MEASURING POVERTY AS A FUZZY AND MULTIDIMENSIONAL CONCEPT

THEORY AND EVIDENCE FROM THE UNITED KINGDOM

by

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B.A., Seoul National University, 2002

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Submitted to the Graduate Faculty of

the Graduate School of Public and International Affairs

in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

University of Pittsburgh

2012

UNIVERSITY OF PITTSBURGH

GRADUATE SCHOOL OF PUBLIC AND INTERNATIONAL AFFAIRS

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Previous research shows that poor people define poverty not only in material terms, but also in psychological and social terms, though it has been consistently characterized by economic resources in social sciences. Using a method based on 'fuzzy-set' theory can be uniquely placed to answer the question as it allows us not only to tackle the problem of arbitrary poverty line, but also to integrate multiple dimensions into one index in an intuitive way. It can avoid the problem of poverty line entirely by introducing the concept of 'membership function' which represents a degree of inclusion in a fuzzy subgroup *poor*.

I therefore argue that the fuzzy measures of poverty can be a strong multidimensional alternative for the measures centered around income. To support the argument, two crucial points are clarified. Firstly, the difference between traditional measures and the fuzzy measures needs to be discussed further since the discussions on the new measures so far lean more toward the fresh insights from the measures, so that the distinction in policy-relevant information has not been emphasized enough. From the comparison, I present that the fuzzy measures can provide a richer description of the social phenomenon, enabling a more acceptable distinction between different subpopulations. Secondly, how the measures behave statistically should be considered in depth because one of the most frequent critiques for poverty measurements is that present methods depend too much on arbitrary decisions like setting a poverty line. Utilizing a Monte Carlo simulation, I find that the measures (*Totally Fuzzy*, *Totally Fuzzy and Relative*, and *Integrated Fuzzy and Relative*) acknowledge

two points quite well: (i) poverty is a multidimensional concept, and (ii) the 'poor' and 'non-poor' are not two mutually exclusive sets and the distinction can be 'fuzzy'. It also turns out that the sampling distribution of the fuzzy measures is well-behaved, and they are robust to arbitrary choice in the estimation as well as reliable with relatively small sample size. Besides, I show that they are robust to measurement errors. Finally, I investigate the identification performance of each measure and show that the measures have a strong consistency.

Keywords: poverty, fuzzy, multidimensional, measurement, capability, simulation, Monte Carlo method, Bootstrap.

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PREFACE

Just after I finished writing, I was really surprised at the fact that I could finish it at all. I have never realized before that the whole process of writing a dissertation is loaded with a variety of obstacles, which, honestly, has made me to think about giving it up in every step along the way. However, I also came to understand what the saying "every could has a silver lining" really means because everytime I made mistakes (never been singular), I could learn something I might not be able to learn without those. But more importantly, I find out that I can learn something valuable from mistakes ONLY when I have very good guides (never been singular either). All my dissertation committee members have done exactly that to me, and I am very sorry that I do not know how to express my appreciation enough. My advisors, Dr. Picard and Dr. Maertens have shown an incredible understanding of my poor knowledge on the subject and poorer English. Without their help, I don't think I can even imagine this day. Of course, I cannot forget the warm support from the other members. The critical but supportive comments from Dr. Finkel and Dr. Reynolds have encouraged me so much, and Dr. Stone's incisive discussion on my work has provided me the crucial chance to look back on my whole approach.

As a recipient of tremendous amounts of support, I cannot mention the indebtedness to my family enough. Especially to my uncle Eun-Kyu Kim and aunt Mi-Ae Kim, who literally make this dissertation a reality, I don't know how I can possibly appreciate their wholehearted help. Last but not the least, I am extremly happy to be able to dedicate this dissertation to my grandmother, Jung-Sook Kim, who have been the rock for me whenever I just want to give in to the hardships. Of course, I will never forget my father who must be very happy in Heaven, and......God.

1.0 INTRODUCTION

The question on the nature of poverty cannot be answered by simple sentence because it really opens up a variety of debates. These include the fundamental debates on the informational basis of the phenomenon[1](Grusky & Kanbur, 2006) as well as the differences in perspectives, such as, the contention between absolute and relative perspective (Seidl, 1988). In addition, the controversy between objective and subjective view is still ongoing (Hagenaars, 1991; Sen, 1979b). However, those diverse opinions on poverty themselves seem to imply one unequivocal statement: poverty is a complex, multidimensional phenomenon (Atkinson, 2003; Kolm, 1977).

1.1 STANDARD APPROACH TO MEASURING POVERTY

Traditional measures of poverty, such as, the headcount ratio[2] or poverty gap index[3] focus exclusively on a 'money metric', i.e., income. As income relates to welfare, captured by the utility function in standard economics, this is indeed an intuitive approach. Due to the quality of easy acceptability, most empirical studies and policy decisions regard an increase in income as an indication of an increase in welfare (Boadway & Bruce, 1984), especially in

[1] The title of Sen (1979a)'s Tanner lecture at Stanford University - "Equality of what?" is very suggestive in this regard.

[2] The headcount ratio is a fraction of people whose income goes below a specific standard - the so-called poverty line. It began with the studies of Booth and Rowntree (Seidl, 1988), and basically it is a measure of the incidence of poverty.

[3] Poverty gap index, firstly suggested by U.S. Social Security Administration, is the sum of the differences between poor people's income and the poverty line. Essentially this is the measure of the intensity of poverty. Let x_i indicate income of individuals, z a poverty line, and n population size, then $G = \sum_{i=1}^{n} (z - x_i)$.

United States (Blank, 2008).

Using income as a measure of welfare provides decision makers with an easy-to-understand description of welfare in a particular country. For example, headcount ratio, the most widely accepted poverty measure, can be interpreted as the number (or ratio, often) of poor people. Relatedly, the poverty gap can be easily understood as the total amount of money needed to make every poor person's income equal to a poverty line. In fact, this is one of the reason why these traditional measures are preferred to their unidimensional alternatives, such as Sen's poverty index[4] or the Watts index[5] which have more desirable properties (Blackorby & Donaldson, 1980; Haughton, 2009; Kakwani, 1984) or satisfy more 'axioms' (Kakwani, 1980; Sen, 1976). For instance, the headcount ratio gives no information on the 'intensity' of poverty, or the inequality present within the set of poor people. Also, in terms of axioms, though this measure can satisfy a 'focus'[6] axiom, it does not meet 'monotonicity' and 'transfer'[7] axioms, which Sen (1976) argues are crucial for poverty measurements (Brady, 2003; Clark & Hulme, 2005; Foster, Greer, & Thorbecke, 1984). On the contrary, Sen's poverty index incorporates the information on inequality within the poor and on their extent of poverty. Sen's index also satisfies the basic axioms. Despite these advantages, Sen's index is rarely used outside academia (Atkinson, 1999; Ravallion, 1996).

[4]Sen (1976) proposed a new measurement of poverty which was derived from a set of axioms. Let H be the headcount ratio, I the poverty gap ratio (poverty gap index divided by the number of poor), and G the Gini index for poor people, Sen index can be expressed as follows: $S = H[I + (1 - I)G]$

[5]Watts (1968) uses a ratio of the measure of permanent income to poverty threshold as approximation. His measurement is the following: $P = \sum_{i,L} N_i log(W_i)$, where N is family size, L is an indicator of a family having W less than an arbitrary threshold (usually one), and W is a family's "welfare ratio" as the ratio of its permanent income to the appropriate poverty threshold.

[6]Focus axiom requires that a poverty measurement should be only sensitive to the change in poor people's welfare indicator (Sen, 1976).

[7]Monotonicity axiom says that a poverty index should increase when a poor person experiences a drop in his welfare indicator, and transfer axiom is satisfied if a transfer from a poor person to anyone richer makes an index increases (Zheng, 1993).

1.2 PROBLEMS OF PREVIOUS APPROACH

Using income as the sole indicator in poverty measurement is a major concern for the measurement of poverty because the relationship between income and utility is of an ordinal nature and income is just one of many variables determining overall utility (Zheng, 1997; Sen, 1979c). In a way, one could argue that these income based measures are necessary, but not sufficient indicators of poverty (S. Anand, 1977). Sen (1985a) even argues that the income-based perspective is not enough to represent the complex nature of well-being because utility fundamentally refers to a psychological phenomenon which may or may not be influenced by other objective conditions, such as, quality of housing or fresh air. In addition, these measures ignore individual characteristics like gender or race which might interact with income in determining well-being[8] (Sen, 1979a, 1983, 1985a). For instance, identity-based imperfect markets might influence what one can achieve and do with a given income (Bourguignon & Chakravarty, 2003)[9]. Empirically, Callan, Nolan, and Whelan (1993) find, using Irish household survey data, that the correlation coefficient between income and their deprivation measure that includes 24 necessity items[10] is -0.51. This is consistent with what Townsend (1979a) finds using British data. Although Klassen (2000) reports a rather high correlation of 0.85 between expenditure poverty and deprivation measurement using South African household data, this number drops to 0.50 for the most deprived groups.

Second, these traditional measures also share the problem of setting a poverty line. This is questionable because it classifies population into two mutually exclusive groups, poor and non-poor, which implies that anyone who has income less than the line is poor. Easy as it is to understand, the simple division is problematic. For example, how can we say that a person whose income is one dollar more than poverty line is entirely non-poor, when a person with one dollar less income than the line is definitely poor?[11] In other words, is

[8] This disadvantage is called "valuation neglect" by Sen (1985a), since the utility approach does not consider how each person values the same levels of well-being differently according to his/her unique characteristics.

[9] If a person cannot get enough health care because there is no hospital nearby, then having high income does not necessarily indicate high well-being of that person.

[10] Using dummy indicators (one for possession, zero for no possession) for socially needed necessities like having refrigerator, heating for the living rooms, or the ability to save, Callan et al. (1993) construct an index by making the sum of 24 indicators.

[11] The issue of "poverty trap" that people just under the poverty line do not have a strong incentive to

poverty really a state or property which one can have or not? As Watts (1968) states:

> Poverty is not really a discrete condition. One does not immediately acquire or shed the afflictions we associate with the notion of poverty by crossing any particular income line (Watts, 1968).

As a response, scholars have looked for better ways to conceptualize poverty (Atkinson, 1987; Cheli & Lemmi, 1995; Halleröd, 2006; Betti, Cheli, & Cambini, 2004; Maasoumi & Lugo, 2006; Silber, 2007). However, even if the binary distinction has to be accepted, still there is no easy answer to how to set a poverty line[12]. For example, the most widely accepted definition of poverty line - "minimum necessities of merely physical efficiency" (Kakwani, 1984) - can be easily challenged by asking what 'minimum' means (S. Anand, 1977). Even Orshansky (1965) who suggested U.S. poverty line based on income-food expenditure relationship admits that "even for food, social conscience and custom dictate that there be not only sufficient quantity but sufficient variety to meet recommended nutritional goals and conform to customary eating patters", and Townsend (1979a) points that in U.S., "rough and arbitrary judgments are made at the really critical stages of fixing the level of the poverty line." Also, it has been suggested that relatively less attention is given to the methods used in drawing the poverty line (Foster, 1984; Ravallion, 1998).

Third, we know that measuring income might not be an easy exercise. First, one needs to decide which components to consider as part of income. For example, should only labor earnings be considered? or benefits derived from social programs included? If so, then how can we deal with the difference between cash and in-kind benefits? (S. Anand, 1977; Kangas & Ritakallio, 1998) In this regard, Seidl (1988) points out that income from the black economy or transfer from wealth can change the income measurement significantly. The problem here, however, is not that there can be many variations of income definition, but that we do not have any specific reason to choose one of them. Second, one needs to decide which time period to consider. As monthly income is generally more variable compared to yearly income, an income distribution in population using monthly data can show a entirely different picture compared to one derived from yearly data (Wagle, 2008a).

escape from poverty may be seen as a problem caused by this dichotomous distinction (Sen, 1995).

[12]See S. Anand, Segal, and Stiglitz (2010a) for more detailed discussions on the problem of setting poverty line in the context of 'global poverty measures.'

Third, an understatement of true income often occurs in household surveys (Deaton, 1997). First of all, it is very difficult for individuals to recall income information perfectly (S. Anand, Segal, & Stiglitz, 2010b), and secondly, individuals may have tendency to hide information about income. In a recent study, Hurst, Li, and Pugsley (2010) find that household surveys in which anonymity is one of the biggest principles can affect self-employed to under-report their income substantially as if they are tax reporting forms.

1.3 SUGGESTED ALTERNATIVES

Sen (1985a, 1985c) suggests 'functionings' and 'capabilities' as fundamental sources of information instead of income. Put differently, he argues for a change from how much income one has to how much one can achieve with this income. The idea of functionings brings a dramatic change of the perspective on well-being because it refers to the realized achievements of a person, in other words, "what a person manages to do or be" (Basu, 1987; Sen, 1985a), while income represents only the possibility of achieving some goals. However, since there can be infinite numbers of functionings that a person "has reason to favor or promote" (Williams, 1987), he moves on with the idea and finally suggests the idea of capabilities as the information basis of well-being, which embodies freedom into the notion of well-being (Brandolini & D'Alessio, 1998). Since the two suggested concepts indicate diverse activities and choices of human beings by definition, multidimensional concept of poverty is now a logical conclusion.

Applying this conceptualization, several multidimensional measures of poverty have been suggested[13]. The first set of measures is based on econometric analysis of the data. The main idea is to compute 'appropriate' weights to create an index as the weighted average of multiple dimensions. So, the important question is how to get the weights. Proponents of this approach answer this question as 'let the data talk.' In other words, they apply a statistical

[13]Though all three measures below are referring to the capability approach as their theoretical basis, it is hard to say that these measures are developed to test the approach empirically. More or less, the capability approach is considered as just one of the justifications for multidimensional approach. There certainly exist, however, several attempts to make the connection between the capability approach and the measurement methods more explicit. See Chiappero-Martinetti (2000), Kuklys (2005), and Bibi (2004).

technique that reveals the weight structure in the data, and use the weights to calculate an index. Lelli (2001) uses factor analysis on the Panel Study of Belgian Households (PSBH)[14]. He concludes that there are seven dimensions of poverty among 54 indicators in the data. Objective as it looks, this approach has the problem of interpretation, for it is not at all clear what the weights really imply (Robeyns, 2006)[15]. Also this approach to multidimensional poverty does not give a solution to the 'unrealistic' binary distinction between poor and non-poor (Miceli, 1998).

The second set of measures is derived from a set of desirable axioms. For instance, Bourguignon and Chakravarty (2003) show that one simple functional form for multidimensional poverty measurement that satisfies *focus*, *transfer*, and *monotonicity* axioms as well as *subgroup decomposability* axiom[16] is the multidimensional extension of the Foster-Greer-Thorbecke (FGT) measure[17]. However, though the axioms are hard to argue with, this approach is not that popular because the interpretation depends on the nature of the relationship between the dimensions, and on the correlation between them. An example will make this clear. Assume that we calculate a multidimensional index using two indicators, income and health. Now, if income is more beneficial (in terms of their welfare) for people

[14]For different attempts using factor analysis, see Lemmi, Pannuzi, Valentini, Cheli, and Betti (2004) or Kuklys (2005). Dewilde (2004) adopts the latent class analysis which is a categorical equivalent of factor analysis.

[15]Following is a part of rotated factor loading table from Lelli (2001). Though it can be concluded that being able to go to a sports match, café, and restaurant are significant components of factor 2, the analysis itself does not show what the factor 2 means and how the three components are related.

