SCHOOL STARTERS' EARLY STRUCTURE SENSE

Miriam M. Lüken

I analyse low and high achieving children's competences regarding pattern and structure at the beginning of formal schooling comparatively. The aim is to evaluate the range of school starters' early structure sense. The results suggest overall high pre-instructional competences which, however, differ strongly between the mathematical high and low achievers. I name cognitive milestones for the development of a sound early structure sense.

Keywords: Early mathematical development; Patterning; Structure sense

Sentido estructural temprano de estudiantes al inicio de su escolarización

Analizo comparativamente las competencias relacionadas con patrones y estructura al comienzo de la escolaridad formal en escolares con bajo y alto rendimiento académico. El objetivo es evaluar la gama de sentido estructural de niños que se inician en la escuela. Los resultados sugieren en general altas competencias pre-instruccionales las cuales, no obstante, difieren considerablemente entre los escolares de bajo y alto rendimiento académico. Identifico hitos cognitivos para el desarrollo de un sentido estructural sólido.

Términos clave: Desarrollo matemático temprano; Identificación de patrones; Sentido estructural

From the first day at school children have to deal with mathematical patterns and structures. In patterning activities they encounter repeating patterns in order to identify regularity, recognize relations, abstract rules, build sequences or make predictions (Economopoulos, 1998; Threlfall, 1999). Spatial patterns are often used as (standard) number presentations to visualize numerical structures in a specific geometrical way. In doing so particular characteristics of numbers can be illustrated and are used to develop mental representations of numbers. The perception of pattern and the ability of structuring also are the basis for subitizing (Mulligan, Prescott, Papic, & Mitchelmore, 2006). Pattern and structure are an important part of mathematics lessons at the beginning of primary school. There-

Lüken, M. M. (2012). School starters' early structure sense. *PNA*, *7*(1), 41-50.
Handle: http://hdl.handle.net/10481/21534

fore the ability to perceive and use pattern and structure—in short: having an early structure sense—is a significant precondition.

Thus said, it immediately raises the question if children already have a structure sense at the beginning of formal schooling or if it has to be developed instructionally. Over the last couple of years there have been a few studies addressing this topic. Mulligan and Mitchelmore (2009) tested 103 Australian Grade 1 students (5.5 to 6.7 years) on 39 pattern and structure items. Responses were categorised according to the degree of structure and four stages of structural development could be identified. According to these results patterning competences evolve from a pre-structural stage where "representations lack any evidence of numerical or spatial structure" to an emergent, than to a partial structural stage, and finally to the stage of structural development, where children's "representations correctly integrate numerical and spatial structural features" (p. 41). Mulligan and Mitchelmore also believe that children from an early age onward have an—as they call it—"awareness of mathematical pattern and structure" (p. 44).

Van Nes (2009) interviewed 38 Dutch Kindergarteners (4 to 6 years old) on tasks about counting, subitizing, repeating and spatial structure patterns. She also identified four phases in the development of spatial structuring ability. In the lowest unitary phase a child is not able to recognize spatial structures. This competence characterises the next recognition phase. In the usage phase a child furthermore uses and in the highest application phase even applies spatial structure.

All existing studies describe rather general characteristics of young children's structural development. To enable teachers to understand and support this development more specific and clearer descriptions of abilities regarding pattern and structure are needed. In an initial attempt I described early structure sense as a collection of abilities, which includes recognizing a configuration as a familiar structure or pattern (e.g., dots on dice, finger pattern), in particular recognizing a familiar structure both in its simplest form and as part of a more complex pattern. Further abilities are dividing a pattern into sub-structures, recognizing mutual connections and relationships between sub-structures (e.g., find regularity, detect similarities and differences…) and integrating sub-structures to see a pattern as an entity (e.g., in order to determine its quantity, extend, etc.) (Lüken, 2010). To become more specific, this paper addresses the following two questions: How is the range of patterning competences, in particular what are the competences of low compared to high achieving children at the beginning of school? What are the cognitive milestones in the development of a sound early structure sense?

