A Review of Research Trends in Mathematics Teacher Education

Mario Sánchez

This article presents a literature review in the field of mathematics teacher education research. The review focuses on identifying the main research trends in this field, that is, the main research topics that the mathematics teacher educators' community is currently addressing, and the main theoretical concepts used to study these topics. One of the contributions of the review is that it identifies new research trends that have not been previously reported. Some of these trends are online mathematics teacher education, and the design and role of tasks in mathematics teacher education.

Keywords: Literature review; Mathematics teacher education research; Research trends

Una Revisión de las Tendencias de Investigación en la Formación de Profesores de Matemáticas

Este artículo presenta una revisión de la literatura en el campo de la investigación en formación de profesores de matemáticas. La revisión se centra en identificar las principales tendencias de investigación en este campo, es decir, los principales temas de investigación que la comunidad de formadores de profesores está abordando actualmente, así como los principales conceptos teóricos usados para estudiar esos temas. Una de las contribuciones de esta revisión es que se identifican tendencias de investigación que no han sido documentadas previamente. Algunas de estas tendencias son la formación en línea de profesores de matemáticas y el diseño y rol de las tareas en la formación de profesores de matemáticas.

Términos clave: Investigación en formación de profesores de matemáticas; Revisión bibliográfica; Tendencias de investigación

The purpose of this review was twofold. On the one hand and at a personal level this review allowed me, as newcomer to the field, to obtain a general idea of the

Sánchez, M. (2011). A review of research trends in mathematics teacher education. *PNA, 5*(4), 129-145.

essence of this research area and an overview of the theoretical landscape it has produced. In other words, I have tried to understand the main research topics that are being addressed or investigated by the mathematics teacher educators' community. I have also tried to identify the main theoretical concepts that have been used to address those topics.

I think that this review will be useful to the newcomers to the field of mathematics teacher education research, but also to the most experienced people in the area. I claim this, since the review will provide them with an updated overview of the current stage of development of the field.

LIMITATIONS

This review will provide the reader with a general overview that only highlights the major trends in the field. I am overlooking some areas that are relevant but not having a large representation in the field as the major trends do. I am referring to areas like research dealing with reform processes and policy issues, curriculum-based studies, constructivism and the use of history in teacher development.

Another limitation of my review is that it does not capture the main trends within the field regarding empirical methods of research. For instance, I am aware that the use of video cases is widespread; nevertheless, this is not reported in the review.

Due to space restrictions, it has not been possible to include all the bibliographical references in which this review is based. Thus, a selection of the most relevant references has been undertaken. This limitation could imply that the reader feels that the review is too general. Nevertheless, I have tried to include specific examples that illustrate the discussed research trends whenever possible.

METHOD

The first step before starting the review was to establish some limits. These limits were defined through three questions that guided the development of this review: What to look for? How far should I look? Where to look?

What to Look for?

My interest was to identify two elements in the consulted literature. On the one hand, the main research topics in which the mathematics teacher education community is interested in; and on the other hand, the main theoretical concepts used by this research community. In order to grasp these two elements I decided to include in the review, literature concerning the development of in-service mathematics teachers as well as literature related to the education of pre-service mathematics teachers.

There are three necessary conditions in order to qualify as a trend any topic or concept: (a) the volume condition, (b) the socio-geographical condition, and (c) the temporal condition. The volume condition refers to the number of investigations conducted on a particular issue. I considered as trend those themes that are being investigated from different theoretical angles and by several different people. The socio-geographical condition means that, in addition to requiring different people working on the same research topic or using the same theoretical concept, I sought for research being developed and communicated in different regions of the world. This condition ensures that there is genuine international interest about a particular topic or concept. The last condition is called *temporal*, and it refers to a particular subject that has remained as a focus of interest within the community or has been constantly researched for at least five years. I included this condition trying to avoid including ephemeral research trends in this review.

How Far Should I Look?

Because I wanted to produce a more or less extensive but also updated review, I initially opted for narrowing my search to a ten years interval. Thus, the review mainly included references published between 1999 and 2009. However, as I will explain later, it was difficult to keep this period of time as a limit during all the review stages.

Where to Look?

Four layers determined the literature search. Three of them are explicitly defined while the fourth is somewhat subjective. The descriptions of the layers are as follow.

