THE ROLE OF CONTEXT-RELATED PARAMETERS IN ADULTS' MENTAL COMPUTATIONAL ACTS

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Researchers who have carried out studies pertaining to mental computation and everyday mathematics point out that adults and children reason intuitively based upon experiences within specific contexts; they use invented strategies of their own to solve real-life problems. We draw upon research areas of mental computation and everyday mathematics to report on a study that investigated adults' use of mental mathematics in everyday settings. In this paper, we report on one adult's use of mental computation at work and highlight the role of context and context related parameters in his mental mathematical activities.

Keywords: Adults' mental mathematics; Ethnomathematics; India; Mental computation; Workplace mathematics

El papel de los parámetros relacionados con el contexto en actos de cálculo mental en adultos

Los investigadores que han realizado estudios relacionados con el cálculo mental y las matemáticas cotidianas señalan que los adultos y los niños razonan intuitivamente basándose en las experiencias de contextos específicos; usan estrategias inventadas por sí mismos para resolver problemas de la vida real. Nos basamos en las áreas de investigación del cálculo mental y las matemáticas cotidianas para informar sobre un estudio que investigó el uso que hacen los adultos de la matemática mental en el entorno cotidiano. En este artículo, informamos sobre el uso que hace un adulto del cálculo mental en su trabajo y destacamos el papel del contexto y de los parámetros relacionados con el contexto en sus actividades matemáticas mentales.

Términos clave: Cálculo mental; Etnomatemáticas; India; Matemática mental en adultos; Matemáticas del trabajo

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Mental computation, the process of carrying out arithmetic calculations without the aid of external devices, is an indispensable tool that helps both adults and children estimate and compute quickly and reasonably (Reys & Barger, 1994, p. 31). Researchers argue that people compute more efficiently mentally when problems are embedded in a certain context than when posed as a plain computational problem, and they have called for investigation of everyday practices that involve mental mathematics (Carraher, Carraher, & Schliemann, 1987; Saxe, 1991).

In the last two decades several researchers have analyzed and documented the mathematical practices of adults as well as children, which take place outside the school settings (e.g., Carraher, et al., 1987; D'Ambrosio, 1985; Gerdes, 1996; Saxe, 1991). Although the past two decades have seen a surge in research dealing with the everyday mathematical practices of adults, many of these studies were conducted in the western hemisphere and developed nations of the world. Very few studies have investigated the nature of workplace mathematics in developing nations. We devised and completed a research study that attempted to fill in part of this gap. The research reported in this paper is part of a larger project that investigated bus conductors' use of mental computation in everyday settings in Chennai, India. In this paper, we examine one bus conductor's mental computational acts as he solved mathematics tasks at work and outside work settings.

THEORETICAL BACKGROUND

One can adopt two different approaches with respect to mental computation, traditional and contemporary views. According to the traditional view, mental computation is considered a skill that only a few individuals possess and that is available to them. The contemporary view associates mental computation with higher order thinking processes. According to this view, mental computation is considered a vehicle for promoting thinking, conjecturing, and generalizing based on conceptual understanding rather than as a set of skills (Trafton, 1986). We adopt the latter view. A traditional information-processing perspective has been significantly used in research involving mental computation. This perspective involves two assumptions, decomposition and decontextualization (Silver, 1994). Decomposition implies that competence or skill can be broken down into individual elements of skill or knowledge. Decontextualization implies that knowledge exists in the mind of the individual and is independent of the situation in which it is used. This theory has been criticized mainly because research evidence (e.g., Carraher, et al., 1987) suggests that, contrary to the view that an individual possesses a set of skills and competencies that can be decontextualized, research participants were able to apply these in certain everyday situations and not in certain school settings. Thus, researchers have argued that it would be worth considering researching mental computation from an alternate perspective, one that would take into consideration the influence of context and context related parameters on an individual's use of mental computation (Reys & Barger, 1994; Silver, 1994).

