Buoyancy and thermal radiation effects for the Blasius and Sakiadis flows with a convective surface boundary condition

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Abstract

This study is devoted to investigate the Buoyancy and thermal radiation effects on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Using a similarity variable, the governing nonlinear partial differential equations have been transformed into a set of coupled nonlinear ordinary differential equations, which are solved numerically by using shooting technique along side with the sixth order of Runge-Kutta integration scheme and the variations of dimensionless surface temperature and fluid-solid interface characteristics for different values of Prandtl number *Pr*, radiation parameter N_R, parameter a and the local Grashof number Gr_x, which characterizes our convection processes are graphed and tabulated. Quite different and interesting behaviours were encountered for Blasius flow compared with a Sakiadis flow. A comparison with previously published results on special cases of the problem shows excellent agreement.

Keywords: Heat transfer; Blasius/Sakiadis flows; Thermal radiation; Thermal Grashof number; Convective surface boundary condition.

1. Introduction

Investigations of boundary layer flow and heat transfer of viscous fluids over a flat sheet are important in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Among these studies, Sakiadis [1] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows. Tsou et al. [2] analyzed the effect of heat transfer in the boundary layer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis [1]. Erickson et al. [3] extended the work of Sakiadis [1] to include blowing or suction at the stretched sheet surface on a continuous solid surface under constant speed and investigated its effects on the heat and mass transfer in the boundary layer. The related problems of a stretched sheet with a linear velocity and different thermal boundary conditions in Newtonian fluids have been studied, theoretically, numerically and experimentally, by many researchers, such as Crane [4], Fang [5-8], Fang and Lee [9]. The classical problem (i.e., fluid flow along a horizontal, stationary surface located in a uniform free stream) was solved for the first time in 1908 by Blasius [10]; it is still a subject of current research [11,12] and, moreover, further study regarding this subject can be seen in most recent papers [13,14]. Recently, Aziz [15], investigated a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Very more recently, Makinde & Olanrewaju [16] studied the effects of thermal buoyancy on the laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition. Olanrewaju & Makinde [17] presented the combined effects of internal heat generation and buoyancy force on boundary layer over a vertical plate with a convective surface boundary condition.

On the other hand, convective heat transfer with radiation studies are very important in process involving high temperatures such as gas turbines, nuclear power plants, thermal energy storage, etc. In light of these various applications, Hossain & Takhar [18] studied the effect of thermal radiation using Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Furthermore, Hossain et al. [19,20] have studied the thermal radiation of a gray fluid which is emitting and absorbing radiation in a non-scattering medium. Moreover, Bataller [21] presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow). This study is an extension of those analyses. It is aimed at analysing the effect of buoyancy parameter Gr_x, radiation parameter N_R on both Blasius and Sakiadis thermal boundary layers over a horizontal plate with a convective boundary condition. This boundary condition scarcely appears in the pertinent literature. The most recent attempt for the Blasius and Sakiadis flows but without buoyancy parameter has been developed by Bataller [21] whose results we used for comparison including Aziz [15] and Makinde & Olanrewaju [16] which discussed Blasius flow. Interaction of thermal radiation and thermal Grashof number with wall convection is included. Our results have been displayed for range of given parameters.

The aim of the present paper is to report the effects of thermal radiation and thermal Grashof number as well as Prandtl number Pr and convective parameter a on both Blasius and Sakiadis thermal boundary layers under a convective boundary condition.

2. Problems formulation

Taking into account the buoyancy and the thermal radiation terms in the momentum and energy equations, the governing equations of motion and heat transfer for the classical Blasius flat-plate flow problem can be summarized by the following boundary value problem [15-16,21]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \tag{3}$$

The boundary conditions for the velocity field are:

$$\begin{aligned}&u=v=0 \;\; at \;\; y=0; \;\; u=U_\infty \quad at \;\; x=0,\\ &u\to U_\infty \;\; as \;\; y\to\infty,\end{aligned} \tag{4}$$

for the Blasius flat-plate flow problem, and

$$\begin{aligned}&u=U_w; \quad v=0 \quad at \quad y=0,\\ &u\to 0 \;\; as \;\; y\to\infty,\end{aligned} \tag{5}$$

for the classical Sakiadis flat-plate flow problem, respectively.

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$\begin{aligned}&-k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right],\\ &T(x,\infty)=T_\infty.\end{aligned} \tag{6}$$

Here u and v are the velocity components along the flow direction (x-direction) and normal to flow direction (y-direction), υ is the kinematic viscosity, k is the thermal conductivity, c_p is the specific heat of the fluid at constant pressure, ρ is the density, g is the acceleration due to gravity, β is the thermal volumentric-expansion coefficient, q_r is the radiative heat flux in the y-direction, T is the temperature of the fluid inside the thermal boundary layer, U_∞ is a constant free stream velocity and U_w is the plate velocity. It is assumed that the viscous dissipation is neglected, the physical properties of the fluid are constant, and the Boussinesq and boundary layer approximation may be adopted for steady laminar flow. The fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium.

The radiative heat flux q_r is described by Roseland approximation such that

$$q_r = -\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ^* *and* K' are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Bataller [21], we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{8}$$

Using (7) and (8) in (3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3K'}\frac{\partial T^4}{\partial y}. \tag{9}$$

In view of eqs. (9) and (8), eq. (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty^3}{3\rho c_p K'}\right)\frac{\partial^2 T}{\partial y^2}, \tag{10}$$

where $\alpha = \dfrac{k}{\rho c_p}$ is the thermal diffusivity.

From the equation above, it is clearly seen that the influence of radiation is to enhance the thermal diffusivity. If we take $N_R = \dfrac{kK'}{4\sigma^* T_\infty^3}$ as the radiation parameter, (10) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\alpha}{k_0}\frac{\partial^2 T}{\partial y^2}, \tag{11}$$

where $k_0 = \dfrac{3N_R}{3N_R + 4}$. It is worth citing here that the classical solution for energy equation, eq. (11), without thermal radiation influences can be obtained from the above equation which reduces to $u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \alpha\dfrac{\partial^2 T}{\partial y^2}$, as $N_R \to \infty (i.e., k_0 \to 1)$.

We introduce a similarity variable η and a dimensionless stream function f(η) as

$$\eta = y\sqrt{\frac{U}{\upsilon x}} = \frac{y}{x}\sqrt{\mathrm{Re}_x}, \frac{u}{U} = f', v = \frac{1}{2}\sqrt{\frac{U\upsilon}{x}}(\eta f' - f), \tag{12}$$

where prime denotes differentiation with respect to η and Re_x is the local Reynolds number ($= \frac{Ux}{\upsilon}$), we obtain by deriving eq. (12)

$$\frac{\partial u}{\partial x} = -\frac{U}{2}\frac{\eta}{x} f''; \frac{\partial v}{\partial y} = \frac{U}{2}\frac{\eta}{x} f'' \tag{13}$$

And the equation of continuity is satisfied identically.

$$\frac{\partial u}{\partial y} = Uf''\sqrt{\frac{U}{\upsilon x}}; \ \frac{\partial^2 u}{\partial y^2} = \frac{U^2}{\upsilon x} f'''. \tag{14}$$

Nothing that in eqs. (12)-(14) $U = U_\infty$ represents Blasius flow, whereas $U = U_w$ indicates Sakiadis flow, respectively. We also assume the bottom surface of the plate is heated by convection from a hot fluid at uniform temperature T_f which provides a heat transfer coefficient h_f.

