of tasks presented in textbooks (%).

| | | Portugal | Spain | Brazil | USA |
|---|---|---|---|---|---|
| Cognitive demand | Reproduction | 34 | 29 | 22 | 41 |
| | Connection | 62 | 68 | 68 | 47 |
| | Reflection | 4 | 3 | 10 | 12 |
| Structure | Open | 0 | 0 | 4 | 1 |
| | Semi open | 1 | 0 | 9 | 9 |
| | Closed | 99 | 100 | 89 | 90 |
| Context | Mathematical | 35 | 26 | 39 | 83 |
| | *Same topic* | *26* | *26* | *19* | *66* |
| | *Between topics* | *9* | *0* | *20* | *17* |
| | Non mathematical | 65 | 74 | 61 | 17 |
| | *Daily life situations* | *5* | *2* | *3* | *2* |
| | *School situations* | *12* | *1* | *6* | *1* |
| | *Professional situations* | *13* | *11* | *8* | *0* |
| | *Life in society* | *34* | *58* | *32* | *9* |
| | *Other areas of knowledge* | *1* | *2* | *10* | *5* |
| | *Imagination/fiction* | *0* | *0* | *2* | *0* |

*Structure*

All the textbooks analysed present few tasks with an open or semi open structure. Such tasks do not exist in the Spanish textbooks. The textbook that stands up because it offers tasks with a more open structure is the Brazilian, with 9% of semi open tasks and 4% of open tasks (Annex 8). The American textbook, that presents a high level of closed tasks (90%), proposes 9% of semi open tasks (Annex 9) and 1% of open tasks. The Portuguese textbook is very similar in this regard to the Spanish, with no open tasks and only 1% of semi open tasks. In summary, the overwhelming majority of the tasks have a closed structure. Open tasks can only be identified in the Brazilian and American textbooks.

*Context*

In most textbooks analysed, the majority of tasks of the proportion chapters are framed in terms of non mathematical contexts. The exception is the American textbook, in which 83% of the tasks have mathematical contexts. The other textbooks contain less than 40% of tasks relative to mathematical contexts, concerning the same chapter or related to other chapters. From these tasks framed in mathematical contexts, those that concern the exploration of concepts within the same mathematical chapter are in larger number, especially in the American textbook (66% of the tasks are of this kind). This textbook presents a very low percent of tasks in non mathematical contexts (17%), when compared with the remaining textbooks in which most tasks are related to students' daily activities, school situations, professional situations, life in society, other areas of knowledge and imagination. The most prominent subcategory of non mathematical context is life in society. The trend to stress mathematical contexts by American textbooks was already noted in the study by Li (2000), and a similar result was reported by Shield and Dole (2002) regarding Australian textbooks.

## Discussion

As we have seen, the textbooks analyzed share several features. The kind and localization of the tasks is very similar. All textbooks have introductory, application, and consolidation tasks at the end of each section and again at the end of the chapter. The distribution of the cognitive demand of tasks is similar (with emphasis in connections) and the structure is also similar (with emphasis in closed tasks). Furthermore, all textbooks make little references to the use of technology, including the calculator.

We also note some differences among the textbooks. The most striking one is the strong emphasis of the American textbook in mathematical contexts whereas the textbooks of the three other textbooks clearly emphasize non-mathematical contexts. The American textbook is also noteworthy by its emphasis on revision tasks, not only at the end of sections and chapters, but also at the beginning of sections. A textbook with so few references to non mathematical contexts and with so a high stress on revision is certainly addressed to teachers using a different sort of pedagogy and to students with quite different learning habits.

Another important difference concerns the linear or spiral approach. The Portuguese and Spanish textbooks concentrate in a single chapter the topic of proportion, seeking to do a comprehensive discussion, thus adopting a linear conception of curriculum. In contrast, the Brazilian and American textbooks approach concepts at different stages of the textbook, presenting therefore a spiral organization.

There is another important difference in the way the textbooks approach the mathematical notions and procedures. In one case (Portugal) the study of proportion is based in the study of rational numbers, in another (Spain) is based on rational numbers and percents and in another cases (Brazil, USA) it is based in the study of rational numbers, equations, and patterns. In the Portuguese, Brazilian, and American textbooks the notion of ratio plays in important role, but it does not happen in the Spanish textbook that uses the notion of “proportional series”. Only two textbooks mention the “fundamental property of proportions”.