The main assumption underlying our study is that adults' mental computational activities in everyday situations—in particular, at work—are influenced by context-related parameters. Hence, we needed a theoretical framework that acknowledged the influence of context on their mathematical practices. We chose Saxe's four-parameter model (1991) to explore the overall research purposes of this study. This model helped us bring out the context related parameters that influenced bus conductors' mathematical goals related to their work activities. In this paper, we will zoom in on one bus conductor's mental mathematical activities as he completed work-related and non work-related tasks. In particular, we claim that context and context related parameters were crucial in this participant's successful adaptation and application of mental computational strategies in everyday situations. Thus, the question investigated is as follows: What role do context and related parameters play in one bus conductor's mental mathematics activities?

Methodology

The overall research study was qualitative in nature. An instrumental case study approach was used to select participants for this study. The bus conductors are employees of the government organization, Metropolitan Transport Corporation (MTC). After gaining entry into the organization, the first author used convenience sampling to choose a bus depot for investigation. Purposive sampling was used to select the five participants, who were carefully and appropriately chosen based on their years of service with the MTC, educational qualifications, service records, and their willingness to participate in the study. In this paper, we focus on data pertaining to one bus conductor, Vira (pseudonym) chosen because of salient elements in his case study.

Data collected for this study include official documents, field notes, summary of observations and informal conversations, transcriptions of formal and semi structured interviews and personal reflections. The primary researcher (PR) accompanied and observed Vira during his work shift three times a week, observing a total of ten trips. Based on the first few observations, the PR singled out several episodes that were most helpful in examining Vira's use of mental computation at work. Snippets of Vira's work-related activities were isolated and used to describe his work-related mathematical activities. For further analysis, we scrutinized his work-related mathematical activities, and identified and described solution strategies he used to complete these activities.

In order to investigate the influence of context-related parameters on Vira's mental computational activities, he was asked to solve two types of computational tasks outside work settings, contextual (CT) and plain (PT). The PR administered these tasks orally in an informal setting and Vira was given 10-15 seconds to complete each task. The CTs were administered first; the PTs were administered a week later in the same setting. The PTs involved the same mathematical computation as the CTs, but were completely stripped of the context. We compared Vira's performances on CTs and PTs to investigate the influence of context on his computational skills.

ANALYSIS AND RESULTS

In this section, first we present data that demonstrate Vira's use of mental computation connected to his work-related activities. Elsewhere we have documented in detail, mental mathematical methods bus conductors used to complete ticket transactions (Naresh & Presmeg, 2008).

Mental Computation-Work Setting

At work, Vira used mental computation (a) to complete ticket transactions, (b) to determine the overall ticket sales amount, and (c) to determine his everyday allowance (*batta*). In this paper, we will highlight our analysis of Vira's mental computational activities related to tasks (b) and (c).

Determination of Ticket Collection Amount

Every day at the end of his work shift, Vira was required to determine the overall ticket sales amount for his shift. He used an official record called a *waybill* to record the number of tickets sold in each denomination and used this information to calculate overall earnings. Corresponding to each ticket denomination, Vira first determined the number of tickets sold, using the beginning and ending numbers on his ticket bundles. Next, he determined the collection amount for each ticket denomination. In Table 1, we present information adapted from Vira's waybill contents—official record—. In this table, we have documented the mental computational strategies Vira used to determine the ticket collection amount. Vira took less than 15 seconds to determine—mentally—the collection amount for each ticket denomination. He recorded the solutions in his waybill and traffic return—official records—to keep track of his calculations from time to time.

Analysis of his mathematical descriptions compel us to infer that he consistently used associative, distributive, and commutative properties of whole numbers, fractions and decimals, and the reformulation, translation and compensation (RTC) strategies to solve mental mathematical tasks (Reys, Reys, Rybolt, & Wyatt, 1982).

