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Integrated Flat Plate Solar Thermoelectric System

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Abstract

A simple flat plate solar collector, which serves as a water heater and integrated to thermoelectric modules, was put in place. The essence was to harness the same solar energy that causes the bulk of heat gains to the building to heat water while at the same time the unit acts as an air-conditioner and generator to drive air circulating fans. A space with six occupants was considered for the study. A heat gain assessment was carried out to determine the power required of the thermoelectric modules to match the space. A collector size to power the modules, was then determined, constructed and the performance assessed. With two glass covers, average maximum temperature of 106^0C was recorded at mid clear sky days on latitude 7°0'49" North, 6°30'14" East in the months of April to September. Each of the thermoelectric element (TE) modules generated a voltage of 2V, enough to power a fan and a number of light emitting diodes (LED). The performance of the system was strongly dependent on the intensity of solar insolation and temperature difference of hot and cold sides for the thermoelectric module. Integrated design suggested that all of the device's features and components were chosen to work in harmony with each other toward the goal of creating a sustainable built environment. This encouraged attention to the materials chosen so that they will age gracefully and require an appropriate amount of maintenance over their lifetime

Keywords: Solar, heating, cooling, electricity, COP, entropy.

1. Introduction

In the tropical and temperate parts of the world, where the sun impinges immense quantities of energy on the surrounding and indoor environment, the energy is quite beneficial, warming our homes and reducing our need for using heating fuel. However, for homes in the warm summer months and for commercial office buildings most of the year, unmanaged solar energy can create a thermal heating load that must be removed by air-conditioning. Nowadays, there is technology (photovoltaics) available to create electricity with the sunlight as by Derrick, Francis and Bokalders [1], but photovoltaic panels are rather very expensive and the efficiency or figure of merit, which is material dependent is presently very low. Air conditioning in buildings is significant users of grid electrical energy and their energy consumption has important implications on social, economic, and environmental issues. Worst still, there is frequent outage of grid electricity and relying on nuclear energy may not be encouraged stemming from the recent Japanese experience. A challenging task today is to design and promote energy efficient air conditioning system in a cost effective and environmentally responsive way [2]. Since the past two decades and due to increased awareness on environment, energy saving has become a major issue and various designs and options have been proposed worldwide [3] Finding the right energy and air conditioner to use is a complicated process that depends on availability, reliability, safety, schedules of operation; internal heat gains, and other factors [5-9]. However, in these climates (with the proper designs) the solar energy, which causes the bulk of the building heat gains can be used to heat or cool buildings and at the same time meet sundry needs such as providing hot water and electricity for lighting purposes. At the moment, there is no known record of a designed flat plate solar collector, which serves as water/air heater and forms the hot sink of thermoelectric module that acts as air conditioner and generator. The primary motivation in this study, therefore, is to make a first foray into this territory.

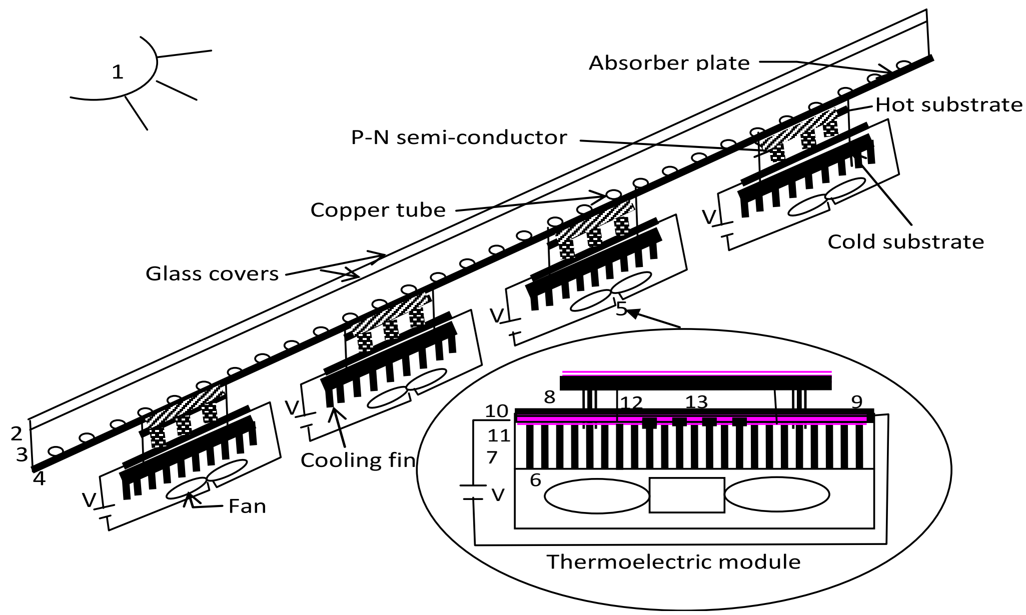
2. Material and Method

As shown in Figure 1, the design employed a box-shaped flat plate solar collector with tempered phosphate glass covers (1) known for their transmission of a wide range of wavelengths. This is the unique component of the solar equipment driving the air conditioner, generating electricity and serving as water/air heater. The absorber employed aluminum plate (4) painted with a wavelength-selective coating (3-black emulsion) that maximizes absorption of radiation in the solar energy spectrum of ultra-violet while it minimizes losses in the infra-red spectrum. It is housed in an insulated weatherproof case with a glazed front. The casing (2) was made of seasoned wood with aluminium cover to reduce conduction and convection heat losses to the surrounding air and cushion the system from weather effects. The side insulations were made up of wood, saw dust and foil paper. At the inner inside of the box, the copper tubes, which contained water, were embedded into the absorber plate, which in turn was attached at the bottom to thermoelectric modules (5). The thermoelectric module was a solid-state device driven by the solar collector heat absorber; the temperature difference on it creates a potential difference by Seebeck's effect and acting as a cooler by Thompson-Peltier's effect. The typical thermoelectric cooler (TEC) used consisted of alternating blocks of N-type and P-type bismuth telluride semiconductors (12) sandwiched between, and soldered to two opposing thin ceramic (alumina) plates (8 & 10). The typical TEC is a few millimeters thick, with surface area of 2.5 sq. cm. Each unit is capable of pumping 11 watts because it was assembled with several cell connected "thermally" in parallel and electrically in series; however this output was limited only by the size of the heat supply form the collector and on the geometry and number of the semiconductor junctions. As a cooler, the heat that was pumped from the load through the TEC to a heat reservoir, along with the resistive heating associated with the Peltier process, was often dissipated to the surrounding air via a finned cold sink (7). A fan (6) incorporated was used to move ambient air through the heat sink to pick up sensible cooling from the module.