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7
Friends	-5	15	-18	5	-1	4	-12
Sport match	-2	45	7	0	1	-5	-5
Café	1	57	2	13	0	-5	4
Restaurant	0	34	24	37	1	-9	-6

[16]If a poverty index satisfies this axiom, it can decompose overall poverty into that of mutually exclusive subgroups according to certain characteristics like race or region. The marginal contribution of a subgroup's poverty is its population share (Zheng, 1997).

[17]FGT measure indicates a general family of poverty measurement by Foster et al. (1984) which is an average of poverty gap raised to the power of α ($FGT = \frac{1}{n}\sum_{i=1}^{l}(z - x_i)^{\alpha}$, where z denotes a poverty line, n population size, x_i individual income, l the number of poor people, and α is an indicator of the sensitivity to poverty). Bourguignon and Chakravarty (2003)'s measure utilizes the same concept to multiple number of dimensions, which can be expressed as $P_{\theta}(X; z) = \frac{1}{n}\sum_{j=1}^{m}\sum_{i \in S_j} a_j \left(1 - \frac{x_{ij}}{z_j}\right)^{\theta_j}$ in an additive function case, where n indicates population size, j each attribute, m the number of dimensions, a_j and θ_j the weight given to each indicator, z_j poverty line for each attribute, S_j a set of poor people in each dimension, and x_{ij} the attribute.

with better health[18], then increasing the correlation between them would make the index decrease. In policy terms, this implies that the best way to decrease poverty is to provide more health service to less poor people, or even non-poor people. On the contrary, if income can somehow compensate for the lack of health[19], then increasing the correlation would not make much difference[20].

Thirdly, the approach considers poverty to be a 'fuzzy'[21] concept (Cerioli & Zani, 1990; Cheli & Lemmi, 1995), meaning that it might be impossible to have a clear-cut distinction between poor and non-poor. This approach uses a membership function to capture each individual's degree of inclusion to the *poor* set. Although there are diverse opinions on the precise interpretation, this method yields an index that has a value between 0 (definitely non-poor) and 1 (definitely poor) for each individual, which can be interpreted as a 'propensity to poverty' (Betti, Cheli, Lemmi, & Verma, 2006). Therefore, this index does not require a poverty line. More importantly, this measure provides a relatively easy way to construct multidimensional measures of poverty. Since the membership functions for each indicator represents the degree of inclusion in the fuzzy subset *'poor'*, even a simple average of the membership functions for each indicator can be considered as expressing the overall level of membership in the *poor* set (Cerioli & Zani, 1990).

1.4 RESEARCH QUESTIONS

Although traditional approach to measuring poverty has provided valuable information for policy, many attempts to improve the approach have rarely been accepted mainly due to the "eye-catching" property of the traditional approach (Streeten, 1994). However, diverse debates on the weakness of it as well as the new approaches adopting both multidimension-

[18]Bourguignon and Chakravarty (2003) see this relationship as *complements*.

[19]If this is true, the two dimensions can be called substitutes (Bourguignon & Chakravarty, 2003).

[20]For detailed explanation, see Appendix B.

[21]The concept of 'fuzziness' originates from the inherent vagueness in any representational system, such as languages, though the external world seems to be continuous (Dubois & Prade, 2000). Based on the graduality principle which extends the two-valued classical logic to a more general case, fuzzy proposition is related to the degree of truth of statement (Fustier, 2006). Rather than saying one statement is either true or false, this proposition mentions a proposition can be "true", "untrue", or "more or less true". By Zadeh (1965)'s fuzzy logic, the degree of truth is assumed to belong to between zero and one.

ality as well as various methodological advances suggest that now is the time to consider new method of measuring poverty more seriously. Applying those new approaches, this study tries to investigate and test fuzzy measures of poverty, and unfold the strengths and weaknesses of the new method.

This study tries to focus on the fuzzy measures of poverty, specifically, their relative strengths and statistical properties. First, I will integrate the following seven dimensions of well-being: economic resources, health, employment, housing, material possession, social capital, and participation in social activities[22], into three fuzzy set based measures: *Totally Fuzzy (TF)*, *Totally Fuzzy and Relative (TFR)*, and *Integrated Fuzzy and Relative (IFR)*.

Then, I propose to address the following questions:

1) How 'poor' is the United Kingdom in terms of the three fuzzy set measures proposed?

This includes calculating the degree of poverty[23] of each individual in the sample, and aggregating this information to compute a poverty index for the country as a whole, and comparing the three measures with one-another as well as with the individual and aggregate results of the traditional poverty measures. In addition, I aim to analyze the results with respect to diverse demographic characteristics.

2) What are the statistical properties of the three fuzzy set measures proposed?

The poverty measures computed in (1) are a statistic, as they are calculated from a sample of the relevant population. The statistical properties of the three fuzzy set measures of poverty proposed are - as of yet - unknown[24]. This implies that we do not know how the measure computed will change if we change the sample (Anderson, 2008). For example, a significant difference between two different measures might be just a statistical

[22] These dimensions are identified from four multidimensional researches on poverty by Allardt (1993), Cummins (1996), Max-Neef (1993), and Narayan, Patel, Schafft, Rademacher, and Koch-Schulte (2000).

[23] As there is some disagreement on this interpretation of fuzzy set measures (Lemmi & Betti, 2006), diverse possibility of interpretation is discussed in chapter 2

[24] Since various assumptions in measurement can considerably influence the results, Filippone, Cheli, and D'Agostino (2001) and Brandolini (2007) mention that a thorough analysis of sensitivity is required for multidimensional measures of poverty.

artifact due to sampling variation. Since multidimensional measures of poverty include diverse indicators that have a complex relationship between them, one cannot understand the statistical properties intuitively. Therefore, I propose to investigate the statistical properties and 'robustness' of the three measures to the diverse variation in data using a Monte Carlo simulation technique[25].

1.5 STRUCTURE OF THE DISSERTATION

This study is structured as follows. In chapter 2, I give an overview of the literature on which this dissertation builds, and in chapter 3, I briefly describe the dataset I use and discuss the research methodology. I will calculate the degree of poverty in U.K. using the three fuzzy set measures and examine the new information which the fuzzy measures of poverty can give as well as the contrasts with traditional measures in chapter 4. Chapter 5 will investigate statistical properties of the fuzzy measures, and Chapter 6 concludes.

[25]Basically, Monte Carlo simulation is an umbrella term that represents any process of extracting several random samples from a simulated data set that retains certain properties defined by a researcher (Mooney, 1997). The name refers to the famous casino in Monaco.

2.0 CONCEPTS AND DEFINITIONS

2.1 CAPABILITY APPROACH

Sen (1979c, 1985a, 1985c) argues that utility cannot be used as the only information base of poverty measurement because it is established on the mental status of a person, thus ignoring the physical condition of a person. In addition, he asserts we need to take into account each person's own 'valuational exercise', meaning that a certain income for person i can have a different meaning compared to a certain income for person j. Sen suggests that the freedom to choose is a better measure of well-being (Saith, 2001; Kuklys & Robeyns, 2004). A theoretical approach based on this argument - the capability approach - will be discussed more in detail below.

2.1.1 Development of the Capability approach

Sen (1979a) investigates three different perspectives on equality - utilitarian equality, total utility equality and Rawlsian equality, he points that the former two concepts of equality relied on a same information basis - utility, which is, he admits, very defensible since the cardinality of utility has made progress of economics possible. The concept of utility to Sen, however, is very incomplete information source specifically for examining equality because interpersonal comparison that is the most indispensable part of studying equality is possible only when a special assumption is met, that is, when the utility function of everyone is same[26] (Klassen, 2000; Sen, 1979c, 1979a).

This understanding leads him to look for new basis for well-being, for which Rawls

[26] Sen (1979a) calls this condition "egalitarianism by serendipity."

(2005)'s emphasis on the 'primary goods' provides good ground[27] (Klassen, 2000; Saith, 2001). Sen (1985a, 1985b) suggests that the achievement that people succeed to obtain from these goods and/or commodities - called 'functioning' - is a better indicator of well-being than the simple command over goods (e.g., income) which does not necessarily imply a better 'standard of living' for individuals (Sen, 1985c). For instance, two people with same income can be entirely different in terms of well-being for many reasons: one may be in situation where freedom to use the income is prohibited, or one might not have a market to use it. In any case, the perspective based on functionings can tell the difference between the two, while a traditional perspective cannot.

The functionings, nonetheless, are not enough to be a basis for the measurement of well-being because the freedom of choice cannot be measured by them. For instance, if two people who has same income cannot use it because both don't have an access to a market. Then we can conclude that two people are same in terms of both utility (since both have same income) and functioning (both cannot use it). However, what if one of them does not have an access to market because the person chooses to stay away from it whereas the other's reason is just the affordability of transportation? Sen (1985a, 1992) might argue that the latter has lower well-being than the former because the intrinsic value of freedom to choose and achieve should be considered in the measurement of well-being[28]. Thus, he concludes that the freedom of choice[29] of functionings which is called 'capability' should be the final information basis for well-being (Ringen, 1995; Comim, 2001; Wagle, 2002). This does not mean, however, that the concept of capability can substitute utility (Clark, 2005). In fact, Sen himself criticized Rawls for dismissing the concept of utility as irrelevant (Saith, 2001; Sen, 1979a). On the contrary, what is emphasized in above discussion is that there is a series of steps from commodity to characteristics through capability and finally to utility, and the third category is the most appropriate concept for the measurement of well-being

[27] However, Sen (1979a) criticizes that the Rawlsian framework has some "fetishism" element in the sense that it understands primary goods as the embodiment of advantage itself rather than taking advantage to be a relationship between persons and goods.

[28] In similar regards, two people who are hungry can be very different if one of them chooses to go on a hunger strike while the other just does not have anything to eat. It can be said that the former has a higher standard of living because he has the freedom to choose hunger.

[29] This does not mean that choice is equal to freedom. In fact, Sen (1985a) criticizes the identification of binary relation underlying choice with a person's ordering of own well-being as a "heroic simplification."

(Sen, 1983).

2.1.2 Details of the capability approach

To conceptualize the freedom to choose, one should begin with 'functionings'. According to Sen (1985a), functionings denote what a person manages to do or to be - thus, 'a part of the state of the person[30]'. Algebraically, denote by x_i a vector of commodities (e.g., a car, a cell phone), $c(\cdot)$ the characteristics of the commodity vector (e.g., car can carry people or cargo, conversation with anyone at any time is possible by cell phone), and b_i the functionings (e.g., being able to transport oneself, having a large social network). Then the functionings b_i can be expressed as follows:

$$b_i = f_i\left(c(x_i)\right) \tag{2.1}$$

where $f_i(\cdot)$ is 'personal utilization function' (Sen, 1985a) creating a mapping between the consumption and functionings. He argues that the well-being of a person is best revealed by the evaluation of b_i, because how well a person is must be seen in terms of the kind of life he or she is living, in other words, 'what the person is succeeding in doing or being' (Sen, 1985a).

This functioning, however, is not enough to measure the standard of living of individuals since it cannot discriminate between different situations, where even same functionings can yield different well-being, like the example in previous section. This leads us to the concept of 'capabilities'. A capability can be defined as capturing the set of functionings that an individual could reach if he chooses to:

$$Q(x_i) = \{b_i | b_i = f_i\left(c(x_i)\right)\} \tag{2.2}$$

Thus, according to Sen's approach, what really needs to be measured is the freedom of choice of functionings people have because it relates to how well a person actually lives his/her life (J. Deutsch & Silber, 2005; Sen, 1985a, 1985c). Thus, one should include information about internal and external conditions of individuals. For instance, previous income-centered measures cannot explain why people of Costa Rica feel more satisfied with life compared to

[30]However, he emphasizes that they should be distinguished from both i) having commodities, and ii) happiness from the use of commodities.

U.S. citizen[31] (English, 2010) in spite of the huge gap in GDP per capita[32]. The capability approach, however, tells us that we might need to take more dimensions of life into consideration, i.e., functionings which are not associated with income could make the Costa Rican feel more satisfied with life[33].

From a theoretical perspective, capability is a more appropriate measure of well-being of a person than income (Alkire, 2002; Comim, 2001; Gasper, 2002; Chiappero-Martinetti, 2000; Ringen, 1995; Sen, 1992; Wagle, 2009). The fact that it includes a notion of opportunity, however, makes it hard to be measured empirically (Rawls, 1999; Robeyns, 2005b; Sen, 1985a, 1992). That is, if we want to measure capability, we should consider and evaluate every possible functioning that a person could achieve in a situation *ex ante*[34] (Klemisch-Ahlert, 1993; Thorbecke, 2007; Tsui, 2002). As this appears close to impossible, many favor using functionings as an empirical basis for well-being (and poverty as a lack thereof). Sen (1992) mentions that "refined functionings" which can be obtained by redefining functionings to account for "counterfactual opportunities[35]" can be used as the empirical basis for the capability approach. Robeyns (2005b) argues that in some cases, it makes more sense to investigate achieved functionings directly instead of capabilities, because the idea of a good life is often influenced profoundly by people's background, so 'it is not choice at all.' Basu (1987) asserts that the notion of opportunities in the capability approach is somewhat misleading because one person's capability set is not independent of other person's. So, he prefers functionings to capabilities as an empirical basis for the measurement of well-being. Gasper (2002) even claims that the normative emphasis on choice is 'more a policy rule

[31]The only possible explanation from that perspective is the so-called 'cheap preference' (Robeyns, 2000), which means that Costa Ricans get satisfied with life more easily than Americans do. In fact, this presents more difficult questions than answers, such as where these different preferences come from (Hausman & McPherson, 1996).

[32]According to the World Bank database, GDP per capita of Costa Rica in 2009 is $6,382 in current U.S. dollar terms, while that of U.S. is $46,436 (`http://data.worldbank.org/indicator/NY.GDP.PCAP.CD/countries/1W?display=default`).

[33]Therefore, it can be argued that the capability approach makes the traditional measurement more insufficient (Basu, 1987) because it becomes clearer that various dimensions of human 'being' and 'doing' cannot be measured even approximately by only one indicator of income or expenditure.

[34]Tsui (2002) clearly says that "the capability of a person is an opportunity set of bundles of functionings and not the functionings achieved."

[35]This means to consider only plausible numbers of options that individuals might have chosen. Sen (1985c, 1993) puts this as "choosing A when B is also available is a different 'refined' functionings... from choosing A when B is not."

to let people make their own mistakes than an evaluative rule that capability is inherently more valuable than functionings', and Thorbecke (2007) mentions "is *ex ante* capability that important for measuring poverty? ··· pragmatic approach would argue that it is the actual outcome that matters and that if *ex ante* capability cannot be ascertained." Following these arguments, I propose to use functionings as an empirical basis of my research.

2.1.3 Operationalizing the Capability approach

Deciding on functionings as empirical basis does not make a poverty measurement any easier as one need to operationalize the approach as to include a multidimensional approach. Since Townsend (1979b) first attempt to construct a non-monetary measure of poverty, the most serious criticism for multidimensional poverty measures is the arbitrariness in the choice of relevant dimensions (Ringen, 1995). Nevertheless, the debates between Nussbaum (2003) and Sen (2004a) show that the agreement on even abstract dimensions, not to mention indicators seems implausible, and some types of value judgment are an inescapable part of this choice process (Booysen, 2002; Esposito & Chiappero-Martinetti, 2008; Wagle, 2008b). Therefore, the approach for more agreeable set of dimensions through a wide literature review seems more realistic than an endeavor for a fixed and 'universal[36]' set (Sen, 2004a). Based on Alkire (2002)'s contribution of comparing fifteen approaches to human development, this study will look into four approaches.

