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The calculation of dynamic ecological footprint on the basis of the dynamic input-output model

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Abstract

The Leontief input-output model is widely used to determine the ecological footprint of consumption in a region or a country. It is able to capture spillover environmental effects along the supply change, thus its popularity is increasing in ecology related economic research. These studies are static and the dynamic investigations are neglected. The dynamic Leontief model makes it possible to involve the capital and inventory investment in the footprint calculation that projects future growth of GDP and environmental impacts. We show a new calculation method to determine the effect of capital accumulation on ecological footprint.

Keywords: Dynamic Leontief model, Dynamic ecological footprint, Environmental management, Allocation method

1. Introduction

The Ecological Footprint (in the following EF) is a resource accounting tool that measures how much biologically productive land and sea is used by a given population or activity, and compares this to how much land and sea is available, using prevailing technology and resource management schemes. (Wackernagel et al.,1996) It measures human demand on nature, by assessing how much biologically productive land and sea area is necessary to maintain a given consumption pattern. As a result the physical areas are expressed in so-called global hectares, making the comparison between regions, nations easier. The Ecological Footprint and biocapacity accounts cover six land use types: cropland, grazing land, fishing ground, forest land, built-up land and carbon uptake land (to accommodate the Carbon Footprint). For each component, the demand for ecological services is divided by the yield for those ecological services to arrive at the Footprint of each land use type.

In this analysis unsustainable nations have a larger ecological footprint than the land available for them. Nations can burden their excess ecological footprint on either other nations or on future generations. Static models cannot capture the whole spectrum of this spillover effects as shown in the table 1.

Bicknell et al. (1998) were the first to introduce static input-output analysis, incorporating it into the method of the ecological footprint calculations. Lenzen and Murray (2001) revised the method of Ecological Footprint. They have made modifications on the original concept in order to make it suitable for input-output analysis. Thus a regional, disturbance-based approach is taken in their study, including actual Australian land use and emissions data.

Wiedmann et al. (2006) have also shown a combination of Ecological Footprint accounting with monetary input–output analysis.

Table 1: Spillover effects to study dynamic ecological footprint

	Static economic models		**Dynamic economic models**	
	Ecological limits are rigid	Ecological limits can be transgressed in short term	Ecological limits are rigid	Ecological limits can be transgressed in short term
Closed economy	National EF ≤ national biocapacity Ecological footprint of consumption = ecological footprint of production Ecological impacts of capital investment on future EF *not* captured	National EF may exceed national biocapacity Ecological footprint of consumption = ecological footprint of production A nation may burden environmental impacts on future generations	National EF ≤ national biocapacity Ecological footprint of consumption = ecological footprint of production Ecological impacts of capital investment on future EF *are* captured	National EF may exceed global biocapacity A nation may not burden environmental impacts on other nations A nation may burden environmental impacts on future generations Ecological impacts of capital investment on future EF *are* captured
Open economy	Global EF ≤ global biocapacity A nation may burden environmental impacts on other nations A nation may NOT burden environmental	Global EF may exceed biocapacity Ecological impacts of capital investment on future consumption and	Global EF ≤ global biocapacity A nation may burden environmental impacts on other nations	Global EF may exceed global biocapacity A nation may burden environmental impacts on other nations

	impacts on future generations Ecological impacts of capital investment on future consumption and EF are NOT captured	EF are *not* captured A nation may burden environmental impacts on future generations or on other nations	A nation may *not* burden environmental impacts on future generations Ecological impacts of capital investment on future EF ARE captured	A nation may burden environmental impacts on future generations Ecological impacts of capital investment on future EF *are* captured

The footprint model can also be made dynamic along the ecological time

The paper is organized as follows. In section 2 the model is posed and we make some assumption about the model. The next section characterizes the properties of the balanced growth path in case of effective environmental regulation of the government. In section 4 we illustrate our results by the help of a simple numerical example, and the last section summarizes the results of the paper and proposes some possibilities for further research.

2. Description of the investigated models

Our model is based on the equations of the dynamic multi-sector input-output model well known in the literature. (Leontief (1986)) We extend this model with a system of inequalities for land requirements in order to analyze the balanced path compared to the environmental standards imposed by the government.

2.1. Dynamic ecological footprint in a closed economy

Suppose that there are n economic industries each industry producing a single economic commodity and m types of lands released by industries, e.g. agricultural, forest or degraded land. The input-output balance of the economy is described by the equation of economic goods and the inequality of lands. The equation of goods describes the balance between the total output of goods of production and the sum of total input of goods of all activities of the economy and the consumed goods.