The assembly was uniformly clamped together with an array of screws, usually thermally insulated from the "cold" and "hot" side of the TEC with nominally-insulating shoulder washers. Belleville washers were placed under the screw heads to provide a spring-like compression of the surrounding components onto the TEC. The cables and tanks use technology found in other heating, ventilation and air conditioning applications. The advantage of the design is that it serves a triple function: proving hot water, space cooling/heating as well as generating electricity.

Integrated design suggests that all of the device's features and components must be chosen to work in harmony with each other toward the goal of creating a sustainable built environment. This encourages attention to the materials chosen so that they will age gracefully and require an appropriate amount of maintenance over their lifetime. The mechanical, electrical and plumbing systems employed in this design are energy efficient with high velocity air-conditioning design strategy for all buildings based on the use of energy efficient, outdoor hot water and air handling equipment. Plumbing fixtures have been specified with water conservation in mind and all electrical systems are monitored from a control centre. With efficiency and conservation driving the design, long term energy savings outweigh short term design, material and installation cost.

Figure 1: Flat Plate Solar Thermoelectric system



The study consists of four phases. The first phase involved assessment of the heat gains to the space under consideration. The second phase dealt with the energy available and collected by the collector. Then data were collected for the electrical power production offered by the system. The data collected was used to formulate a model which estimates the relationship between a space heat gains with six occupants, the power and numbers of thermoelectric modules required to meet the demand of the space cooling and the collector size required to power the module, while serving sundry needs. In the last phase, the model developed is evaluated to determine the level of imperfections in the system using entropy generation minimization method.

The variables that affect the performance of the system were identified for detailed study using the experimental setup. The solar collector was positioned to face southern direction and inclined at an angle of 17^0 (latitude of test location$+10^0$). Measuring instruments used include digital volt-ammeter, a protractor and thermocouples. The tests were carried out between 9 a.m. and 5 p.m. at different days for six months and the average values recorded for the different time duration.

3. Theoretical Frameworks

The theoretical frameworks were begun with the assessment of the heat gains to the space under consideration and the collector greenhouse radiation required to power a generator and an air conditioning thermoelectric module. Some basic parameters, which are necessary to describe and formulate the equations of the performance of the solar thermoelectric air conditioner and generator were presented: namely maximum current (I_{max}), maximum voltage (V_{max}), maximum temperature difference (dT_{max}), maximum heat transfer (Q_{max}), total cooling load required (Q_s), daily absorbed radiation (D_S), collector efficiency (η_C), figure of merit (Z), coefficient of performance (COP) and entropy generation (S_{gen}). As current flows through a material, heat is generated. Thermoelectric (TE) material is no different. There is a point where the heat generated internally offsets the TEs ability to pump heat. Each TE has a limit on how much heat it can pump. This limit is referred to as Q_{max}. The current associated with Q_{max} is referred to as I_{max}. The corresponding voltage across the coolers is referred to as V_{max}. If a TE is completely insulated and isolated from the environment and running at I_{max}, it will produce its maximum temperature difference, dT_{max}. At this point it will also be pumping no heat whatsoever. As heat is applied to the cold side of the TE, the temperature differential is suppressed. Effectively, one trades temperature differential for heat pumping. As such, if the temperature differential is 0, the corresponding heat load is Q_{max}.

3.1. Heat Gains to Building

The design of an air conditioning system depends on the type of structure in which the system is to be placed, the amount of space to be cooled, the number of occupants, and the nature of their activity. A room or building with large windows exposed to the sun, or an indoor office space with many heat-producing lights, requires a system with a larger cooling capacity than an almost windowless room in which cool fluorescent lighting is used. The circulation of air must be greater in a space in which the occupants are allowed to smoke than in a space of equal capacity in which smoking is prohibited. In homes or apartments, most of the cooled or heated air can be re-circulated without discomfort to the occupants; but in laboratories or factories employing processes that generate noxious fumes, no air can be re-circulated, and a constant supply of cooled or heated fresh air must be supplied. Air conditioning demands that all these problems are solved.

The sizing of the air conditioning system in building requires an accurate analysis of the probable heat gains or losses. Heat gains to building include the following;

- Solar heat gains—this is dependent on the location (latitude of the place), the sun position, month, day, hour, existing intensity of radiation (direct and diffused); orientation of building; quality of building partitions (transparent or opaque) and the shading.
- Heat gains through the building structure—which depends upon the materials and fabric properties, form and sizes and the 'sol-air' temperature, T_{amb},. The sol-air temperature is the value of the outside temperature, which would in the absence of all radiation of heat exchange gives the same rate of heat flow in the outer surface wall (or any other type of partition) as the actual combination of air temperature differences and radiation exchanges really does, that is ,

$$h_s = U_w A(T_{amb} - T_i) \tag{1}$$

where $T_{amb} = T_o + {}^\circ J x J$ [w/m^2] and T_o is outside air temperature, T_i=inside air temperature, $^\circ J$=absorption co-efficient, and J=total solar radiation (direct and diffused).

- Internal heat gains—which are human body, electric lighting and appliances, gas appliances, process losses and infiltration, that is, the unwanted heat introduced into the room.

To obtain designed conditions in most cases, as in Nigeria, where no heating is required, it is necessary to supply air at lower temperature and less moisture content for the air in the room. This air will gain heat and moisture as it passes through the room until it obtains the room conditions. Calculations of air temperature quantity are based on the formula:

$$h_s = \dot{m}\, C_p(T_r - T_s) \tag{2}$$

where h, enthalpy, T_r =room air temperature and T_s=supply air temperature.

Each of the steps below was keyed to an example of a 3.6m long by 3.6m wide by 3m high office space with a maximum of one lecturer and five students. Other pertinent information for this example is given as follows:

a. Interior design temperature: 27ºC
b. Exterior design temperature: 30.5
c. Exterior design humidity: 80%

Air-conditioning units are rated in terms of effective cooling capacity, which properly should be expressed in kilowatt units. The following steps outline the procedure for estimating the cooling load (in KJ/hr) for the example problem –office space defined above.

i. Space Cooling Load Estimate

Using the simplified design method and the space floor area, the space cooling load in terms of supply air required was estimated. This accounts for the heat gains from 6 persons, equipment, lights, and space envelope. The space gave a floor area of 12.96m^2 and with a cooling load factor, CLF of 1.2 [10]. The air CLF requirement is equal to floor area x cooling load factor=15.552CLF.and the total space cooling, Q_s

$$Q_s = 1.08 \times CLF \times dT \quad (4)$$
$$= 1.08 \times 15.552 \times (30.5-27) = 58.787 \text{ KJ/hr}$$
$$= 16.330\text{W}$$

where

dT = (Interior Design Temperature – Cooling Coil Temperature.