Defining the non-dimensional temperature $\theta(\eta)$ and the Prandtl number Pr as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \mathrm{Pr} = \frac{\upsilon}{\alpha}, \tag{15}$$

We substitute eqs. (12)-(14) into eqs. (2) and (11) we have:

$$f''' + \frac{1}{2} ff'' + Gr_x \theta = 0, \tag{16}$$

$$\theta'' + \frac{\mathrm{Pr}\, k_0}{2} f\theta' = 0. \tag{17}$$

When $k_0 = 1$, the thermal radiation's effect is not considered.

The transformed boundary conditions are:

$$\begin{gathered} f = 0, \ f' = 0, \theta' = -a[1 - \theta(0)] \ \ at\, \eta = 0, \\ f' \to 1\, as \ \theta \to 0 \ as \ \eta \to \infty \end{gathered} \tag{18}$$

for the Blasius flow, and

$$\begin{gathered} f = 0, \ f' = 1, \theta' = -a[1 - \theta(0)] \ \ at\, \eta = 0, \\ f' \to 0\, as \ \theta \to 0 \ as \ \eta \to \infty \end{gathered} \tag{19}$$

for the Sakiadis case, respectively. Where

$$a = \frac{h_f}{k}\sqrt{\upsilon x / U_\infty}\,,\; Gr_x = \frac{\upsilon x g \beta (T_f - T_\infty)}{U_\infty^2} \tag{20}$$

For the momentum and energy equations to have a similarity solution, the parameters Gr_x and a must be constants and not functions of x as in eq. (20). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$ and the thermal expansion coefficient is proportional to x^{-1}. We therefore assume

$$h_f = c x^{-1/2},\; \beta = m x^{-1} \tag{21}$$

Where c and m are constants. Putting eq. (21) into eq. (20), we have

$$a = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}},\quad Gr = \frac{m g \upsilon (T_f - T_\infty)}{U_\infty^2} \tag{23}$$

Here, a and Gr are defined by eq. (23), the solutions of eqs. (16)-(19) yield the similarity solutions, however, the solutions generated are the local similarity solutions whenever a and Gr_x are defined as in eq. (20).

3. Numerical procedure

The coupled nonlinear eqs. (16) and (17) with the boundary conditions in eqs. (18) and (19) are solved numerically using the sixth-order Runge-Kutta method with a shooting integration scheme and implemented on Maple [22]. The step size 0.001 is used to obtain the numerical solution with seven-decimal place accuracy as the criterion of convergence.

4. Results and discussion

Numerical computations have been carried out for different embedded parameters coming into the flow model controlling the fluid dynamics in the flow regime. The Prandtl number used are 0.72, 1, 3, 5, 7.1, 10 and 100; the convective parameters a used are 0.1, 0.5, 1.0, 5.0, 10, and 20; the radiation parameters N_R used are 0.7, 5.0, 10, and 100; and the Grashof number (Gr_x) used are Gr > 0 (which corresponds to the cooling problem). The cooling problem is often encountered in engineering applications; for example, in the cooling of electronic components and nuclear reactors. Comparisons of the present results with previously works are performed and excellent agreements have been obtained. We obtained the results as shown in Tables 1 - 6 and figures 1-17 below.

Table 1 shows the comparison of Aziz [15] work with the present work for Prandtl numbers (Pr = 0.72, and 10) and it is noteworthy to mention that there is a perfect agreement in the absence of radiation parameter and the Grashof number. Table 2 shows the comparison of Makinde & Olanrewaju [16] work with the present work for Prandtl numbers (Pr = 0.72, 3.0 and 7.1) and Grashof numbers (Gr_x = 0.1, 1.0 and 10) and there is a perfect agreement in the absence of radiation parameter. Table 3 shows the comparison of Bataller [21] work for Blasius and Sakiadis flows for Prandtl numbers (Pr = 0.72, 1.0, 5.0, 10 and 100) and radiation parameter (N_R = 0.7, 5.0, 10 and 100) and it is noteworthy to mention that there is a perfect agreement in the absence of Grashof number. Accurately, the results at a = 0.5, Pr = 5 and N_R = 0.7 for the missed plate temperature θ(0) values were numerically obtained as θ(0) = 0.55489763 for Blasius flow, and θ(0) = 0.44474556 for Sakiadis flow, respectively (see table 3). In table 5, we show the influence of the embedded flow parameters on the temperature at the wall plate for the Blasius and Sakiadis flow. It is clearly seen that when Biot number a increases the wall temperature for Blasius and Sakiadis flow increases while increase in Prandtl number Pr, radiation parameter N_R, and local Grashof number Gr_x decreases the wall temperature for both Blasius and Sakiadis flow. Table 5 shows the influence of the flow parameters on the Nusselt number and the Skin friction for Blasius flow. Increase in the convective parameter a, Prandtl number Pr, thermal radiation parameter N_R, and the local Grashof number Gr_x bring an increase in the Nusselt number. Skin friction increases with an increase in the convective parameter and the local Grashof number while increase in the Prandtl number and the radiation parameter decreases the Skin friction at the wall plate. In table 6, we show the effect of flow embedded parameters on the Nusselt number and the Skin friction for Sakiadis flow. Increase in all the flow parameters brings an increase in the Nusselt number and also in the Skin friction except the local Grashof number.

Table 1: Values of $\theta(0)_{Blasius}$ for different values of a without thermal radiation and thermal Grashof number. Parenthesis indicates results from Ref. [15].

a	Pr = 0.72	Pr = 10
0.05	0.14466116 (0.1447)	0.06425568 (0.0643)
0.20	0.40352252 (0.4035)	0.21548442 (0.2155)
0.60	0.66991555 (0.6699)	0.45175915 (0.4518)
1.00	0.77182214 (0.7718)	0.57865638 (0.5787)
10.0	0.97128537 (0.9713)	0.93212791 (0.9321)
20.0	0.98543355 (0.9854)	0.96487184 (0.9649)

Table 2: Values of $\theta(0)_{Blasius}$ for different values of a with thermal Grashof number and without thermal radiation. Parenthesis indicates results from Ref. [16].