Before we calculate the dynamic ecological footprint in a dynamic input-output model context, we demonstrate the methodology of the determination of the ecological footprint in a static input-output model. The static Leontief model has the following form:

$$x = Ax + c \qquad (1)$$

where

- x is the n-dimensional vector of gross industrial outputs,

- c is the n-dimensional vector of final consumption demands for economic commodities,
- A is the $n \times n$ matrix of conventional input coefficients, showing the input of goods that are required to produce a unit of product.

Throughout this paper we assume that the technological matrix A is productive, i.e. it has a nonnegative Leontief-inverse.

The land requirements of the n sectors can be expressed, as

$$Lx \leq l \qquad (2)$$

where

- L is the $m \times n$ matrix of land input coefficients of producing sectors, showing the quantity of land area required during producing a unit of industrial product,
- l is the m-dimensional vector of carrying capacity or biocapacity of the land areas.

The application of input-output model to calculation of ecological footprint was first initiated by Bicknell et al. (1998). They have examined two types of models: closed and open economy. The open economy is an extended Leontief model with export and import activities. The methodology of Bicknell et al. (1998) was corrected by Ferng with a land multiplier. This paper we follow the results of the last paper.

Let us assume that we analyze only the first type of the land areas l_1, which is the first row of matrix L. Then the use of this land type by industrial sectors can be calculated as

$$FP_{sc} = l_1\, x = l_1\, (I - A)^{-1}\, c, \qquad (3)$$

where value FP_{sc} is the static closed ecological footprint of land area 1.

The input-output balance of the economy is described by the equation of economic goods and the inequality of land use. The equation of goods describes the balance between the total output of goods of production and the sum of total input of goods of all activities of the economy and the consumed goods. The next equation is a dynamic generalization of equation (1).

$$x_t = Ax_t + B(x_{t+1} - x_t) + c_t\,,\ (t = 1,2,\ldots,T) \qquad (4)$$

where

- x_t is the n-dimensional vector of gross industrial outputs in period t,
- c_t is the n-dimensional vector of final consumption demands for economic commodities in period t,
- B is the $n \times n$ matrix of capital coefficients, showing the invested products to increase the output of the producing sectors by a unit,
- T is the length of the planning horizon.

The equation (4) is an extension of (1) with the capital accumulation. The initial value x_1 is known for the difference equation system. Expression $B(x_{t+1} - x_t)$ is the investments of the economy, which contains the inventory investments, as well. This tag is a final demand type for the products of the economy.

The land requirements of the economy can not exceed the carrying capacity of the land

$$Lx_t \leq l \ , (t = 1,2,\ldots,T) \qquad (5)$$

The land requirements to calculate the ecological footprint consists of two elements: land requirements of the final demand and land requirements of the capital accumulation. It can be written, as

$$l_1 x_t = l_1(I - A)^{-1} [c_t + B(x_{t+1} - x_t)] \qquad (6)$$

If the final demands c_t (t=1,2,…T) are known, then we can determine the dynamic land requirements. We assume that accumulation matrix B is invertible, so the total output of the economy can be determined from equations (4):

$$x_t = \left[B^{-1}(I-A)+I\right]^{t-1} x_1 - \sum_{\tau=1}^{t-1} \left[B^{-1}(I-A)+I\right]^{t-\tau-1} Bc_\tau \ .$$

After substitution we have the *dynamic ecological footprint* in closed economy in the t^{th} period:

$$FP_{dc,t} = l_1 x_t = l_1 \left\{ \left[B^{-1}(I-A)+I\right]^{t-1} x_1 - \sum_{\tau=1}^{t-1} \left[B^{-1}(I-A)+I\right]^{t-\tau-1} Bc_\tau \right\}. \qquad (7)$$

In the next section we analyze the open economy.

2.2. Dynamic ecological footprint in an open economy

The model (1) extended with export and import has the following form:

$$x = Ax + e + c \qquad (8)$$

where

- e is the n-dimensional export vector of the economy.

The total output of the economy is calculated as

$$x = (I - A)^{-1} (e + c).$$

Expression (8) must be extended with the import requirements of the examined economy. Import goods are used by the industrial sectors to produced new products and imported products are consumed as final demand. The following equation shows the import use of the economy.

$$i = C_A x + c_i \qquad (9)$$

where

- i is the vector of sum of the imported goods,
- C_A is the $n \times n$ matrix of import input coefficients, showing the input of imported goods that are required to produce a unit of product,
- c_i is vector of the goods imported to the final demand.