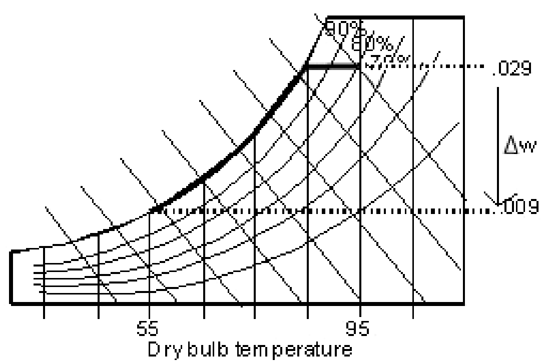
ii. Ventilation Load Estimate

Fresh air has both sensible and latent heat gain components. For an office, the required rate of ventilation is 10 CLF per person. Therefore, for 6 persons 120 CLF of fresh air is required giving the sensible heat gains from ventilation as $Q_{sensible}$ = 1.08 x CLF x dT (Outdoor Design Temp – Cooling Coil Temp) = 236.8kJ/hr=63W.

With a thermoelectric, TE module of 11W, this meant using 6 modules approximately to meet the room demand, only four are shown in Figure 1.

To determine latent heat gains from ventilation, the outside air temperature and relative humidity was plotted on the psychrometric chart to determine the amount of moisture that must be removed from the air at the cooling coil. This is the change in moisture, Δw.

Figure 2: Diagram of psychrometric



For latent heat factor of 4840, [10],

$$Q_{latent} = 4840 \times CLF \times \Delta w \quad (4)$$
$$= 4840 \times 120 \text{ CFM} \times (0.029 - 0.009)$$
$$= 10080 \text{ kJ/hr}$$

and total ventilation load,

$$Q_V = Q_{sensible} + Q_{latent} = 12283.2 \text{ kJ/hr}$$

the total cooling load CLF is now 120+15.552=135.5522 CLF.

Based on the total required CLF for the space, the size of the mixing box for or the handling unit AHU) was found to be: AHU=Q_s+Q_v=151.104kJ/hr=41.933W

3.2. Modelling of the Solar Collector

The daily heat gain is approximated by first calculating the specific daily average absorbed radiation, in J/m^2, for each south-facing surface collector, which according to the isotropic diffuse assumption developed by Liu and Jordan [11] and extended by Anna [8] is:

$$D_S = H_b R_b (\tau\alpha)_b + H_d (\tau\alpha)_d \frac{1+\cos\beta}{2} + H\rho_g (\tau\alpha)_g \frac{1-\cos\beta}{2}$$
$$= H_b \cos i + H_d \frac{1+\cos\beta}{2} + H\rho_g \frac{1-\cos\beta}{2} \qquad (5)$$

where D_S= daily absorbed radiation on tilted surface in J/m^2,

R_b=cosθ/cosz$=\dfrac{\cos(\varphi-\beta)\cos\delta\cos\omega+\sin(\varphi-\beta)\sin\delta}{\cos\varphi os\delta\cos\omega+\sin\varphi\sin\delta}$ is the ratio of beam radiation on a tilted surface to that on a horizontal surface, H=average daily radiation on a horizontal surface in J/m^2, (τα)=average transmittance-absorptance product and ρ_g = ground reflectance, 0.2 $\delta = 23.45\sin 360\left(\dfrac{284+day}{365}\right)\dfrac{\pi}{180}$ is the declination angle which is determined based on the day of the year and is the angle formed between a line drawn from the center of the earth to the center of the sun and the plane of the equator;$-23.45^0 < \delta < 23.45^0$

ω= hour angle from the local meridian in degrees

β=tilt angle of surface from horizontal in degrees,

$\cos\theta=\cos\alpha(\alpha_s-\alpha_w)\sin\beta+\sin\alpha\cos\beta$ or $\theta=\arccos[\cos(L-\beta)\cos\delta\cos\omega+\sin(L-\beta)\sin(\delta)]$ is the incident angle for each hour, the angle between the normal of a surface and the beam radiation. Introducing in the number of daylight hours given as:

$N_{hs} = \dfrac{2}{15}\cos^{-1}\left(-\tan L\tan\delta\right)$, where

ω= (solar time, N_{hs} −12)15 is the hour angle which is defined based on the time of day and ω=−tanLtanδ, which is the sunset hour angle which is the angular displacement of the sun at sunset from the local meridian (it is an indication of how long the sun is up during the day), it is possible to determine the number of sun shine hours for a particular day.

L = latitude, angle of location north or south of equator, north is positive, $-90^o \le L \le 90^o$

α – solar altitude angle, which is the angle between the horizontal and the line to the sun (or the sun's rays), the complement of the zenith angle

sinα = sin$L$sinδs + cos$L$cosδcosh

α_w- the deviation from due south of a horizontal projection of a line normal to surface, east is negative and west is positive; also called orientation $-180^o \le a_w \le 180^o$

a_s- solar azimuth angle, which is the angle between south and the projection of the beam radiation on a horizontal plane, east is negative and west is positive

$$\alpha_s = C_1 C_2 \alpha_s' + C_3 \frac{1-C_1C_2}{2}180$$

$$\alpha_s' = \frac{\sinh\sin\cos\delta}{\sin z}$$

z –zenith angle, which is the angle between the beam radiation and the vertical, the complement of the solar altitude angle; also the angle of incidence on a horizontal surface,

Cosz=cosLcosδ+cosh+cosδsinL

h- hour angle, which is the angle of the sun east or west of the local meridian due to the earth's rotation, based on time of 24 hours required for sun to move 360^{o}around earth and each hour is 15^o with the morning negative and the afternoon positive

Two factors which cause variation in Extraterrestrial Radiation, H on plane normal to radiation on the *n*th day of the year include variation in emitted radiation by sun (small) and variation in earth-sun distance – can be determined from geometry of relative movement of earth around sun. The solar constant, Ho which is the average amount of energy from the sun per unit time that is received on a unit area of a surface perpendicular to the sun's rays outside of the earth's atmosphere at a mean sun-earth distance (d_m) is given as 1377 W/m^2.