Layer 1

When I started the review it made sense to me to use other reviews from mathematics teacher education carried out previously as a support. Thus, the first layer consists on literature reviews on mathematics teacher education research conducted within the latest decade. In this layer I included the following papers: Adler, Ball, Krainer, Lin, and Novotna (2005); Grevholm (2008); Lerman (2001); Llinares and Krainer (2006); Ponte and Chapman (2006); and Sowder (2007).

Layer 2

The second layer of the review is comprised by books specialized in mathematics teacher education, and papers published in proceedings of international conferences. The specialized books included in this layer were: Jaworski, Wood, and Dawson (1999), Lin and Cooney (2001), Strässer, Brandell, Grevholm, and Helenius (2004), the four volumes of the *International Handbook of Mathematics Teacher Education* (Clarke, Grevholm, & Millman, 2009; Even & Ball, 2009; Jaworski & Wood, 2008; Krainer & Wood, 2008; Sullivan & Wood, 2008; and

Tirosh & Wood, 2008). The international conferences included in the review were the International Congress on Mathematical Education proceedings from ICME-9 and ICME-10, the Conference of European Research in Mathematics Education (proceedings from CERME 1 to CERME 5) and the proceedings of the Symposium on the occasion of the 100th anniversary of ICMI in Rome (Menghini, Furinghetti, Giacardi, & Arzarello, 2008).

The research trends present in the proceedings of the International Group for the Psychology of Mathematics Education (PME) conferences were addressed by including the papers of Llinares and Krainer (2006), Ponte and Chapman (2006), and Krainer and Llinares (2010) in the review. These three papers summarize research trends and key issues in mathematics teacher education research, based on the review of the PME proceedings produced during the last three decades. These are quite comprehensive reviews that provide a broad perspective on the research trends produced within the PME community. These general reviews were included in the first layer of my own review. In the case of the CERME proceedings, I mainly focused on reviewing the reports of the mathematics teacher education working groups. In the case of the ICME proceedings I used the same criterion. However, I also included the individual writings—individual papers, plenary lectures—addressing topics related to mathematics teacher education. As for the proceedings of the symposium on the occasion of the 100th anniversary of ICMI, I only included the paper of Grevholm and Ball (2008), but I also consulted some of the papers of the working group "The Professional Formation of Teachers" of the same symposium.

Layer 3

The third layer consists of two research journals: *Educational Studies in Mathematics* (ESM) and *Journal of Mathematics Teacher Education* (JMTE). I decided to include the ESM journal because I consider it one of the most important journals in the field of mathematics education research. The inclusion of JMTE was an obvious choice. JMTE is currently the only specialized journal in the area of mathematics teacher education research. When I tried to apply the ten-year limit to the third layer, I realized that the number of articles to read would be very large and therefore it would be impracticable to go through such amount of papers. So, I decided to reduce the time interval to five years. Thus, in this layer of the review I included articles published in ESM and JMTE during the period 2005-2009.

Layer 4

The fourth layer is a bit subjective because is not focused on a particular type of publication nor limited by a well-defined time interval. The fourth layer refers to all those articles I was familiar with before starting the review, but that were relevant to inform and to shape the review. It also includes those articles that I met through the interaction with fellow researchers during the development of the re-

view. Some of them provided me with bibliographical suggestions that were very important for the progress of this review. Other papers included in this layer were located by going through the references lists of the papers reviewed in the previous layers.

In sum, Table 1 shows an overview of the sources consulted during the preparation of this review.

Table 1
Sources and Number of Papers

Source	Number of papers
Previous reviews	6
Specialized books	9
Proceedings	
ICME9	8
ICME10	9
CERME 1	1
CERME 2	1
CERME 3	4
CERME 4	3
CERME 5	1
Symposium 100th anniversary of ICMI	3
Journals	
ESM	12
JMTE	143
Miscellaneous papers	38

RESULTS

In this section the results of the review are presented. I will divide them into three categories: (a) research concerns, which are the areas of interest that researchers in mathematics teacher education are currently investigating; (b) theoretical concepts, which are the theoretical concepts that are most used in this research field; and (c) new trends, which are emerging research areas that were identified in the literature review.

Research Concerns

In this section, the discussion is centered on the teachers' beliefs, views and conceptions, teachers' practices, teachers' knowledge and skills, relationship between theory and practice, and finally the reflective practice.

