THE CALCULATOR AS AN INSTRUMENT OF VALIDATION OF MATHEMATICAL KNOWLEDGE: A CASE STUDY

Antonio Codina Sánchez*
Jose Luis Lupiáñez Gómez*
CINVESTAV del IPN. México. Universidad de Granada, España

The discussions that arise when demonstration in mathematics is dealt with often cause opposing positions regarding the wisdom of its use. How can it be conducted in the classroom? How to discern which proofs enrich the academic formation of the students? If, in addition, we consider the contribution of new technology to teaching, it is necessary to reflect on how the form of validating mathematical knowledge is affected, if it is at all altered, in the classroom. In this paper several considerations on these ideas are discussed, and an experience with junior high school math teachers is described where mathematical reasoning is taught using a TI-92 calculator.

A recent concern for including, in different present-day educational programs, a specific part related to demonstration. For example, in the Principles and Standards in School Mathematics: Discussion Draft (NCTM, 1998), one of the standards proposed is that of "Reasoning and Proof". Also, many recent studies focus on reasoning in mathematics and the role it plays in the educational environment. Studies of this type distinguish different ways of validating mathematical knowledge in the classroom. Duval (1999), among others, writes of argumentation and demonstration as two types of reasoning that possess distinctive structure, organizational links and cognitive functioning. The first obeys relationships of convincing that do not have to possess the rigor and formality of the second that are provided by a previously fixed theoretical statute.

Balacheff (1999), declares himself forcefully on the subject: *"Argumentation constitutes an epistemological obstacle for learning demonstration [...]. Argumentation is to conjecture as demonstration is to theorem."*

* Work supported, in part, by ALFA Project, FIEMAL net, European Union.

This dichotomy leads one to think about whether either form of reasoning to justify propositions in the mathematics classroom is desirable. Even though argumentation does not have the veracity of demonstration from a mathematical standpoint, this lower degree of rigor could be used to provide the students with the fundamentals they need to assume the truth of certain mathematical properties. Moreover, if technology is to be used in the classroom, it is necessary to establish what kinds of reasoning can be carried out with these tools.

In the experience with teachers, which constitutes the case study, we analyze and evaluate the opinion they have on the role of mathematical reasoning in junior high school mathematics education. More concretely, the objectives of the study were to inquire into teachers' conceptions on mathematical demonstration, and the value they place on it in an educational environment. Also, their conceptions about what new technology can contribute to this topic is examined and validated through work with a calculator.

The methodology employed implicated three phases, each one of which had its respective instruments for collecting information. In the first phase, the five teachers who took part in the study answered an initial questionnaire, which included the following questions:

1. **What do you understand by Demonstration in Mathematics?**
2. **What role do you think it plays in secondary education?**
3. **Is it possible to do demonstrations using technology?**

In the second phase, we exposed the teachers to some examples of certain types of reasoning that can be done with a calculator. Since not all of them were familiar with the TI-92, we reviewed the main functions and commands that it possesses. Immediately afterward, each one was given a calculator and they were asked to propose for the next day an example of their own of a justification of some property and to specify whether it constituted a mathematical demonstration.

The third and last phase included the presentation of the results of the participants' work and another questionnaire. The presentation of the subjects was registered on videotape and the questionnaire asked the following questions:

1. **Conjecture properties, visualize those properties, outline an argument to justify them or demonstrate them formally. Which of these skills can be developed with the use of technology in the mathematics classroom?**
2. **Do you think it is advisable to use a calculator to develop mathematical reasoning?**
3. **Would you modify your answer to any of the questions in the first questionnaire after this experience? Why**

The responses to the first question presented certain analogies, although there was a difference between those who assigned mathematical demonstration the capacity to characterize the scientific discipline—"*It is the activity that permits giving rigor, formality, veracity, and objectivity to certain hypotheses or affirmations within a theory and that which characterizes mathematics*"—and those who gave it a more instrumental role—"*It is the utilization of certain procedures or tools in order to, parting from some hypotheses that are considered true, affirm that said affirmation is also true*." Most of the responses allude to the necessity of some hypotheses or premises from which it is intended to make the veracity of a statement evident, even though, only in some cases, it is considered possible to demonstrate the non-veracity of a statement.

For the second question, all of the answers granted demonstration a role of relevant importance in the teaching of mathematics, basically because "*things are not so just because they are, but because they have their cause*", and also, although in a lesser degree, because it favors the formative development of students. For the last question of the first questionnaire, there was also certain uniformity among the responses. In general, the subjects doubted the viability of demonstrating with the use of technology, although they did not deny the possibility. *"I think so, but I don't know how it would lay out rigor, formality ..." "I imagine it is possible, but I don't really know how ..."*

Only one of the subjects declared that it is possible to do formal demonstrations using technology. This was the only person that defended the thesis that, in fact, the example he presented was reasoning with all the rigor required by mathematics. Indeed, his work changed the manner of thinking of the other subjects, who at first thought it was difficult or impossible to carry out such justifications.

Of the five presentations in the third and last phase, only one of them was not done in the CABRI environment. This one dealt with the geometric interpretation of the derivative of a function. Another subject represented the Central Angle Theorem; while another tried to prove visually the Existence of Euler's Straight Line. It was seen that there was conviction that propositions and problems of geometry lend themselves, more and better, to visual proof, since algebraic treatment, which generally requires reasoning pertaining to other branches of mathematics, can be disregarded. Some of the answers in the questionnaires reinforce this assertion.

After the work of the teachers was presented, they were given the last questionnaire, and they were permitted to return them answered a few days later, as they finally did. The responses to this last group of questions revealed that, for three of the subjects, the use of technology in the classroom allowed the conjecture of properties, their visualization, and even their justification. Although lacking the rigor or formal demonstration, this means that it is a very useful tool from a didactic standpoint. According to their commentaries, the activity allowed them to examine new ways of proving certain mathematical properties. The other two teachers confirmed that is it possible to demonstrate formally.

According to the theory considered for this study, the idea of *argumentation*, proposed by Duval (1999) is present in the opinions of the subjects in terms of it being a type of reasoning that does not have the dose of rigor pertaining to demonstration, although it is a type of justification of important and valid interest in junior high school mathematics education. Also, it can be deduced from the responses of the subjects that they understand that the work with argumentation requires careful handling and that it will not lead rapidly, nor directly, to formal justification. However, we understand that the considerations of

Balacheff (1999) concerning the epistemological obstacle that this mode of reasoning supposes when confronted with demonstration go beyond the intuitions of teachers and require a more profound and detailed discussion.

Moreover, the subjects placed special emphasis on the need to plan which results and properties should, and could, be justified in a junior high course and on how the use of calculators obligates important changes in the mathematics curriculum. They think that this transformation must reflect the way they understand teaching, in terms of a change in activities in the mathematics' classroom and the assessment's process, where new facets of students' knowledge building can be developed.

In this study interesting opinions given by the subjects on demonstration with the use of technology were observed which lead to a reflection on the continuation of this type of study with the idea of correctly implementing the use of new technology in the practice of our teachers.

References

BALACHEFF, N. (1999) ¿Es la Argumentación un Obstáculo? Invitación a un debate. International Newsletter on the Teaching and Learning of Mathematical Proff, May-June 1999.

DUVAL, R. (1999) Argumentar, Demostrar, Explicar: ¿Continuidad o ruptura cognitiva?, México D.F.: Iberoamérica.

NCTM (1998) Principles and Standards in School Mathematics: Discussion Draft. Reston, Virginia: NCTM.