The daily extraterrestrial radiation, D_h on a horizontal surface, integrated from sunrise to sunset is:

$$H_d = \frac{24x3600}{\pi} H_o \left[1 + 0.034\cos\left(\frac{360xday}{365.25}\right)\right]\cos z \tag{6}$$

where cosz = 0.9972cos$L$cosδcosh + cos$L$cosδ $= \left(\cos L \cos\delta \cos\ h + \frac{\pi h}{180}\sin L \sin\delta\right)$

cos h = -tanLtanδ (sun set hour angle)

The total radiation, the H values are obtained from radiation data, but the distribution of this total radiation term into the diffuse and beam components must be estimated based on the sun angle and the clearness of the sky according to the Collares-Pereira and Rable correlation [11]:

$$H_d/H = 0.775 + 0.00606(\omega - 90) - [0.505 + 0.00455(\omega - 90)]\cos(115k_i - 103) \tag{7}$$

where ωs = sunset hour angle

k_i = daily average clearness index

τα=0.6

The values in the adjacent column are determined from H_b=H-H_d. To calculate the heat gain to the collector, the efficiency of the collector must be taken into account. This is done by assuming typical collector characterization values. The collector overall loss coefficient is assumed to be 5 W/m^2-C and the collector heat removal factor to be 0.75. The final useful heat gain can then be calculated with the equation:

$$Q_u = A_c F_R(H - U_L(T_i - T_a) \tag{8}$$

where

Q_u = Useful heat gain in J/day

A_c = Area of collector, in m^2

F_R = Heat removal factor

U_L=Overall loss coefficient of collector in W/m^2-C

The efficiency is calculated according to the equation:

$$\eta_C = 0.525 - 0.8858\frac{T_1 - T_a}{H} + 0.0074\frac{(T_i - T_a)^2}{H} \tag{9}$$

where T_i = Inlet temperature to collector and T_a = Ambient temperature

The dependence of the transmittance-absorptance product on the incident angle is accounted for using the incidence angle modifier:

k_i= 1-0.1441(□$_n$)-0.0948(□$_i$)

□i=(1/cosθ)-1

The hourly heat collected is calculated based on collector efficiency and the incident radiation:

$$Hc = \eta k_i H A_c \tag{10}$$

The parameter, UA value for the air changes per hour accounts for the heat loss from the temperature difference between air leaving the building and air entering the building. It is calculated with the equation:

$$UAeq = NV\rho C_{p/}.3600 \tag{11}$$

where UAeq = UA equivalent for air changes, 2220 W/C

N = number of air changes per hour, 2

V = volume of interior of greenhouse, 0.34 m^3

ρ = density of air at 20 C, 1009 kg/m3

C_p=specific heat of air at 30 C, 0.995 J/kg-C

Solving for the thickness, the equation becomes:

$$C_{th} = \frac{kA_s dTx3.56x10^7}{H_L} \tag{12}$$

The hourly heat loss from the greenhouse is calculated based on the heat loss rate integrated over one hour to give the total hourly heat loss:

$$H_L = UA(T_i - T_a)\Delta t \tag{13}$$

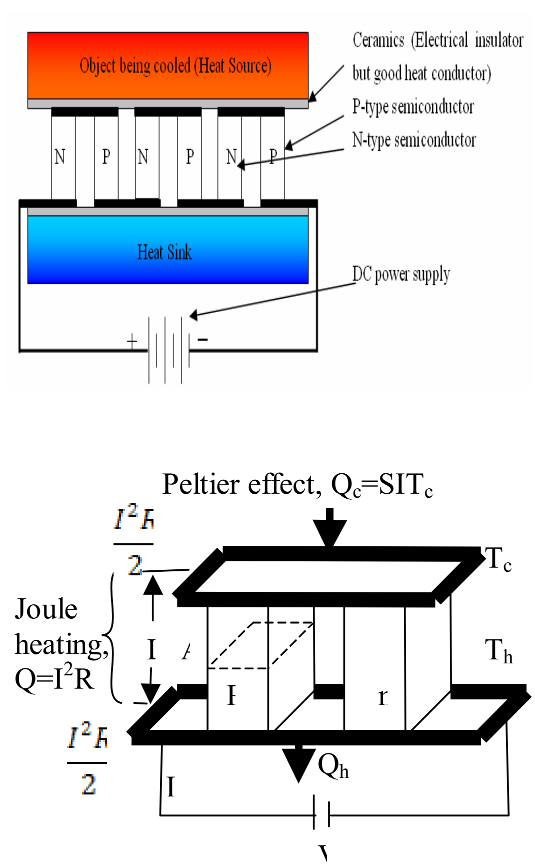
where H_L = Hourly heat loss from greenhouse in J and Δt= Time change per hour time step, 3600 seconds

3.3. Sizing the Dual Purpose TEC Module

The total heating load was calculated by first estimating a temperature difference across the module and assuming an input current for any particular module. This defines the active amount of heat that the module can pump from the ambient. Combining this with the total power input determines how much total heating the module can do. The temperature difference guess based on the thermal resistances to and from the module, the heat loads being transferred and the irreversibility were then obtained.

Figure 2 represents a thermoelectric module couple. The Figure shows some terms used in the mathematical equation: element or p-n module height (L), cross-sectional area (A), algebraic sum of the heat pumped by the Peltier effect (cooling Q_c or heating load, Q_h,) on cold side temperature (T_c), hot side temperature (T_h), temperature difference (dT) = (T_h-T_c), applied current (I) Seebeck coefficient (S)or thermo-power, measured in microvolts /K, voltage (V), electrical resistivity (R), thermal conductivity (k) and number of p-n element pairs, or couples (N).

Figure 3: TEC module



(a) Source: Rowe (1994)

(b)

The critical parameters for thermoelectric materials include figure-of-merit (ZT), the Seebeck coefficient (S), thermal resistance (θ_{TEC}), material electrical resistivity (ρ) and electrical conductivity (σ). The following represent the major formulations that can be made using these parameters .

3.3.1. The Space Cooling and Heating Load

Various TECs were identified with a parameter Q_{max} which were generally agreed to be the maximum amount of heat pumpable at $T_c = T_h = 30^oC$ (303 K). The basic material parameters S_m, k_m, (where m represent average), ρ and the device parameters N_c and λ=L/A, have been substituted for the parameters S, R, I and θ_{TEC}, which are given as [5, 14, 15]:

$$R = 2\rho\lambda N_c \tag{14}$$

$$I = \frac{S_m T_c}{\rho\lambda} \tag{15}$$

The TEC θ_{TEC} in Watts/Kelvin, S in Volts/Kelvin and R in Ohms are dependent both on the materials used within the TEC as well as the "geometry" of the device, given by the number and dimensions of the individual n and p type semiconductor elements. The thermoelectric coooling (TEC) θ_{TEC} is given as [13] :

$$\theta_{TEC} = \frac{\lambda}{2k_m N_c} = \frac{sheet\ thickness,\ \mathrm{L}}{Ak} \tag{16}$$