Bi	Gr_x	Pr	$\theta(0)$
0.1	0.1	0.72	0.24922837 (0.24922)
1.0	0.1	0.72	0.76249384 (0.76249)
10	0.1	0.72	0.96944035 (0.96944)
0.1	0.5	0.72	0.23862251 (0.23862)
0.1	1.0	0.72	0.22955153 (0.22955)
0.1	0.1	3.00	0.16954012 (0.16954)
0.1	0.1	7.10	0.13278839 (0.13278)

Table 3: Values of $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ for different values of a, Pr, and N_R in the absent of thermal Grashof number Gr. Parenthesis indicates results from Ref. [21].

a	Pr	N_R	$\theta(0)_{Blasius}$	$\theta(0)_{Sakiadis}$
0.1	5	0.7	0.19957406 (0.1996265)	0.13807609 (0.1380922)
0.5	5	0.7	0.55489763 (0.5548979)	0.44474556 (0.4447517)
1.0	5	0.7	0.71374169 (0.7137422)	0.61567320 (0.6156583)
10	5	0.7	0.96143981 (0.9614407)	0.94124394 (0.9412387)
20	5	0.7	0.98034087 (0.9803475)	0.96973278 (0.9697438)
1	0.72	0.7	0.83312107 (0.8334487)	0.84297896 (0.8623452)
1	1.0	0.7	0.81555469 (0.8156143)	0.81785952 (0.8281158)
1	5	0.7	0.71374169 (0.7137422)	0.61567320 (0.6156583)
1	10	0.7	0.66301284 (0.6630187)	0.51639994 (0.5163969)
1	100	0.7	0.47592614 (0.4759402)	0.23747971 (0.2374795)
5	5	0.7	0.92574298 (0.9257453)	0.88900927 (0.8890038)
5	5	5	0.90376783 (0.9037694)	0.83172654 (0.8317292)
5	5	10	0.90044458 (0.9004477)	0.82284675 (0.8228368)
5	5	100	0.89700322 (0.8970060)	0.81361511 (0.8136082)

Table 4: Values of $\theta(0)_{Blasius}$ *and* $\theta(0)_{Sakiadis}$ for several values of the parameters entering the problem.

a	Pr	N_R	Gr_x	$\theta(0)_{Blasius}$	$\theta(0)_{Sakiadis}$
0.1	5	0.7	0.1	0.19753138	0.13775550
0.5	5	0.7	0.1	0.54658747	0.44265532
1.0	5	0.7	0.1	0.70501920	0.61292892
10	5	0.7	0.1	0.95932704	0.94027601
20	5	0.7	0.1	0.97922106	0.96920391
1	0.72	0.7	0.1	0.82436476	0.83116411
1	1.0	0.7	0.1	0.80642320	0.80572810
1	5	0.7	0.1	0.70501920	0.61292892
1	10	0.7	0.1	0.65528104	0.51532923
1	100	0.7	0.1	0.47246774	0.23744803
5	5	0.7	0.1	0.92196930	0.88737565
5	5	5	0.1	0.89982474	0.83090242
5	5	10	0.1	0.89649412	0.82209565
5	5	100	0.1	0.89304899	0.81293145
5	5	0.7	0.2	0.91891118	0.88591024
5	5	0.7	0.3	0.91631148	0.88457452
5	5	0.7	0.4	0.91403503	0.88334270

Table 5: Values of $f''(0)_{Blasius}$, $\theta'(0)_{Blasius}$ *and* $\theta(0)_{Blasius}$ for several values of the parameters entering the problem.

a	Pr	N_R	Gr_x	$\theta(0)_{Blasius}$	$-\theta'(0)_{Blasius}$	$f''(0)_{Blasius}$
0.1	5	0.7	0.1	0.19753138	0.08024686	0.35549045
0.5	5	0.7	0.1	0.54658747	0.22670626	0.39549180
1.0	5	0.7	0.1	0.70501920	0.29498079	0.41312720
10	5	0.7	0.1	0.95932704	0.40672956	0.44083687
20	5	0.7	0.1	0.97922106	0.41557870	0.44297550
1	0.72	0.7	0.1	0.82436476	0.17563523	0.47982849
1	1.0	0.7	0.1	0.80642320	0.19357679	0.46710681
1	5	0.7	0.1	0.70501920	0.29498079	0.41312720
1	10	0.7	0.1	0.65528104	0.34471895	0.39495048
1	100	0.7	0.1	0.47246774	0.52753225	0.35527077
5	5	0.7	0.1	0.92196930	0.39015346	0.43680984
5	5	5	0.1	0.89982474	0.50087626	0.41431353
5	5	10	0.1	0.89649412	0.51752937	0.41156519
5	5	100	0.1	0.89304899	0.53475500	0.40886337
5	5	0.7	0.2	0.91891118	0.40544409	0.53184089
5	5	0.7	0.3	0.91631148	0.41844259	0.62020358
5	5	0.7	0.4	0.91403503	0.42982480	0.70358410

Table 6: Values of $f''(0)_{Sakiadis}$, $\theta'(0)_{Sakiadis}$ and $\theta(0)_{Sakiadis}$ for several values of the parameters entering the problem.

a	Pr	N_R	Gr_x	$\theta(0)_{Sakiadis}$	$-\theta'(0)_{Sakiadis}$	$-f''(0)_{Sakiadis}$
0.1	5	0.7	0.1	0.13775550	0.08622444	0.43350645
0.5	5	0.7	0.1	0.44265532	0.27867233	0.41073121
1.0	5	0.7	0.1	0.61292892	0.38707107	0.39814732
10	5	0.7	0.1	0.94027601	0.59723989	0.37420826
20	5	0.7	0.1	0.96920391	0.61592164	0.37210806
1	0.72	0.7	0.1	0.83116411	0.16883588	0.30704987
1	1.0	0.7	0.1	0.80572810	0.19427189	0.32277934
1	5	0.7	0.1	0.61292892	0.38707107	0.39814732
1	10	0.7	0.1	0.51532923	0.48467076	0.41618829
1	100	0.7	0.1	0.23744803	0.76255196	0.43957601
5	5	0.7	0.1	0.88737565	0.56312172	0.37805517
5	5	5	0.1	0.83090242	0.84548788	0.40210478
5	5	10	0.1	0.82209565	0.88952171	0.40463142
5	5	100	0.1	0.81293145	0.93534270	0.40703672
5	5	0.7	0.2	0.88591024	0.57044879	0.31474240
5	5	0.7	0.3	0.88457452	0.57712737	0.25349452
5	5	0.7	0.4	0.88334270	0.58328646	0.19398242

A. Velocity profiles

The effects of various thermophysical parameters on the fluid velocity are illustrated in Figs. 1 to 8. Fig. 1 depicts the effect of local thermal Grashof number on the fluid velocity and we observed an increase in the fluid velocity as the Grashof number increases which resulted to thickening the boundary layer thickness across the plate. It is interesting to note that as $Gr_x > 0$, there is sudden increase in the fluid velocity before satisfying the boundary condition for Blasius flow. Fig. 2 depicts the influence of convective parameter a on the velocity profiles. It is interesting to note that increasing the convective parameter increases the fluid velocity also. Fig. 3 depicts the influence of Prandtl number Pr on the velocity boundary layer thickness. It decreases the velocity boundary layer thickness. Similar thing happens in fig. 4 for Blasius flow. It is also interesting to note that the same effect was observed in the Sakiadis flow (see figs. 5-8).