The equations (8)-(9) are well known from the input-output literature. Table 2 shows the connection between the expressions in equations (8) and (9). In this table vectors v and v_c are the value added of the economy. The next equation holds in input-output literature, but we will not use to calculate the footprints:

$$1^T A\langle x\rangle + 1^T C_A \langle x\rangle + v^T = x^T.$$

Vector 1^T is he summation vector with elements one and the vectors are transposed.

Table 2: Transaction table for the static economy

	Sectors	Final demand	Exports	Total output
Sectors	$A\langle x\rangle$	c	e	x
Imports	$C_A \langle x\rangle$	c_i	-	i
Value added	v	v_c		
Total output	x			
Land input	$L\langle x\rangle$			

To calculate the static footprint for the first type of land, we must sum up the land requirements:

- land requirements of the final demand: $l_1 (I - A)^{-1} c$,
- land embodied in goods produced for export: $l_1 (I - A)^{-1} e$,
- land embodied in goods imported for final demand: $l_1 (I - A)^{-1} c_{i,}$
- land embodied in goods imported by industrial sectors: $l_1 (I - A)^{-1} C_A (I - A)^{-1} (e + c)$.

The land requirements (ecological footprint) of the static open economy are

$$FP_{so} = l_1\, x + l_1\, (I - A)^{-1}\, i = l_1\, (I - A)^{-1}\, [c + e + c_i + C_A\, (I - A)^{-1}\, (e + c)].$$

After determination of the ecological footprint in a static closed economy, we are going to study the land requirements in an open dynamic economy. Let us have a look at table 3, being similar to table 2.

Table 3: Transaction table for the dynamic economy in the t^{th} period

	Sectors	Final demand	Capital accumulation	Exports	Total output
Sectors	$A\langle x_t\rangle$	c_t	$B(x_{t+1} - x_t)$	e_t	x_t
Imports	$C_A\langle x_t\rangle$	$c_{i,t}$	$C_B(x_{t+1} - x_t)$	-	i_t
Value added	v_t	$v_{c,t}$			
Total output	x_t				
Land input	$L\langle x_t\rangle$				

The difference to the case of the static model is now that we have introduced the import requirements coefficient of capital accumulation. In this model the system equations are the following:

$$x_t = Ax_t + B(x_{t+1} - x_t) + c_t + e_t\,,\ (t = 1,2,\ldots,T) \qquad (10)$$

and

$$i_t = C_{\mathrm{A}}\, x_t + C_{\mathrm{B}}(x_{t+1} - x_t) + c_{i,t}\,,\ (t = 1,2,\ldots,T) \qquad (11)$$

where

- i_t is the vector of sum of the imported goods in the t^{th} period,
- C_{B} is the $n\times n$ matrix of capital coefficients of imported goods to accumulation, showing the invested products to increase the output of the producing sectors by a unit,
- e_t is the n-dimensional export vector of the economy in the t^{th} period,
- $c_{i,t}$ is vector of the goods imported to the final demand in the t^{th} period.

The explicit solution of model (10) can be written in the following way:

$$x_t = \left[B^{-1}(I - A) + I\right]^{t-1} x_1 - \sum_{\tau=1}^{t-1} \left[B^{-1}(I - A) + I\right]^{t-\tau-1} B(c_\tau + e_\tau).$$

We are now able to calculate the dynamic ecological footprint for the open economy. The footprint, i.e. the land requirements consist of six parts in a period:

- land requirements of the final demand: $l_1\,(I - A)^{-1}\,c_t$,
- land requirements of capital accumulation: $l_1\,(I - A)^{-1}\,B(x_{t+1} - x_t)$,
- land embodied in goods produced for export: $l_1\,(I - A)^{-1}\,e_t$,
- land embodied in goods imported by industrial sectors: $l_1\,(I - A)^{-1}\,C_{\mathrm{A}}\,x_t$,
- land embodied in goods imported to final demand: $l_1\,(I - A)^{-1}\,c_{i,t}$,
- land embodied in goods imported for accumulation: $l_1\,(I - A)^{-1}\,C_{\mathrm{B}}(x_{t+1} - x_t)$.

The cumulated land requirements (ecological footprint) are in the t^{th} period

$$FP_{do,t} = l_1 x_t + l_1 (I - A)^{-1} i_t$$

After some elementary operations, we have

$$FP_{do,t} =$$
$$l_1\left\{I+(I-A)^{-1}\left[C_A+C_BB^{-1}(I-A)\right]\right\}\left[B^{-1}(I-A)+I\right]^{t-1}x_1+l_1(I-A)^{-1}\left[c_{i,t}-C_BB^{-1}(c_t+e_t)\right]$$
$$-l_1\left\{I+(I-A)^{-1}\left[C_A+C_BB^{-1}(I-A)\right]\right\}\sum_{\tau=1}^{t-1}\left[B^{-1}(I-A)+I\right]^{t-\tau-1}B(c_\tau+e_\tau)$$

It is not easy to explain the dynamic ecological footprint in this last form.