Table 1
Illustration of Vira's Mental Computational Strategies

Ticket Denomination	Tickets sold	Mental computational strategy
2.0	431	To find 431×2, double 431. $400+400=800$; $31+31=62$; Total: $800+62=862$
3.0	293	First find 300×3 as 900; Subtract $7\times 3=21$ from 900; $293\times 3=(300\times 3)-(7\times 3)=900-21=879$
3.5	61	To find 61×3.5, first find 61×3 as 183. Then find half of 61 as 30.5 $183+30.5=213.5$
4.0	139	To find 139×4, first find 140×4 using $(100\times 4)+(40\times 4)$ to get 560. Subtract 4, to get 556
4.5	66	$66\times 4.5=$ $(66\times 4)+(6\times 4)+66/2=$ $(60\times 4)+(6\times 4)+33=$ $240+24+33=297$
5.0	28	$28\times 5=(30\times 5)-10=$ $150-10=140$
5.5	1	5.5

Determination of Everyday Allowance

Vira received a daily allowance for every shift on which he worked. This amount depends on the overall ticket collection amount. It is Vira's responsibility to determine his batta at the end of his shift. The batta is comprised of a fixed allowance (45 rupees) and a variable allowance. For every 100 rupees (Rs. 100) collected, the variable allowance is fixed at Rs. 2. We use waybill contents presented in Table 2 to illustrate his batta calculation methods.

Table 2
Waybill Contents from Vira's Official Records

Ticket denomination	Tickets sold	Ticket collection amount
5.0	47	235
7.0	98	686
8.0	25	200
9.0	133	1197
10.0	141	1410
11.0	90	990
12.0	18	216
13.0	115	1495
14.0	34	476
15.0	13	195

The waybill entries indicate the total collection amount as Rs. 7100. They also indicated that Vira claimed Rs. 177 as his batta and remitted Rs. 6953 as the total ticket collection amount to the accountant. Vira explained his method as given below.

> *Total ticket collection amount is 7100. In an express bus for every Rs. 100 collected, the variable allowance is Rs. 2; for every Rs. 1000, the allowance is Rs. 20. For Rs. 7000 it is 140. Since I added 7 thousands to get 7100, I added 20 seven times to get 140. I added another two to this amount to account for the extra 100. The total variable allowance is 142. The fixed allowance is Rs. 45. Normally, I would add this amount to the variable allowance to get the total allowance. But, on that day, we lost a complete trip to traffic delays and we completed only 6 trips instead of the stipulated 8 trips. Because of this, I had to forfeit a part of the fixed allowance, the steering allowance (Rs. 10). Thus, I deducted Rs. 10 from the fixed allowance. Thus the actual allowance that I claimed was:* $142 + 35 = 177$.
>
> *Before I remitted the collection amount to the cashier, I added Rs. 30 (advance amount that I collected in the morning) to 7100 to get Rs. 7130, and deducted Rs. 177. I first took away 200 and then added 23 to compensate for the extra amount taken away.*

Vira noted that the use of monetary units greatly helped him complete mental calculations quickly. Official records such as the waybill and the traffic return helped Vira keep track of the number of tickets sold in each denomination and also to document his solutions to mental math tasks. In brief, Vira used several context-related aspects effortlessly to complete work-related mental mathematical tasks. He used a special term, *conductors' mathematics*, to describe his workplace mathematics. Below, using his own words, we highlight context-related parameters that helped him efficiently complete computational tasks at work.

> *I know how to solve problems without using money. But when I use money, I calculate very quickly. There is also another benefit. How can you be so sure your math is correct? This is where money can help. You can check if you are doing correct calculations by looking at the money in hand. I think I use lot of different ideas here, sometimes passengers help, money is a big factor, math that I already knew from school, use of bus tickets, and much more. The math I do at work is possible because of what is available at work. I adapt and use all of these factors to be an efficient conductor. All conductors do this.*

Mental Computation. Non-Work Setting

We now present findings related to Vira's use of mental strategies as he solved mental computational tasks outside work settings. Table 3 presents a summary of his responses to the CTs and PTs. Details of Tasks 9 and 3 are subsequently presented in order to illustrate the contrasts in his thinking.

Table 3
Summary of Vira's Responses

Task	Context	Plain
1	✓	×
2	✓	×
3	×	×
4	✓	×
5	✓	×
6	✓	×
7	✓	×
8	✓	×
9	✓	×
10	✓	×
11	✓	✓
Total right	10 (91%)	1 (9%)

Note: ✓ = right; × = wrong.

