

State-Space Feedback Linearization for Depth Positioning of a Spherical URV

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Abstract

Variable ballast, a common mechanism in underwater vehicle, is utilized as vertical motion actuator of a spherical URV in order to control its depth positioning. Since the model of this system is nonlinear and controllable therefore state-space feedback linearization is utilized in this depth positioning. The idea of state-space feedback linearization is to algebraically transform all state variable of nonlinear systems dynamics into (fully or partly) linear ones, so that linear control techniques can be applied. This method can stabilize the equilibrium point of this system which is unstable in open loop system. From the control analysis and simulation results, it can be observed that the asymptotical stabilization is achieved by tracking the error. Hence, state-space feedback linearization can also be applied for tracking a trajectory of desired depth position.

Keyword: variable ballast, spherical URV, feedback linearization

1. Introduction

Demands on exploring undersea environments are being increased. Many equipments of oil and gas companies or power system and communication companies which are located at undersea, need to be maintained and monitored regularly. Underwater robotic vehicles (URVs) have long been applied in this application. The URVs also have been used for gathering bathymetry data for oceanographic research. The URVs are used to perform task in depths where it would be too hazardous or impractical for humans to do. Kinds of task that performed by URV will decide the proper shape's design of URV's body/hull.

If URV is applied for tracking or surveying that should travel in a long distance, torpedo-like or airplane-like is suitable, because in this shape the URV can be easier to move in high speed. If the URV doesn't need many maneuvers, the hull in box frame is suitable, e.g. JHUROV [1]. If the URV need to move in omni-direction, a sphere shape is suitable, e.g. ODIN

[2]. ODIN is an URV with sphere shape and closed frame. The sphere shape design has axially symmetric and it gives advantage in providing uniform drag in any direction of its movement, therefore it is easy to develop the algorithm to control the motion of the URV. By this advantage, a sphere URV is suitable for a test-bed. In this paper, the sphere shape of URV is used.

In order to be able to move, URV must be equipped with thrusters as motion actuators. If the URV moves at horizontal plane, it should be in zero buoyancy condition therefore the thrusters can work optimum. Zero buoyancy of the URV is not easy to be hold if the URV has fixed mass, because sometime the density of the water is uncertain from one place to another. Hence, a mechanism that can maintain the zero buoyancy condition which is known as variable ballast is needed. This mechanism will maintain the difference between buoyancy and the weight of URV. Hence, this mechanism can also be used as motion actuator in vertical plane.

Many designs of variable ballast mechanism have been proposed. A variable ballast mechanism by utilizing water pump in order to control water in the ballast tank was developed by [3, 4]. High pressure air compressor to control amount of water in the ballast tank was used in [5]. All of these mechanisms used fixed volume of the ballast tank. They just controlled the volume of the water in the tank. The other variable ballast mechanism was presented in [6] which is utilized variable ballast tank in order to control the URV's buoyancy.

In this paper, the variable ballast mechanism presented in [6] is used as motion actuator for depth positioning of a spherical URV. A control strategy is proposed to locate the spherical URV at certain depth. Since the model presented in [6] is a nonlinear model, then feedback linearization control strategy is designed.

Feedback linearization is one of the methods in designing feedback controller for nonlinear control systems. The main idea of this method is to algebraically transform nonlinear systems dynamics into (fully or partly) linear ones, so that linear control techniques can be applied. This differs entirely from conventional linearization method because feedback

linearization is achieved by exact state transformation and feedback, rather than by linear approximations of the dynamics. The feedback linearization can be viewed as a method of transforming original system models into equivalent models of a simple form.

2. Modeling

The model of URV used in this paper was presented in [6]. This model describes depth positioning of a spherical URV by using variable ballast mechanism as the actuator. The design of this spherical URV is shown in Fig. 1.