Methodology

The data here presented come from a 2-year longitudinal study. Below I describe the data collection and analysis performed.

Data Collection

This longitudinal study was conducted in two state primary schools in a large German city. The sample comprised 74 children, 38 girls and 36 boys, ranging from 5.8 to 7.2 years of age at the beginning of the study. The children in both primary schools came from low to high socioeconomic families, with 31% having a migrant background (at least one parent not born in Germany).

In two out of three assessments the children's mathematical competences were measured with standardised tests. For the first part of the assessment, that took place in Kindergarten two to three months before school enrolment, I used the German version of the Utrecht Early Numeracy Test (Van Luit, Van de Rijt, & Pennings, 1994). For re-assessing the mathematical competences after two years of schooling (third part of assessment) I employed the standard German mathematics test DEMAT 2+ (Krajewski, Liehm, & Schneider, 2004). The data, which is in the main focus of this paper, come from the second part of the assessment where I assessed school starters' competences in patterning and structure-perception. The interviews took place during the seventh week after school enrolment. Pattern and structure tasks were developed based on a theoretical framework and tested in a pre-study. Six task-categories with each comprising several items were designed to explore the children's ability to conceive, reproduce, copy from memory, use, extend and create repeating and spatial patterns. The pattern and structure tasks were administered as semi-structured individual interviews. The children were asked to think aloud, and concrete objects were used in every task. All interviews were video recorded.

Data Analysis

In a first step quantitative analyses were conducted which showed a significant correlation between school starters' early structure sense and their mathematical competences. These results are subject of a former paper (Lüken, 2010). This article focuses on the qualitative analyses, which I conducted in a second step on the basis of the quantitative outcome. To assess the range of patterning competences and in particular to compare the structure sense of mathematically low and high achieving children, the sample was divided in quartiles according to the children's results in the Utrecht Early Numeracy Test and the pattern and structure tasks. Twenty interviews of children who scored in the same quartile for both tests were analysed. The interviews were evenly distributed among the four quartiles. The data analyses of the interviews followed a grounded theory framework, called *thematisches kodieren* (thematical coding; Flick, 1999). This method compares groups that are established in advance. Thus it meets my interest to describe the similarities and differences in the structure sense of children with different mathematical abilities. An analysis template was set up for each task and all students' responses (the videotapes had been fully transcribed) were categorized accordingly. Strategies were then empirically verified for the four quartiles and the relevant categories/strategies for the discrimination of the quartiles further analysed.

RESULTS AND DISCUSSION

This paper focuses on the comparison of low and high achieving children's strategies (first versus fourth quartile) while dealing with pattern and structure tasks. The analyses of the second and third quartiles are left out here but can be found in Lüken (2011). The presentation of the results is structured into the following significant pattern and structure abilities: pattern recognition, grasping structure, making use of structure, ability of spatial structuring and awareness of/attention to mathematical pattern and structure.

Pattern Recognition

Recognizing a pattern is shown in two different ways. On the one hand one can recognize a familiar pattern in the sense of a well known picture (most common with spatial structure patterns). On the other hand, one can detect regularity (e.g., in a repeating pattern). The competences of low and high achieving children vary strongly according to either meaning of pattern recognition. Both aspects are addressed here.

The children were shown several flash cards with spatial dot patterns, asked to determine the quantity and to reproduce the pattern from memory. The following transcript[1] is taken from the interview with Moskan, a low achieving child, while dealing with the pattern of seven dots shown in Figure 1.

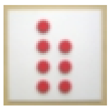

Figure 1. Pattern shown to a low achieving child

Interviewer: How many counters did you see?

Moskan: Mhm, there was the six. And one is missing.