Vira's performance on the CTs was exemplary. He answered all but one of these correctly. He commented that he found all of the problems in this category easy because the contexts were good and familiar. The context of the problem guided his solution process and he was able to complete the CTs within the stipulated time. However, Vira was completely perplexed when he was presented with the PTs. He attempted to use traditional school-taught strategies to solve problems that involved multiplication and division of decimals and this posed him a severe challenge. The only correct solution he gave was for the following problem: Determine $2\frac{1}{4} + 0.5$. To solve this problem, he added 0.5 and 2.25 in Tamil (local language), and gave the answer as $2\frac{3}{4}$.

While he used efficient mental strategies—similar to those he used at work— to solve contextual tasks, he failed to apply similar strategies to solve the PTs. Here is an example to illustrate the difference in strategies he employed to solve CTs and PTs.

CT 9: A typist types 25 words per minute. How long will it take the typist to type a document that contains 430 words?

Vira: I know that I had to do $430/25$. When you listen to the problem, I know that in four minutes, 100 words can be typed, and for 400 words, it is 4×4, which is 16; add one more minute to type 25 words. So the answer is 18 minutes if you want full minutes, or 17 minutes and few seconds.

However, when the same problem was presented devoid of context, to determine $430/25$, Vira resorted to the long division method without any success. Vira acknowledged that the context of the problem greatly influenced his choice of mental computational strategies. He said that the plain computational problems reminded him of school mathematics and he found it difficult to recall appropriate school-taught strategies to solve them. However, after he explained his solution strategies to contextual tasks, he acknowledged that he could have used similar strategies to solve the PTs as well. Vira lamented that at school he had been an average student of mathematics, and that now he had trouble solving school-type problems.

The influence of the context of money was evident in Vira's case.

CT 3: The cost of a dozen bananas is Rs. 7.20. How much would $2\frac{1}{2}$ dozens cost?

Vira: In order to determine the cost of $2\frac{1}{2}$ dozen bananas, I first found the cost of one banana. 12 bananas is 720 paise; one banana is 60 paise, because there are 12 60-paise in 720 paise. Now $2\frac{1}{2}$ dozen bananas is 30 bananas; so 30 bananas cost 1800 paise, which is Rs. 18. [Notice that he first determined the cost of one banana as 60 paise]

PT 3: Compute: $7.2\times 2\frac{1}{2}$

Vira: I changed $2\frac{1}{2}$ to 2.5 because the other number is in decimals. I need both numbers in the same form. After this, usually I would write one below

> the other in line with the decimals and multiply. I could not do this mentally.

Vira pointed out that the use of money in the former case helped him solve the problem easily. When attempting to solve PTs, he resorted to the use of school-taught algorithms, which he could not instantly access mentally.

Vira was quick to notice the connection between the two types of problems. As he explained his solution strategies for the computational tasks, he noted that he had failed to solve similar problems on an earlier occasion. He remarked that he found the contextual tasks easy to solve compared to the plain tasks. However, he mentioned that there was a difference between these problems and the problems that he solved at work. He called the problems that I presented to him as readymade problems as opposed to the ones that emerged at work.

Discussion

For our research project, we drew upon research on mental computation and everyday mathematics and extended it to a new context, that of bus conductors in India. In this paper, we reported on one bus conductors' mental mathematical acts and highlighted the role of context and context related parameters in his everyday mathematical actions. Vira's workplace setting demanded that he perform mental mathematical calculations quickly and efficiently. We have demonstrated that Vira successfully availed himself of the use of several context-related parameters to complete work-related mathematical tasks. We have documented that the use of context and context-related parameters not only helped Vira acquire, adapt and use efficient mental strategies to complete tasks at work, but also helped him use similar strategies to solve problems outside work settings. However, the lack of context severely affected his ability to compute mentally. Thus, to answer the research question, we conclude that context, such as association with money or work settings, and context-related parameters, such as memories associated with school mathematics, played a significant role in Vira's mental mathematical activities. The difference between his success in tasks within which he could identify elements similar to those he used in his work practices, and tasks in which he could not, is quite striking. On the theoretical level, our research evidence supports a model of mental computation as a vehicle for promoting thinking, conjecturing, and generalizing based on conceptual understanding (Trafton, 1986), rather than the traditional view of such computation as a set of skills to be mastered.

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