(i) In *Voices of the Poor*, Narayan et al. (2000) introduce diverse dimensions of poverty that are important to poor people themselves, based on 78 Participatory Poverty Assessment (PPA) reports covering 47 poor countries around the world.. They identify four dimensions of poverty: i) *material well-being*, including food security and employment, ii) *psychological well-being*, including hopelessness and humiliation, iii) *state-provided infrastructures*, or services, such as, transportation or dependable water supply, iv) *assets of poor*, including physical, human, social capital, and environmental assets. Following table 2.1 includes the dimensions as well as indicators for them.

[36]Commenting on the study of deprivation begun by Townsend (1979b)'s approach to non-monetary poverty index, Veit-Wilson (1987) poses a question on how a selected list of indicators by a researcher can be justified.

Table 2.1: List of dimensions from Narayan et al. (2000).

Dimension	Sub-dimension	Indicators
Material Well-being	Food security	food / water / shelter
	Employment	dependable, formal wage labor
Psychological well-being		distress at being unable to feed children shame, stigma and humiliation lack of cultural identity / social solidarity
	Powerlessness	experience with officers of the state experience within market mechanisms (lack of choices & resources)
State services		water / roads and bridges electricity / school teachers
Assets	Physical capital	access to land / ability to self-provision / housing personal or household properties (car, jewelry, electronic equipment)
	Human capital	illness / disability literacy / education (access to education)
	Social capital	kinship networks professional networks
	Environmental assets	seasonal fluctuation in food and water rainy season / extreme weather conditions environmental fragility / resource degradation scarce affordable housing (urban case)

(ii) Describing the basic principles of the state of well-being in the Comparative Scandinavian Welfare Study, Allardt (1993) arranges basic human needs according to the three necessary conditions of human existence - having, loving, and being[37], as below table 2.2. *Having* refers to material conditions necessary for survival of an individual, and it includes the consideration of economic resources, housing conditions, employment, working conditions, health, and education. *Loving* is the need to interact with other people and to participate in social relationships, which covers attachments to family, kin, or communities, and patterns of friendship. Finally, *Being* indicates the need for integration into society, possible indicators of which are political activities, opportunities for leisure-time activities, or the opportunities for a meaningful work life.

(iii) Cummins (1996) integrates 173 different dimensions from the literature on life satisfaction into seven 'headings' used by the Comprehensive Quality of Life Scale. He finds that 68% of the dimensions can be integrated under seven headings: *material well-being*, *health*, *productivity*, *intimacy*, *safety*, *community*, and *emotional well-being.* Table 2.3 details the dimensions.

(iv) Max-Neef (1993) advocates "Human Scale Development" and focuses on basic human needs, self-reliance, and organic articulation with environment. He organizes human needs into two categories: existential and axiological[38]. For exploring diverse human needs related to poverty, the axiological classification seems useful, which consists of nine different dimensions in table 2.4: *subsistence*, *protection*, *affection*, *understanding*, *participation*, *idleness*[39], *creation*, *identity*, and *freedom*[40].

On the basis of above studies, table 2.5 can be constructed and the following seven dimensions are identified: *economic resources, health, employment, housing, material possession, social capital, and participation in social activities*:

[37]Measuring these conditions, the author strongly recommends using both objective and subjective indicators. While objective indicators refer to the observation of factual conditions, subjective indicators stand for "measurement of attitudes" (Allardt, 1993). For example, the ratio of students to teachers can be an objective indicator for an educational environment, whereas subjective indicators can be obtained by asking students' opinion about the educational environment.

[38]"Existential" categories indicate four aspects of human existence: being, having, doing, and interacting, each of which corresponds to personal or collective attribute, institutional context, actions, and locations and milieus (as times and spaces), respectively. On the other hand, "axiological" categories denote nine dimensions of human needs.

[39]Alkire (2002) replaces this term as "leisure", but I will use the original term since Max-Neef (1993) argues that this term has some productive meaning, and therefore is totally different from laziness.

[40]Specific meanings of these dimensions are not elaborated by the author, but indicators of the dimensions are fully provided.

Table 2.2: List of dimensions from Allardt (1993)

Dimension	Sub-dimension	Indicators
Having	economic resources	income wealth
	housing conditions	space available amenities
	employment	occurrence or absence of unemployment
	working conditions	noise temperature physical work routine measure of stress
	health	pain and illness availability of medical aid
	education	years of formal education
Loving		attachments and contacts in the local community attachments to family and kin active patterns of friendship attachments with fellow members in association relationships with work-mates
Being		the extent a person can participate in decisions political activities opportunities for leisure-time activities opportunities to enjoy nature, either through contemplation or activities

(a) Note that, strictly speaking, economic resources are not functioning per se (Brandolini & D'Alessio, 1998). However, since economic resources can be directly linked to diverse

Table 2.3: List of dimensions from Cummins (1996)

Dimension	Indicators
Material well-being	car / clothes economic situation food / home material possession living situation socio-economic status
Health	health intellectual performance
Productivity	achieve success / available activity employment / house-work / school
Intimacy	child interaction / friends contact with family / living partner
Safety	amount of privacy / control / legal safety financial security / how handle problems
Community	area live in / education / helping others acquaintance and contacts / services and facilities
Emotional well-being	beautiful well-being / overall comfort comfort from religion / life opportunities emotional adjustment / free-time activity

functionings (e.g., buying healthy food), this dimension is usually included (B. J. Whelan, 1993; C. T. Whelan, 1993; Kangas & Ritakallio, 1998; Lelli, 2001). Certainly the term does not indicate income or consumption exclusively. On the contrary, as the concept of functioning includes appropriate control over the resources, various forms of economic resources can be included as indicators.

Table 2.4: List of dimensions from Max-Neef (1993)

Dimension	Indicators
Subsistence	food, shelter, work
Protection	insurance, savings, social security, health system
Affection	friendship, family, partnership
Understanding	literature, teachers, method, education, communication
Participation	rights, responsibilities, duties
Idleness	games, spectacles, parties
Creation	abilities, skills, work
Identity	symbols, language, religion, habits, values, norms
Freedom	equal rights

(b) Health is one of the most basic functionings of human beings because without it proper 'function' of an individual in any society is impossible (S. Anand & Sen, 1997; Doyal & Gough, 1991; Duclos, Sahn, & Younger, 2006a; Federman & Garner, 1996). Therefore, this functioning is included in almost every research adopting the capability approach[41].

(c) Employment can be considered as an important functioning because it does not just imply having a job, but also having an opportunity to participate in social interactions ("the life of the community", according to S. Anand and Sen (1997)). Also, the importance of employment in obtaining proper economic resources cannot be ignored.

(d) Housing is regarded as an inevitable factor even in consumption-based traditional approach. From Orshansky (1965) to Citro and Michael (1995), the cost for housing constitutes an important part of minimum cost-of-living. In the capability approach, not only the cost but also the conditions of housing matter because housing indicates a crucial functioning of "security" or "protection" (Blank, 2008; Doyal & Gough, 1991).

[41]Robeyns (2000) reviews twelve researches adopting the capability approach, and all of them regard health as an important functioning.

Table 2.5: Diverse dimensions of functionings

Narayan et al. (2000)	Max-Neef (1993)	Allardt (1993)	Cummins (1996)
Material well-being	Subsistence	Having	Material well-being
- food security	Protection	- economic resources	Health
- employment	Affection	- housing conditions	Productivity
Psychological well-being	Understanding	- employment	Intimacy
State services	Participation	- working conditions	Safety
Assets	Idleness	- health	Community
- physical capital	Creation	- education	Emotional well-being
- human capital	Identity	Loving	
- social capital	Freedom	Being	
- environmental assets			

(e) Though it is certain that material possession itself is not a functioning[42], some part of it - for example, having a telephone or a refrigerator - can be included as a functioning. Bauman (2003) understands those specific possessions as "minimum standards of functioning in modern American society", and Boarini and d'Ercole (2006) also consider the possession of durable goods as "essential to perform every-day life activities."[43] According to Townsend (1979b), the lack of possession for certain goods can even be understood as a manifestation of poverty. Therefore, for certain types of goods, material possession can be understood as a functioning.

(f) Social capital is broadly understood to be the extent of participation in social networks (Narayan et al., 2000). This functioning emphasizes that human well-being can increase

[42] Tomer (2002) puts it in this way, "It is not about how much food one consumes; it is about eating tasty food and being well-nourished."

[43] These phrases indicate that there is still a room for inevitable arbitrariness in terms of choosing specific indicators, because the concept of "modern American society" or "every-day life activities" implies cultural or relative aspects of poverty.

through relationships that make individuals more capable (Tomer, 2002).

(g) The participation in social activities can be considered as one important functioning. Though this overlaps with social capital, here the functioning represents something more general[44]. In a sense, the underlying motivation for this functioning comes from the social exclusion perspective which emphasizes the importance of participation in major social opportunities of the society (Dagum, 2002).

Since all dimensions above are abstractly defined, more concrete indicators for the dimensions need to be selected. Here, it should be clearly noted that this choice process of indicators cannot completely rule out arbitrariness. However, this does not mean that the scientific rigor of the study is weakened[45]. On the contrary, this presence of arbitrariness needs to be understood as unavoidable due to the basic plurality and ambiguity that surrounds the concept of poverty[46] (Foster, 1984; Sen, 1981). Therefore, accepting Sen (1997)'s advice on the problem that "Openness to critical scrutiny, combined with public consent, is a central requirement of non-arbitrariness of valuation in a democratic society", this study chooses each indicator based on previous empirical researches without assuming that this is a 'universal' list[47]. Detail list of the variables used in this dissertation is provided in chapter 4.

[44]Within the general concept of social participation, political participation is especially emphasized by many theorists, including Sen (1999) and Nussbaum (2003) (see also Robeyns (2005a); Wagle (2008b); Clark and Hulme (2010); P. Anand, Krishnakumar, and Tran (2010)). Since many problems associated with poverty can be attributed to the lack of political voice of the poor (Sen, 1983), including this dimension seems appropriate. However, considering both the context of U.K., a developed country with long history of democracy, and the lack of proper indicators (only a variable of political party preference exists in the dataset), this study will not specify political dimension. Still it is important dimension especially when multidimensional measures are to be applied to developing countries' contexts, where often democratic political system is weak or nonexistent.

[45]Foster and Shorrocks (1988) point that arbitrary decisions also exist in traditional poverty measurements. They identify two main sources of arbitrariness: 1) the precise functional form adopted to aggregate influences the results eventually obtained, and 2) how to set a poverty line. See also Ringen (1988); Haughton (2009)

[46]For more detailed discussion on the arbitrariness in multidimensional poverty measurement, see Qizilbash (2004).

[47]Clark and Qizilbash (2008) find that their 'supervaluationist' approach to the choice of indicators that is based on the rule of unanimity cannot yield robust results empirically. See also discussion between Sen (2004a) and Nussbaum (2003)

2.2 FUZZY SET MEASURES OF POVERTY

Cerioli and Zani (1990) try to address the problem of sharp distinction between poor and non-poor (Watts, 1968) by applying the fuzzy set theory which expresses the characteristic of an object as its grade of membership to the characteristic[48]. Algebraically, let X be a set of people. Then a fuzzy subset A (e.g., the 'tall' people) of X is a set of pairs:

$$A = \{(x, \mu_A(x))\} \qquad \forall x \in X \tag{2.3}$$

where $\mu_A(x)$, a membership function, is a mapping from X to the closed interval [0,1] so that each value represents the grade of membership of x in A. So, it can be said that every person in X belongs to the subset A ('tall') with different degree with zero indicating no membership and one full membership[49]. Following this logic, we can measure people's degree of membership in the 'poor' set with a number between zero and one. As the membership function is defined for every element in X, the numbers can be regarded as an individual index of a 'propensity to poverty' (Verma & Betti, 2002), or the average of membership function in a population can be considered as the aggregate index for the society. As such, note that a fuzzy set measure of poverty consists of two elements: (i) a membership function for 'poor' fuzzy subset and (ii) a weight function for aggregating each dimension. In addition, multidimensionality can be easily integrated in this measurement. One can capture the degree of inclusion to 'poor' group for each dimension, and then compute the final index as a weighted average of the degree of membership. I propose to focus on three specific measures within this set of measures.

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[48] Although there have been several proposals that try to embody this "gradation" concept in pre-existing measurements (Atkinson, 1987), still no one alternative stands out.

[49] According to the fuzzy set theory, most of the concepts we use in social science (Keefe & Smith, 1996) or in language (Dubois & Prade, 2000; Zadeh, 1965) actually do not have clear and sharp borderline of application. For example, we can always say one person is either tall or small, but it is very hard to show the exact height of the person to be qualified as either tall or small - so-called "Sorites paradox" (Klir & Yuan, 1995).

2.2.1 The interpretation of fuzzy set measures of poverty

Before further introduction, it has to be examined how we can interpret the measures. This is not a small problem because a measurement based on fuzzy-set theory is "intuitively less conspicuous and thus more difficult to defend" (Wagle, 2009). Also, Qizilbash (2003) points that the meaning of fuzzy set is not so clear especially "when it is applied for measuring vertical vagueness."

Smithson (2006) classifies four kinds of interpretation for membership function. Firstly, formalist interpretation considers membership functions solely in mathematical terms by mapping an underlying support variable into the membership scale. So membership function represents the characteristic of the underlying variable, which often is very difficult to understand concretely. Secondly, probabilist interpretation simply asserts that a degree of membership of object x in set A is the probability that x belongs to A. Despite the intuitiveness, many scholars reject this interpretation because it implies that fuzziness comes from imperfect knowledge or information, which is not always the case in fuzzy logic. Thirdly, the proponents of decision-theoretic viewpoint argue that the degree of membership corresponds to the utility (payoff) of asserting that x is in A, which is related to the degree of truth in asserting that x belongs to A. However, this view begs the question of where a utility scale comes from. Lastly, axiomatic measurement theory approach considers that a numerical membership assignment shows the structure of qualitative axiomatic conditions that should be investigated empirically.

According to previously suggested interpretations, the general interpretations of fuzzy measures of poverty have followed the first one. For example, the fuzzy measures are understood as "the propensity to poverty" (Verma & Betti, 2002), "degree of deprivation" (Brandolini & D'Alessio, 1998), "level of poverty" (Dagum & Costa, 2004) or "vulnerability[50]" (Qizilbash, 2002). On the contrary, the second interpretation is also supported by the initial proponents of the measurement methods. Cerioli and Zani (1990) mention that

[50] Qizilbash (2002) explains that this term is not used as a terminology of economics, which indicates the probability of being poor. Rather, he argues that it is a concept of distance from being definitely poor, or "being classified as poor" (Qizilbash, 2003). His intuition is that the closer the relevant person is to being definitely poor, the larger the number of possible ways of further specifying a borderline between the poor and non-poor in the vague zone which would result in that person classifying as 'poor'.

aggregate level TF measure can be interpreted as "the probability of the fuzzy event "being in poverty" in the reference population", and Cheli (1995) also argues that TFR measure can be understood as "a fuzzy generalization of the headcount ratio of the poor."[51] Besides, Filippone et al. (2001) propose that we should understand the fuzzy measures as the relative social position of an average household in the population, which is close to the fourth interpretation.

Considering the intuitive appeal and the fundamental principles of fuzzy set theory, the first interpretation seems to be appropriate. The probabilistic interpretation still retains the problem of seeing fuzziness as imperfect information. That is, it can be understood as the probability of being 'right' when we say someone is poor, which presumes that we know what is being poor for sure, and the fuzziness does not originate from the concept of poverty itself but our lack of information. However, it should be noted that the former cannot be understood as denoting the intensity or depth of poverty. In fact, the interpretations by Brandolini and D'Alessio (1998) or Dagum and Costa (2004) suggest exactly that way. But Qizilbash and Clark (2005) oppose this interpretation because it also presupposes that we already know a non-fuzzy concept of poverty, which is we try to avoid by this method. Therefore, this study understands the fuzzy measures as indicating the propensity to being definitely poor.