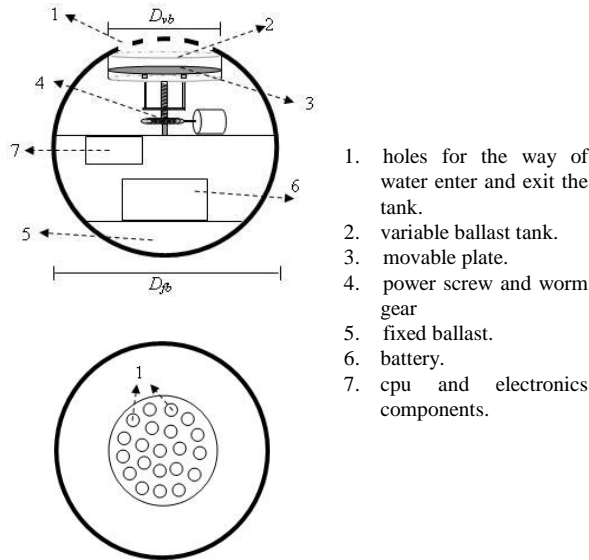


Figure 1. Shape of spherical URV and its parts

Since we just consider to the vertical motion of the URV in order to control the depth position of this URV, the forces work in this system are illustrated in Fig. 2.

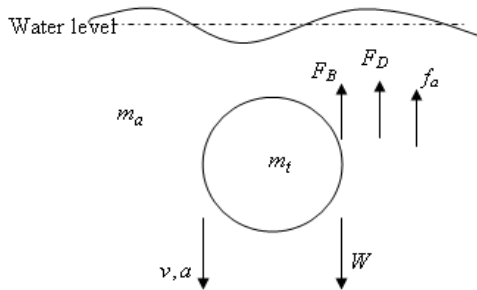


Figure 2. Forces act at URV's body

From Fig. 2, $W = m_t g$ is the gravitation force, $F_B = \rho_w V_{fb} g$ is buoyancy force, $F_D = \frac{1}{2} C_D A_{fb} \rho_w |v| v$ is drag force, m_t is total mass of URV, g is gravitational acceleration, ρ_w is density of water, V_{fb} is volume of the hull, A_{fb} is projected area of the hull, v is velocity, C_D is drag coefficient and $m_t = m_s + \Delta m$, where Δm is mass changes due to the change of volume of water in the ballast and m_s is the initial total mass of URV's hull. Then by solving all the forces and motion equation of the URV, the dynamic model for depth positioning a spherical URV is presented as [6]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{x_3}{(m_s + m_a + \frac{x_3}{g})} - \frac{\text{sign}(x_2) C_D A_{fb} \rho_w x_2^2}{2(m_s + m_a + \frac{x_3}{g})} \\ \dot{x}_3 &= \frac{\rho_w g A_{vb} u}{k_m k_{gc} \left(W_{bs} + x_3 + \rho_w g A_{vb} x_1 - \frac{A_{vb} P_a x_3}{\rho_w g V_{ih} - x_3} \right)} \end{aligned} \quad (1)$$

where $x_1 = z$, $x_2 = v$, and $x_3 = \Delta W$ are depth position, velocity of vertical motion, and weight change of the variable ballast system respectively, and are known as state variables, $u = P_m$ is the power needed to change the weight of variable ballast in ΔW and is known as input. m_a is the added mass, W_{bs} is initial weight of water in ballast tank, A_{vb} is the area of variable ballast tank, V_{ih} is initial volume of air inside the URV's hull, P_a is air pressure at water surface, k_m is coefficient of worm gear and power screw couple, and k_{gc} is the transmission ratio or velocity reduction of worm gear and the power screw, and all are constant.

3. Stability of a Point

Consider Eq. 1, if $u = x_2 = x_3 = 0$ then the URV will remain at the last depth position, $x_1 = z$. This condition is known as equilibrium condition. Therefore, the equilibrium condition can occur at any depth position. To analyze the stability of this equilibrium condition, Lyapunov provides a method which is known as Lyapunov direct method [7]. By analyzing the possible Lyapunov function,

$V(\mathbf{x})$ and $\dot{V}(\mathbf{x})$, the stability of the equilibrium point can be determined. The possible Lyapunov function can be obtained by using gradient method [7, 8]. Since the possible Lyapunov function of Eq. 1 is obtained as

$$V(\mathbf{x}) = \frac{x_2^2}{2} - x_2 x_3 + \frac{x_3^2}{2}, \quad (2)$$

$$\dot{V}(\mathbf{x}) = \frac{(2x_3 - \text{sign}(x_2)C_D A_{fb} \rho_w x_2^2)(x_2 - x_3)}{2(m_s + m_a + \frac{x_3}{g})}, \quad (3)$$