Temperature profiles

The influences of various embedded parameters on the fluid temperature are illustrated in Figs. 9 to 17. Fig. 9 depicts the effect of thermal Grashof number on the temperature profile for Blasius flow and it is seen that increase in the local Grashof number decreases the thermal boundary layer thickness across the plate. We can see also that the same effect was seen for Sakiadis flow (see fig. 10). Fig. 10 depict the curve of temperature against spanwise coordinate η for various values of convective parameter a. It is clearly seen that increases in the convective parameter increases the temperature profile and thereby increase the thermal boundary layer thickness. Similar effect was seen also in fig. 14 for Sakiadis flow. Fig. 11also represents the curve of temperature against Spanwise coordinate η for various values of Prandtl number. Increase in Prandtl number leads to a decrease in the thermal boundary layer thickness. At high Prandtl fluid has low velocity, which in turn also implies that at lower fluid velocity the specie diffusion is comparatively lower and hence higher specie concentration is observed at high Prandtl number. In fig. 15 the same effect was observed. In fig. 12, there is a decrease in the thermal boundary layer thickness when the thermal radiation parameter is increased (see fig. 16). Finally in fig. 17, we display the curve of the temperature gradient against Spanwise coordinate η for various values of thermal Grashof number for Sakiadis flow and we observed that as $Gr_x > 0.1$, the thermal boundary layer thickness decreases which leads to a reverse flow.

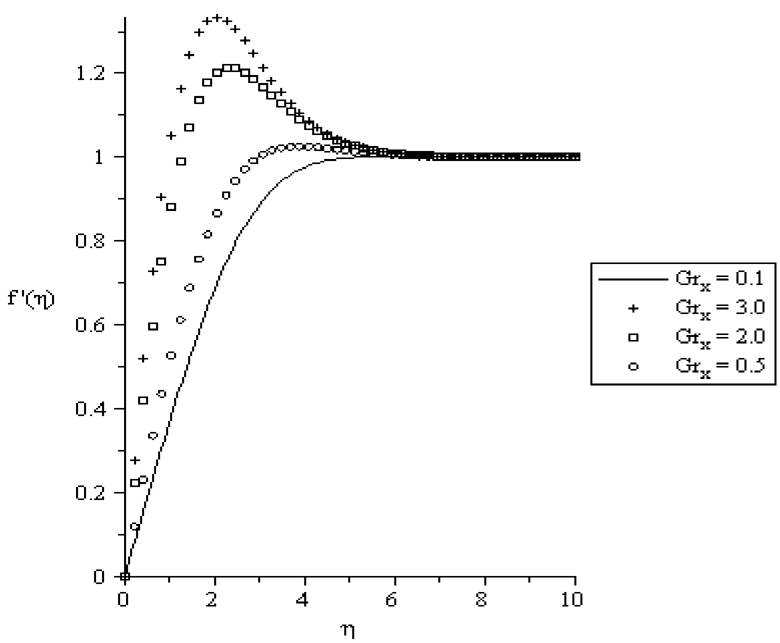

Figure 1: Velocity profiles for embedded parameter *Pr* = 0.72, *a* = 0.1, N_R = 0.7 for **Blasius flow**

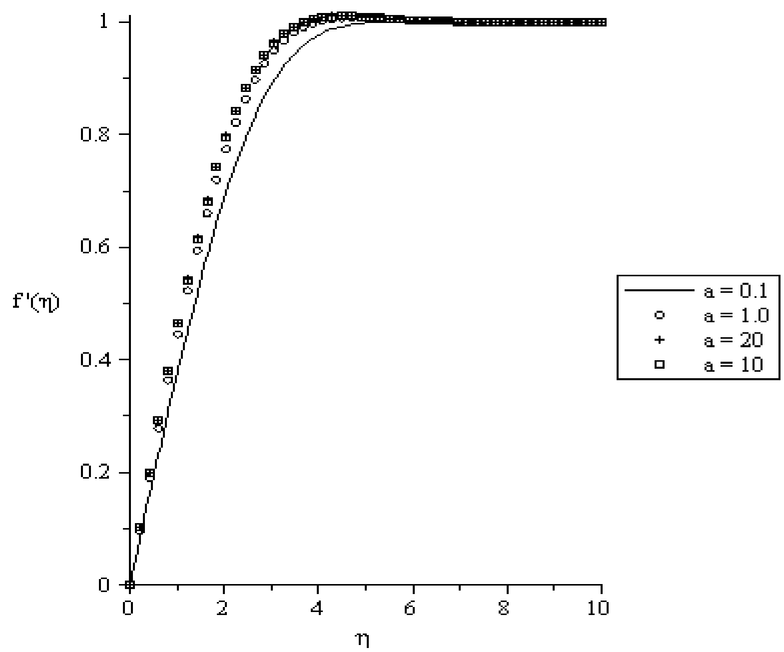

Figure 2: Velocity profiles for embedded parameter Pr = 0.72, Gr_x = 0.1, N_R = 0.7 for **Blasius flow**

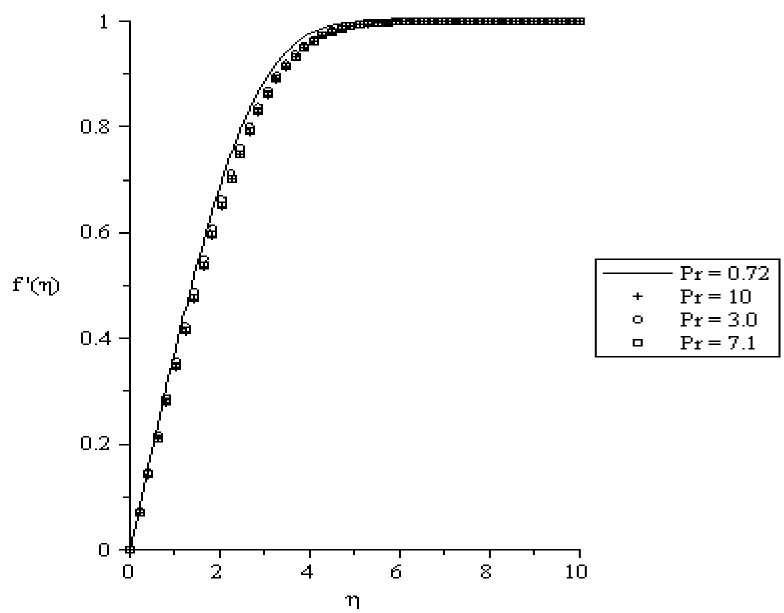

Figure 3: Velocity profiles for embedded parameter a = 0.1, Gr_x = 0.1, N_R = 0.7 for **Blasius flow**