Moskan seems to recognize the die-pattern of the six in this pattern, although it is not quite clear if she attends to eight dots with one missing as the six (8-1) or six dots with one extra at the top (6+1). Anyway, Moskan is one of the few (but still there are some!) low achieving children that are able to recognize familiar patterns both in their simplest form and as part of a more complex pattern. High achieving children are all able to do this.

Furthermore, high achievers, in contrast to the low achievers, can recognize regularity. Lukas, a high achiever, was asked to extend a repeating pattern out of

[1] The transcripts have been translated from German to English for the purpose of this paper. Names are anonymized.

five red and five blue pearls (shown in Figure 2 as white and grey pearls) and explain his action.

Figure 2. Pattern shown to a high achieving child

Lukas: Every time I make five.

High achieving children show a sound understanding of pattern as unit of repeat and are thus able to extend a repeating pattern according to the given regularity; sometimes they can even explain the rule. No regularity was found in the low achieving children's extensions.

Grasping Structure

For grasping structure there are always relations to be discovered or established. The two following transcripts show the way, low and high achievers differently grasp the structure of the twenty fields shown in Figure 3. Celina and Joshua were asked to explain the picture.

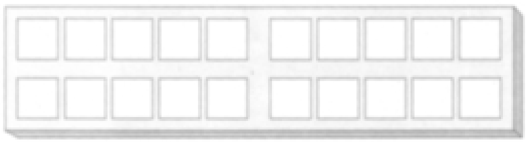

Figure 3. Pattern shown for ten counters

Celina: [Counts silently the ten squares on the right side while pointing a finger on each.] These are seven, here. [Points somewhere in the middle of the ten squares on the right side.] One, two, three, four, five, six, seven, eight, nine, ten and here there are ten. [Counts the ten squares on the left side of the twenty field]

Interviewer: Something else?

Celina: [Looks at the backside of the twenty field.] One can also, well, I see seven counters.

Interviewer: Mhm, and something else?

Celina: And ten counters.

Joshua: [Counts the squares on the left top silently and without pointing.] Five whites in this corner [describes with his finger a circle around the five squares on the left top] five whites in the other [describes with his finger a circle around the five squares on the right top]. I assume, and thus five in every corner.

Celina, like other low achieving children who decompose the twenty fields in two parts, is only able to count up to around ten. She has to structure and count the sub-structures in order to answer the question of the interviewer. The bigger vertical gap serves her as external, visual-geometrical stimulus for the decomposition. During the interviews I often watched children stressing the vertical or horizontal gap of the twenty field by putting a hand on it. Joshua, as a high achieving child, shows his shift of focus by stressing the groups of five instead of the gap. His attention as a high achieving child lies on the sub-structures, which he relates to each other. He is sure of the equal cardinality of his established sub-structures; he does not even have to count the squares in more than one of his sub-structures. Celina decomposes and determines the quantity of each established sub-structure by counting but does not relate them to each other. She does not perceive that her sub-structures are equal and consequently her counting (with two different subsets) cannot be correct.

To sum the competence of grasping structure up: Low achieving children are able to grasp the structure of a pattern by the help of Gestalt principles (Goldstein, 2002). They structure a pattern mentally by decomposing it rather subconsciously along geometric clues for grouping, like different colours, similarity or, like in the twenty field above, bigger and smaller distances between the objects. In that way the focus of their perception lies on the figural aspects and the visual impression of the pattern. The established sub-structures stay unrelated from each other and the external grouping is not related to numerical aspects. In contrast, high achieving children structure a pattern consciously by decomposing it flexibly along external, geometric clues for grouping into sub-structures that can be perceived simultaneously. They relate the sub-structures to each other while linking the structural with the numerical aspects. The focus of high achievers lies on numerical aspects of the sub-structures rather than on figural features.

Making Use of Pattern and Structure

High achievers use the spatial structure of an arrangement as well as familiar patterns explicitly to abbreviate numerical procedures (see also Van Nes, 2009). We introduce the pattern shown in Figure 4 to high achievers and in what follows we can observed Lukas' responses.