The "cooling or heating effect" is the rate of the heat removal from the cold reservoir and is the sum of three terms [7, 12]:

$$Q = SIT_{c\ \mathrm{or\ h}} \pm \frac{I^2 R}{2} - \frac{kA}{L} dT \tag{17a}$$

Generally, the heat pumping capacity of a module is proportional to the current, I and is dependent on the element geometry, (A, L) number of couples, N and the material. The heat transferred into the cold sink (T_c) and hot sink (T_h) is represented respectively by the minus and plus signs.

$$Q = N\left[SIT_{c\ \mathrm{or}\ h} \pm \frac{I^2 R}{2} - (T_h - T_c)/\theta_{TEC}\right]$$

$$= N\left[SIT_{c\ \mathrm{or}\ h} \pm \frac{I^2 R}{2} - K(T_h - T_c)\right] \tag{17b}$$

where $K=\theta_{TEC}$

The first Q term, $SIT_{c\ \mathrm{or}\ h}$ is the Peltier cooling or heating effect. The second term, $I^2R/2$, represents the Joule heating effect associated with passing an electrical current through a resistance. The Joule heat is distributed throughout the element, so 1/2 the heat goes towards the cold side, and 1/2 the heat goes towards the hot side. The last term, $kA(T_h-T_c)/L$, represents the Fourier effect in which heat conducts from a higher temperature to a lower temperature. So, the Peltier cooling or heating is reduced or increased respectively by the losses associated with electrical resistance and thermal conductance.

It is useful to display the function of Equation (8b) in a graphical format and especially in a way that is normalized to TEC parameters that depend only on the basic materials used in the TECs and not on the size or number of TEC couples. This makes one graph useful for all TECs that use the same basic materials, regardless of model number.

The heat pumped Q reaches a maximum with current and this maximum depends on the cold side and hot side temperatures. The maximum value of Q is given as:

$$Q_{\max} = \frac{S_m^2 T_c^2 N_c}{\rho\lambda} + \frac{T_c - T_h}{\lambda / 2k_m N_c} \tag{17c}$$

A minimum achievable cold side temperature, T_{Cmin} is identified which is achieved by pumping no heat (Q = 0). The value of T_{Cmin} is given by:

$$T_{c\min} = \frac{\rho k_m \left(-1 + \sqrt{1 + \frac{2S_m^2 T_h}{\rho k_m}}\right)}{S_m^2} \tag{18}$$

T_{Cmin} depends only on the TEC material parameters and the hot side temperature. For a hot side temperature of 305.8K, T_{Cmin} = 300K [7]

The total voltage drop across the TEC is then,

$$V = IR + S(T_h - T_c)$$
$$= 2N\left[S(T_h - T_c) + IRL/A\right] \quad (19)$$

consisting of a purely resistive component on the left and a thermoelectric component on the right generated by the temperature difference between the two sides. For the voltage, the first term, $S(T_h-T_c)$ represents the Seebeck voltage. The second term, IRL/A represents the voltage related by Ohm's law.

To determine the current and voltage needed from a power supply to pump a given heat load, equation (8b) for a single module (N=1) is re-arranged resulting in the equation:

$$I = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (20a)$$

where $A = R\theta_{TEC}/2$

$B = -\theta_{TEC} S T_{c\text{ or }h}$

$C = Q_c\theta_{TEC} + T_h - T_c$

and the voltage drop follows from equation (10). There are two solutions indicated by the +and - signs and both are valid. The lower current (- sign) represents the stable solution (from a control theory standpoint).

Next we consider the effect of the finite thermal resistance θ_{load} of the heat conductor between the thermal load and the actual TEC cold side, and another thermal resistance θ_{hS} represented by the heat sink between the TEC hot side and the final heat reservoir. As a result of this, the current is now given as:

$$I = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (20b)$$

where $A = R(\theta_{HS} + \theta_{TEC}/2)$

$B = -\theta_{TEC} S(T_{load} - Q_c\theta_{load})$

$C = Q_c(\theta_{load} + \theta_{HS} + \theta_{TEC}) + T_{amb} - T_{load}$

and note that it is in the same form as the original equation (11a) with additional components associated with the temperature drops across the real-world thermal resistances. The voltage drop is now given by:

$$V = IR + S(\theta_{hS}Q_c + \theta_{hS}I^2R + Q_c\theta_{load} + T_{amb} - T_{load}) \quad (21)$$

which is the same equation as equation (19) with T_c and T_h replaced with the appropriate dependencies on ambient temperature, T_{amb} and temperature load, T_{load}.

Qualitatively, the problem caused by the thermal resistances of the heat sink and load is that they cause an increased temperature drop across the TEC for a given T_{amb} and T_{load}. This, in turn, causes an increase in the current required to pump heat through the TEC.

These equations are very simplified and are meant to show the basic idea behind the calculations that were involved. The actual differential equations do not have a closed-form solution because S, R, and k are temperature dependent. Unfortunately, assuming constant properties can lead to significant errors, but the approximate solutions obtainable are within acceptable results for engineering practice.

The power at the PV panel is:

$$\dot{Q}_{out} = VI = SIdT + I^2R = \frac{V(V - SdT)}{R} \quad (22a)$$

At the TEC module,

$$\dot{Q}_{in} = \frac{kAdT}{L} \quad (22b)$$

3.3.2. The Air conditioner Figure-of-Merit and Co-efficient of Performance

As knowledge of thermoelectrics increased, the most important discoveries were related to material properties. In 1911, Altenkirch derived the energy of thermoelectric conversion efficiency or coefficient of performance (COP), now known simply as thermoelectric dimensionless figure of merit, ZT [13-16]. When a current is applied across the junction, some heat is absorbed in order to compensate for the heat generated by thermal conductance at equilibrium as well as by Joule (resistive) heating. It is this balancing condition which is characterized by each material's efficiency coefficient known as figure of merit, Z.

The parameters which are interesting in evaluating the performance of a cooling refrigeration system are the COP, the heat pumping rate, $\dot{Q}$ and dT_{max} which the device will produce. Technically, the word efficiency relates to the ratio of the amount of work one gets out of a machine to the amount of power input. In heat pumping applications, this term is rarely used because it is possible to remove more heat than the amount of power input it takes to move that heat. For thermoelectric modules, it is standard to use the term "coefficient of performance" rather than "efficiency." The power conversion efficiency or the coefficient of performance, COP is defined as the amount of heat pumping one gets for each unit of electrical power supplied, that is:

$$COP = \frac{\dot{Q}_p}{\dot{Q}_h} = \frac{SL}{kA} I \left(1 - \frac{I}{I_{\max}}\right) \tag{23}$$

where $\dot{Q}_h$ is the rate of heat removal from the hot body and $\dot{Q}_p$ is the electrical power output.