Teachers' Beliefs, Views and Conceptions

Undoubtedly this is one of the most popular research areas in mathematics teacher education. Probably the interest of this community in investigating mathematics teachers' beliefs and conceptions is associated with the prevailing idea that teachers' beliefs and conceptions inform and define their teaching practices (Skott, 2009). This could explain why there is a big interest in identifying teachers' beliefs, conceptions and views about different aspects of their teaching.

The interest in this research area has not decreased over the ten-year period covered by the review. On the contrary, researchers' interests in this area have become more specialized and their research reports and studies reflect this specialization. We can find studies related to teachers' beliefs about their role as mathematics teacher, the concept of computational estimation, gender, the use of computers for mathematical learning, a new educational reform, teachers' views of mathematics, etc.

Although research on teachers' beliefs may seem very diverse, there are prevailing trends. According to Philipp (2007), research on mathematics teachers' beliefs is focused on: (a) understanding teachers' beliefs, (b) investigating the relationship between teachers' beliefs and practices, and (c) changing teachers' beliefs (p. 306).

Teachers' Practices

This is another dominant research area in mathematics teacher education. Primarily, researchers in this area are trying to characterize the actions that teacher performs within the classroom, and understand the factors shaping and promoting their development. The kind of studies belonging to this category reports different aspects of teaching practice within the classroom. For example, the types of questions asked during their lessons, the way teachers manage their time during a particular lesson, or teachers' choice of examples in the classroom.

It is important to note that a small group of researchers has begun to focus on the mathematics teachers' work outside the classroom. They are particularly focused on the kind of resources used by teachers in order to define the content of their lessons. The argument for focusing on the interaction between a teacher and the external resources he/she uses to plan his/her lessons is that this type of activity is at the core of teacher's professional activity and development (Gueudet & Trouche, 2009, p. 199).

Teachers' Knowledge and Skills

At the centre of this research area we found the following question: What kind of knowledge and skills a person needs in order to be a "good" mathematics teacher? There are many studies that underline the importance of mathematical knowledge but there is widespread recognition that to possess mathematical knowledge is a necessary, but not a sufficient condition, to be a good mathematics teacher. It is argued that other kind of knowledge and skills are required, such as mathematical knowledge for teaching or mathematical pedagogy, knowledge of students' cognition in mathematics, and attention-dependent knowledge or awareness.

I think this discussion about mathematics teacher's knowledge should be shaped by the context in which the teacher develops his or her work. In other words, I think there must be some basic knowledge and skills that any mathematics teacher should have, but I also believe that there are other skills and abilities that are especially needed in particular contexts. Just as Adler (2000) pointed out: "What knowledge bases (are necessary) for teaching culturally and linguistically diverse learners? And for teaching across urban and rural under-resourced schools?" (p. 210)

The Relationship Between Theory and Practice

The relationship between theory and practice is an academic consideration that has been present in the mathematics teacher educators' community for many years. One concern that is at the heart of this discussion is that theoretical knowledge—the one produced by researchers— is usually perceived as something different and disconnected from practical knowledge —the one that teachers acquire through their experience—. Researchers are trying to show that both types of knowledge are mutually informed, but they are also trying to explain the nature of this relationship, how to support it, and what its consequences are. An example of this is the work of Skott (2006). This author claims that there is actually a relationship between theory and practice but such relationship is not a linear one. He uses the concept of theoretical loop to illustrate how the practice can serve as a basis for theorising and, in turn this theorisation can inform practice, although not necessarily the practical context in which the theorisation was based.

It is notable that the discussion of the relationship between theory and practice has been of particular interest to the CERME community of teacher educators. In fact, at the CERME 3 conference there was a working group called "Inter-Relating Theory and Practice in Mathematics Teacher Education" (Jaworski, Serrazina, Koop, & Krainer, 2004). One of the conclusions of this working group was that more collaboration between teacher educators and teachers was needed in order to strengthen the relationship between theory and practice. This collaborative trend is reflected in the special issue also entitled *Inter-Relating Theory and Practice in Mathematics Teacher Education* in 2007, which was published in

the JMTE. In this issue the papers written by Scherer and Steinbring (2007) and Jaworski (2007) report results of research projects that were developed through a close collaboration between researchers and teachers. This type of collaborative research in which teachers are regarded as professionals investigating their own practice, is known as action research.