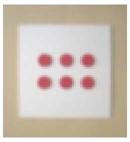

Figure 4. Pattern shown to a high achieving child

Lukas: Six.

Interviewer: How did you see it so fast?

Lukas: Three plus three!

To determine a quantity they decompose complex patterns flexibly and consciously and relate them either with a familiar partition of number or identify the quantity through relating and comparing. Low achieving children do not possess these abilities.

Ability of Spatial Structuring

To assess children's ability to apply structure to an unordered amount of objects I asked them to put five counters on the table in such a way that a hand puppet (which I presented) could easily see how many there are. The following transcripts show three examples of arrangements with the child's respective explanation.

Helene: [Arranges all counters in a horizontal line, touches each one with a finger, counts, thinks, nods.] Okay.

Lion: [Arranges the counters as the die-pattern.] Because a five looks like this.

Lukas: [Arranges the counters as the die-pattern.] Because you can see it immediately that here (tips the four outer counters) are four on each side and, here are two and, here, four [tips the top two and then the bottom two counters] and in the middle is the fifth.

Helene shows a very common strategy for the low achieving children—the arrangement in a line for convenient counting. Except only one child that arranged the counters randomly on the table and seemed to have no idea at all that counters can be organized, all the low achievers did structure their counters spatially. Their arrangement, however, reflect their mathematical abilities. They have learned that objects that are ordered in a row can be more easily counted than a random arrangement. They are not aware of criteria for a quick and easy number perception, only for easy counting.

The most common strategy for all quartiles except the first was to arrange the five counters into a die-pattern, like Lion and Lukas did. What makes the difference in the structuring ability is the explanation of this configuration. The children in the quartiles between the high and low achievers (e.g., Lion) interpret the die-five as a number (the five). The high achievers (e.g., Lukas) interpret it as a partition of a number ($4+1;\ 2+2+1$). High achievers have an awareness of the spatial structure and function of particular configurations. Sometimes they even hold metacognitively criteria for an easy and quick number perception. Their already developed mathematical abilities reflect their structural ability. They have an idea of numbers as appropriately grouped quantities and know that the spatial structure of a pattern represents its mathematical structure.

Awareness of and Attention to Mathematical Pattern and Structure

The study addressed no special tasks for assessing children's awareness of mathematical pattern and structure. This point derives from the complete analyses regarding the tasks altogether. Low achieving children tend to a pre-assigned perspective on and way of structuring patterns. They have difficulties to relate sub-structures. Number presentations are more often seen with a daily framework than from a mathematical perspective. While perceiving structure external characteristics, spatial dimension und figural aspects are most important. On the contrary, high achieving children have an insight into the convenience of structure for determining, comparing and operating with small quantities. They are able to flexibly structure a pattern and shift their focus on different aspects of pattern and structure. They relate established sub-structures in more than one way. Number representations are also seen with a mathematical framework. They understand that some configurations support numerical procedures. External aspects are less important, the focus lies on the decomposing and the established sub-structures. Figural aspects and arithmetical knowledge, mathematical abilities and structuring competences are integrated with high achieving children and are used naturally while solving problems.

Concluding Remarks

Children at the beginning of school show high pre-instructional competences regarding pattern and structure. The qualitative analyses, however, reveal that these competences differ strongly between the lowest and the highest achievers. This fact has consequences for further mathematics learning especially for the low achieving children. Quantitative analyses found that the 25% school starters with the lowest patterning competences after two years of schooling also belonged to the 25% children with the lowest mathematical competences (Lüken, 2011). Despite the overall high patterning competences, some children obviously need stimulating and supporting instruction to further develop their abilities regarding pattern and structure. To put it in Radford's words: Some children need support in "the domestication of their eye" (Radford, 2010). Radford describes the domestication of the eye as "a lengthy process in the course of which we come to see and recognize things according to 'efficient' cultural means" (p. 4). Transferred to patterning activities and perception of structure at the beginning of school the "cultural means" refers to the mathematical perspective. A learner has to organize the perception of things in a particular, mathematical way, for instance learning to relate geometric clues to numerical matters. Perceiving the different colors and succession in a repeating pattern or the visual gaps that groups a spatial pattern is not the problem with low achieving children. Relating the figural, external aspects with mathematical aspects is the step they obviously cannot take alone but have to be instructionally supported with.