2.2.2 ***Totally Fuzzy (TF) method***

The first measurement based on the fuzzy set theory was Totally Fuzzy (TF) method suggested by Cerioli and Zani (1990). In determining membership function of individual i on indicator j - $\mu_j(i)$, they suggest to define two thresholds values j_{min} and j_{max} such that if j for an individuals is smaller than j_{min} the person would be defined as definitely poor while if j is higher than j_{max} then the person is definitely not poor. This logic can be applied to both continuous and ordinal variable cases. However, in the latter case, the maximum and minimum values can be determined more easily by assuming the value of the lowest category as minimum, the highest as maximum. For example, if 'health status' variable would take

[51] Miceli (1998) also accepts this interpretation. He argues that the fuzzy measures represent the proportion of individuals belonging in a fuzzy sense to the poor subset.

five values from "very good" (five) to "very bad" (one), then one can be the minimum value, and five the maximum. Here, the order by the membership function is reversed because poor health indicates people to be closer to 'poor' group, which means bigger membership function value. Finally for dichotomous variables, it can be easily inferred that the subset 'poor' would not be a fuzzy set because the membership function will have only one of two values - 0 or 1. For instance, if ownership of house is one dimension, then membership function will have value zero if a person owns house, one otherwise[52]. Still, it is entirely possible to include these variables in constructing poverty measurement because traditional set can be considered as just a special case of fuzzy set (Dubois & Prade, 2000; Fustier, 2006; Klir & Yuan, 1995). The membership function for each individual can be calculated by following equation 2.4[53]:

$$\begin{cases} \mu_j(i) = 1 & \text{if } 0 < j_i < j_{min} \\ \mu_j(i) = \frac{x_{j,max} - x_{ij}}{x_{j,max} - x_{j,min}} & \text{if } j_i \in [j_{min}, j_{max}] \\ \mu_j(i) = 0 & \text{if } j_i > j_{max} \end{cases} \tag{2.4}$$

To aggregate various dimensions, the simplest idea would be an average of the dimensions. However, it seems more reasonable to assume that some dimensions are more important than others in determining individual well-being. Thus, they propose a weight function that sees poverty essentially as a matter of relative deprivation, so that people can be considered to be poor if they fail to meet the standard of living that is customary in their own society. In particular, they propose the frequency of 'definitely poor' phenomenon for each dimension as a weight. Thus, the weight for dimension j among K dimensions can be computed by the following equation 2.8:

$$w_j = \ln(\frac{1}{f_j}) / \sum_{j=1}^{K} \ln(\frac{1}{f_j}) \tag{2.5}$$

where f_j denotes the frequency of 'definitely poor' phenomenon for dimension j. Basically, this weight function is built to assign lower weight to the dimension in which many people

[52]Simple algebraic expression for 'color TV' is as follows:

$$\begin{cases} \mu_j(i) = 1 & \text{if a person does not own a color TV} \\ \mu_j(i) = 0 & \text{if a person owns a color TV} \end{cases}$$

[53]This notation is a little bit confusing because while important variable such as income or expenditure usually moves in the opposite direction to the membership function, membership function value of 1 indicates 'poor' state.

turn out to be ‘definitely poor’, because high frequency of ‘definitely poor’ in a dimension indicates that people highly belong to the fuzzy subset ‘poor’[54] in the dimension, which can be translated into lower relative deprivation in the dimension (Cerioli & Zani, 1990). Besides of this relative deprivation interpretation, this weight can be understood as an attempt to minimize influence from irrelevant dimensions. For example, consider an indicator, ‘having twenty-carat size diamond ring’. Since most of the people in a population do not have it, the frequency for the indicator would be very close to 1. However, as the indicator is very unlikely to reflect the social phenomenon of poverty, it is reasonable to assign very small weight on the indicator, and the equation 2.5 does that.

2.2.3 *Totally Fuzzy and Relative (TFR) method*

Cheli and Lemmi (1995) argue that the TF method has two weaknesses. First, the choice of two threshold values is arbitrary. Second, the choice of a linear functional form for the membership function lacks both a theoretical basis and empirical evidence. They argue instead to use a cumulative distribution function as the basis of membership function. This method can be called ‘totally relative’ because the membership function value is entirely determined by the relative position of individual in population distribution.

$$\mu_j(i) = 1 - F(j_i) \quad \text{or} \quad \mu_j(i) = F(j_i) \tag{2.6}$$

where $F(j)$ indicates cumulative distribution[55] of indicator j.

For ordinal variables, fundamentally the same idea is applied. However, this specification can be sometimes problematic if one of the frequencies of extreme modalities is very high. For example, following table 2.6 shows that by equation 2.6 the membership function for the ordinal variable x with five scales cannot be zero, though the membership function should be between zero and one. In order to overcome this problem, they suggest following membership function formula 2.7 for ordinal variables which has k categories in them ($j^{(k)}$ indicates k-th category of indicator j):

[54] Cerioli and Zani (1990) make the metaphor of “experts” for the weight function because the opinion of the people who appear to be close to fuzzy subgroup ‘poor’ - “experts” in that sense - is weighted heavily in the function.

[55] The choice of appropriate formula depends on the specific property of the dimension. For income which decreases poverty conceptually, the former part is appropriate.

Table 2.6: An example of evaluation of the membership function

Categories	relative frequencies	M.F according to 2.6	M.F from 2.7
$x^{(1)}$	0.6	0.6	0
$x^{(2)}$	0.15	0.75	0.375
$x^{(3)}$	0.1	0.85	0.625
$x^{(4)}$	0.1	0.95	0.875
$x^{(5)}$	0.05	1.00	1.00

$$\mu_j(i) = \mu_{j^{(k)}}(i) = \begin{cases} 0 & \text{if } k = 1 \\ \mu_{j^{(k-1)}}(i) + \frac{F(j_i^{(k)}) - F(j_i^{(k-1)})}{1 - F(j_i^{(1)})} & \text{otherwise} \end{cases} \tag{2.7}$$

For binary variables, Cheli and Lemmi (1995) adopted the same functions as in the TF method (see footnote 52). However, for weight function, they propose to use the mean of membership function value for each dimension as a weight, instead of the frequency of 'definitely poor' phenomenon. Thus, the weight for dimension j among K dimensions can be computed by the following equation 2.8:

$$w_j = \ln(\frac{1}{\mu_j}) / \sum_{j=1}^{K} ln(\frac{1}{\mu_j}) \tag{2.8}$$

where u_j denotes the average membership function for dimension j. The interpretation of the weight is fundamentally same to the weight of TF method. However, since the average membership function can include information on the whole distribution of categories in each indicator, not just the lowest category, it can be considered as more generalized relative weight.

2.2.4 *Integrated Fuzzy and Relative (IFR) method*

The Integrated Fuzzy and Relative (IFR) measurement is presented by Betti, Cheli, Lemmi, and Verma (2005b). They argue that the TFR method has two pitfalls: 1) since the measure uses cumulative distribution function, it is not sensitive to the actual disparities in any continuous variables, and 2) relatedly, the mean of the membership function for any continuous variable is always 0.5, regardless of its distribution (Cheli, 1995; Betti & Verma, 1998; Betti, Cheli, Lemmi, & Verma, 2005a). For example, assuming that there is a population that includes only four people and their ID number is equal to the order of their income, then a cumulative distribution and the membership function of the population according to TFR method is as following table 2.7, which clearly indicates that the mean of the membership function is 0.5[56], and only their *order* is counted in the measurement, not their actual difference in income.

Table 2.7: Example of membership function in TFR method

ID	1	2	3	4
Cumulative distribution	0	1/3	2/3	1
Membership function	1	2/3	1/3	0

In order to address the problem, they introduce Lorenz function to combine the information of the share of individuals less poor than a person concerned with the information of the share of the total equivalent income received by all individuals less poor than the person. In other words, this measure weights the distance between the line of perfect equality and the Lorenz curve by a function of the individual's position in the distribution, giving more weight to its lower end[57]. Expressed algebraically, the membership function is as follows:

$$\mu_j(i) = [1 - F(j_i)][1 - L(j_i)] \tag{2.9}$$

[56]Also, it is evident that the problem is caused by the fact that cumulative distribution only takes an order of the people with respect to income into account. The amount of each person's income that is crucial information as well in poverty measurement is not regarded at all.

[57]For graphical presentation, see Figure A11 in Appendix A

where $F(j)$ is cumulative distribution function and $L(j)$ is the Lorenz function.

The authors claim that this measure is a more sensitive with regard to the actual disparities in a dimension (e.g. income) compared to the simple cumulative distribution function which is just the proportion of individuals less poor than the person concerned. Furthermore, this measurement has one important advantage over previous methods: it has a close relationship with the Gini coefficient since the mean of the membership function is $(2+G)/6$. So, it can be concluded that IFR measure as an aggregate index is sensitive to the distribution of each indicator, which means it can satisfy transfer axiom[58].

In addition, they argue that separate measures should be estimated for monetary and non-monetary dimensions because monetary dimensions still have a 'fundamental role' in poverty research (Betti, D'Agostino, & Neri, 2002; Betti & Verma, 2008). Still, integrating into one index is more attractive strategy for policy makers, they introduce the concept of "manifest" and "latent" poverty. The former indicates a subgroup of population who is poor for both of the dimensions, the latter being who is poor for either one of the dimensions[59] (Betti & Verma, 1998, 2008). For non-monetary dimensions that mainly consist of ordinal and dichotomous variables, they first calculate a deprivation indicator for each indicator, d_{ji} (based on the TF method) where j indicates each dimension and i denotes each individual[60], and then integrate each indicator into one index using a weight function that is discussed below.

With regard to the weight function, they suggest: (i) the weight is determined by how well a dimension can differentiate individuals in the population, for example, if only 10% of a population appear to be poor in a dimension, it should be weighted heavier than another dimension in which 90% is poor[61], and 2) it takes the correlation between the dimensions into consideration to limit the influence of indicators which are highly correlated - in other

[58]For any resource transfers within a population that makes the Gini coefficient closer to one, such as, transfer from relatively poor to rich, aggregate IFR index will also become closer to one, since it is equivalent to $(2+G)/6$. For transfer axiom, see footnote 7.

[59]Precise meaning of the two concepts depends on the fuzzy set operation method applied (Betti & Verma, 2008). The interpretation here follows 'standard' operation, in which the intersection of two fuzzy set a and b can be written as $i(a,b) = min(a,b)$ and union $u(a,b) = max(a,b)$ (Klir & Yuan, 1995).

[60]If there are continuous indicators in non-monetary dimension, equation 2.9 is used.

[61]This shows 'relative' characteristic of the measure clearly. If everyone is poor in a dimension, its contribution to individual's poverty - its weight - should be zero in a relative point of view.

words, to avoid redundancy of information (Betti & Verma, 2008). In particular, the weight can be defined as follows:

$$w_j = w_j^a * w_j^b \tag{2.10}$$

Specifically, Betti and Verma (1998) suggest that the coefficient of variation of each dimension's membership function value can be used as the first term, and as the second term the following can be used:

$$w_j^b = \left(\frac{1}{1 + \sum_{j'=1}^{K} \rho_{j,j'} | \rho_{j,j'} < \rho_H} \right) \left(\frac{1}{\sum_{j'=1}^{K} \rho_{j,j'} | \rho_{j,j'} \geq \rho_H} \right) \tag{2.11}$$

where $\rho_{j,j'}$ is the correlation coefficient between two different indicators, ρ_H is a pre-determined value, and K is the total number of dimensions. The underlying motivations of 2.11 are: (i) the weight is not affected by the inclusion of irrelevant dimensions, (ii) the weight is only marginally changed by small correlations, and (iii) the weight is reduced proportionately to the number of redundant variables[62].

2.2.5 Fuzzy set measurements and the capability approach

The capability approach has attracted many researchers mainly due to its comprehensive consideration of individual diversity and the freedom of choice (Gasper, 1997; Kuklys, 2005; Szeles, 2004). However, this strength is often regarded as the weakness simultaneously because the informational basis that is necessary to implement Sen's notions is very hard to operationalize, not to mention its philosophical difficulty (J. Deutsch & Silber, 2005; Lelli, 2001; Rawls, 1999). Kuklys (2005) summarizes the difficulty of the capability approach in operationalization as fourfold: 1) the selection of relevant functionings, 2) measurement of functionings at the individual level, 3) aggregation across individuals, and 4) choice between capabilities and functionings as fundamental information source.

[62]High correlation between indicators implies that they basically measure the same underlying phenomenon. In other words, some variables can be 'redundant' in terms of information. The authors also reported that in most of the application the second factor in this weight goes to one

In the context of measurement study, the primary concern is the first difficulty because it is natural to conclude that a concept is immeasurable when we have too many ideas of it that we don't have a necessary ground on which we can make a judgment. Sugden (1993) notes that

> "Given the rich array of functionings that Sen takes to be relevant, given the extent of disagreement among reasonable people about the nature of the good life, and given the unresolved problem of how to value sets, it is natural to ask how far Sen's framework is operational"

However, "there is more than one way in which an idea of this kind can be operationally effective" (Atkinson, 1999). At least, it enables us to have a different perspective on poverty, away from the exclusive focus on the monetary dimension, and at the same time, helps us to recognize the ethical aspect of the poverty study by introducing concepts like freedom and right into it (Gasper, 1997; Sen, 2004b). The problem is, in a sense, not that the capability approach does not have a theoretical soundness, but that it does not provide a basis of judgment that is universally acceptable.

As Sen (2004b) argues, however, there is no reason to believe that the cogency of a theory should be judged only by the complete feasibility[63]. Still, in spite of Sen (2004a)'s refusal to provide basic guideline for the approach (Nussbaum, 2003), many researchers have showed various reasonably acceptable lists of functionings and capabilities (P. Anand, Santos, & Smith, 2009; P. Anand, Hunter, et al., 2009; Nussbaum, 2000; Nussbaum & Glover, 1995), and more importantly, the fuzzy set theory has been suggested as one good theoretical basis to address the indecisiveness of the capability approach.

Contrary to the common skepticisms for the capability approach (Roemer, 1996; Srinivasan, 1994; Sugden, 1993), the lack of specificity of the approach comes from the intrinsic complexity and vagueness of the concept of poverty (Chiappero-Martinetti, 2008). As Keefe and Smith (1996) find out from Cicero, "nature has permitted us no knowledge of limits such as world enables us to determine, in any case, how far to go." That is, we can say someone is poor by any meaning, but we cannot say what exactly makes the person poor because it is impossible to draw a clear border between the poor and the others, and according to the

[63]It has been pointed that utility is not measurable (Dutta, 1994; Varian, 1999), but still utility-based approaches have been operationalized "in more or less accepted ways" (Gasper, 1997).

fuzzy set theory, this is not a problem that can be solved by more information or knowledge (Klir & Yuan, 1995; Smithson, 2006; Zadeh, 1965). Thus, the best way to address this inescapable vagueness of the concept in measurement is to model the vagueness itself, not to attempt to eliminate it (Fusco, 2003). Sen (2005) points that "if an underlying idea has an essential ambiguity, a precise formulation of that idea must try to capture the ambiguity rather than attempt lose it."

The fuzzy measures of poverty I investigate in this dissertation provide one way to model the vagueness of poverty. By taking into account the intrinsic vagueness of poverty[64], it can provide two important information about poverty: depth and the breadth of the dimensions (Chiappero-Martinetti, 2008).

[64] Qizilbash (2002, 2003) makes a distinction between horizontal and vertical vagueness. The former indicates a case where it is not clear whether a predicate ('poor') applies, while the latter relates to the difficulty of finding exact point of distinction between two states.