The maximum obtainable temperature difference is between the cold and hot side of the thermoelectric elements within module when I_{max} is applied and there is no heat load applied to the module. This parameter was measured with the hot-side of an element at a temperature of 300 K. In reality, it is virtually impossible to remove all sources of heat in order to achieve the true dT_{max}. Therefore, the number only serves as a standardized indicator of the cooling capability of a thermoelectric module

The conversion efficiency, η is related to the figure of merit, which was determined by three main material parameters: the thermo-power S, the electrical resistivity ρ, and the thermal conductivity k. Heat is carried by both electrons (k_e) and phonons (k_{ph}), and $k = k_e + k_{ph}$. The quantity, figure of merit, ZT_m itself is defined as [14, 16-19]:

$$ZT_m = \frac{S^2 \sigma}{k} = \frac{S^2}{k\rho} \frac{(T_c + T_h)}{2} \tag{24}$$

The coefficient of performance (COP) can be expressed as the quotient of Q_C by W:

$$COP = \frac{Q_c}{W} = \frac{(\sigma_p - \sigma_n) IT_c - kdT - 0.5 I^2 R}{I\left[(\sigma_p - \sigma_n) dT + IR\right]} \tag{25}$$

By setting dI/dQc=0, the maximum cooling power can be obtained:

$$Q_{\max} = \frac{(\sigma_p - \sigma_n)^2 T_c^2}{2R} - kdT \tag{25}$$

Similarly, by setting Q_c=0, the maximum temperature difference dT_{max} can be calculated,

$$dT_{\max} = \frac{(\sigma_p - \sigma_n)^2 T_c^2}{2kR} \tag{27}$$

The figure of merit Z with a dimension of k^{-1}, (and a dimensionless figure of merit ZT) is usually employed and is derived as follow [7, 13, 14]:

$$ZdT = \frac{SL}{kA} I_{\max} = \frac{S^2}{UR} dT \tag{28a}$$

where U=kA/L=1/R (electrical), k=σ (thermal) and $I_{max}=\frac{V_{max}}{R}=\frac{SdT}{R}$ so that the figure of merit of the thermoelectric element is defined as:

$$Z=\frac{S^2\sigma}{k}=\frac{S^2}{Rk}=\frac{S^2}{\left[\left(\rho_n k_n\right)^{\frac{1}{2}}+\left(\rho_p k_p\right)^{\frac{1}{2}}\right]^2}$$

$$=\frac{\left(\sigma_p-\sigma_n\right)^2}{\left[\left(\rho_n k_n\right)^{\frac{1}{2}}+\left(\rho_p k_p\right)^{\frac{1}{2}}\right]^2} \tag{28b}$$

where σ is the electrical conductivity in the direction of current flow and ρ, density of the semi-conductor.

By setting dη/dI=0, we can get the value of I at which η reaches its maximum value,

$$I_{max}=\frac{\left(\sigma_p-\sigma_n\right)\left(T_h-T_c\right)}{R\left(1+ZT_m\right)^{\frac{1}{2}}-1} \tag{29}$$

The corresponding COP is therefore given as:

$$COP=COP_C\left[\frac{\sqrt{1+ZT_m}-\frac{T_h}{T_c}}{\sqrt{1+ZT_m}+1}\right] \tag{30a}$$

where $COP_C = T_c/(T_h-T_c)$ is the Carnot COP.

By assuming a common Seebeck's, S for the p and n elements,

$$COP=\frac{Q_c}{Q_s}=\frac{SIT_c-0.5I^2-K(T_h-T_c)}{SI(T_h-T_c)+I^2R} \tag{30b}$$

where Q_s is the energy supplied. For $\frac{d(COP)}{dI}=0$

$$I=\frac{S(T_h-T_c)}{R(\sqrt{1+ZT_m}+1)} \tag{31}$$

For maximum cooling from the condition dQc/dI=0,

$$I_{opt}=\frac{ST_c}{R}=ZT_c\sqrt{K/R} \tag{32}$$

$$Q_{c\max}=K\left[\tfrac{1}{2}ZT_c^2-(T_h-T_c)\right] \tag{33}$$

$$COP_{opt}=\frac{\tfrac{1}{2}ZT_c^2-(T_h-T_c)}{ZT_hT_c} \tag{34}$$

The COP of N-stage (N>1) thermoelectric cooling system as given by Bao [6] can be expressed in the following form, assuming that each stage operates over a temperature difference of $(T_h- T_c)/N$

$$\eta_n=\frac{1}{(1+\frac{1}{\eta'})^N-1} \tag{35a}$$

where η' is the COP of a single-stage module and

$$\eta'=n(\eta_1+\frac{1}{2})-\frac{1}{2} \tag{35b}$$

Here η_1 is the coefficient of performance for a single-stage Peltier module that operates over a temperature difference of $(T_h-T_c)/N$. For example, the COP of a 2-stage module is

$$\eta_{12} = \eta_1 + \frac{1}{8(2\eta_1 + 1)} \tag{35c}$$

and the COP of a 3-stage module is

$$\eta_3 = \eta_1 + \frac{(2\eta_1 + 1)}{27\eta_1^2 + 27\eta_1 + 7} \tag{35d}$$

η_c is the Carnot efficiency = $(T_h–T_c)/T_h$, $T_m=1/2(T_h+T_c)$ and T_h and T_c are the hot and cold temperatures, respectively.

The optimum geometry couple COP or ZT is:

$$ZT = \frac{mT_c - \frac{m^2}{2} - \frac{T_h - T_c}{Z}}{m(T_h - T_c) + m^2} \tag{36}$$

where m=IR/S.

There exists, therefore, an optimum heat sink design derived from the balance between the reduction in the plate-to-plate thermal conductance and the TE module's operation at a larger temperature differential. The basis for the optimization of the heat sink is the minimization of the generation of entropy. In effect, the design that uses an optimized heat sink will need less power to achieve a given set of heat pumping conditions than designs using non-optimized heat sink.

Thus, a significant difference in temperature (large dT) is also needed to generate sufficient electrical energy, and the infrared (IR) region of the solar spectrum can supply the needed hot temperature, T_h. This is important because IR radiation generates only waste heat in conventional semiconductor-based solar photovoltaic cells.

The main challenge in improving thermoelectric materials is that the three relevant properties (electrical conductivity, thermal conductivity, and Peltier coefficient) are interrelated. The equation dictates that one must maximize ZT. To increase ZT requires a material that is highly electrically conductive, but thermally insulative, with a large S. The Z is a direct measure of the cooling performance of a thermoelectric module. Z is temperature dependent though, so, when comparing one module to another, they must be based on the same hot-side temperatures.