3. Properties of the balanced growth path of the dynamic footprint models

In this section we demonstrate the functioning of the dynamic ecological footprint for the two cases, i.e. closed and open dynamic economies. We analyze the motion of the economy along the balanced growth path.

3.1. Balanced growth path and ecological footprint in the closed economy

Assumption 1.
Throughout the paper it is assumed that the matrices A, B and L are nonnegative, B is nonsingular and c_t is a nonnegative vector. In a previous work Dobos and Floriska (2007) have already studied the balanced growth solution of the system (4) for recycling products corresponding to a given growth rate α ($\alpha \geq 0$) supposing that both production and consumption increase at the same rate α. A similar investigation was made by Schoonbeek (1990). Under these assumptions the balanced growth solution of the model (4) has the form

$$x_t = (1+\alpha)^t \cdot x_0 \text{ and } c_t = (1+\alpha)^t \cdot c_0 \qquad (12)$$

where $\alpha \geq 0$. After substituting the former expressions for x_t and c_t in the equation (4) we have got the following relation

$$(I-A-\alpha \cdot B) \cdot x_0 = c_0 . \qquad (13)$$

After that we have established conditions for the existence of nonnegative output configuration x_0. The output configuration x_0 corresponding to equation (13) exists and it is nonnegative if $\alpha \in [0, \alpha_0)$, where α_0 is the marginal growth rate such that $\lambda_1(A + \alpha_0 B) = 1$, i.e. it is the balanced growth rate of the closed dynamic Leontief model. Where $\lambda_1(M)$ denotes the Frobenius root of an arbitrary nonnegative square matrix M, it is the nonnegative real dominant eigenvalue of M. If the former condition for the existence of nonnegative x_0 is fulfilled then the output configuration x_0 has the following form:

$$x_0\,(\alpha, c_0) = (I - A - \alpha B)^{-1} \cdot c_0. \qquad (14)$$

We assume that the carrying capacity of the land is constant in the planning horizon. Then the vector of biocapacity is a known vector l. Let us substitute this expression and the relations (12) and (14) in the inequality (5) then we obtain the following inequality

$$(1+\alpha)^t L(I - A - \alpha B)^{-1} c_0 \le l. \qquad (15)$$

***Lemma* 1.** The growth rate of production and consumption α is limited by an upper bound α^* due to biocapacity. That is the following limitation must hold $0 \le \alpha \le \alpha^*$ where

$$\max_i \left(\frac{(L(I - A - \alpha^* B)^{-1} c_0)_i}{(l)_i} \right) = 1 \qquad (16)$$

and $(\cdot)_i$ denotes the i^{th} component of the respective vector.

Proof. We assume that we are at the beginning of the examined time period i.e. $t = 0$. Using the relation (15), we determine the maximal growth rate α^* for which the quantity of the land use generated is not more than the allowed limit. Then for this α^* must hold the equality (16).

Remark 1.
We should impose more strict restriction on the chosen growth rate α than we have made previously (according to Dobos and Floriska (2009) the upper bound for α is the marginal growth rate α_0, i.e. the balanced growth rate for the closed dynamic Leontief model). Considering α_0 for the value of α^* in the equality (7), the left-hand side of it will be an unbounded function for α_0 . This implies that α^* should be less than α_0. That is the following inequalities must hold: $0 \le \alpha \le \alpha^* < \alpha_0$.
Under these assumptions, there will be come a time t^* such that the amount of one type of land use generated by industries will be equal to the allowed carrying capacity.

Lemma 2. The time t^* can be calculated by the following formula:

$$t^* = \frac{1}{\ln(1+\alpha)} \min_i \left(\ln \frac{(l)_i}{(L)_i (I - A - \alpha B)^{-1} c_0} \right) \qquad (17)$$

where $(\cdot)_i$ denotes the i^{th} component of the respective vector and the i^{th} row of the respective matrix.

Proof. By a simple mathematical calculation we express t^* from the inequality (15).

This lemma gives estimation for the time interval without an adjustment process on land use. After this point of time the economy must change either production level or consumption rate, or both. In our model we assume that first the production rate is adjusted to the carrying capacity and then the consumption level. It can be proven that this kind of adjustment process leads to a higher consumption level then another choice, i.e. first adjusted consumption and than production.