Finally, we note differences in the discourse. As we indicated above, the textbook from Brazil presents problematic situations and use their discussion for the presentation and systematization of concepts whereas the discourse adopted in the other three textbooks assumes that students must first understand the explanations and examples solved and next solve a battery of reproduction and connection tasks. In accordance, the way the students are addressed varies in the textbooks, from a questioning/problem solving style (Brazilian textbook) to an explaining/practicing style in the other three textbooks. These different styles support rather different kinds of student activity.

Having described several features of the way textbooks present proportion, one may consider what the reasons for such similarities and differences are. One strong factor that influences textbooks is certainly the national curriculum documents. The Brazilian textbook is noteworthy as it emphasizes non mathematical contexts, presents a relatively higher quantity of cognitively demanding tasks and, most especially, follows an exploratory and informal approach, maintaining an extended dialogue with pupils before any formalization. These features are well aligned with the orientations of the

Brazilian curriculum documents *Parâmetros curriculares nacionais*. The Spanish textbook, in contrast, moves quickly towards formalization and insists in tasks with low level of cognitive demand. It presents proportion without using the notion of ratio, but when it comes to explaining how to solve proportion tasks it suggests the use of the rule of three. This is well in line with the *Reais Decretos*, the Spanish official documents that stress a rather formal and procedural approach to mathematics. These two cases are examples of strong alignment of textbooks and curriculum documents.

The American and Portuguese textbooks studied represent a different situation. In the American textbook there are features consonant with the NCTM (2000) *Principles and standards*, such as some attention to tasks with high level of cognitive demand and to patterns and regularities, but there are features that depart from the orientations of this document such as the emphasis in mathematical contexts and in reproduction tasks. The high number of pages and tasks of this textbook suggest that teachers are expected to devise a specific trajectory within it for their students to follow. In the Portuguese textbook there are also features strongly related to the curriculum documents, such as the wide use of non mathematical situations, but there are others that are not in line with these documents such as the high emphasis on low level cognitive demand tasks and the complete absence of reference to the calculator. These two textbooks provide examples of weak alignment with at least some aspects of the curriculum documents.

The reasons for strong or weak alignment deserve our attention. First, they may relate to editorial options, aiming to achieve high level sales. The concern to adjust to the preferences of a wide audience of teachers is quite visible in the American, Spanish and Portuguese textbook publishers, whereas the Brazilian publisher decided to run the risk of adopting a more innovative style, targeted to a more specific teacher group.

Second, strong or weak alignment with curriculum orientations may have to do with how the textbook is meant to be used by students. In Portugal and Spain the textbook is bought by families, is property of the student, and one of its most important roles is to be used as study resource at home. Therefore, textbooks travel between home and school and must not be very heavy. In the USA, in contrast, the textbook is property of the school and is loaned to each student who keeps it in his/her school locker or at home. Therefore, there is no problem that textbooks have extra material that allows teachers more freedom to devise teaching trajectories with them.

And third, pedagogical traditions regarding mathematics education at each country also constitute a strong influence in textbooks. This is quite apparent in the Spanish textbook. It follows the national documents by presenting proportion through the study of proportional series. However, instead of using this to connect to the notion of function and its representations (namely, tables and graphs), it goes on teaching students about how to using the rule of three. The same happens with the Portuguese textbook that presents the fundamental property of proportions but later emphasizes the use of the rule of three to solve tasks involving proportions.

## Conclusion

Teachers have to consider many issues regarding textbooks, both when they analyse textbooks for adoption and when they prepare their classes and think about what to

recommend their students to do. Does the textbook include a variety of tasks in terms of cognitive demand, structure and context? Does the textbook promote a proper use of technology? The concepts and procedures presented in each chapter are related to the material of the previous chapters? The students are offered with tasks to do and stimulated to reflect on their work or are mostly presented with rules to practice?

This article shows how the study of mathematics textbooks may use a frame of analysis based in didactical notions. In this paper, the focus was in the teaching approach and in the tasks proposed, taking into account the current curriculum orientations. Using this methodology, we found notable similarities but also striking differences in the textbooks analyzed in several respects. It may be that within each country we would found also great variations among textbooks. However, in some cases, we see some connection between the curriculum documents that stress an innovative point of view and the nature of the approach and the activity proposed in the textbooks. In other cases, we see how textbooks adjust to and reinforce conservative elements of curriculum documents and traditional teaching practices. For future studies, it would be interesting to know how different kinds of textbooks are used in practice by students and teachers. In particular, it would be interesting to know in what measure the learning opportunities provided by innovative textbooks gets enacted in classroom practice and, most especially, in student learning.

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Annex 1 – Presentation of the fundamental property of proportions, rule of three and reduction to the unit in the Portuguese textbook, p. 50.