3.0 METHODOLOGY

As this study has two distinct research questions, the methodology is twofold. First, the basic logic of comparison and contrast is adopted. Since the first research question is about investigating the representation by the two different approaches - traditional measures and the fuzzy set measures, their differences and similarities can be clarified by the logic. For the second question, Monte Carlo method is used because the method is very effective in identifying the statistical properties of a statistic which has no clear analytic solution (Manno, 1999; Metropolis & Ulam, 1949; Mooney, 1997; Robert & Casella, 2004, 2010).

3.1 METHOD

3.1.1 Comparative analysis

In order to compare two different approaches to measuring poverty, the first point that should be decided is the object of comparison[65]. Since the measurement of poverty should demonstrate two levels of reality - aggregate and individual - at the same time[66] (Sen, 1992, 1997; Ravallion, 1992), it is a good strategy to follow the two aspect for the comparison.

The aggregate comparison looks relatively simple because it is just to look at two numbers from different approaches. However, since the two indices denote their own concept of 'poverty', a simple comparison of numbers cannot yield useful information. For example, if we get the headcount ratio 0.15 and fuzzy set measure 0.18 over a population, what does

[65]Paraphrasing the title of Sen (1979a)'s famous Tanner lecture, "Comparison in terms of what?"

[66]However, Callan and Nolan (1991) and Hagenaars (1991) argue that the poverty measurement studies since Sen (1976) have only focused on the 'aggregation' part.

it mean that fuzzy set measure is bigger? Simple answer for this question is not possible because they are referring to different aspects of reality. The former indicates that 15% of the population has income less than a poverty line, whereas the latter shows that the propensity to poverty of the population is 0.18, the meaning of which requires more discussions for clarification (see section 2.2.1.). On the other hand, if we use the poverty gap index, we might be able to compare the 'intensity' of poverty in its common-sense meaning. However, this interpretation is said to be problematic for both measures (Haughton, 2009; Kakwani, 1984; Qizilbash & Clark, 2005). Therefore, reasonable comparison is only possible by examining possible discrepancy between the two perspectives in terms of theoretical expectation in detail.

Specifically, the first comparison looks into the two distinct groups in population that are identified by headcount ratio, that is, *income-poor* and *income-nonpoor*, and see how the fuzzy set measures describe the two groups. In traditional welfare economic theory, the people in each group are qualitatively different in the sense that the poverty line is assumed to be a line between two distinct "properties of a situation"[67] (Watts, 1968). Thus, to accept the theoretical assumption, the fuzzy set measures of poverty should also provide a consistent conclusion for it: fuzzy set measures for *definitely nonpoor* population should be clearly different from those for *definitely poor* population. Comparing the fuzzy set measures for the two group can provide an empirical basis for affirming or refuting the theoretical expectation. In addition, same comparisons for subgroups according to several demographic variables (gender, region, employment, and so on) would help to investigate whether the theory concurs with empirical evidence. Conversely, the comparison based on the fuzzy set measures is also done as follows: 1) sort the population into the decreasing order of fuzzy set measures, 2) choose a group of people the number of which is same to the poor people by headcount ratio[68], and 3) see whether the group chosen in the second step correspond with the poor group by headcount ratio. If it turns out that the two groups are similar, then it can be concluded that the two measures are measuring same social phenomenon. Otherwise,

[67] Foster (1984) says that it is "no small assumption" that the studies of poverty measurement since Sen (1976) assumes a common poverty line for all individuals to be given.

[68] This is not an arbitrary decision since fuzzy set measures can be interpreted as "the probability of the fuzzy event "being in poverty" in the reference population" (Cerioli & Zani, 1990; Smithson, 2006).

the strength and weakness of the fuzzy set measures need to be discussed further.

Secondly, in order to investigate why there is a difference between the two approaches, it is required to look into the differences in individual level, because ultimately it is individual conditions that can either validate or reject a poverty measurement. In fact, the biggest motivation for multidimensional poverty measurement is the simple idea that the variables that only measure utility in indirect ways cannot be the only determinants for an individual's condition in poverty (Ayala, Jurado, & Pérez-Mayo, 2010; Muffels, Berghman, & Dirven, 1992; Nolan & Whelan, 2010; Stiglitz, Sen, & Fitoussi, 2009; B. J. Whelan & Whelan, 1995). Therefore, I directly compares individual indicators of households who are in contradictory position by the two approaches, i.e., households who appear to be *income-poor* but closer to *definitely non-poor* in fuzzy sense, or *income-nonpoor* but closer to *definitely poor*. Although this comparison is unlikely to provide a clear answer for the question who is poor due to the *fuzzy* nature of the concept of poverty, it certainly shows what kind of discussion we need to have a consensual definition of poverty.

3.1.2 Monte Carlo simulation

The second part of this study focuses on the statistical behaviors of the fuzzy measures of poverty. For the purpose, I adopts a *Monte Carlo* approach, which is grounded on randomly generated variables and iterative calculation (Metropolis & Ulam, 1949; Shreĭder, 1964; Robert & Casella, 2004, 2010). This approach is especially useful when there is no strong theory regarding a statistic's behavior, because it provides researchers the ability to track the behavior of a given statistic by extracting several random samples that have certain properties (Davison & Hinkley, 1997; Gentle, 2003; Mooney, 1997; Robert & Casella, 2010). In the perspective of the social sciences, Gilbert and Troitzsch (2005) also mention that simulation approach is appropriate when the complexity of social phenomenon does not allow an "elegant" analytic solution.

According to Mooney (1997), the basic procedure for a Monte Carlo simulation is as follows:

1. Specify the data generation process[69] in symbolic terms
2. Sample by the process
3. Calculate the statistic and store it in a vector
4. Repeat step 2 and 3 t times, where t is the number of trials
5. Construct a relative frequency distribution of the resulting values

Thus, as the first and foremost step, an appropriate data generation process needs to be decided. Since the generated data should include multiple dimensions which in turn contain various indicators in different levels of measurement, it would be very difficult to consider each indicator's exact generation process. Also, strict theoretical position requires us to have clear assumptions on human choice of multiple dimensions of life, which sounds almost impossible. However, we do not have to have all the information on the data generation processes in order to examine the properties of the measurement methods, because the information is related to only whether the resulting index makes sense, not how it behaves. Thus, I conduct two kinds of analysis according to different assumptions about the data generation process. Firstly, I assume that the data is *identically and independently drawn (i.i.d.)* from a multivariate normal distribution with mean and variance-covariance matrices computed from the 16th wave of the BHPS.[70] The resulting simulated dataset would contain 6,339 cases across 39 indicators. Using symbolic notation, this data generation process can be expressed as equation 3.1.

$$X = \{X_1, X_2, \ldots, X_{39}\}, \qquad X \sim MN(\mu, \Sigma) \tag{3.1}$$

, where μ is a vector of means (39×1), and Σ is a variance-covariance matrix (39×39).

This approach, however, has one serious problem that the generated variables can have either negative value or value over one, both of which are beyond the domain of the membership function in fuzzy set theory. As a solution, two methods are conceivable: 1) using

[69] Mooney (1997) actually uses the word 'pseudo-population'.

[70] This assumption corresponds to the 'parametric bootstrapping method'(Chernick, 2007; Efron, 1982; Efron & Tibshirani, 1993). Technically, the bootstrap and Monte Carlo methods are separate techniques with different traditions. But a clear distinction between the two concepts does not provide much information. In fact, Hall (1992) argues that Efron's biggest contribution is to recognize the benefits of combining the two techniques.

a truncated multivariate normal number generator[71] (Wilhelm & Manjunath, 2010), and 2) utilizing a multivariate normal number generator and then 'truncate'[72] the generated variables from zero to one. Although the latter looks quite arbitrary, this is fundamentally same to one proper way of generating variates from a truncated multivariate normal distribution - 'rejection', which accepts a sample when it exists only inside the support region[73] (Gentle, 2003; Lemieux, 2009; Robert & Casella, 2004). In addition, since the truncation taken into consideration by the former significantly reduces the variance and changes the covariances (Wilhelm & Manjunath, 2010), there is no reason to prefer one to the other. So, I apply the Monte Carlo method for the choice. After 3,000 datasets by each method are obtained, I calculate the first and second moments for each indicator. As it can be seen in Appendix C, it turns out that the latter method can provide more similar datasets to the original data in terms of the moments, which is used analyses below. Still, one more challenge needs to be dealt with: the different levels of measurement. While some of the indicators are measured as continuous variables, others are as binary variables, which implies that using a pseudo-random number generator from multivariate normal distribution generally does not provide a properly simulated dataset. In addition, since the calculation process of the three fuzzy measures is essentially a weighted average, replacing originally binary variables with continuous variables from random number generator would change the result significantly. So, instead of generating datasets similar to the original dataset, I generate the membership functions of each indicator, which by definition always ranges from zero to one. Further, to make the simulated dataset as similar as possible to the original data, I recode some of simulated continuous indicators back into binary indicators, considering each indicator's mean and covariance with other indicators.

Secondly, I adopt the method of non-parametric bootstrapping, which considers a sample as a "pseudo-population"(Mooney, 1997). This method is fundamentally based on the

[71]Using the notations in equation 3.1, this can be expressed as follows:

$$X \sim TN(\mu, \Sigma, a, b)\,, \text{ where } \quad a \leq X \leq b \tag{3.2}$$

.

[72]This means that making values over one as one and negative values as zero.

[73]Thus, the problem of rejection is that it becomes quite inefficient as the number of dimension increases (Wilhelm & Manjunath, 2010).

"plug-in principle," where known sample values are taken as simple estimates of the entire population. Let an empirical distribution $\hat{F} \sim \{x_1, x_2, x_3, \cdots, x_n\}$ is known, random *i.i.d* (independently and identically distributed) sample taken to estimate the entire population distribution $F \sim \{X_1, X_2, X_3, \cdots, X_n\}$, and let $\hat{\theta}$ a point estimate of an unknown population parameter θ, then we can estimate the variance of $\hat{\theta}$ from $\hat{F}$ by the plug-in principle, that is, establish a numerical approximation of the variance through repeated simulations employing Monte Carlo method. This approximation works as follows. First, draw a random sample $F^* \sim \{x_1^*, x_2^*, x_3^*, \cdots, x_n^*\}$ from $\hat{F}$ with replacement. Next, calculate $\hat{\theta}_b^*$ from F^*, a bootstrap replication of $\hat{\theta}$ computed from the b^{th} bootstrapped sample. If above processes are repeated B times, then the variance of $\hat{\theta}$ can be calculated as following equation 3.3:

$$\mathrm{Var}_{boot}(\hat{\theta}) = \frac{1}{B-1}\sum_{b=1}^{B}[\hat{\theta}_b^* - \overline{\hat{\theta}^*}]^2 \tag{3.3}$$

where $\overline{\hat{\theta}^*} = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}_b^*$ is the mean of the parameter estimates obtained in the B resamples. Based on the law of large numbers, as the number B approaches to infinity, the bootstrap variance (3.3) converges to the estimate of the variance $\hat{\theta}$ found in the sample $\hat{F}$, and by the assumption of random *i.i.d.*, this can be considered as a consistent estimate of θ, a population parameter.

From the generated data, the weight functions for each index are calculated and finally, fuzzy indices for each individual are computed a number of times. Since there is no guideline for 'right' number of iterations (Efron & Tibshirani, 1993), I adopt several numbers of simulations, i.e., 100, 1,000, or 5,000 times to see whether the number of iterations can make a difference in results.

3.2 DATA DESCRIPTION

The data used in this study is 16th wave of the British Household Panel Survey *(BHPS)*, collected in 2006. Since 1991, it has been conducted as an annual survey of each adult (older than sixteen years old) member of a nationally representative sample of more than 5,000 households, over 10,000 individuals (M. F. Taylor, Brice, Buck, & Prentice-Lane, 2010). At Wave 9 (1999) two additional samples - Scotland and Wales samples, 1,500 households respectively - were recruited in order to assess the impacts of the public policy in the two regions, and at Wave 11 (2001) new Northern Ireland sample (2,000 households) was added to the original sample for the same reason. In sum, at Wave 18 (2008) which is the latest release, there are 11,415 households - 20,177 individuals - in the dataset. The 16th Wave is chosen because several additional questions on social satisfaction were asked in the wave. Eliminating households with any missing values in the variables from 11,507 households in the original dataset, the data of 6,339 households is analyzed in this study.

The data is collected by a package of instruments: household coversheet, household composition form[74], household questionnaire, individual schedule, self-completion questionnaire, proxy schedule[75], and telephone questionnaire[76]. Most of the time is assigned for the individual schedule, in which diverse information on each adult member of the household (aged 16 or over) is obtained. Specifically, the questionnaire includes individual demographics, residential mobility, health and caring, current employment and earnings, neighborhood, values and opinions, and household finances and organization (M. F. Taylor et al., 2010). The household questionnaire complements the information on the accommodation and tenure and some household level measures of consumption, which is often not identified clearly in individual survey. Self-completion questionnaire includes subjective or attitudinal questions the answers to which are usually sensitive to the presence of other people. The questions included are a reduced version of the General Health Questionnaire (GHQ)[77] that is often used as an indicator of subjective well-being.

[74]Preceding two instruments are needed to preserve panel characteristics of the data set. They collect information on the change of address or family composition.

[75]This is a abbreviated version of individual questionnaire that can be asked for the people absent or unavailable due to health or other conditions.

[76]If all other methods for face-to-face interview fail, an experienced interviewer gathers information by a telephone interview.

[77]This questionnaire was originally developed as a screening instrument for psychiatric illness (M. F. Taylor et al., 2010).

3.2.1 Income dimension

In this dimension, four variables are adopted: household income, financial situation, saving, and inheritance. More detailed information of the variables is in table 3.1.

Table 3.1: Variables in income dimension

Name	Description
Household income	Annual household income for year 2005
Financial situation	Self-evaluation of personal financial situation
Saving	Amount saved each month
Inheritance	Amount of inheritance / bequest

Household income: The mean income of the sample is £28,261[78]. Compared to the average gross household income of £32,779 from *Annual Abstract of Statistics* from the Office for National Statistics (Macrory, 2008), the average income of the sample is about 14% low[79]. Additionally, median income is about 84% of mean income, and maximum income is £268,049. Following histogram shows that it is severely skewed to the right.

Financial situation: This variable is measured as an ordinal variable with five different degrees of self-evaluation on each person's financial situation. As it turns out in table 3.2, over 70% of the sample tell that they are financially in good shape. Among the rest, only 6.7% mention that they have financial difficulty.

Saving: The money amount saved each month is asked for this variable. It is distributed from 0 to £6,000, and average saving is about £65. However, the interpretation should be cautious since it is shown that 64.4% of the sample does not save at all. Figure A1 (in appendix A) indicates that the distribution is even more skewed than income.

[78] For the purpose of comparing the result to national statistic, income is equivalized by the eqivalence scale variable included in the BHPS dataset. Also, sampling weight is applied to compensate for both heterogeneous response rate and over-representation of low income class (M. F. Taylor et al., 2010). As a result, mean income reduces by 5.2% - from £29,800 to £28,261 -, compared to unweighted results.

[79] All income figures mentioned here are gross income. Since BHPS provides separated net household income file, it would be more desirable to use net household income in terms of comparison. However, using net household income reduces sample size by 770 - over 10% - due to missing value.