***Lemma* 3**. After the time t^* (i.e. for $t \geq t^*$), the *maximum* growth rate of production is zero.

Proof. Denote γ the growth rate of production after the time t^*. Then the balanced growth path of production has the form $x_t = (1+\gamma)^{t-t^*} x_{t^*}$ for $t \geq t^*$. Substituting these expressions for x_t and l the inequality (5) we obtain

$$\left(\frac{1}{1+\gamma}\right)^{t-t^*} l \geq L x_{t^*} \text{ for } t \geq t^*.$$

If $\gamma > 0$, for $t \to \propto$ the former inequality will be $0 \geq L x_{t^*}$. This obviously is not fulfilled for every $x_{t^*} \neq 0$. If $\gamma = 0$, the former inequality will be $l \geq L x_{t^*}$. This is obviously fulfilled for the time t^*. This concludes that the maximal value for γ, i.e. the maximum growth rate of production is zero.

This lemma allows us to construct the path of the production level. The production level is grown with a growth rate α until point of time t^* and after this point the growth rate is 0. The growth rate can be determined as follows:

$$x_t = \begin{cases} (1+\alpha)^t \cdot x_0 & t < t^* \\ (1+\alpha)^{t^*} \cdot x_0 & t^* \leq t \leq T \end{cases}.$$

The next proposition makes it possible to calculate the consumption levels along the planning horizon. Let us now define the growth rate of the production level in the planning horizon as function of the time:

$$\gamma_t = \begin{cases} \alpha & t < t^* \\ 0 & t^* \le t \le T \end{cases}.$$

The results of lemmas 1, 2, and 3 can be summarized in the next

Proposition 1. In case of a balanced growth solution of the model (4) and (5), for a given rates of growth, the following must hold

$$(I - A - \gamma_t \cdot B)\, x_t = c_t, \text{ for } t = 1, 2, \ldots, T. \qquad (18)$$

Proof. This relation can be proved in similar way as we have got the relation (14).

The consumption rate can be constructed as

$$c_t = \begin{cases} (1+\alpha)^t \cdot c_0 & t < t^* \\ (1+\alpha)^{t^*} \cdot (I - A - \beta \cdot B) \cdot (I - A - \alpha \cdot B)^{-1} \cdot c_0 & t^* \le t \le T \end{cases}.$$

Remark 2. An overview of the open economy model.
The growth rate α of the balanced growth path of the system (1) could be at most α^* according to the Lemma 1. In so far as this rate of growth is greater than the biocapacity, then this balanced path with rate of growth α, can be continued at most to the time t^* according to Lemma 2. After the time t^* the *maximal* growth rate of the balanced path is 0, according to Lemma 3. The production corresponding to such a path is growing with a rate of growth α until the time t^*, and with a rate of growth 0 after this time. In case of different growth rates, to a given level of production correspond different levels of consumption in a given time.

In the next lemma we analyze this change of the consumption level.

***Lemma* 4.** The consumption level at the time t^* is not less than it was at the time t^*-1. That is $c_{t^*} \ge c_{t^*-1}$.

Proof. For $\alpha \ge 0$ the next inequality is obviously fulfilled:

$$c_{t^*} - c_{t^*-1} \ge c_{t^*} - (1+\alpha) c_{t^*-1} \qquad (19)$$

Using the formula (18), Lemma 3 and the Remark 2 we get that

$$c_{t^*} - (1+\alpha) c_{t^*-1} \ge (I - A) x_{t^*} - (1+\alpha)(I - A - \alpha B) x_{t^*-1}. \qquad (20)$$

For the rate of growth α we have

$$(1+\alpha)x_{t^*-1} = x_{t^*} \qquad (21).$$

By substituting the equation (21) into the inequality (20) finally we obtain the inequality (19) in the following form:

$$c_{t^*} - c_{t^*-1} \geq \alpha B x_{t^*}$$

The right-hand side of the previous inequality is nonnegative for $\alpha > 0$, B nonnegative matrix and x_{t^*} nonnegative vector. This concludes that $c_{t^*} - c_{t^*-1} \geq 0$.

Remark 3.
The decrease of the growth rate of production from the value α to the value 0 results an excess supply of economic products in the year t^*. Because the less growth rate of production induces less investments in capital goods. This surplus of goods results a sudden growth of the consumption level in the year t^*.

In the closed dynamic Leontief model the carrying capacity can be not exceeded, as we have presented for this model type. In the next section we introduce the import and export in the model. For this case the biocapacity of the land is not an upper bound of the economic growth of a country. The necessary "land" could be imported from oversea to satisfy the final demand and accumulation of the economy.