The discussion on how to address the relationship between theory and practice is still alive in recent international reports (Even & Ball, 2009, p. 3; Grevholm & Ball, 2008, p. 268). I interpret this as an indication that the teacher educators' community continues being interested in seeking ways of reducing the gap between research and practice.

Reflective Practice

Under the label of reflective practice I have grouped all the research that deals with teachers or teacher educators reflecting on and learning from their own practices and experiences. This kind of research has been strongly influenced by the work of Dewey (1933) and Schön (1983), and it has remained in constant development over the ten years covered by this review.

It is clear that there is a general agreement in the mathematics teacher educators' community on considering reflection as a key element in the education and development of mathematics teachers. Nevertheless, we can also see that the meanings attributed to the concept of reflection are varied. In the literature one can find a variety of terms such as reflective thinking, reflective stance, critical reflection, joint reflection, self-reflection, etc., that refer to different nuances and meanings of the concept of reflection. As Mason and Spence (1999) have stated: "The term reflection has become too broad and diffuse in meaning to carry significance in itself" (p. 153).

The sort of arguments provided to justify the relevance of reflection in teacher education are also varied, but they could be organized into three groups: (a) the researchers who claim that reflection is a means to gain knowledge, (b) others claim that reflection can serve as a trigger for changes in beliefs and practices, and (c) some perceive reflection as a link between theory and practice (see section discussed previously).

Theoretical Concepts

Research on mathematics teachers has been based on a variety of theoretical concepts. No single theory or framework dominates scholarship in this area (Grevholm & Ball, 2008, p. 268). However, there are theoretical concepts with a remarkable influence on the research community. In this section I will mention these theoretical concepts, which I also classified as research trends.

Pedagogical Content Knowledge and Others Forms of Knowledge

As mentioned in the previous section, one of the main concerns in the mathematics teacher education community has been to identify the kind of knowledge and

skills that a teacher needs to possess in order to produce good teaching. The categories proposed by Shulman (1986) have been useful to conceptualize the kind of knowledge that teachers require in order to do so. The categories to which I refer are Subject Matter Knowledge (SMK), Pedagogical Knowledge (PK), and Pedagogical Content Knowledge (PCK).

According to Ponte and Chapman (2006), the notion of PCK was introduced in the 1990s into the field (p. 469). Since then, this one and the rest of the categories proposed by Shulman have influenced the research on mathematics teachers' knowledge. Although the categorization proposed by Shulman has been criticized —see for example Mason (1998), who claims that Shulman's taxonomy is rather unstable in practice (p. 224)—, this categorization has stimulated the development of new theoretical concepts better suited to the mathematics teacher's reality. One example of this is the concept of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), which is defined as the mathematical knowledge needed to carry out the work of teaching mathematics. According to Ball y colaboradores, this kind of knowledge could not be captured by the categories proposed by Shulman: "Teaching may require a specialized form of pure subject matter knowledge —'pure' because it is not mixed with knowledge of students or pedagogy and is thus distinct from the PCK identified by Shulman and his colleagues and 'specialized' because it is not needed or used in settings other than mathematics teaching. This uniqueness is what makes this content knowledge special" (p. 396).

Reflection-in-Action and Reflection-on-Action

The work of Schön (1983) has also significantly influenced the development of mathematics teacher education research. I am referring here to the concepts of reflection-in-action and reflection-on-action. Reflection-in-action refers to the kind of reflection that practitioners perform on their own practice while actively engaged in it. This kind of reflection could lead to modifications in their practice in order to meet the immediate needs of the situation. A reflection-on-action takes place when practitioners reflect on their practice after it has occurred. Through this sort of reflection they identify their decisions and their consequences. Then it is possible to consider the actions explicitly that did not work well, and use this experiences for future planning. A basic assumption behind these two concepts is that they represent mechanisms that practitioners use to develop themselves and to learn from their own working experiences.

The categories proposed by Schön also served as a basis for proposing new reflection categories like that of reflection-for-action (Jaworski, 1998; Scherer & Steinbring, 2007). The relevance of the concept of reflection-for-action lies in the fact that it captures the kind of reflections that educators do before implementing a particular teaching strategy or a didactical design. It also includes the idea that this kind of reflection should be productive, that is, it should influence teachers' practice.