then by considering the design parameter of the spherical URV in [6], it is known that x_2 and x_3 are upper and lower bounded, $m_s + m_a > \frac{x_3}{g}$. Therefore, from the above condition of the system, and if condition $\text{sign}(x_2)C_D A_{fb} \rho_w x_2^2 > 2x_3$ and $x_2 < x_3$ or if $\text{sign}(x_2)C_D A_{fb} \rho_w x_2^2 < 2x_3$ and $x_2 > x_3$ are satisfied then the gradient $\dot{V}(\mathbf{x})$ at Eq. 3 is positive definite, but this condition cannot always be hold therefore $\dot{V}(\mathbf{x})$ is not positive definite nor negative definite or semidefinite. By considering the possible Lyapunov function at Eq. 2, if $\forall \mathbf{x} \neq 0$, $\frac{x_2^2}{2} + \frac{x_3^2}{2} > x_2 x_3$, is valid then $\forall \mathbf{x} \neq 0$ the Lyapunov function is positive definite, but this condition cannot always be hold thus $V(\mathbf{x})$ is not positive definite nor negative definite. Hence, from characteristic of $\dot{V}(\mathbf{x})$ and $V(\mathbf{x})$, it can be concluded that the origin as one of the equilibrium point is unstable in Lyapunov sense [7]. Next, in order to stabilize this nonlinear system, state-space feedback linearization will be used.

4. Controllability

An affine nonlinear system with single input and single output (SISO) can be expressed as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \\ y &= h(\mathbf{x}) \end{aligned} \quad (4)$$

To simplify checking controllability of nonlinear system at Eq. 4, local analysis is done, *i.e.* the results are valid only in neighborhood of operating point, but global results are available elsewhere [9]. Local controllability can be determined by examining the rank of the controllability matrix which is analogous to the linear controllability matrix. The controllability

matrix of nonlinear system can be obtained by using Lie brackets which is expressed as [9]

$$\mathbf{C}(\mathbf{x}) = \begin{bmatrix} ad_{\mathbf{f}}^0 \mathbf{g}(\mathbf{x}) & ad_{\mathbf{f}}^1 \mathbf{g}(\mathbf{x}) & \dots & ad_{\mathbf{f}}^{n-1} \mathbf{g}(\mathbf{x}) \end{bmatrix} \quad (5)$$

where n is order of the system, and

$$ad_{\mathbf{f}}^0 \mathbf{g}(\mathbf{x}) = \mathbf{g}(\mathbf{x})$$

$$ad_{\mathbf{f}}^1 \mathbf{g}(\mathbf{x}) = [\mathbf{f}, \mathbf{g}] = \nabla \mathbf{g} \mathbf{f} - \nabla \mathbf{f} \mathbf{g}$$

$$ad_{\mathbf{f}}^i \mathbf{g}(\mathbf{x}) = [\mathbf{f}, ad_{\mathbf{f}}^{i-1} \mathbf{g}(\mathbf{x})] \quad \text{for } i = 1, 2, \dots$$

Revisit nonlinear model at Eq. 1, then vector $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ can be express as

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} x_2, & \frac{x_3 - \text{sign}(x_2)B_2 x_2^2}{(B_1 + \frac{x_3}{g})}, & 0 \end{pmatrix}^T \quad (6)$$

$$\mathbf{g}(\mathbf{x}) = \begin{pmatrix} 0, 0, & \frac{B_3}{\left(W_{bs} + x_3 + B_4 x_1 - \frac{B_5 x_3}{B_6 - x_3} \right)} \end{pmatrix}^T \quad (7)$$

where

$$B_1 = m_s + m_a; \quad B_2 = \frac{C_D A_{fb} \rho_w}{2}; \quad B_3 = \frac{\rho_w g A_{vb}}{k_m k_{gc}};$$

$$B_4 = \rho_w g A_{vb}; \quad B_5 = A_{vb} P_a; \quad \text{and } B_6 = \rho_w g V_{ih}.$$

The controllability matrix $\mathbf{C}(\mathbf{x})$ is obtained by using Lie brackets. This controllability matrix has full rank, 3, which is equal to the order of the system. Thus, the nonlinear model at Eq. 1 holds the condition to be controllable.