*O automóvel do António gasta 6 litros de gasóleo em cada 100 km.*
*Quanto gasta para andar 20 km?*

**Proporções**

A letra $x$ representa o número desconhecido.

| | | | |
|---|---|---|---|
| $\frac{\text{litros}}{\text{quilómetros}} = \frac{\ }{\ }$ | $\frac{\text{litros}}{\text{quilómetros}}$ $\frac{6}{100} = \frac{x}{20}$ | $6 \times 20 = x \times 100$ ou seja: $x = \frac{6 \times 20}{100}$ | $x = 1{,}2$ |
| 1. Forma-se uma proporção. | 2. Escreve-se os dados e representa-se pela letra $x$ o número desconhecido. | 3. Aplica-se a propriedade fundamental das proporções. | 4. Determina-se $x$. |

Para andar 20 km o António gasta 1,2 litros de gasóleo.

**Regra de três simples**

O problema anterior pode ser resolvido usando uma regra muito prática que tem a designação de **regra de três simples**.

Repara que nesta coluna estão os litros. Repara que nesta coluna estão os km. Repara que o número que cruza com $x$ fica no denominador.

| | | |
|---|---|---|
| litros quilómetros | litros quilómetros<br>6 $\xrightarrow{\text{corresponde}}$ 100<br>$x$ $\xrightarrow{\text{corresponde}}$ 20 | $x = \frac{6 \times 20}{100}$ ; $x = 1{,}2$ |
| 1. Escreve-se as variáveis em questão. | 2. Escreve-se os dados e usa-se a letra $x$ para representar o número desconhecido. | 3. Determina-se $x$, colocando no denominador o valor que cruza com $x$ e no numerador o produto dos outros dois valores conhecidos. |

**Redução à unidade**

Outra forma de resolver o problema seria usar o método "redução à unidade".
Calcula-se quanto se gasta para andar 1 km e multiplica-se o resultado por 20.

$$6 : 100 = 0{,}6 \; ; \quad 0{,}6 \times 20 = 1{,}2 \, .$$

António's car needs 6 litters of gas for each 100 km. How much does it need for 20 km?

**Proportions**
(The letter x represents and unknown number)
1. We construct a proportion.
2. We write the data and represent by the letter x the unknown number.
3. We apply the fundamental property of proportions.
4. We find x.

**Rule of three**
The above problem may be solved by a very practical rule that is named rule of three.

1. We write the variables of the problem.
2. We write the data and use the letter x to represent the unknown number.
3. We find x, putting in the denominator the value that crosses wit x and at the denominator the product of the other two known values.

**Reduction to the unit**
Another way of solving the problem would be using the method of "reduction to the unit" We calculate how much we need to go 1 km and multiply the result by 20.

Annex 2 – Rule of three in a solved example in the Spanish textbook, p. 159.

*PROBLEMA: Tres cajas de caramelos de café con leche cuestan 15 €. ¿Cuánto cuestan 5 cajas?*

Si 3 cajas $\xrightarrow{\text{cuestan}}$ 15 € | $3 \rightarrow 15$
5 cajas $\xrightarrow{\text{costarán}}$ ? | $5 \rightarrow x$

Con los tres datos conocidos y el desconocido formamos una pareja de fracciones equivalentes:

$$\frac{3}{5} = \frac{15}{x} \rightarrow 3 \cdot x = 5 \cdot 15 \rightarrow 3 \cdot x = 75 \rightarrow x = 25 \text{ €}$$

*Solución:* Cinco cajas de caramelos cuestan 25 €.

*PROBLEM: Three boxes of coffee and milk candies cost 15 €. How much cost 5 boxes?*

If 3 boxes cost…

With the three known data and the unknown datum we make a pair of equivalent fractions:

*Solution*: Five boxes of coffee and milk candies cost 25 €.

## Annex 3 –Fundamental property of proportions in the Brazilian textbook, p. 258.

Igualdades do tipo $\frac{a}{b} = \frac{c}{d}$ são chamadas de proporção.

Lê-se: "**a** está para **b**, assim como **c** está para **d**".

Os "elementos" **a**, **b**, **c** e **d** são chamados termos da proporção.

Não se esqueça que é proibido dividir por zero. $b \neq 0$ e $d \neq 0$

Como, na leitura da proporção $\frac{a}{b} = \frac{c}{d}$, iniciamos por **a** e terminamos por **d**, chamamos esses termos de extremos. Nessa leitura **b** e **c** vêm entre **a** e **d**, por isso **b** e **c** são termos conhecidos como meios.