Communities of Practice

A community of practice can be defined as a group of people who share an interest or a profession (Wenger, 1998). Members of the group can learn and develop through participation. This concept has helped mathematics teachers' educators to conceptualize teachers' learning as a social process: "Instead of defining learning as the acquisition of knowledge of a propositional nature, learning is conceptualized as being situated in forms of co-participation in the practices of teachers" (Matos, Powell, Sztajn, Ejersbø, & Hovermill, 2009, p. 170).

The influence of this theoretical concept is remarkable, not only because of the high number of studies that have employed this theoretical concept over the past ten years, but also because the concept has served as a foundation for the development of new theoretical concepts such as community of learning and community of inquiry (Jaworski, 2003; Schoenfeld, 1996). The concept of community of inquiry, for example, has allowed researchers to describe a particular community ruled by a critical mode of reflective practice that favours that the practices of the community could be continually scrutinized and reconceptualised to benefit the entire community.

New Trends

One of the main contributions of this review is the identification of new research trends in the area of mathematics teacher education. These are the new research trends identified.

Online Mathematics Teacher Education

Research on online mathematics teacher education is not large when compared with other areas of mathematics teacher education research. However, this sort of research has been on the rise in recent years. The first studies about this issue that I came across when doing this review are included in the proceedings of CERME 3 conference published in 2004 (Santos & Ponte, 2004). Nevertheless, the reader should be aware of the fact that during 2002 and 2003 some studies discussing the use of online communication tools for communication and interaction among preservice mathematics teachers began to appear in the literature (Weigand, 2002).

Thus in general we can say that from 2002, papers from different regions of the world related to the use of the internet for the education and development of teachers, started to appear in different settings. These studies have been reported in specialized books (Borba & Villarreal, 2006), in international conferences (Llinares, 2004) and in research journals (McGraw, Lynch, Koc, Budak, & Brown, 2007).

I agree with the observation that the work in this area is still scarce and is in its beginning stages (Llinares & Olivero, 2008; Ponte, Zaslavsky, Silver, Borba, van den Heuvel-Panhuizen, Gal et al., 2009). But I can also see that researchers are opening up avenues of investigation in this area that seem very promising. I

am pretty sure that in the coming years we will witness the development of these and other lines of research within the online mathematics teacher education.

The Design and Role of Tasks in Mathematics Teacher Education

One of the fundamental premises that underlie this research trend is that "what students learn is largely defined by the tasks they are given" (Hiebert & Wearne, 1993, p. 395). This idea can be extended to other types of learners, such as mathematics teachers or even teacher educators.

One of the first persons that started to highlight the importance of tasks in mathematics teacher education was Zaslavsky (1995). It is important to note that her interest has been focused on mathematics-related tasks. This focus in mathematics-related tasks is clearly manifested in the triple special issue of the JMTE in 2007, where Zaslavsky participated as editor and author. However, it is also important to note that although the type of tasks reported in this special issue are mathematically based tasks, it is also discussed the kind of learning they generate, which goes beyond the mathematical content.

After the special issue on JMTE, the interest on the design, form and function of tasks in teacher education has continued increasing. An evidence of this are the sections that have been devoted to this topic on the latest books on mathematics teacher education (Ponte et al., 2009; Tirosh & Wood, 2008), and particularly the book *Tasks in Primary Mathematics Teacher Education* (Clarke, Grevholm, & Millman, 2009), which provides us with an international overview on the types of tasks that are currently being used in the preparation of primary mathematics teachers.

The Education and Development of Mathematics Teacher Educators

Clearly, the area of mathematics teacher education is concerned with the training and development of mathematics teachers. However, it is possible to perceive that at least during the last five years, it has been present a persevering interest in understanding the particular type of knowledge needed by mathematics teacher educators and how they acquire and develop that knowledge.