5. State-Space Linearization

A SISO nonlinear model given as Eq. 4 is to be state-space linearizable if and only if it satisfies the below conditions [8, 10]:

- Controllable, the matrix $\begin{bmatrix} \mathbf{g}(\mathbf{x}) & ad_{\mathbf{f}} \mathbf{g}(\mathbf{x}) & \dots & ad_{\mathbf{f}}^{n-1} \mathbf{g}(\mathbf{x}) \end{bmatrix}$ has rank n or it has full rank.
- The vector fields $(\mathbf{g}(\mathbf{x}), ad_{\mathbf{f}} \mathbf{g}(\mathbf{x}), \dots, ad_{\mathbf{f}}^{n-2} \mathbf{g}(\mathbf{x}))$ are involutive.

A set of vector field $\{X_1(x), \dots, X_p(x)\}$ is involutive if there is scalar function $\delta_{ijk}(x)$ such that Eq. 8 is satisfied.

$$ad_{X_i} X_j(x) = \sum_{k=1}^p \delta_{ijk}(x) X_k(x), \quad 1 \leq i, j \leq p, i \neq j \quad (8)$$

Therefore when Lie bracket is taken with in this vector field, a new vector will not be generated. Hence the rank of $\{X_1(x), \dots, X_p(x), [X_i, X_j], \dots\}$; $1 \leq i, j \leq p, i \neq j$ is equal to p .

If both condition are satisfied, then new state variable $\mathbf{z} = \phi(\mathbf{x})$ and new input v are determined in such that satisfy a linear time-invariant relation

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}v \quad (9)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The feedback control law can be designed as

$$u = \psi(\mathbf{x}) + \gamma(\mathbf{x})v \quad (10)$$

where

$$\psi(\mathbf{x}) = -\frac{L_{\mathbf{f}}^n z_1}{L_{\mathbf{g}} L_{\mathbf{f}}^{n-1} z_1}; \quad \gamma(\mathbf{x}) = \frac{1}{L_{\mathbf{g}} L_{\mathbf{f}}^{n-1} z_1} \quad (11)$$

where n is order of the system. The new state \mathbf{z} is called the linearizing state, and the control law at Eq.10 is called linearizing control law. The $\phi(\mathbf{x})$ is diffeomorphism in such that $\mathbf{x} = \phi^{-1}(\mathbf{z})$ is satisfied.

In order to determine the linearizing state \mathbf{z} , the first state z_1 must be determined by considering the following conditions [8]:

$$\begin{aligned} \nabla_{z_1} ad_{\mathbf{f}}^i \mathbf{g} &= 0; \quad i = 0, \dots, n-2 \\ \nabla_{z_1} ad_{\mathbf{f}}^{n-1} \mathbf{g} &\neq 0; \quad n \text{ is order of the system} \end{aligned} \quad (12)$$

Then the state transformation

$$\mathbf{z}(\mathbf{x}) = \begin{bmatrix} z_1 & L_{\mathbf{f}} z_1 & \dots & L_{\mathbf{f}}^{n-1} z_1 \end{bmatrix}^T.$$

Consider to designing the state-space feedback linearization for dynamic model given at Eq. 1, it must satisfy the conditions to be state-space linearizable before continuing the controller designing. As mention

before, this dynamic model is controllable thus it holds first condition to be state-space linearizable. In view of the second condition of nonlinear system to be state-space linearizable, since the dynamic model at Eq. 1 is 3rd order system then the set of the vector fields be examined for its involutivity are $\{\mathbf{g}(\mathbf{x}), ad_{\mathbf{f}} \mathbf{g}(\mathbf{x})\}$. By using m file in MATLAB (Appendix), the involutivity of these vector fields are analyzed. Since the rank of set of vector $\{\mathbf{g}(\mathbf{x}), ad_{\mathbf{f}} \mathbf{g}(\mathbf{x}), [\mathbf{g}(\mathbf{x}), ad_{\mathbf{f}} \mathbf{g}(\mathbf{x})]\}$ is equal to 2, then these vector fields are involutive. Therefore the dynamic model for depth positioning of the spherical URV given at Eq. 1 is state-space linearizable then state-space feedback linearization controller can be designed.