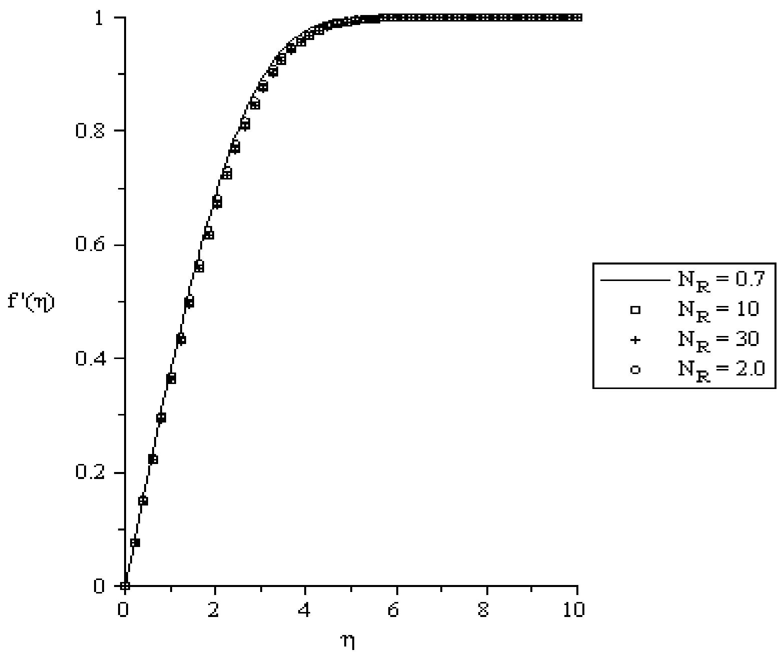

Figure 4: Velocity profiles for embedded parameter a = 0.1, Gr_x = 0.1, Pr = 0.72 for **Blasius flow**

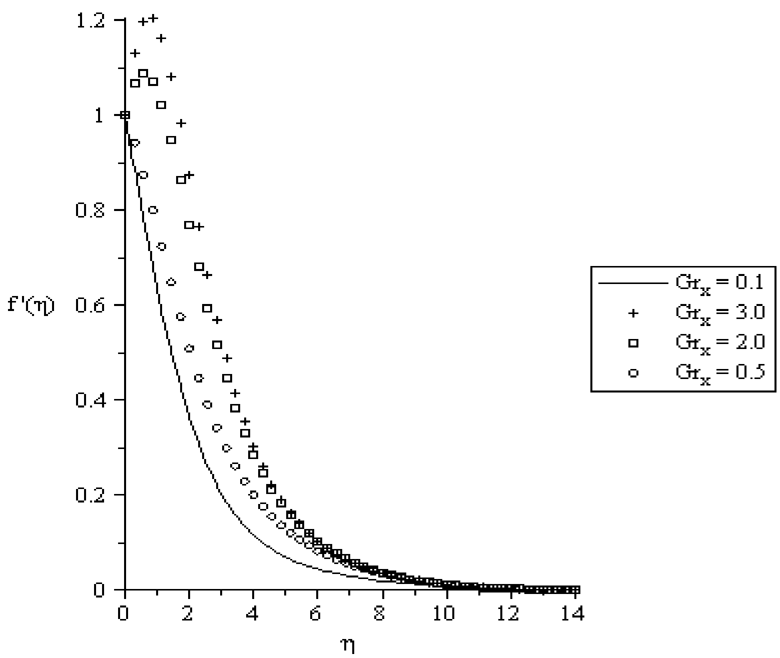

Figure 5: Velocity profiles for embedded parameter Pr = 0.72, a = 0.1, N_R = 0.7 for **Sakiadis flow**

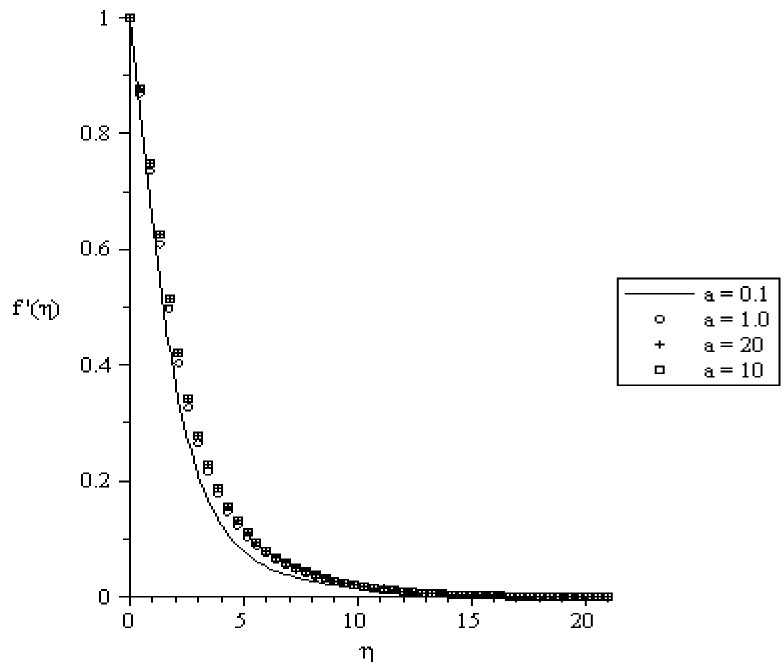

Figure 6: Velocity profiles for embedded parameter *Pr* = 0.72, Gr_x = 0.1, N_R = 0.7 for **Sakiadis flow**

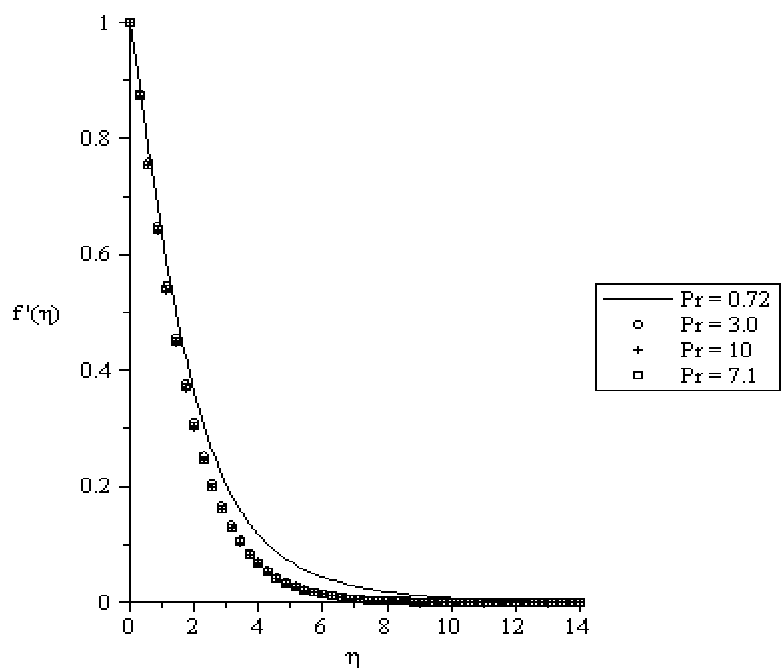

Figure 7: Velocity profiles for embedded parameter *a* = 0.1, Gr_x = 0.1, N_R = 0.7 for **Sakiadis flow**