One of the first indicators of this interest in the education of mathematics teacher educators is found in the introduction of the book *Educating for the Future* (Strässer, Brandell, Grevholm, & Helenius, 2004). There the editors stated that there were three major issues that guided the preparation of the symposium itself and the editing process of the book. One of those issues is the education of teachers and their educators. In this respect, the editors formulated questions such as: Would it be rewarding to have a special education for teacher educators? How could such an education be designed and carried out? What is the difference in the knowledge of a teacher' educator at university or at teacher training college and a "normal" teacher at school? (p. 5)

Although the number of publications related to the education and development of teacher educators is scarce, I consider it as a research trend because of

the interest that this topic has awoken in leader researchers in this area. This interest can be noted by simply giving a look at the recent publications in the area. An example is the 15th ICMI study (Even & Ball, 2009), in which in spite of the fact that none of the papers presented during this conference in Brazil were related to this issue, the topic was included in the book.

Another example is the *International Handbook of Mathematics Teacher Education*, where the eighteen papers that constitute the fourth volume of this handbook are related to the development and learning of mathematics teacher educators (Jaworski & Wood, 2008). A final example is the report written by Grevholm and Ball (2008), who suggested that a possible future study foci within the ICMI organization could be studies focused on teacher educators: "Who they are, what they do, what they know, how they learn their work" (p. 274).

There are already studies in which mathematics teachers educators reflect on their own development as teacher educators (Tzur, 2001). Nevertheless, I think there is a lot more to do in this new research area since many of the questions that have been made in research on mathematics teachers could be applied to teachers' educators —what kind of knowledge and abilities do they need? How do they acquire that knowledge and abilities? What beliefs do they have and how do they impact their practice? What type of tasks should be used in order to support their professional development?—. My prediction then is that we are in front of another fertile field of research, which in the upcoming years will produce a growing amount of research related to the education and development of teacher educators.

Social Justice in Mathematics Teacher Education Research

When I use the term "social justice in mathematics teacher education research", I am referring to those studies that explore the approaches and the conditions that can help us to foster and develop socially just and equitable teaching practices in mathematics teachers and mathematics educators in general. These practices should aim at ensuring a more plural mathematics teaching in the classroom, where all students, regardless of their abilities, social background, religion, gender, race and other social differences, could have access to a quality mathematics education.

I am aware that the number of research on social justice in mathematics teacher education is not large. In fact I would like to be cautious and say that currently this topic is not a well-established trend, but there are indications that it could become one. For example, besides the articles that can be sporadically found in the literature, such as Vithal (2003), I have noticed the constant presence of papers related to teacher education in journals' special issues devoted to social justice and mathematics education. Here I refer particularly to the papers of Jere Confrey and Fiona Walls included in the special issue 20 on social justice in the *Philosophy of Mathematics Education Journal* (2007), and to the papers

wrote by Libby Knott and Eric Gutstein included in the special issue 1 on social justice of the *Montana Mathematics Enthusiast* (2007).

Another factor that made me think of this topic as an emerging trend in mathematics teacher education research was the double special issue on social justice published in the JMTE in 2009, in which theoretical and empirical issues on research on this type are discussed. My interpretation of these facts is that the community of teacher educators—or at least part of it—begins to recognize the fundamental role played by the teacher in the implementation of a critical and socially just education.

A Final Reflection

One of the things I have learned after conducting this review is that one characteristic of the field of mathematics teacher education is the lack of consensus regarding the meaning of key theoretical concepts. It is possible to find many definitions of the concept of belief, many definitions of the concept of reflection and so on. You cannot take any concept for granted. It is even possible to find explicit statements about it. For example, Furinghetti, Grevholm, and Krainer (2002) mentioned that one of the questions that focused the discussion in the working group "Teacher Education between Theoretical Issues and Practical Realization" at the conference CERME 3 was: "How precisely should we define (in our papers etc.) central concepts like reflection, improvement, changes, development?" (p. 266). In a more recent publication, Grevholm and Ball (2008) refer to the central concepts and constructs used in research on teachers and teacher education. In this respect they claim: "Not always are these central concepts explained or defined generally in studies where they are used" (p. 268).

In this situation, my advice to the newcomers to this research field is to identify the main theoretical concepts or ideas playing a role in their own research, and then look into the literature to understand what these concepts mean or how researchers interpret them. This will provide the newcomers with a basis of awareness that in turn will allow them to establish their own position regarding those concepts.

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, *3*(3), 205-224.

Adler, J., Ball, D., Krainer, K., Lin, F. L., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics*, *60*(3), 359-381.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Borba, M. C., & Villarreal, M. E. (2006). *Humans-with-media and the reorganization of mathematical thinking. Information and communication technologies, modeling, visualization and experimentation*. New York: Springer.