By considering the conditions given in Eq. 12, the first component z_1 of the new state vector \mathbf{z} should satisfy

$$\frac{\partial z_1}{\partial x_3} = 0 \quad \frac{\partial z_1}{\partial x_2} = 0 \quad \frac{\partial z_1}{\partial x_1} \neq 0 \quad (13)$$

Thus z_1 must be a function of x_1 only. The simplest solution to this equation is

$$z_1 = x_1 \quad (14)$$

The other states can be obtained by considering function $\mathbf{f}(\mathbf{x})$ given in Eq. 6.

$$\begin{aligned} z_2 &= \nabla_{z_1} \mathbf{f} = x_2 \\ z_3 &= \nabla_{z_2} \mathbf{f} = \dot{x}_2 = \frac{x_3 - \text{sign}(x_2) B_2 x_2^2}{(B_1 + \frac{x_3}{g})} \end{aligned} \quad (15)$$

Then the state space of state transformation is written as

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v, \quad (16)$$

where v as new input of state transformation is the solution of $\dot{z}_3(\mathbf{x})$ that is

$$\dot{z}_3 = v = u \nabla_{z_3} \mathbf{g} + \nabla_{z_3} \mathbf{f} \quad (17)$$

If we compare the transformed state variables to the original state variables, it is clearly seen that the transformed state variables have physical meaning that are depth position, velocity, and acceleration for z_1 , z_2 and z_3 respectively.

By considering system in Eq. 16 as linear system, then linear feedback control strategy can be applied in

order to stabilize the depth positioning system of the spherical URV. If feedback gain $\mathbf{K} = [k_1 \ k_2 \ k_3]$ is applied to the closed loop system of model in Eq.16, and the desired depth position is given as z_{1d} , then the new input v can be obtained as

$$v = k_1 z_{1d} - \mathbf{K}\mathbf{z} \quad (18)$$

By locating the eigenvalues, λ , of this closed loop system in left of half-complex plane, this feedback gain will asymptotically stabilize the system. The eigenvalues of the closed loop system of Eq. 16 can be obtained from the characteristic equation that is

$$\lambda^3 + k_3 \lambda^2 + k_2 \lambda + k_1 = 0 \quad (19)$$

and if the desired characteristic equation of the closed loop system for depth positioning of the spherical URV is

$$(\lambda + 2\xi a)(\lambda^2 + 2\xi a \lambda + a^2) = 0, \quad \xi, a > 0 \quad (20)$$

Thus the system is asymptotically stabilized, then by matching the coefficient of Eq. 19 and Eq. 20, the gain of the feedback can be expressed as

$$k_1 = 2\xi a^3; \quad k_2 = 4\xi^2 a^2 + a^2; \quad k_3 = 4\xi a \quad (21)$$

Since this controller asymptotically stabilizes the system then for $t \rightarrow \infty$, the output $y = z_1 = x_1 \rightarrow z_{1d}$.

6. Simulation

The control strategy obtained in section 5 is simulated in MATLAB/Simulink. The simulation is performed based on schematic diagram given in Fig. 3. Some parameters used in simulation are given in Table 1.

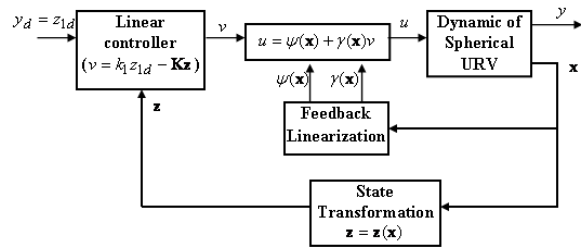


Figure 3. Schematic diagram of control system

By giving desired depth position y_d as step input, then the response of the control system is shown in Fig. 4. To get optimal response, a proper value of ξ and a must be chosen. By choosing $\xi = 0.77$ and $a = 0.036$

the performances of the control system with are shown in Table 2.

Table 1. Parameters of URV and water environment

| | |
|--|--|
| P_a : 1 atm | D_{vb} : 0.18 m |
| ρ_w : 998 kg/m ³ | A_{vb} : 0.0254 m ² |
| μ : 10 ⁻³ Ns/m ² | h : 0.08 m |
| g : 9.81 m/s ² | W_{bs} : 9.96 N |
| | k_{gc} : 8.164x10 ⁴ rad/m |
| m_s : 22.39 kg | k_{mL} : 4.601x10 ⁻⁵ |
| m_a : 11.2 kg | k_{mU} : 1.122x10 ⁻⁴ |
| D_{fb} : 0.35 m | P_{m_max} : 100 watt |
| A_{fb} : 0.09616 m ² | ω_{m_max} : 157 rad/s |
| V_{ih} : 50 % of V_{fb} | |

where μ is dynamic viscosity of water, P_{m_max} is maximum power of motor, and ω_{m_max} is maximum angular velocity of motor.