Numa equação, se multiplicamos os dois membros pelo mesmo número, não nulo, a igualdade não se altera.

Para eliminar os denominadores, na igualdade $\frac{a}{b} = \frac{c}{d}$ multiplicamos o primeiro e o segundo membros por bd.

$$\frac{a \cdot \cancel{b}d}{\cancel{b}} = \frac{c \cdot b\cancel{d}}{\cancel{d}}$$

No primeiro membro os "bês" do numerador e do denominador se cancelam; no segundo membro os "dês" do numerador e do denominador também se cancelam.

O resultado é a igualdade ad = bc.

Se $\frac{a}{b} = \frac{c}{d}$, então $a \cdot d = b \cdot c$.

Esta é a propriedade fundamental das proporções.

*O produto dos extremos é igual ao produto dos meios.*

Equalities of the kind $\frac{a}{b} = \frac{c}{d}$ are called proportions.

We read: "a is for b, so that c is for d". The "elements" a, b, c and d are called terms of the proportion.

Reading the proportion $\frac{a}{b} = \frac{c}{d}$ we begin by a and end by d, so we call these terms extremes. In such reading b and c are between b and c, and therefore b and c are known as middle terms.

In an equation, if we multiply the two members by the same non zero number, the equality does not change.

To eliminate denominators, in the equality $\frac{a}{b} = \frac{c}{d}$, we multiply the first and second number by bd.

…

In the first member the "bb" of the numerator and denominator cancel out; in the second member the "dd" of the numerator and denominator also cancel out.

The result is the equality ad=bc.

If $\frac{a}{b} = \frac{c}{d}$, then a . d = b . c *(This is the fundamental property of proportions)*

The product of the extreme terms is equal to the product of the middle terms.

## Annex 4 – Solved example in the American textbook, p. 536.

Solve the proportion.

$$\frac{5}{2} = \frac{50}{a}$$

$\frac{5}{2} = \frac{50}{a}$ *Find the cross products.*

$5 \times a = 2 \times 50$ *Multiply.*

$5a = 100$

$\frac{5a}{5} = \frac{100}{5}$ *Divide to solve the equation.*

$a = 20$

$\frac{5}{2} = \frac{50}{a}$ *Check your solution*

$\frac{5}{2} \stackrel{?}{=} \frac{50}{20}$ *Replace a with 20. Find the cross products*

$5 \times 20 \stackrel{?}{=} 2 \times 50$ *Multiply.*

$100 = 100$ ✓ *The solution checks.*

## Annex 5 - Connection task in the American textbook, p. 544.

**13. Art** The famous painting known as "Red Interior, Still-life on a Blue Table" was painted by Henri Matisse in 1947. The copy at the left measures 1 in. tall by $\frac{3}{4}$ in. wide. The height of the real painting is 32 in. Find the width of the real painting.

Annex 6 - Reproduction task in the American textbook, p. 560.

**Write as a percent.**

**2.** 0.8 **3.** 0.25 **4.** 1.2 **5.** $\frac{3}{5}$ **6.** $1\frac{1}{2}$

Annex 7 - Reflection task in the American textbook, p. 565.

**48.** ***REASONING*** Find 28% of 75 and 75% of 28. Explain why the answers are the same. Make up another problem where it is easier to find the changed problem than the given one.

Annex 8 – An open task in the Brazilian textbook, p. 268.

14. Na maioria dos atlas e livros de Geografia se encontram as bandeiras dos países componentes da Organização das Nações Unidas (ONU). Porém, em geral, as bandeiras *não são representadas na proporção correta:*


Consultando o quadro ao lado, encontre as proporções verdadeiras e reproduza numa escala de sua escolha o contorno das bandeiras seguintes.

| País | Razão entre as medidas |
|---|---|
| Brasil | 7 × 10 |
| Portugal | 2 × 3 |
| Itália | 2 × 3 |
| França | 2 × 3 |
| Espanha | 2 × 3 |
| China | 2 × 3 |
| Japão | 2 × 3 |
| Uruguai | 2 × 3 |
| Canadá | 1 × 2 |
| Cuba | 1 × 2 |
| Rússia | 1 × 2 |
| Estados Unidos | 10 × 19 |
| Costa Rica | 3 × 5 |
| México | 4 × 7 |

In most atlas and Geography books we find the flags of the countries that belong to the United Nations. However, in general, the flags are not represented in t e correct proportion.

Looking at the table on the side, find the correct proportion and, using our own scale, show the contour of the following flags.

Annex 9- Semi open task in the American textbook, p. 564.

1. **Explain** how you can easily find 50% of a number if you know 25% of the number.