Clarke, B., Grevholm, B., & Millman, R. (Eds.). (2009). *Tasks in primary mathematics teacher education. Purpose, use and exemplars*. New York: Springer.

Dewey, J. (1933). *How we think: a restatement of the relation of reflective thinking to the educative process*. Boston, MA: D. C. Heath.

Even, R., & Ball, D. L. (Eds.). (2009). *The professional education and development of teachers of mathematics. The 15th ICMI Study*. New York: Springer.

Furinghetti, F., Grevholm, B., & Krainer, K. (2002). Teacher education between theoretical issues and practical realization. In J. Novotná (Ed.), *Proceedings of the Second Conference of the European Society for Research in Mathematics Education* (pp. 265-268). Prague, Czech Republic: Charles University.

Grevholm, B. (2008). *Some historical moments in the ICMI history of formation of mathematics teachers.* Paper presented at the Symposium on the Occasion of the 100th Anniversary of ICMI. Rome, Italy. Retrieved on the 10th of March 2011 from http://www.unige.ch/math/EnsMath/Rome2008/WG2/Papers/OVERVW2.pdf

Grevholm, B., & Ball, D. L. (2008). The professional formation of mathematics teachers. In M. Menghini, F. Furinghetti, L. Giacardi, & F. Arzarello (Eds.), *The first century of the International Commission on Mathematical Instruction (1908-2008). Reflecting and shaping the world of mathematics education* (pp. 265-276). Rome, Italy: Istituto della Enciclopedia Italiana fondata da Giovanni Treccani.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, *71*(3), 199-218.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and student learning in second grade. *American Educational Research Journal*, *30*(2), 393-425.

Jaworski, B. (1998). Mathematics teacher research: process, practice and the development of teaching. *Journal of Mathematics Teacher Education, 1*(1), 3-31.

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics*, *54*(2-3), 249-282.

Jaworski, B. (2007). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, *9*(2), 187-211.

Jaworski, B., Serrazina, L., Koop, A. P., & Krainer, K. (2004). Inter-relating theory and practice in mathematics teacher education. In M. A. Mariotti (Ed.), *Proceedings of the Third Conference of the Society for Research in Mathematics Education* (pp. 1-11). Bellaria, Italy: University of Pisa.

Jaworski, B., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education, volume 4. The mathematics teacher educator as a developing professional*. Rotterdam, The Netherlands: Sense Publishers.

Jaworski, B., Wood, T., & Dawson, S. (Eds.). (1999). *Mathematics teacher education. Critical international perspectives*. London: Falmer Press.

Krainer, K., & Llinares, S. (2010). Mathematics teacher education. In P. Peterson, E. Baker, & B. McGaw (Eds.), *International encyclopedia of education* (pp. 702-705). Oxford, United Kingdom: Elsevier.

Krainer, K., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education, volume 3. Participants in mathematics teacher education: individuals, teams, communities and networks*. Rotterdam, The Netherlands: Sense Publishers.

Lerman, S. (2001). A review of research perspectives on mathematics teacher education. In F. L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 33-52). Dordrecht, The Netherlands: Kluwer.

Lin, F. L., & Cooney, T. J. (Eds.). (2001). *Making sense of mathematics teacher education*. Dordrecht, The Netherlands: Kluwer.

Llinares, S. (2004). *Building virtual learning communities and the learning of mathematics teacher student.* Regular lecture presented in the 10th International Congress on Mathematical Education. Copenhagen, Denmark. Retrieved on the 10th of March 2011 from http://www.icme10.dk/proceedings/pages/regular_pdf/RL_Salvador_Llinares.pdf

Llinares, S., & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: past, present and future* (pp. 429-459). Rotterdam, The Netherlands: Sense Publishers.

Llinares, S., & Olivero, F. (2008). Virtual communities and networks of prospective mathematics teachers. In K. Krainer & T. Wood (Eds.), *The international handbook of mathematics teacher education. Volume 3. Participants in mathematics teacher education: individuals, teams, communities and networks* (pp. 155-179). Rotterdam, The Netherlands: Sense Publishers.

Mason, J. (1998). Enabling teachers to be real teachers: necessary levels of awareness and structure of attention. *Journal of Mathematics Teacher Education*, *1*(3), 243-267.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: the importance of knowing-to act in the moment. *Educational Studies in Mathematics*, *38*(1-3), 135-161.