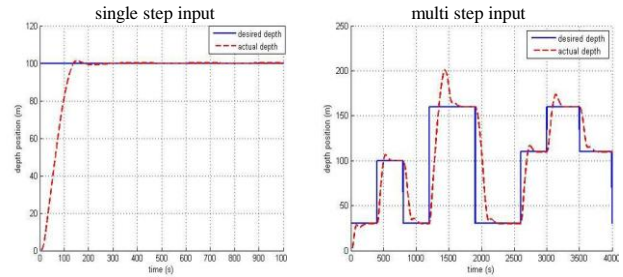


Figure 4. Response of step input reference

Table 2. Performances of the controller in multi step input reference of depth position.

| Step change (Depth position) (m) | T_r (s) | T_s (s) | Over-shoot (%) | $e_{ss}/RMSE$ (m) |
|----------------------------------|-----------|-----------|----------------|-------------------|
| 0-30 | 25.6 | 224 | 0.234 | 0.069 |
| 30-100 | 65.2 | 194.2 | 6.775 | 0.213 |
| 100-30 | 87 | 267.3 | 0.322 | 0.085 |
| 30-160 | 106.7 | 432.8 | 25.537 | 0.357 |
| 160-30 | 127.1 | 342.2 | 0.307 | 0.051 |
| 110-160 | 57.1 | 223.8 | 8.507 | 0.293 |
| 160-110 | 87.1 | 224.8 | 0.077 | 0.219 |

From Table 2, it can be seen that the downward motion is faster than the upward motion, because at the deeper position the hydrostatic pressure is bigger than the shallower one.

Fig. 4 shows that the controller can locate the URV at the desired depth position. The desired depth position behaves as the equilibrium point and the controller asymptotically stabilize this equilibrium point.

Since the asymptotical stabilization is performed by tracking the error, then this strategy can be expected to be applied in tracking a trajectory. By applying the desired depth position as a trajectory, the response of the controller is shown in Fig. 5.

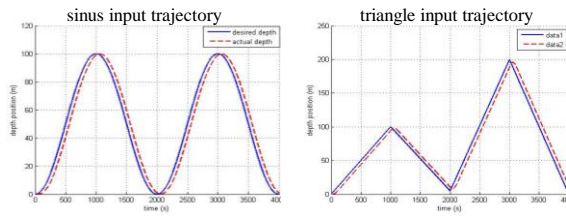


Figure 5. Response for trajectory input

From Fig. 5, it is seen that the actual depth of the URV keeps following the trajectory given as input reference. The output lagged to the desired trajectory thus the error occurs. The Root Mean Square Error (RMSE) for each trajectory is obtained as 9.213 m, and 6.741 m respectively for triangle and sinus input.

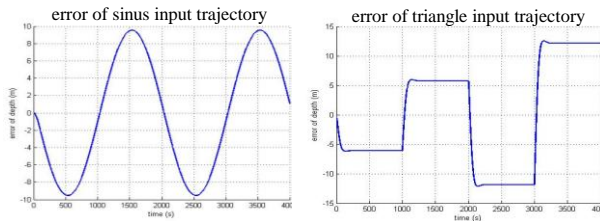


Figure 6. Error for trajectory input.

If the change of the trajectory input is simply constant, such as triangle, the error converges to a constant value. This controller cannot make the error converge to zero when the input is given as trajectory.

7. Conclusion

The state-space feedback linearization that is utilized for depth positioning of a spherical URV, can stabilize the equilibrium point of this system which can occur at any depth position since velocity and input is zero. In open loop system, the stability of the equilibrium point is unstable which is analyzed by using Lyapunov direct method.

Since the stabilization of the equilibrium point is performed by tracking the error then this strategy can also be used for tracking a trajectory, but the controller can not make the error converge to zero since the error

converge to a constant value if the change of trajectory is constant.

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