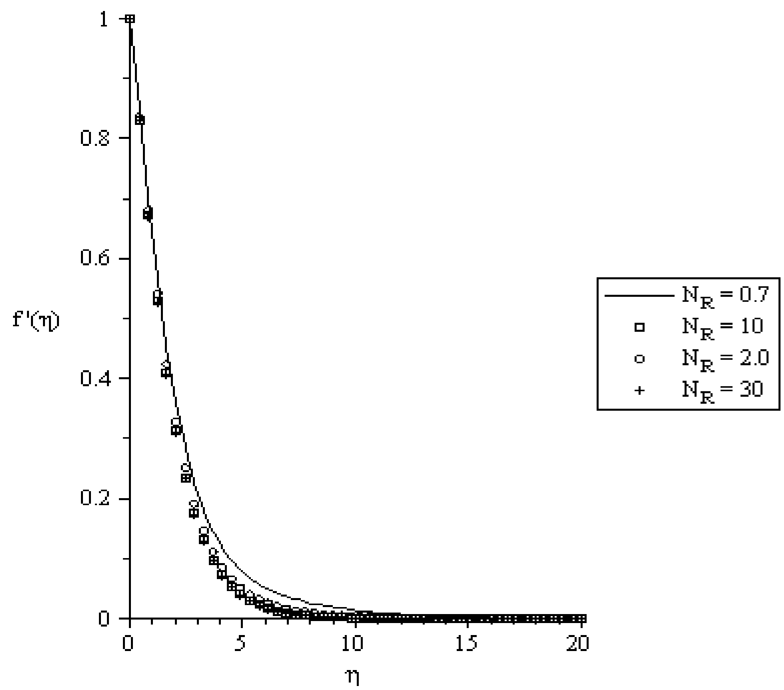

Figure 8: Velocity profiles for embedded parameter a = 0.1, Gr_x = 0.1, Pr = 0.72 for **Sakiadis flow**

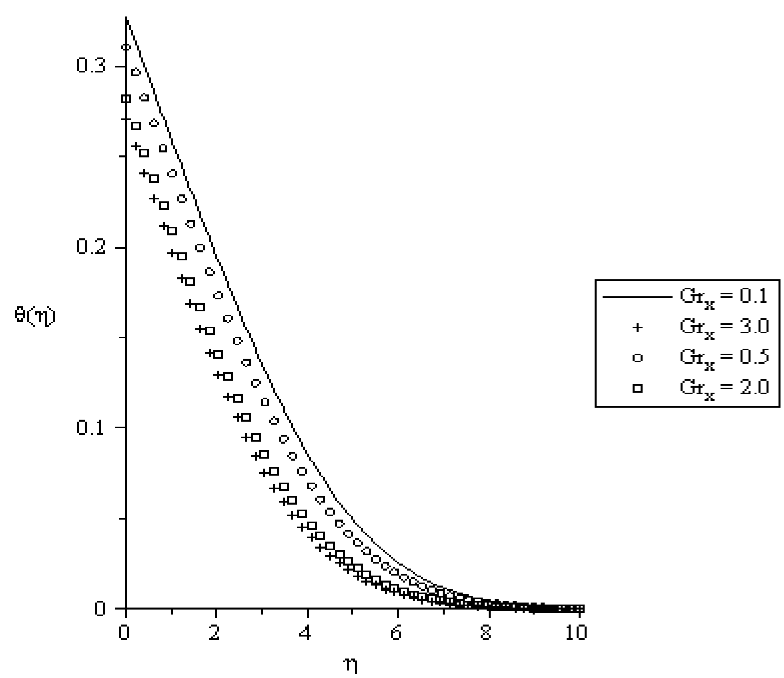

Figure 9: Temperature profiles for embedded parameter Pr = 0.72, a = 0.1, N_R = 0.7 for **Blasius flow**

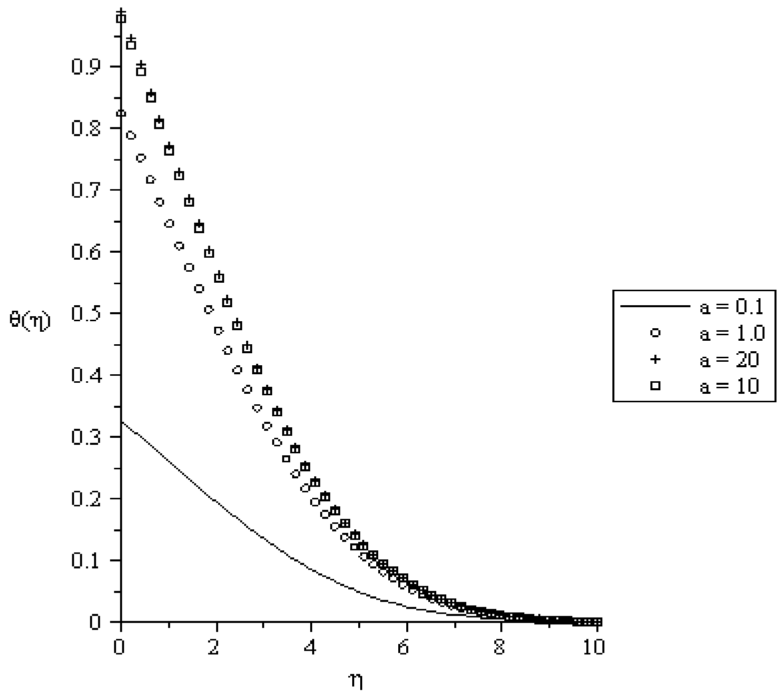

Figure 10: Temperature profiles for embedded parameter Pr = 0.72, Gr_x = 0.1, N_R = 0.7 for **Blasius flow**