Matos, J. F., Powell, A., Sztajn, P., Ejersbø, L., & Hovermill, J. (2009). Mathematics teachers' professional development: processes of learning in and from practice. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics. The 15th ICMI Study* (pp. 167-183). New York: Springer.

McGraw, R., Lynch, K., Koc, Y., Budak, A., & Brown, C. A. (2007). The multimedia case as a tool for professional development: an analysis of online and face-to-face interaction among mathematics pre-service teachers, in-service teachers, mathematicians, and mathematics teacher educators. *Journal of Mathematics Teacher Education*, *10*(2), 95-121.

Menghini, M., Furinghetti, F., Giacardi, L., & Arzarello, F. (Eds.). (2008). *The first century of the International Commission on Mathematical Instruction (1908-2008). Reflecting and shaping the world of mathematics education*. Rome, Italy: Instituto della Enciclopedia Italiana fondata da Giovanni Treccani.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 257-315). Charlotte, NC: Information Age Publishing.

Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: past, present and future* (pp. 461-494). Rotterdam, The Netherlands: Sense Publishers.

Ponte, J. P., Zaslavsky, O., Silver, E., Borba, M. C., van den Heuvel-Panhuizen, M., Gal, H., et al. (2009). Tools and settings supporting mathematics teachers' learning in and from practice. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics. The 15th ICMI Study* (pp. 185-209). New York: Springer.

Santos, L., & Ponte, J. P. (2004). An experience in distance in-service teacher education. In M. A. Mariotti (Ed.), *Proceedings of the third Conference of the European Society for Research in Mathematics Education* (pp. 1-10). Bellaria, Italy: University of Pisa.

Scherer, P., & Steinbring, H. (2007). Noticing children's learning processes-teachers jointly reflect on their own classroom interaction for improving mathematics teaching. *Journal of Mathematics Teacher Education*, *9*(2), 157-185.

Schoenfeld, A. H. (1996). In fostering communities of inquiry, must it matter that the teacher knows the answer? *For the Learning of Mathematics*, *16*(3), 11-16.

Schön, D. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

Shulman, L. (1986). Those who understand, knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Skott, J. (2006). The role of the practice of theorising practice. In M. Bosh (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1598-1608). Barcelona, Spain: FUNDEMI IQS-Universitat Ramon Llull.

Skott, J. (2009). Contextualising the notion of "belief enactment". *Journal of Mathematics Teacher Education*, *12*(1), 27-46.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 157-223). Charlotte, NC: Information Age Publishing.

Strässer, R., Brandell, G., Grevholm, B., & Helenius, O. (Eds.). (2004). *Educating for the future. Proceedings of an international symposium on mathematics teacher education*. Göteborg, Sweden: Royal Swedish Academy of Sciences.

Sullivan, P., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education, volume 1. Knowledge and beliefs in mathematics teaching and teaching development*. Rotterdam, The Netherlands: Sense Publishers.

Tirosh, D., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education, volume 2. Tools and processes in mathematics teacher education*. Rotterdam, The Netherlands: Sense Publishers.

Tzur, R. (2001). Becoming a mathematics teacher-educator: Conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education*, *4*(4), 259-283.

Vithal, R. (2003). Teachers and "street children": on becoming a teacher of mathematics. *Journal of Mathematics Teacher Education*, *6*(2), 165-183.

Weigand, H. G. (2002). New ways of communicating in mathematics teacher education: linking to the Internet. In H. G. Weigand, N. Neill, A. Peter-Koop, K. Reiss, G. Törner, & B. Wollring (Eds.), *Developments in mathematics education in German-speaking countries. Selected papers from the Annual Conference on Didactics of Mathematics, Potsdam, 2000* (pp. 164-179). Berlin, Germany: Verlag Franzbecker.

Wenger, E. (1998). *Communities of practice: learning, meaning, and identity.* Cambridge, United Kingdom: Cambridge University Press.

Zaslavsky, O. (1995). Open-ended tasks as a trigger for mathematics teachers' professional development. *For the Learning of Mathematics*, *15*(3), 15-20.

Mario Sánchez
National Polytechnic Institute
marios@ruc.dk