3.2. Balanced growth path and ecological footprint in the open economy

Assumption 2.
Under the assumption 1 the balanced growth solution of the model (10) has the form

$$x_t = (1+\alpha)^t \cdot x_0\ ,\ \ c_t = (1+\alpha)^t \cdot c_0,\ \text{ and }\ e_t = (1+\alpha)^t \cdot e_0 \qquad (22)$$

where $\alpha \geq 0$. After substituting the former expressions for x_t, c_t and e_t in the equation (10) we have got the following relation

$$(I - A - \alpha \cdot B) \cdot x_0 = c_0 + e_0\,. \qquad (23)$$

Let us assume that the vector of the goods imported to the final demand increases at the same growth rate, as the total output, consumption level, and export:

$$c_{i,t} = (1+\alpha)^t \cdot c_{i,0}. \qquad (24)$$

The import vector of the economy can be written with these assumptions following equation (11), as

$$i_t = (1+\alpha)^t C_A \cdot x_0 + \alpha(1+\alpha)^t C_B \cdot x_0 + (1+\alpha)^t c_{i,0},$$

or using relation (23)

$$i_t = (1+\alpha)^t \left[(C_A + \alpha C_B)\cdot(I - A - \alpha \cdot B)^{-1} \cdot (c_0 + e_0) + c_{i,0}\right],$$

The dynamic ecological footprint for the open economy can be calculated using inequalities (10) and (11) in the following way:

$$EF_{do,t} = (1+\alpha)^t L\left\{\left[I + (I-A)^{-1}(C_A + \alpha C_B)\right]\cdot(I - A - \alpha \cdot B)^{-1} \cdot (c_0 + e_0) + (I-A)^{-1} c_{i,0}\right\}. \tag{25}$$

For this case the economy uses the land capacity if $EF_{do,t} \leq l$, i.e. the dynamic ecological footprint is under the carrying capacity of the land. If $EF_{do,t} > l$, then the economy uses overseas land, as well. In this case the country imports $EF_{do,t} - l$ land to satisfy the final demands of the closed economy.

4. A numerical examples

4.1. Balanced growth in a closed economy

In this section we will demonstrate the functioning of the proposed models. Let us assume that the investigated economy produces three goods and uses two types of land. The matrices of input coefficients, capital coefficients and land use coefficients are the following:

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.1 \\ 0.2 & 0.3 & 0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.07 & 0.03 & 0.02 \\ 0.06 & 0.07 & 0.04 \\ 0.07 & 0.06 & 0.03 \end{bmatrix} \text{ and}$$

$$L = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.4 & 0.5 & 0.3 \end{bmatrix}.$$

Using the results of assumption 1 the marginal growth rate of the model is *0.376* (α_0 = *0.376*). It means that a rational growth rate must be lower than this growth rate.

Let us assume next that the balanced growth rate α is equal 0.10 i.e. 10% and the vectors of initial consumption level c_0 and biocapacity of land l are

$$c_0 = \begin{bmatrix} 5 \\ 7 \\ 6 \end{bmatrix} \text{ and } l = \begin{bmatrix} 200 \\ 300 \end{bmatrix}.$$

The planning horizon of the economy is T =35 years. Applying the equation (14) the initial output of the economy is

$$x_0(0.10) = \begin{bmatrix} 28.215 \\ 35.597 \\ 32.616 \end{bmatrix}.$$

The balanced growth path for the economy is as follows:

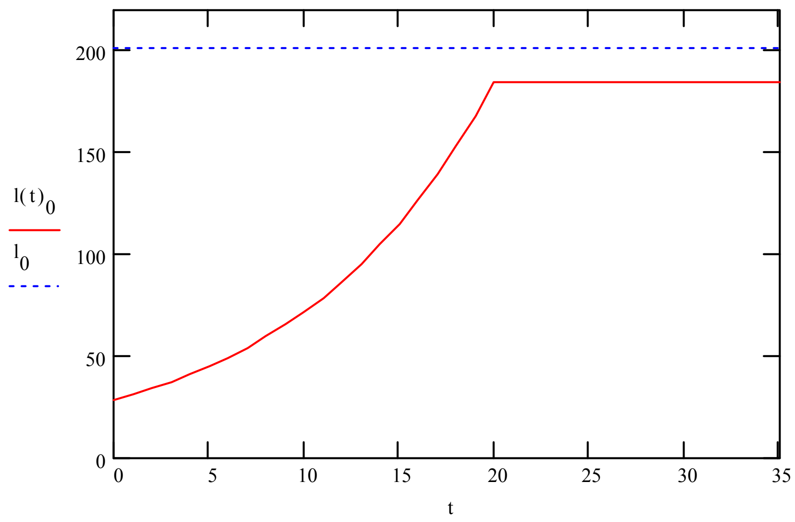
$$c_t = \begin{cases} (1+\alpha)^t \cdot c_0 & t < 21 \\ c_e & 21 \le t \le 35 \end{cases},$$

where the new initial consumption rate $c_e = (1+\alpha)^{20} \cdot c_0$ is

$$c_e = \begin{bmatrix} 36.123 \\ 50.785 \\ 43.789 \end{bmatrix}$$

The land use path for the first type of land is depicted in Figure 1. with the biocapacity as an upper bound. (Biocapacity is depicted with a dotted line.)