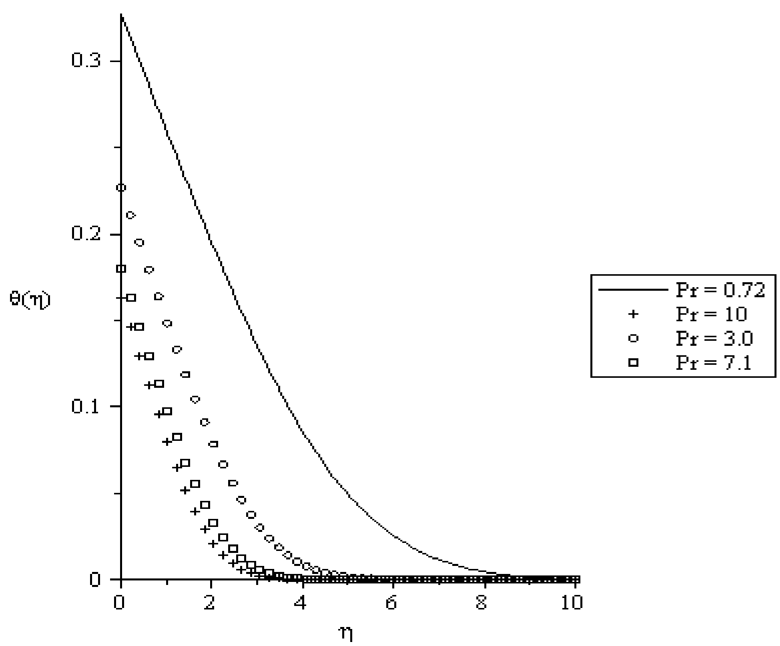

Figure 11: Temperature profiles for embedded parameter a = 0.1, Gr_x = 0.1, N_R = 0.7 for **Blasius flow**

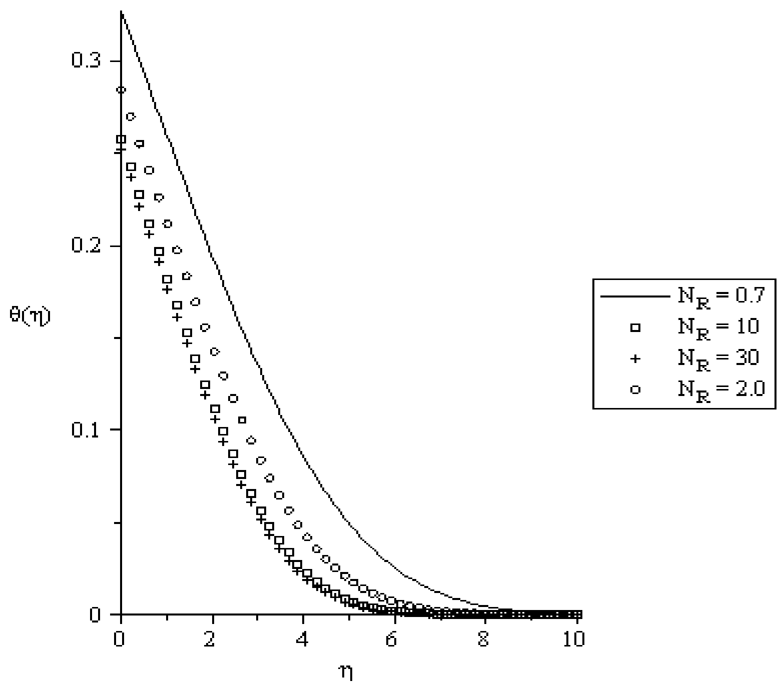

Figure 12: Temperature profiles for embedded parameter a = 0.1, Gr_x = 0.1, Pr = 0.72 for **Blasius flow**

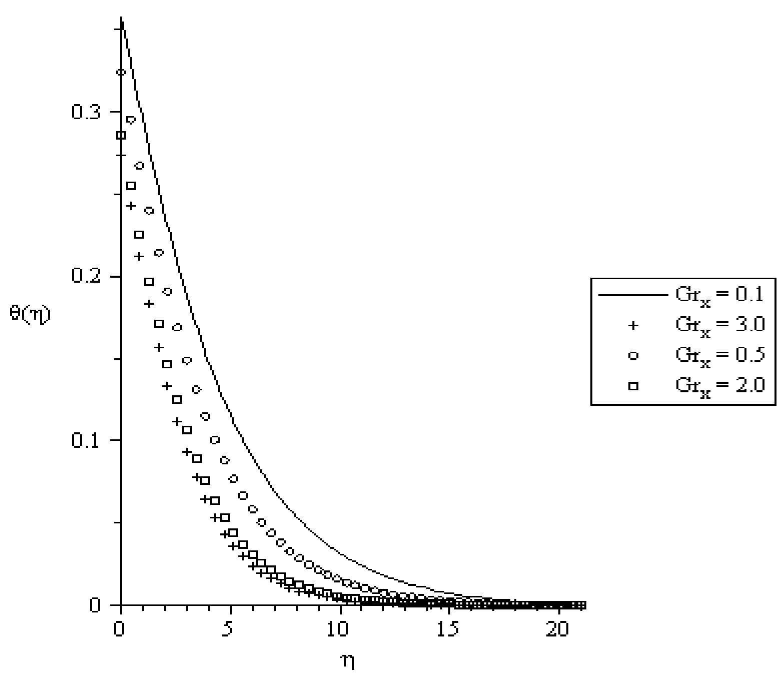

Figure 13: Temperature profiles for embedded parameter Pr = 0.72, a = 0.1, N_R = 0.7 for **Sakiadis flow**

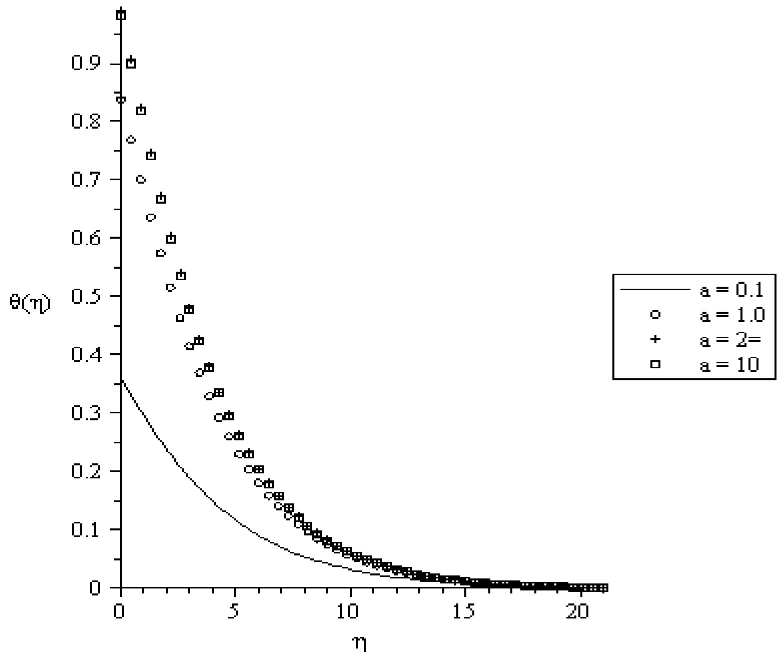

Figure 14: Temperature profiles for embedded parameter Pr = 0.72, Gr_x = 0.1, N_R = 0.7 for **Sakiadis flow**