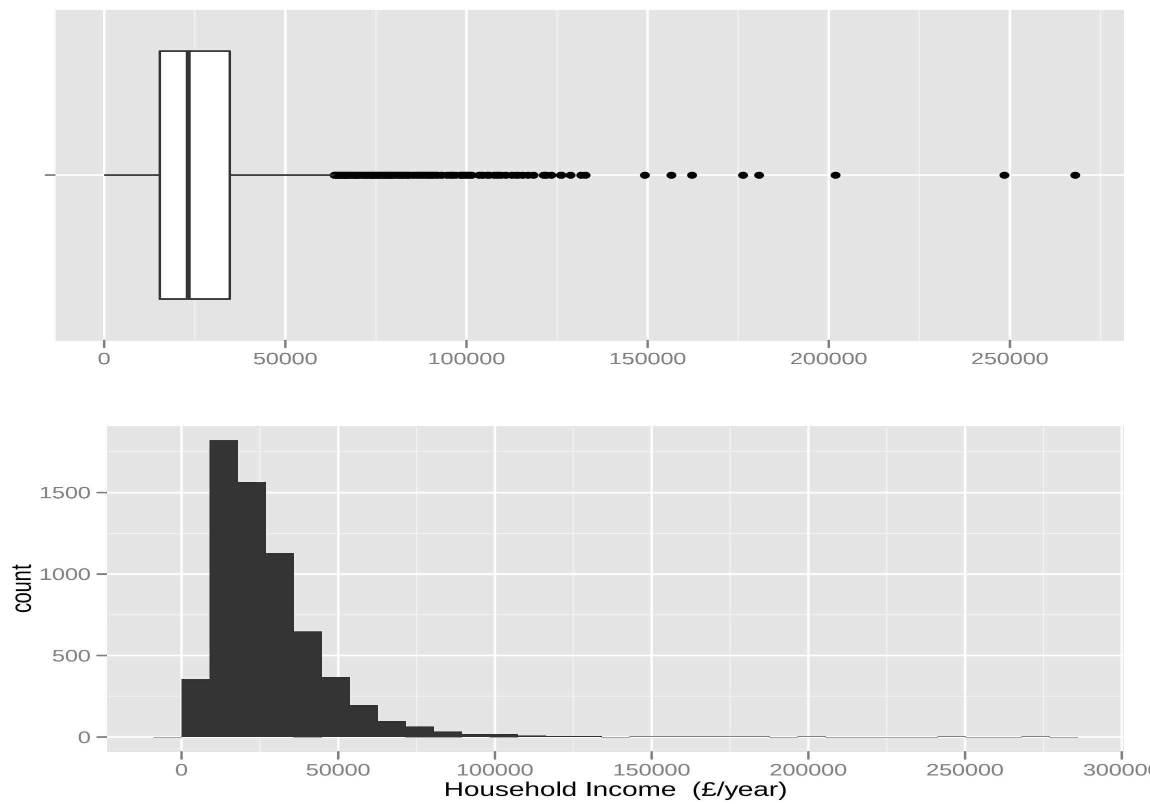

Figure 3.1: Household income distribution

Table 3.2: Self-evaluated financial situation

	Frequency	Percent	Cumulative perc.
Living comfortably	2,011	31.72	31.72
Doing alright	2,432	38.37	70.09
Just about getting by	1,459	23.02	93.11
Finding it quite difficult	293	4.62	97.73
Finding it very difficult	144	2.27	100
Sum	6,339	100	

Inheritance: It is the money amount of inheritance. As it can be easily expected, 6,214 (98.0%) people in the sample do not have any inheritance, but the highest inherited money amount goes up to £154,000. So, the mean amount of £449.9 is not so meaningful for interpretation. Following figure A2 (in appendix A) shows the extreme skewness of inheritance distribution. Even though all zeros are taken out from the sample for the figure, still the graph shows an extreme skewness.

3.2.2 Health dimension

Following three variables are utilized for this dimension. Though all the variables are measuring the subjective evaluation of individuals, it is pointed in many studies that subjective evaluation is q quite widely accepted measurement method for health condition (Brandolini & D'Alessio, 1998; van Doorslaer et al., 1997). Certainly, often it is not clear what a respondent really means (R. G. Wilkinson, 1996), but I can argue that the very 'fuzzy' nature of the indicators is the basic reason for the fuzzy set measures of poverty.

Table 3.3: Variables in health dimension

Name	Description
Health status	Health status over last 12 months
Satisfaction with health	How satisfied with current health
Health inhibits activities	Whether health prohibits respondents from doing things they want to do

Health status: The question asks respondents to evaluate their own health status over the last 12 months, compared to people of their own age. Table 3.4 shows that almost 90% of the sample express that they feel healthy. However, the mean level is 2.22 which is less than 'fair health'. As a natural phenomenon in a developed country, it implies that health status may not be a significant indicator of multidimensional poverty. However, in terms of relative comparison, health can be a crucial indicator since people with poor or very poor health would feel more deprived due to their condition.

Table 3.4: Self-evaluated health status

	Frequency	Percent	Cumulative perc.
Excellent	1,509	23.80	23.80
Good	2,777	43.81	67.61
Fair	1,390	21.93	89.54
Poor	506	7.98	97.52
Very poor	157	2.48	100
Sum	6,339	100	

Satisfaction with health: Unlike the health status variable, this variable measures more subjective aspect of health. It just measures how much the respondents are satisfied with their health regardless of their health status. Also, this variable is measured as seven-point Likert scale, 1 being "Not satisfied at all" and 7 being "Completely satisfied". According to the distribution in table 3.5, about 63% of the sample is shown to be satisfied with their health. The average satisfaction is 4.79, which indicates that satisfaction with health is more positive than neutral.

Health inhibits activities: The questionnaire asked each respondent whether health condition keeps them from doing what they want to do. From the average 2.94 of the four-point Likert item ranging from one (often) to four (never), it can be seen that the sample is not often limited to participating in the activities they want due to health. (Table A1 in appendix A)

3.2.3 Employment dimension

Three variables are used for this dimension: Job status, job satisfaction with security, and overall job satisfaction. More detailed information on the variables is available in table 3.6. In fact, in order to measure well-being from employment, more diverse variables should be considered in analysis, such as, types of industry people are employed, types of work the

Table 3.5: Satisfaction with health

	Frequency	Percent	Cumulative perc.
Not satisfied at all (1)	278	4.39	4.39
2	348	5.49	9.88
3	674	10.63	20.51
4	1,028	16.22	36.73
5	1,575	24.85	61.57
6	1,659	26.17	87.74
Completely satisfied (7)	777	12.26	100
Sum	6,339	100	

really do (Ramos & Silber, 2005), or working environment (Muffels et al., 1992). However, for employment-related variables, there is one unavoidable as well as hard-to-solve question: how can we handle unemployed? So, the number of variables in this analysis cannot but be limited.

Table 3.6: Variables in employment dimension

Name	Description
Permanent job	Whether one's current job is permanent, temporary or no job
Job security satisfaction	How satisfied with job security
Overall job satisfaction	Overall, how satisfied with job

Permanent job: This categorical variable is measuring fundamental difference among sample in terms of labor market participation. Permanent job can be interpreted as having more stable life pattern than temporary employment, which can be translated into more well-being. In order to incorporate the information on the unemployed, the variable is regarded as three-item ordinal variable with 'No job' as the lowest category. Table 3.7 shows the distribution in the sample. It turns out that people with permanent job is under-represented in the sample (53.73%) since national statistic shows that the proportion of people with permanent job is 74.6% (Macrory, 2008).

Table 3.7: Permanent job

	Frequency	Percent	Cumulative perc.
No job	2,798	44.14	44.14
Temporary job	135	2,13	46.27
Permanent job	3,406	53.73	100
Sum	6,339	100	

Job satisfaction with security: To the question, "How satisfied or dissatisfied with your job security", only 6% of the sample answer that they are dissatisfied with their job security, and the average level of satisfaction is 5.49, which indicates that people are quite satisfied with their job in terms of job security. But this interpretation requires caution because the responses exclude 2,798 people who do not have job. Under the assumption that the people without job have less well-being than those who are completely dissatisfied with job, I include people with no job as the lowest rank in this variable. It makes sense if it is reasonable to assume that the variable measures qualitative difference of well-being between two types of people, employed and unemployed.(Table A2 can be found in appendix A)

Overall job satisfaction: People without job are also included in this question for the same reason. Like satisfaction with job security, only a small proportion of sample (5.3%) answers negatively, and the average level (5.37) is quite higher than neutral position.

3.2.4 Housing dimension

All variables included in this dimension are binary because it is quite difficult to measure well-being improvement from housing in ordinal sense, not to mention cardinality. Measur-

Table 3.8: Overall job satisfaction

	Frequency	Percent	Cumulative perc.
No job	2,798	44.14	44.14
Completely dissatisfied (1)	43	0.68	44.82
2	97	1.53	46.35
3	194	3.06	49.41
Neither satisfied nor dissatisfied (4)	263	4.15	53.56
5	865	13.65	67.20
6	1,720	27.13	94.34
Completely satisfied (7)	359	5.66	100
Sum	6,339	100	

ing satisfaction with housing can be one way to address the difficulty [80], but since many researchers go with objective approach first[81], this thesis follows the objective path.

The list of the variables is as follows: lack of adequate heating, leaky roof, shortage of space, noise from neighbors, street noise, not enough light, condensation, damp walls or floors, and rot in windows and floors. From table 3.9, it can be known that about 8% of the sample is experiencing some type of deprivation in housing dimension. Overall, it turns out that physical conditions of housing do not cause much trouble, but there are some problems in housing environments, such as, noise from neighbors or street. Also, almost 19% of the sample tell that their housing does not have enough space.

[80]Muffels (1993) argues that subjective approach does not mean the concept is purely subjective, but it does imply that common perceptions are important.

[81]Ringen (1988) put it a simple sentence: "poverty is not a question of how people feel, but of how they live".

Table 3.9: Housing-related variables

Variable	Yes	No	Variable	Yes	No
Lack of adequate heating	195	6,144	Not enough light	291	6,048
	(3.08)[a]	(96.92)		(4.59)	(95.41)
Leaky roof	201	6,138	Condensation	519	5,820
	(3.17)	(96.83)		(8.19)	(91.81)
Shortage of space	1,187	5,152	Damp walls, floors	381	5,958
	(18.73)	(81.27)		(6.01)	(93.99)
Noise from neighbors	684	5,655	Rot in windows, floors	248	6,091
	(10.79)	(89.21)		(3.91)	(96.09)
Street noise	906	5,433	Sum	4,612	52,439
	(14.29)	(85.71)		(8.08)	(91.92)

[a]Numbers in parenthesis are percentage

3.2.5 Durable goods dimension

Total 12 items are considered in this study. All the variables are dichotomous except the number of car variable[82]. Considering the context of U.K., a developed country, most of the numbers are not surprising, except dish washer.

3.2.6 Social capital dimension

In this dimension, variables are measuring people's interactions with other people or their neighbors. Since the dataset does not have indicators that gauge the quality of the interactions, most of them are about the frequency of the interactions. Also, one variable measuring satisfaction with social life is included for subjective aspect of this dimension. Five variables are included in this analysis as follows:

[82]Table for this variable can be found in appendix A, table A3

Table 3.10: Durable goods

Goods	Don't have	Do have	Goods	Don't have	Do have
Color TV	65	6,274	Home computer	2,031	4,308
	(1.03)[a]	(98.97)		(32.04)	(67.96)
VCR	379	5,960	CD player	1,097	5,242
	(5.98)	(94.02)		(17.31)	(82.69)
Freezer	316	6,023	Phone	604	5,735
	(4.99)	(95.01)		(9.53)	(90.47)
Washing machine	292	6,047	Cell phone	792	5,547
	(4.61)	(95.39)		(12.49)	(87.51)
Dish washer	3,882	2,457	Internet	2,622	3,717
	(61.24)	(38.76)		(41.36)	(58.64)
Microwave	527	5,812	Sum	12,607	57,122
	(8.31)	(91.69)		(18.08)	(81.92)

[a]Numbers in parenthesis are percentage

Feed visitors once a month: This variable actually asks the intention of feeding visitors, not experience. So, it is considered as measuring psychological aspect of a respondent. However, more important aspect of this variable is that it also asks whether the reason of no intention is due to preference or affordability. If one has no intention just because the person does not like having visitors, then there is no reason to think of lower well-being for the person. But if one has no intention because the person cannot afford it, then it is natural to conclude that the person has lower well-being than other people. 235 people with no intention because of preference will be coded as the same to the people who have intention. (Table 3.12)

Talking to neighbors / meeting people[83]: The frequency of interactive behavior is

[83]In questionnaire, it is clearly mentioned that the 'people' indicates friends or relatives who do not live

Table 3.11: Variables in social capital dimension

Name	Description
Feed visitors once a month	Intention of feeding visitors once a month
Talking to neighbors	Frequency of talking to neighbors
Meeting people	Frequency of meeting people (friends or relatives) at home or elsewhere
Satisfaction with social life	How satisfied / dissatisfied with social life
Contact with the closest friend	Frequency of getting in touch with the closest friend either by visiting, writing or by telephone

Table 3.12: Intention of feeding visitors

		Frequency	Percent	Cumulative perc.
Yes		4,955	78.17	78.17
No	Preference	235	3.71	81.88
	Affordability	1,149	18.13	100
Sum		6,339	100	

measured in the two variables (see appendix A, table A4). Almost 80% of the sample says that they talk to neighbor / meet people more than once or twice a week. If once or twice a month is considered enough, then at least 93% of the sample are having some interactions with other people. On the contrary, about 7% says that they talk to neighbors less often than once a month, and less than 3% mentions that they meet people less often than once a month.

Satisfaction with social life: According to table 3.13, about 64% of the respondents

with the respondent.

says that they are satisfied with their social life. On the other hand, 17% answers that they are not satisfied.

Table 3.13: Satisfaction with social life

	Frequency	Percent	Cumulative perc.
Not satisfied at all (1)	167	2.63	2.63
2	262	4.13	6.77
3	632	9.97	16.74
4	1,298	20.48	37.21
5	1,797	28.35	65.56
6	1,355	21.38	86.94
Completely satisfied (7)	828	12.06	100
Sum	6,339	100	

Contact with the closest friend[84]: This variable measures the frequency of getting in touch with friends, but it can be interpreted as the intensity of their relationship. Table A5 (in appendix A) shows that more than 80% of the people comes in to contact with the closest friend at least once a week, while a little less than 20% has communication once a month or less often.

3.2.7 Social participation dimension

The variables in this dimension contain the information on the frequency of people's participation in social activities, such as voluntary works or local groups. Also, one question asks about trade union membership which is an important indicator of social inclusion (Giorgi & Verma, 2002; Rankin & Quane, 2000).

Local group activities: It turns out that three quarters of respondents have never participated in local group activities like attending meetings. On the contrary, 15% answers that they frequently attend meetings, more than once a month.

[84]'Friend' here does not include people who live with the respondent, but it can include relatives.

Table 3.14: Variables in social participation dimension

Name	Description
Local group activities	Frequency of attending meetings for local groups / voluntary organizations
Voluntary work	Frequency of doing unpaid voluntary work
Union membership	Whether a respondent is a member of trade union at work

Table 3.15: Local group activities

	Frequency	Percent	Cumulative perc.
At least once a week	419	6.61	6.61
At least once a month	517	8.16	14.77
Several times a year	391	6.17	20.93
Once a year or less	266	4.20	25.13
Never	4,746	74.87	100
Sum	6,339	100	

Voluntary work: It would not be far-fetched to assume that doing unpaid voluntary work can be related to more active social participation than attending meetings. Table 3.16 supports this speculation by showing that even less people (about 18%) are involved in this type of activity somehow than people attending meetings of local groups.