The magnitude of the difference between the thermo-powers of the two materials is directly proportional to the Peltier coefficient of the junction. In a physical sense, the Peltier coefficient can be thought of as the amount of energy each electron carries across the junction relative to the Fermi energy. To find the most efficient thermoelectric cooling device, it is necessary to optimize each material's figure of merit, making Z as large as possible. This is a difficult task since the thermo-power, electrical conductivity, and thermal conductivity are each determined by the specific electronic structure of the material; it is not possible to change one parameter without changing the others.

3.3.3. Entropy Generation Analysis

The nature of the effort to minimize the entropy generation can be seen by examining the following equation. The operating entropy generation, $S_{gen,}$ equation is as follows [15, 20, 21, 22a, 22b]

$$S_{gen} = \frac{ds}{dt} - \sum \frac{Q}{T} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s \geq 0 \tag{37a}$$

Certain assumptions are made regarding the use of exergy in modeling a TE system:

- The system is at steady state. Hence, ds/dt**,** the rate of entropy accumulation is zero.
- Heat transfer through a heat sink occurs solely from the sink interface to the sink-TE module interface.
- The heat and cold sink plates are isothermal.
- The heat sink footprint matches the TE module footprint.

Equation (28a) can be applied to the heat sink particulars as in Figure 1 and then reduces to the following by substituting the parameter of heat flow [13]:

$$S_{gen}=\frac{\left(\dot{Q}_{hot}-\dot{Q}_{cold}\right)}{\frac{kA(T_{cold}-T'_{cold})}{L}} \tag{37b}$$

This loss represents the imperfection in the system.

4. Results and Discussion

Using Microsoft Excel, equations (5) to (12), (16) to (19) (22), (23) to (25), (27), (30), (34) to (36) and (37b) were used to compute the solar system parameters as presented in tables 2-3 with their corresponding characteristic plots in Figures 4 and 5. The following design specifications were made as follow for the thermoelectric module:

Table 1: Design Specifications for the Thermoelectric Module

dT_{max} °C	Q_{max} W	S_m V/K	ρ ohm-cm	k_m watts/m-K
10.5	11	2.0×10^{-4}	1.10×10^{-3}	1.55 x 10

Giving from equation (18), Z=0.002346
Base width= 30mm
Base Length=34mm
Top width=30mm
Top length=30mm
Height=4mm
Giving on average, U=kA/L=0.00372
Solar collector size=$2m^2$

Table 2: Solar Radiation at Latitude 7°0'49" North, 6°30'14" East

Time in mins	30	60	90	120	150	180	210	240
H in kJ	10491	14225	15841	15927	17630	31223	33722	31223
Time in mins	270	300	330	360	390	420	450	480
H in kJ	37630	35927	33841	24225	21673	15368	13690	12861

Table 3: Integrated Solar Energy Heater, Cooler and Generator Performance Data

Time in mins	dT_{max} ^0C	Q_{in} kJ	Q_u kJ	Q_h kJ	Q_c kJ	Q_{out} kJ	η_c	COP_{TE}	COP_{opt}	S_{gen} kJ/kgK
30	3.0	975.45	802.56	8.85	8.22	30.029	0.525	0.48	0.801	13.19
60	3.6	975.45	802.6	8.99	8.35	30.498	0.525	0.48	0.809	13.24
90	4.1	977.19	739.99	9.81	9.12	33.079	0.525	0.482	0.911	13.11
120	4.4	977.19	724.74	10.03	9.33	33.78	0.525	0.482	0.939	13.09
150	5.8	977.54	694.75	10.62	9.88	35.66	0.525	0.483	1.003	13.09
180	6.6	977.54	672.65	10.99	10.23	36.83	0.525	0.484	1.049	13.06
210	6.9	977.54	650.92	11.36	10.58	38.01	0.525	0.484	1.095	13.04
240	7.1	977.89	650.88	11.22	10.45	37.54	0.525	0.485	1.088	12.99
270	7.2	978.93	650.88	11.22	10.45	37.54	0.526	0.484	1.088	12.99
300	7.0	978.93	650.88	11.22	10.45	37.54	0.526	0.485	1.088	12.99
330	6.6	978.93	658.08	11.07	10.31	37.07	0.526	0.485	1.071	12.99
360	6.4	978.93	665.28	10.78	10.04	36.13	0.526	0.484	1.046	12.94
390	6.2	978.59	679.88	10.49	9.76	35.19	0.526	0.483	1.013	12.94
420	6.0	978.24	629.3	10.38	10.07	30.03	0.526	0.484	1.094	12.69
450	5.5	977.54	709.54	9.9	9.21	30.50	0.525	0.483	0.947	12.94
480	5.2	977.54	724.62	9.6	8.94	33.08	0.525	0.482	0.915	12.94

Figure 4: Time Dependent Energy (Q_{in} Q_u Q_h Q_c and Q_{out}) of the Integrated Solar Energy Collector

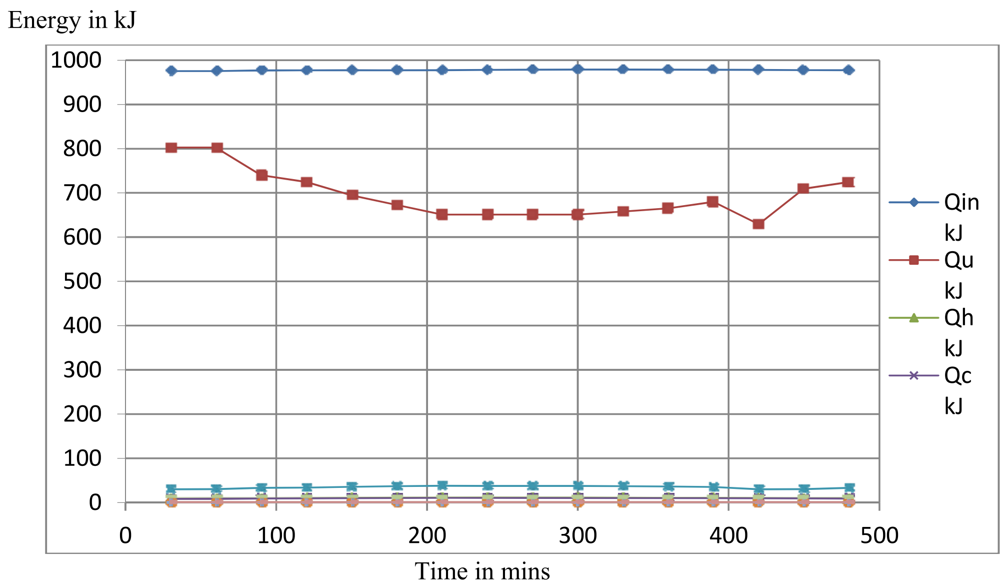
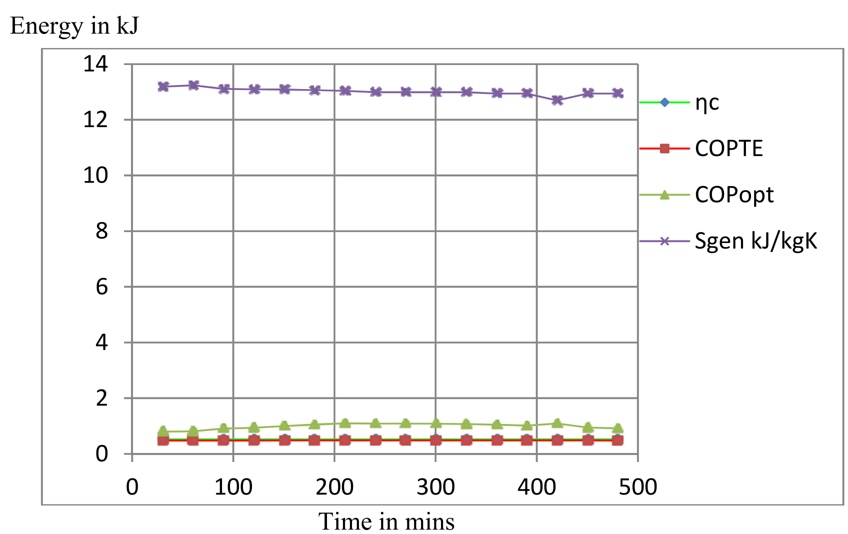