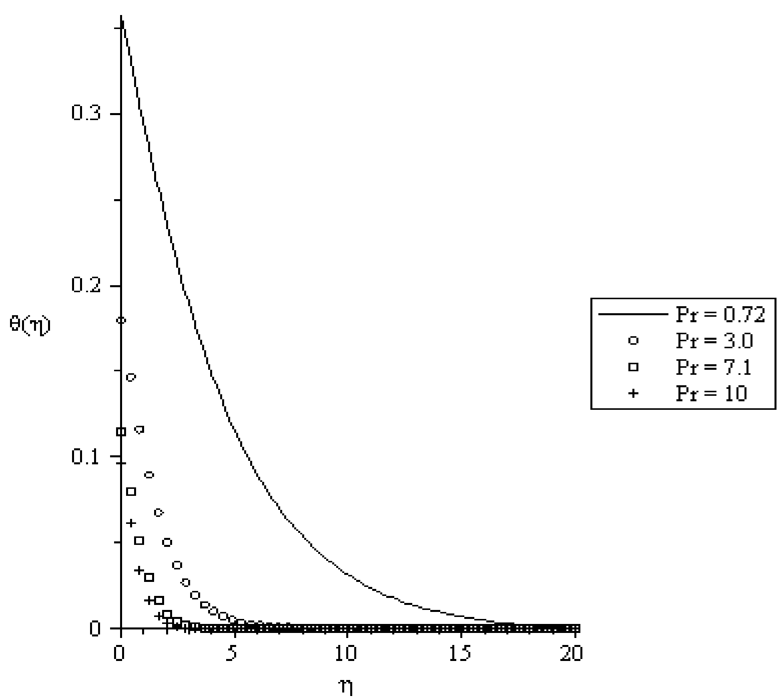

Figure 15: Temperature profiles for embedded parameter a = 0.1, Gr_x = 0.1, N_R = 0.7 for **Sakiadis flow**

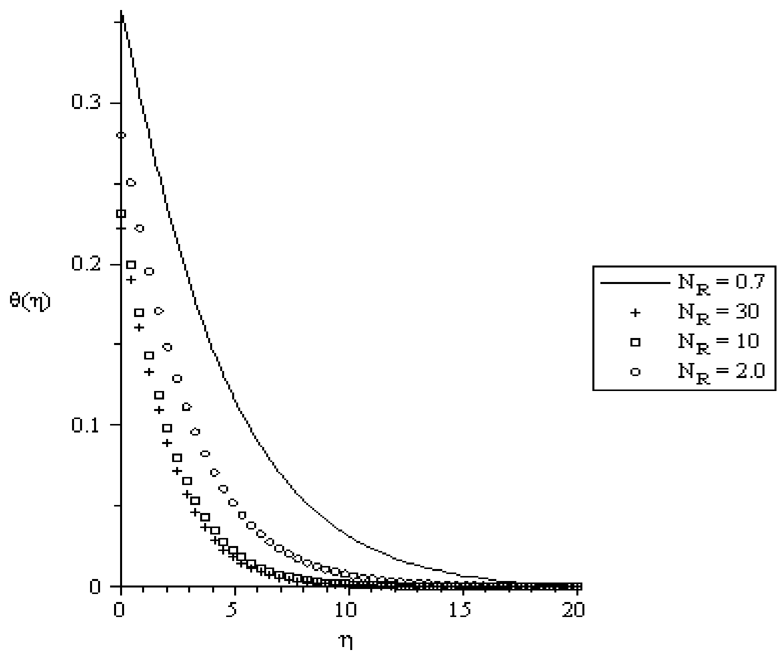

Figure 16: Temperature profiles for embedded parameter $a = 0.1$, $Gr_x = 0.1$, $Pr = 0.72$ for **Sakiadis flow**

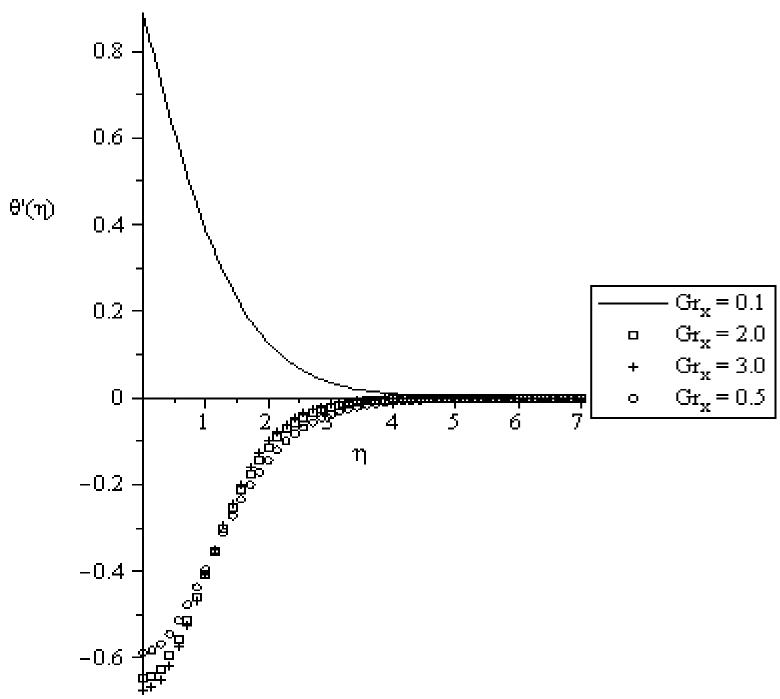

Figure 17: Temperature – gradient profiles for embedded parameter $Pr = 5$, $a = 5$, $N_R = 0.7$ for **Sakiadis flow**

5. Conclusions

In this article an IVP procedure is employed to give numerical solutions of the Blasius and Sakiadis momentum, thermal boundary layer over a horizontal flat plate and heat transfer in the presence of thermal radiation and the thermal Grashof number under a convective surface boundary condition. The lower boundary of the plate is at a constant temperature T_f whereas the upper boundary of the surface is maintained at a constant temperature T_w. It is also noted that the temperature of the free stream is assumed as T_∞ and also we have $T_f > T_w > T_\infty$. The transformed partial differential equations together with the boundary conditions are solved numerically by a shooting integration technique alongside with 6th order Runge-Kutta method for better accuracy. Comparisons have been analyzed and the numerical results are listed and graphed. The combined effects of increasing the thermal Grashof number, the Prandtl number and the radiation parameter tend to reduce the thermal boundary layer thickness along the plate which as a result yields a reduction in the fluid temperature. On the contrary, the values of $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ increase with increasing a and decreases with increasing Gr_x. In general, the Blasius flow gives a thicker thermal boundary layer compared with the Sakiadis flow, but this trend can be reversed at low values of embedded parameters controlling the flow model. Finally, in the limiting cases, $N_R \to \infty (i.e., k_0 \to 1)$ the thermal radiation influence can be neglected.

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