Union membership: For this variable, the distinction of non-union members is required between people who just do not want to be a member when they have a trade union and people who do not have one at work, since it is reasonable to assume that the latters are experiencing some loss of capability. On the contrary, the former people can be considered

Table 3.16: Voluntary work

	Frequency	Percent	Cumulative perc.
At least once a week	457	7.21	7.21
At least once a month	220	3.47	10.68
Several times a year	251	3.96	14.64
Once a year or less	201	3.17	17.81
Never	5,210	82.19	100
Sum	6,339	100	

same to the union members because they also have the capability.[85] So, this variable is considered as having three categories, one being those people who do not have job, others who do not have trade union, and the others who have trade union at their job because non-union members with job also have the capability to become a union member.

Table 3.17: Union membership

	Frequency	Percent	Cumulative perc.
No job	2,798	44.14	44.14
Not union members	1,721	27.15	71.29
Union members	1,820	28.71	100
Sum	6,339	100	

[85] In this sense, this indicator can be considered more a measurement of capability than that of functionings.

4.0 FUZZY POVERTY MEASURES CALCULATION AND COMPARISON

Using BHPS data set, each fuzzy measure is calculated in this chapter. For each calculation, detailed formulas for membership function as well as weight function are described and discussed. After the calculations for each method, a comparison with traditional measures is performed to clarify the new insights that are possible from the fuzzy measures.

4.1 *TOTALLY FUZZY* MEASURE

Totally Fuzzy method (TF) is the first attempt to apply the concept of 'fuzziness' to poverty[86]. Cerioli and Zani (1990) argue that it is more appropriate to consider poverty as a *fuzzy* concept because it is not certain where the boundary between poor and nonpoor can be drawn. Besides, research on poverty measurement shows that it is also not clear how to set the line (S. Anand, 1977; Townsend, 1979a; Foster, 1984). Using the degree of inclusion to the fuzzy subgroup *poor*, they think poverty can be measured within range of zero to one, that is, from 'definitely nonpoor' to 'definitely poor'[87].

The first suggestion from the idea, TF measure, has one unique trait: it needs two criteria, above which people can be regarded as definitely non-poor and below which they are definitely poor (Cerioli & Zani, 1990), for each variable to calculate the membership function for individual i to an attribute j, expressed as $\mu_j(i)$. Although there have been

[86]Kundu and Smith (1983) discusses the possibility of using the concepts of fuzzy set theory in poverty measurement study ahead of Cerioli and Zani (1990), though they do not provide a concrete method.

[87]These two terms indicate two extreme values of the membership function, zero and one, in fuzzy set theory. Originally, membership function value zero means that an element does not belong to a set at all, and one that an element belongs to a set completely.

criticisms on the inevitable arbitrariness of setting two poverty lines (Miceli, 1998), this method has strength in the sense that it can explicitly include a common-sense consensus of the society on the concept of poverty[88].

Formulas for TF measure are different for each level of measurement of variables included in the study. So, the discussions of the formulas for specific levels are in order. Also, an weight function that is indispensable for aggregating different dimensions is investigated.

4.1.1 Formulas for membership function

Continuous variables: Assuming that two criteria for each indicator j are set as $x_{j,max}, x_{j,min}$ and saying $x_j(i)$ is individual's value on variable j, then following formulas 4.1 can be used:

$$\mu_j(i) = \begin{cases} 1 & \text{if } x_j(i) < x_{j,min}, \\ \dfrac{x_{j,max} - x_j(i)}{x_{j,max} - x_{j,min}} & \text{if } x_{j,min} < x_j(i) < x_{j,max}, \\ 0 & \text{if } x > x_{j,max}. \end{cases} \tag{4.1}$$

Ordinal variables: Basic formulas for ordinal variables are same to those for continuous variables. The only difference is that $x_{j,min}$ indicates the lowest level of well-being, while $x_{j,max}$ the highest level of poverty. For example, for five-step ordinal variable for health condition, 1 being the poorest health, 1 is $x_{j,min}$. By the same logic 5 being the most healthy, $x_{j,max}$ is 5.

Binary variables: For binary variable, the logic of traditional set theory is directly applied. So, if individual i owns attribute j, then it is natural to conclude that that individual has zero membership for fuzzy subset 'poor', in other words, the person does not belong to the subset at all, and vice versa. This can be expressed as following 4.2.

$$\mu_j(i) = \begin{cases} 1 & \text{if } x_j(i) = 0, \\ 0 & \text{if } x_j(i) = 1. \end{cases} \tag{4.2}$$

[88]It is certain that the consensus cannot be achieved without debate in political nature. However, the concept of relative deprivation (usually as a half or 60% of median income) which is commonly used as an important criterion for social policy in Europe is also built on that kind of consensus.

Formula for weight function: For aggregating various information from multidimensions, weight function is needed. For TF measure, Cerioli and Zani (1990) suggest a frequency-based weight function as following 4.3.

$$w_j = \frac{\ln\left(\frac{1}{f_j}\right)}{\sum_{k=1}^{K} \ln\left(\frac{1}{f_k}\right)} \tag{4.3}$$

where f_j denotes the proportion of people who appear to be 'completely poor'[89] on indicator j, and K is the number of indicators.

The basic idea of the weight is 'relative deprivation': for attributes that most of the population have, the weight gets bigger because people without those attributes would feel more severe sense of relative deprivation than people who don't have relatively rare items. This implies that TF method reflects the relative nature of poverty in the calculation.

TF measure: Applying above formulas, TF measure for an individual i is calculated as follows:

$$\mu(i) = \frac{\sum_{j=1}^{K} w_j \times \mu_j(i)}{\sum_{j=1}^{K} w_j} \tag{4.4}$$

where K is the number of indicators, and the aggregate TF measure is the average of $\mu(i)$.

$$\mu = \frac{1}{n} \sum_{t=1}^{n} \mu(t) \tag{4.5}$$

where n is population size.

[89] Cerioli and Zani (1990) use a different term, "people who have poverty symptom."

4.1.2 Setting criteria

Before the actual calculation of the TF measure, two criteria for continuous variables should be determined explicitly since some level of arbitrariness is inevitable. Among the variables chosen in previous chapter, three variables are measured as continuous variables: household annual income, amount saved, and amount inherited. For the variables, criteria for calculation is set as table 4.1.

Table 4.1: Two criteria for continuous variables

Variable	x_{min}	x_{max}
Income	£13,919	£37,673
Saving	£0	£1,200
Inheritance	£0	£6,000

x_{min} for household income is set at 60% of sample median income, which is considered as the relative poverty line in traditional welfare economic approach (Atkinson, Cantillon, Marlier, & Nolan, 2002; Bradshaw & Finch, 2003), while x_{max} is 150% of median income. The latter value is determined by the previous research, specifically by Brandolini and D'Alessio (1998) and Miceli (1998). For x_{max} for saving amount variable, the level of £1,200 is set roughly as 10% of the minimum income level, based on the assumption that if people can save more than 10% of their income, they may not be considered as poor. Finally, x_{max} of inheritance variable is decided at £6,000 that corresponds with the median. Since it is very hard to find theoretical ground for the maximum, I simply decide to use the central tendency, considering the variable shows extremely skewed distribution.

Though all these decisions cannot escape the criticisms for arbitrariness, still it is possible to know how much difference the arbitrariness makes by comprehensive sensitivity analysis. If the decisions does not change the conclusions from analysis, then arbitrariness itself does not have to be a weakness of the method. This examination is administered in section 4.1.6.

4.1.3 General interpretation

Calculated from the seven dimensions[90] previously mentioned, the average of TF measure for the 16th wave of BHPS is 0.193. For more information, the histogram for TF measure is pictured as figure 4.1, and other descriptive statistics is in table 4.2. Just considering the

Table 4.2: Descriptive statistics for TF measure

Statistic	Value	
Mean	0.193	
Median	0.180	
Range	0.016	0.671
Quartiles	(1st) 0.130	(3rd) 0.243
S.D.	0.086	

fact that the measure ranges from zero (definitely nonpoor) to one (definitely poor), it can be said that the propensity to poverty of the U.K. according to the TF measure is relatively low (Betti & Verma, 1998; Lelli, 2001). Or, following the interpretation of its proposers (Cerioli & Zani, 1990), it can be concluded that the sample belongs to the fuzzy set *poor* by 19.3% on average. Based on the 'proportional cardinality'[91] interpretation (Smithson, 2006), the number can be compared to the headcount ratio - the proportion of poor people - in a fuzzy sense (Cerioli & Zani, 1990). Other proponents of fuzzy set measures even argue that anyone belongs to the poor fuzzy subset more than this level can be considered poor, in its traditional sense since the measure can be understood as an indicator of the distance from being *definitely poor* (Filippone et al., 2001; Dagum, 2002).

Additionally, the relationship between membership functions for different dimensions can be very helpful to understand how this new measurement method works since that

[90] Economic resources, health, employment, housing, durable goods, social capital, and social participation.

[91] In fuzzy set theory, cardinality denotes the size of a fuzzy set, which can be translated as the number of the elements of a traditional set. Mathematically, the proportional cardinality is defined as the sum of membership function over all elements divided by the total number of elements. Thus, this can be interpreted as probability, though it is not the same property. For example, if a *poor* fuzzy set is changed into a traditional set, then the proportional cardinality becomes equal to the probability of being poor, the number of poor people divided by the total number of people.

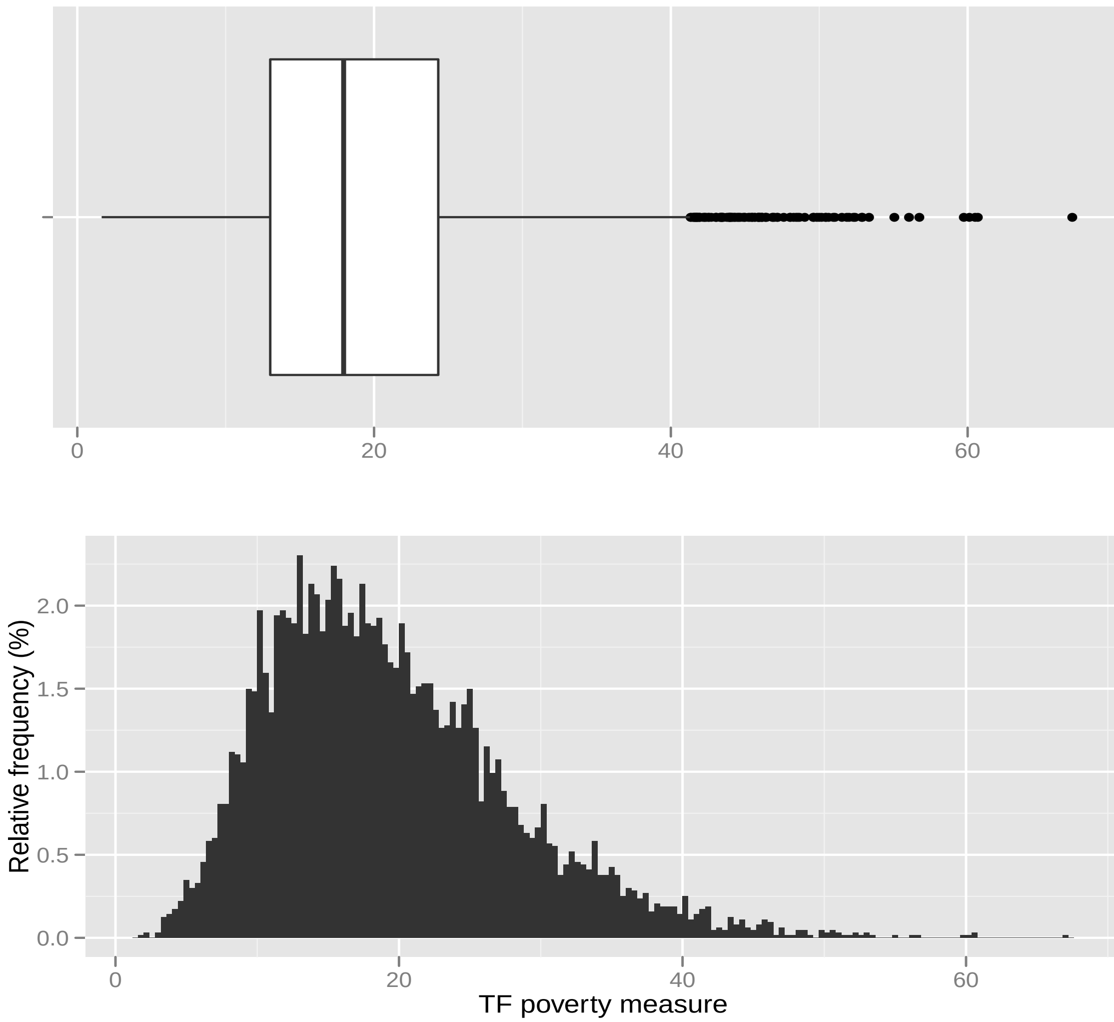

Figure 4.1: TF measure distribution

is a good indication of how the measure reflects reality. As considering every indicator simultaneously is not an efficient way to investigate the relationships, table 4.3 presents the correlation coefficients[92] between the seven dimensions. Simple reading of the table can provide several interesting points: 1) the relationships between economic resources and other dimensions are not particularly strong, 2) social capital is more related to health than

[92]Since the membership functions range from zero to one, the computation for logistic-transformed membership functions is conducted to check the influence of limited range. There is no substantive change in the results (see table A6 in Appendix A).

Table 4.3: Correlation coefficients for each dimension's membership functions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) Economic resources	1.000						
	(0.000)						
(2) Health	0.287	1.000					
	(0.000)	(0.000)					
(3) Employment	0.325	0.381	1.000				
	(0.000)	(0.000)	(0.000)				
(4) Housing	0.208	0.093	0.016	1.000			
	(0.000)	(0.000)	(0.209)	(0.000)			
(5) Durable goods	0.253	0.174	0.368	0.048	1.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		
(6) Social capital	0.014	0.146	-0.053	0.089	-0.025	1.000	
	(0.257)	(0.000)	(0.000)	(0.000)	(0.043)	(0.000)	
(7) Social participation	0.330	0.332	0.802	0.024	0.347	-0.045	1.000
	(0.000)	(0.000)	(0.000)	(0.055)	(0.000)	(0.000)	(0.000)

* The numbers in parenthesis are $p - values$ from the significance test.
* All dimensions' membership functions range from zero ("definitely non-poor") to one ("definitely poor").

to economic resources, 3) the relationship between economic resources and social capital is statistically insignificant and very weak in strength, and 4) social participation is very strongly correlated with employment dimension. The first observation seems to confirm the necessity of the multidimensional perspective since it implies that monetary variables are not necessarily a good proxy for well-being, let alone the best, which has been argued in many research (S. Anand, 1977; Callan et al., 1993; Ringen, 1988, 1995). The second and third points also can be understood as supporting multidimensional perspective in that social capital dimension, which represents good social relationships between people, is capturing an independent domain from economic resources. The fourth interpretation looks reasonable since the relationship is emphasized in the *social exclusion* perspective (Burchardt, Le Grand, & Piachaud, 1999; Jenkins & Micklewright, 2007; Paugam, 1996; Room, 1999).

4.1.4 Poverty profile for subgroups

More detailed poverty profile is in order. Table 4.4 is constructed by computing TF measure for age and country categories. It turns out that age group between 25 and 49 is in the least level of poverty, while people older than 65 is the poorest as can be seen in figure A3 in Appendix A. Also, it appears in table 4.4 that the differences in poverty among countries

Table 4.4: Poverty profile for age and country

Variable	Categories	Value	Variable	Categories	Value
Age	16~24	0.229	Country	Britain	0.191
	25~49	0.171		Wales	0.196
	50~64	0.188		Scotland	0.194
	65 or more	0.232		Northern Ireland	0.200

are no more than 5%[93], though Britain is slightly less likely to be classified as poor than the other three countries.