At the end of this paper some cognitive milestones in the development of a sound structure sense are described, drawn from the comparison of low and high achieving children's patterning strategies. Recognizing a configuration as a familiar pattern both in its simplest form and as part of a more complex pattern is something a lot of the low achievers are able to accomplish. To connect the pattern's spatial structure with its numerical structure, however, is a milestone. Similarly, almost all low achieving children can divide a pattern into sub-structures, the difficulty lies in recognizing and establishing mutual connections and relationships between the sub-structures and in integrating the sub-structures for example to abbreviate numerical procedures. The milestone in the work with repeating pattern constitutes the perception of regularity, to "see" the unit of repeat. A milestone only very few children have accomplished at the beginning of formal schooling is the ability to flexibly decompose and relate sub-structures, to intentionally reframe the structures of a pattern. This competence seems not to be the least important although it is the least developed. It is assumed that special instructional support is needed to develop a flexible pattern perception.

At this point further research is needed to develop well-founded instructional actions which can help to support the development of a child's structure sense that consequently might also lead to improved mathematical competences.

References

Economopoulos, K. (1998). What comes next? The mathematics of pattern in kindergarten. *Teaching Children Mathematics, 5*(4), 230-234.

Flick, U. (1999). *Qualitative forschung. theorie, methoden, anwendungen in psychologie und sozialwissenschaften.* Reinbek bei Hamburg, Germany: Rowohlt Taschenbuch Verlag.

Goldstein, E. B. (2002). *Wahrnehmungspsychologie*. Heidelberg, Germany: Spektrum, Akademischer Verlag.

Krajewski, K., Liehm, S., & Schneider, W. (2004). *DEMAT 2+. Deutscher mathematiktest für zweite klassen. Manual.* Göttingen, Germany: Beltz.

Lüken, M. M. (2010). The relation between early structure sense and mathematical development in primary School. In M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 241-248). Belo Horizonte, Brazil: PME.

Lüken, M. M. (2011). *Das erkennen von mustern und strukturen im mathematischen anfangsunterricht. Theoretische grundlegung und empirische forschung zur mustererkennungs- und strukturierungsfähigkeit von schulanfängern* (Unpublished dissertation). Leibniz Universität, Germany.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33-49.

Mulligan, J. T., Prescott, A., Papic, M., & Mitchelmore, M. (2006). Improving early numeracy through a pattern and structure mathematics awareness program (PASMAP). In P. Grootenboer, R. Zevenbergen, & M. Chinnappan

(Eds.), *Identities, cultures and learning spaces. Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia* (pp. 376-383). Adelaide, Australia: MERGA.

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics, 30*(2), 2-7.

Threlfall, J. (1999). Repeating patterns in the early primary years. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (pp. 18-30). London, United Kingdom: Cassell.

Van Luit, J., Van de Rijt, B., & Pennings, A. (1994). *The Utrecht Early Numeracy Test. Manual.* Doetinchem, The Netherlands: Graviant Publishing Company.

Van Nes, F. (2009). *Young children's spatial structuring ability and emerging number sense.* Utrecht, The Netherlands: All Print.

This document was originally published as Lüken, M. M. (2011). School starters' early structure sense. In B. Ubuz (Eds.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, p. 153-160). Ankara, Turkey: PME.

Miriam M. Lüken
Leibniz Universität Hannover
miriam.lueken@t-online.de