Figure 5: Time Dependent Efficiency of solar collector, COP and Entropy of the Integrated Unit



The calculated total heating demand for the year was used to provide an estimate of the required collector size, but this does not accurately reflect how the integrated heating system works. In reality, the collector will continuously increase the tank temperature, and the greenhouse heating demand will continuously decrease the temperature. The relationship between the variables, namely solar radiation, solar altitude, solar azimuth and temperature and the power generated per unit area by the collector were explored in detail using data collected from the experimental study.

Experimental results demonstrated that the unit can maintain a temperature difference (dT_{max}), in the system at a range of 3–7.2 °C, and have a range of COP_{TE} and COP_{opt} of 0.0.048-0.485 and 0.801-1.095 respectively within the test period and the heating effect is slightly higher than the cooling effect. Maximum figure of merit, Z and exergy, S_{gen} range were analyzed and found to be 0.002346 (therefore, ZT is very low) and 12.94-13.09kJ/kgK respectively. When these values of S_{gen} are compared to the values of Q_h, Q_c and the total Q_{out} in table 3, Figures 4 and 5, it shows that the amount of energy eventually harvested using this system is low. The irreversiblities or degraded energy is high with this conversion of energy from solar to thermal and then electrical energy. In fact the efficiency, which was found to be almost a constant at 0.525 is very low, but against the background that the energy is harvested from a renewable source (solar) and at no cost, this is worth reaping.

The results show that the thermoelectric air-conditioning system can indeed be used for very effective and efficient cooling/heating and generation of electricity though the efficiency is very low. Since thermoelectric coolers are solid-state heat pumps, they can actively pump heat from the ambient in addition to the heating effect that comes from the electrical resistance of the cooler itself. So, the thermoelectric cooler can be more efficient than a resistive heater (within limits). However, the value of the entropy generation, S_{gen} is indicative that the imperfection in the system is very high. A high ZT is required before any significant impact can be made by solar thermoelectric air-conditioning system.

5. Conclusion

The solar air conditioner produced dT_{max} of 3-7.2°C within the test period. Analysis indicated that the performance of the system is strongly dependent on intensity of solar insolation, the figure of merit, Z and temperature difference of hot and cold sides for the thermoelectric module, which are very low because of the level of imperfection of the system. A very high ZT value is needed in order for integrated solar flat plate and thermoelectric cooler/generator to compete with components which either act as sole thermal solar flat plate collector, solar thermoelectric cooler or solar flat plate thermoelectric generator earlier developed in the literature cited.

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Appendix

Explanation of Nomenclature			
$\dot{Q}$	input electric power of the thermoelectric module, W;	$\dot{Q}_{solar}$	output electric power of solar cells, W;
$^{a}{}_{b}$,	battery efficiency	t,	time
$^{a}{}_{dc\text{-}ac}$,	power-conditioning unit	T_a	ambient temperature
$^{a}{}_{pv}$,,	PV panel efficiency	T_{amb}	ambient temperature
A	area of solar array, m^2;	T_c	temperature of cold side, K, °C;
A_c	area of collector, 929 m^2	T_h	temperature of hot side, K, °C;
ac,	alternating current	T_h	hot sink temperature
CA	Storage capacity, Ah;	T_i	inlet temperature to collector
CFA	cooling load factor	T_{load}	temperature load
COP	coefficient of performance;	T_m	average temperature K, °C;
C_p	specific heat of air	U_L	overall loss coefficient of collector
dc,	direct current	V	electric voltage, V;
D_S	daily absorbed radiation on tilted surface in MJ/m^2	V	volume of interior of greenhouse
dT_{max}	maximum temperature difference	V,	voltage
E	daily electric power consumption,	W,	Watt
E,	system efficacy	Z	figure of merit, 1/K;
ET	equation of time	ZT	dimensionless figure of merit
F_R	heat removal factor	α	Seebeck coefficient , V/K;
H	monthly average daily radiation on a horizontal surface in MJ/m^2	β	tilt angle of surface from horizontal in degrees
H_L	Hourly heat loss from greenhouse in MJ	δ	declination, angular displacement of the sun at solar noon

I	electric current, A;	Δt	time change per hour time step, 3600 seconds
K	thermal conductivity	Δw	change in moisture
ki	monthly average clearness index	η_{pv}	energy efficiency of solar cells;
K_m	material thermal conductivity	θ_{max}	maximum degree of electric discharging;
K_t	total thermal conductivity of thermoelectric module, W/K;	λ	ratio of the element length L to area A
L	length of module	ρ	density of air
N	number of p-n pair of couple;	ρ_g	ground reflectance
N_a	number of air changes per hour	σ	electrical conductivity
PV,	photovoltaic		
Q_c	cooling production, W;	$T\alpha$	monthly average transmittance-absorptance product
Q_u	useful heat gain in MJ/day	φ	latitude
R	resistance, Ω;	ω	hour angle from the local meridian in degrees
R_b	beam radiation	ωs	sunset hour angle
S	Solar insolation rate, W/m^2;	η_C	collector efficiency