Figure 1: The land use and the biocapacity of land 1



The second figure presents production level for the first activity. The dotted line shows balanced growth path in case of no carrying capacity constraints. It can be seen that the growth path will be lower after biocapacity is attained.

Figure 2: The balanced growth path for the first sector without and with biocapacity, as upper bound

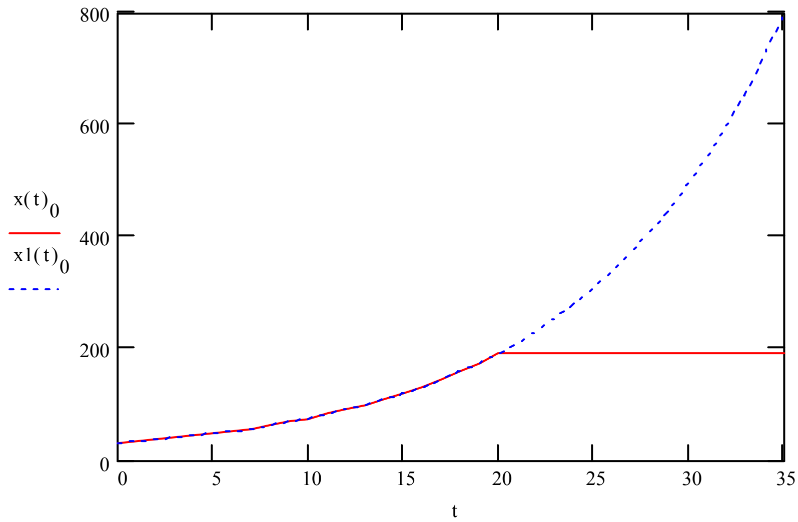
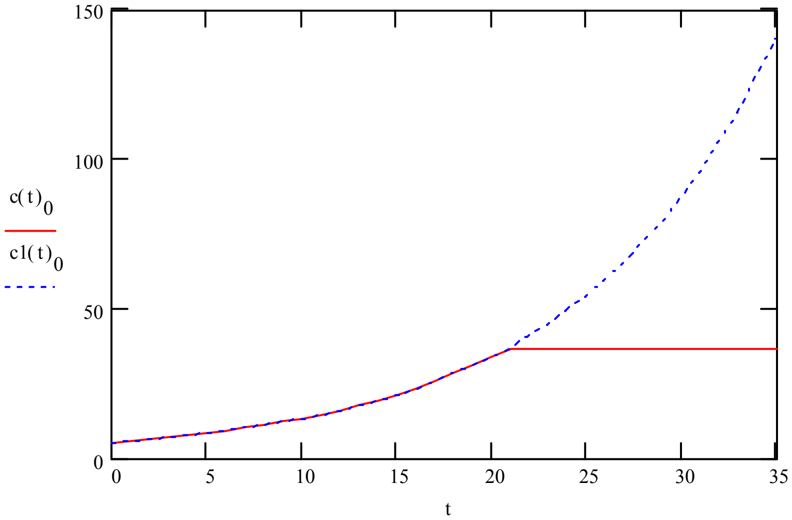


Figure 3 shows the development of the consumption level in time. The dotted line represents the consumption level in case of no upper bound on the land use. This numerical example supports the result of remark 3. After the carrying capacity is achieved, the consumption level is higher then without it. But after three time period the consumption level is lower then with environmental standard. The consumption increases because less goods will be invested to increase the production level. It is a positive effect of biocapacity on the consumption.

Figure 3: The path of the consumption level for the first good without and with carrying capacity constraints



4.2. Balanced growth in an open economy

Let us continue the example of the investigated economy with three goods and two types of land. The matrices of input coefficients of import and capital coefficients of import are the following:

$$C_A = \begin{bmatrix} 0.02 & 0.03 & 0.02 \\ 0.05 & 0.03 & 0.01 \\ 0.02 & 0.03 & 0.03 \end{bmatrix}, \text{ and}$$

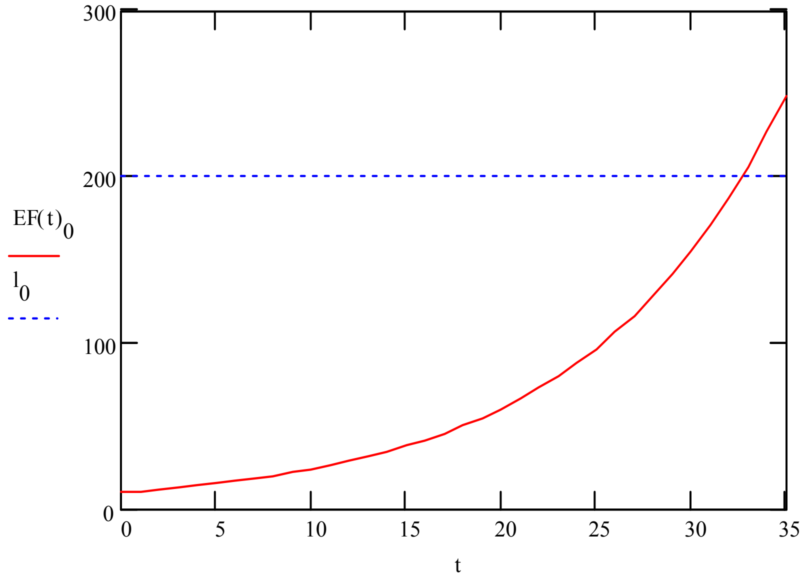
$$C_B = \begin{bmatrix} 0.007 & 0.003 & 0.002 \\ 0.006 & 0.007 & 0.004 \\ 0.007 & 0.006 & 0.003 \end{bmatrix},$$

i.e. we have assumed that the import matrices are equal to $C_A = 0.1 \cdot A$, and $C_B = 0.1 \cdot B$. The initial import level to final demand and export level are known

$$c_{i,0} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{ and } e_0 = \begin{bmatrix} 7 \\ 7 \\ 8 \end{bmatrix}.$$

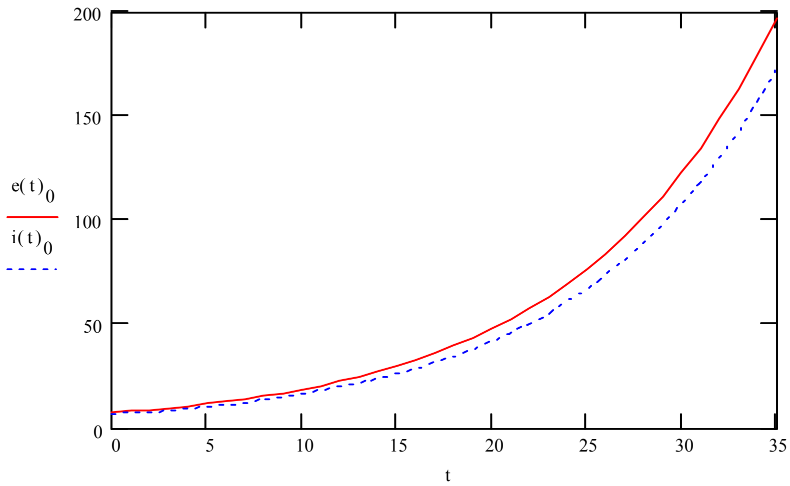
The land use path for the first type of land is depicted in Figure 4. with the biocapacity as an upper bound. (Biocapacity is depicted with a dotted line.) This economy exceeds the carrying capacity after the 33 period. It means that the economy after this period uses the land of other countries.

Figure 4: The dynamic ecological footprint with carrying capacity constraints for the first type of land



We have depicted the balance of the export and import for the analyzed economy. It can be seen that the export of first product of the economy exceeds the imports along the planning horizon. This economy imports more product from the second product than she exports.

Figure 5: Export and import path for the first product of the economy



5. Conclusion and further research

In this paper we have investigated the effect of ecological footprint on production and consumption in a dynamic Leontief model in case of balanced growth path. If the carrying capacity of land is effective, i.e. it is a constraint on the production then the growth rate of the production and consumption will be lower after the allowed levels are attained. Of course, we can not offer the restriction of consumption in this process.

The investigated model assumes no technological development in the economy. In a further research we could introduce technological development into the economic model, i.e. the matrices of the model could be changed in time. In a modern economy the research and development will develop new technologies to save the environment.

A second extension is to investigate the path of the ecological footprint in a dynamic empirical model. To do so, we can apply the Leontief generalized inverse to this dynamic input-output model. The dynamic analysis makes it possible to examine the changes of the land use in a known time interval.

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