In terms of labor force status, table 4.5 shows that people with disability have a little less than two times bigger propensity to poverty than people having job, and they are even more likely to be poor than the unemployed or retired. It seems to suggest that people with diability suffer strongly from factors other than employment, when it comes to the lack of capability. Since labor force status can be highly correlated to age, further analysis might be helpful to see whether the pattern provides an useful information. Following table 4.6 shows the decomposition by labor force status and age[94]. Generally speaking, people in the primary working age are better-off than any other age groups with respect to labor force

[93] Assuming Britain's TF measure is one, Northern Ireland is 1.04.

[94] There are blanks in table 4.6 because no sample exists in that category. No one retires at age between 16~25, and every student is younger than 65 years old in the data.

status, though the difference from 50~64 age group is not substantial. However, even within same age group, quite a variation of TF measure according to labor force status can be observed. For marital status, the fact that widowed people are in the poorest condition is

Table 4.5: Poverty profile for labor force and marital status

Labor force status	Value	Marital status	Value
Self-employed	0.158	Married	0.173
Employed	0.156	Widowed	0.239
Unemployed	0.241	Divorced	0.217
Retired	0.228	Separated	0.203
Student	0.245	Never married	0.224
Disabled	0.292	Civil partnership	0.126

Table 4.6: Subgroup decomposition by age and labor force status

	Self-employed	Employed	Unemployed	Retired	Student	Disabled
16~24	0.198	0.194	0.270		0.258	0.337
25~49	0.147	0.153	0.237	0.221	0.212	0.295
50~64	0.156	0.157	0.240	0.203	0.269	0.285
65+	0.223	0.141	0.242	0.234		0.302

noticeable, while 'married[95]' are better-off than people in any other marital status[96].

Gender inequality is evidenced in table 4.7, but the TF calculation shows that the gap between gender is not so wide as usually expected (Pressman, 2002, 2003). This rather small gap between genders is likely to be the results of the compensation by including "socially

[95] In original questionnaire, 'married' and 'living as a couple' are different categories. Since the difference is not meaningful for this research, I merge the two groups as one.

[96] In fact, 'Civil partnership' has the lowest propensity to poverty in the table. However, since the sample size is too small, I exclude the group in this interpretation.

perceived necessities"(Millar, 2003). For example, following table 4.8 shows that female has more functionings in social capital dimension than male, though female has more propensity to poverty in 'satisfaction with social life' indicator. The classification by regions in Britain shows that there are not much gaps in poverty. The gap between the smallest propensity region, Yorks & Humberside, and the largest propensity region, East Anglia, is only 9%[97]. However, as it can be seen in figure 4.2, most of the regions in Britain show lower TF measure values than the other three countries. The poverty profile by housing tenure is not easy to interpret in that people who owns a house with mortgage have much smaller propensity to poverty than homeowners without one. In order to compare these two groups as well as with other groups, the analysis without housing dimension is more informative since homeowners are more likely to manage their house properly, which results in better functionings in housing dimension. As table 4.9 indicates, it turns out that homeowners' situation does not change in the calculation excluding housing dimension.

Table 4.7: Poverty profile for gender, housing tenure and regions

	Housing tenure	Value	Regions	Value
	Owned	0.200	London	0.191
	Owned with mortgage	0.154	South East	0.184
	Rented from local authority	0.254	South West	0.192
	Rented from housing association	0.254	East Anglia	0.197
	Rented from employer	0.207	East Midlands	0.190
	Rented private housing unfurnished	0.223	West Midlands	0.186
	Rented private housing furnished	0.244	North West	0.195
Gender	Male	0.187	Yorks & Humberside	0.180
	Female	0.199	North East	0.186

For different occupations people are employed[98], 'Managers' show the least degree of

[97] $0.197/0.180 \times 100 \approx 1.09$

[98] This classification of occupation follows ISCO-08, endorsed by the Governing Body of the International Labor Organization (ILO) in March 2008. Details of the system can be found on ILO website (`http://www.ilo.org/public/english/bureau/stat/isco/isco08/index.htm`)

Table 4.8: Difference in social capital between gender

Social Capital	Male	Female
Intention to feed visitors once a month	0.187	0.176
Frequency of talking to neighbors	0.228	0.208
Frequency of Meeting people	0.204	0.155
Satisfaction with social life	0.342	0.377
Contact with the closest friend	0.332	0.226

Table 4.9: Group comparison according to housing tenure without housing dimension

Housing tenure	TF measure without housing dimension
Owned	0.260
Owned with mortgage	0.191
Rented from local authority	0.309
Rented from housing association	0.310
Rented from employer	0.230
Rented private housing unfurnished	0.262
Rented private housing furnished	0.289

propensity to poverty, whereas agriculture-related careers show the highest degree. The profile for different household types in table 4.10 reveals complex relationships between household structure and poverty: 1) couples show the least level of poverty no matter they have children or not, 2) lone parents (single-headed households) are better-off than single-person household regardless of child, 3) age does make difference in single-person household's poverty level[99], and 4) having non-dependent children increases the propensity to poverty

[99]To confirm this conjecture, table A7 in Appendix A is constructed. This shows that age affects TF measure significantly not only for single-person household, but also for all other types of households.

Figure 4.2: TF measure by Regions & Countries

for couple, but not for lone parents.

For metropolitan areas, one fact draws attention: 'Other regions' which include all non-metropolitan regions in England show lower average degree of poverty than most of the metropolitan areas. Following table 4.11 and figure 4.3 shows that only Greater Manchester has lower median poverty degree than non-metropolitan regions[100].

Finally, the profile for household size shows non-linear relationship between household size and poverty degree. It can be known from the right panel of table 4.11 that the median propensity to poverty decreases until household size equals to four, and increases with

[100] Dotted line in figure 4.3 indicates the median level of 'Other regions'.

Table 4.10: Poverty profile for occupations and household type

Occupations	Value	Household type	Value
Managers	0.139	Single non-elderly	0.216
Professionals	0.147	Single elderly	0.251
Technical Profession	0.150	Couple: no child	0.180
Clerks	0.155	Couple: dependent child	0.164
Service workers	0.167	Couple: non-dependent child	0.170
Agriculture	0.215	Lone parent: dependent child	0.204
Craft related	0.158	Lone parent: non-dependent child	0.204
Machine operators	0.170	2+ unrelated adults	0.205
Elementary occupations[a]	0.172	Other types	0.211

[a] "Elementary occupations" involve the performance of simple and routine tasks which may require the use of hand-held tools and considerable physical effort, such as cleaning, basic maintenance of apartments.

Table 4.11: Poverty profile for metropolitan areas and household size

Metropolitan area	Value	Household size	Value
London	0.191	1	0.235
West Midland Conurbation	0.213	2	0.185
Greater Manchester	0.184	3	0.173
Merseyside	0.208	4	0.166
South Yorkshire	0.198	5	0.176
West Yorkshire	0.208	6	0.204
Tyne & Wear	0.212	7	0.250
Other regions	0.188	8	0.298

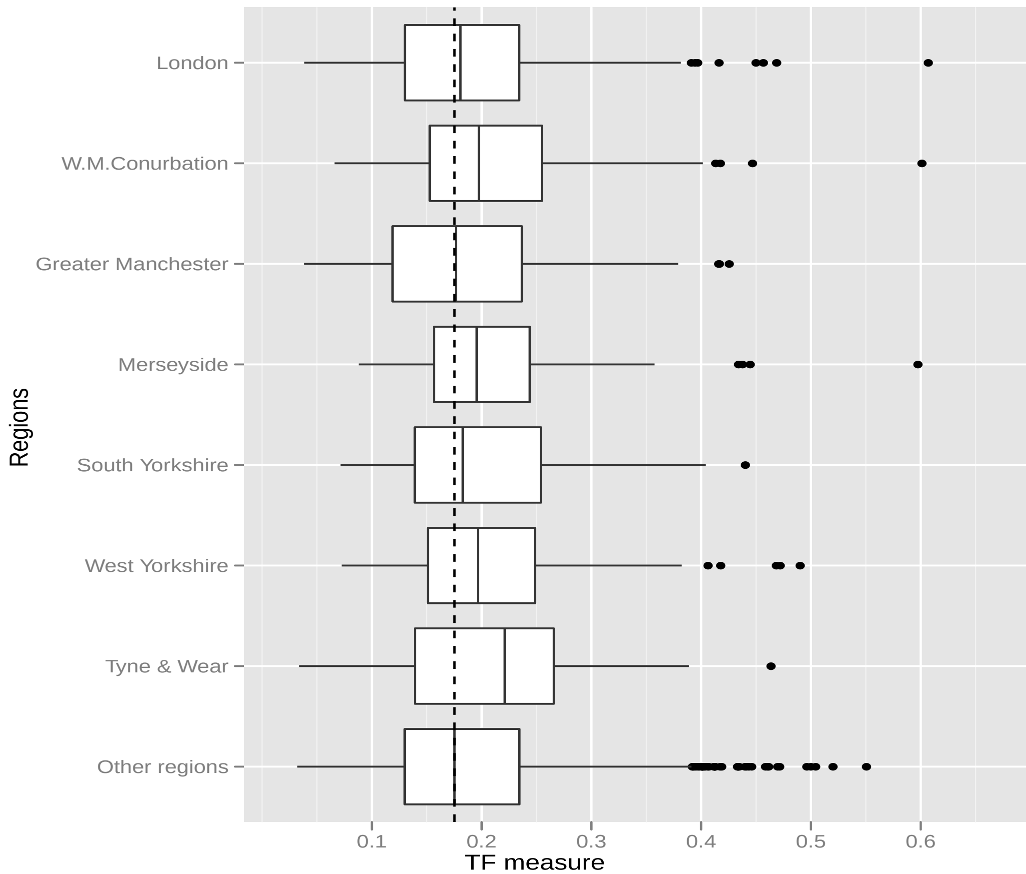

Figure 4.3: TF measure distribution for metropolitan areas

household size after that level. The former can be attributed to the scale of economy, while the latter implies that bigger household can be related to poorer household. Using mean instead of median does not change the result from figure 4.4.

4.1.5 Poverty profile for dimensions

Although the poverty profile in previous section is one good way to examine poverty in a country, it is not enough for this multidimensional approach in that it cannot show the contribution of each dimension to the aggregate poverty index. In fact, this poverty decomposition by dimensions can be very informative to policymakers because it could provide an

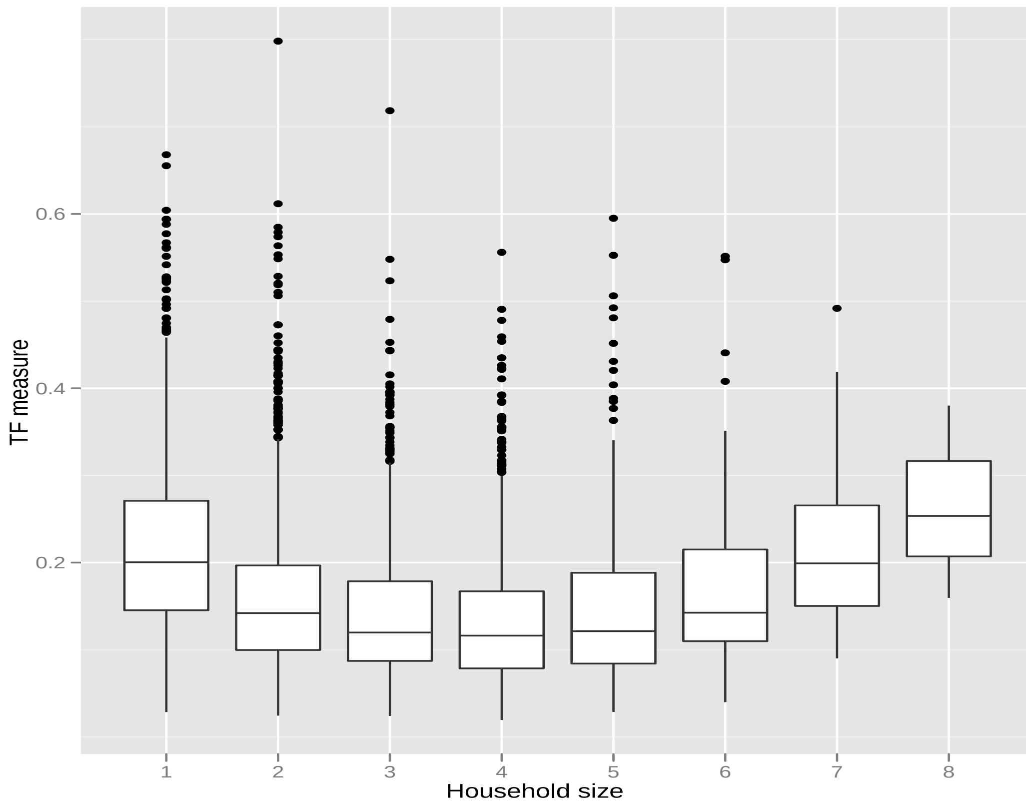

Figure 4.4: Median TF measure distribution for household size

answer for the important policy question: the priority. If one dimension determines overall level of poverty index, then it is natural to take an action to address that dimension first. This decomposition is possible for TF measure of poverty because it is an weighted average of multiple dimensions. Previous calculation of TF measure is done by applying following weights shown in table 4.12 in percentage term. Generally, a dimension with high weight indicates that many people are not 'poor' in that particular dimension. For instance, 'Amount inherited' has weight very close to zero because only a few people (120 out of 6,339) have inheritance[101]. In other words, the variable is treated as if only the rich can have it, and

[101] One justifiable objection to this interpretation is that the old are more likely to inherit, therefore, most of the feeling of relative deprivation might affect only young people. However, it is shown in figure A4 in Appendix A that 'the amount inherited' indicator is quite evenly distributed across age, though extremely high levels of inheritance take place mainly in relatively old age.

therefore not so relevant to measuring poverty. On the contrary, the case of color TV shows that it is very important to measure poverty, because almost everybody has it[102], which can be interpreted that people who don't have it feel severe deprivation.

Table 4.12: Weights used in TF measure calculation

Dimension	Variable	Weight(%)	Dimension	Variable	Weight(%)
Economic resources	HH income	1.8	Durable goods	Color TV	5.2
	Amount saved	0.5		VCR	3.2
	Amount inherited	0.02		Freezer	3.4
	Financial situation	4.3		Washing machine	3.5
Health	Health status	4.2		Dish washer	0.6
	Satisf. with health	3.5		Microwave	2.8
	Inhibits activ.	2.4		Home computer	1.3
Employment	Perm./temp. job	0.9		CD player	2.0
	Security satisf.	0.9		Telephone	2.7
	Overall satisf.	0.9		Cellphone	2.4
Housing	No heating	4.0		Internet	1.0
	Leaky roof	3.9		Cars	1.7
	Short space	1.9	Social capital	Feed visitors	1.9
	Neighb. noise	2.5		Talking to neighb.	4.5
	Street noise	2.2		Meeting people	7.6
	Not enough light	3.5		Satis.social life	4.1
	Condensation	2.8		Meeting friends	3.4
	Damp walls	3.2	Social participation	Attend groups	0.3
	Rot in floors	3.7		Voluntary works	0.2
				Union member	0.9

The first thing that is easily read from the table 4.12 is that the weight of income is 1.8, which means that the contribution of income to overall propensity to poverty is 1.8%

[102]Descriptive statistics in Chapter 4 shows that 98.97% of the sample have color TV.

in this particular calculation, while the weight for color TV is about 5.2%. Therefore, it is easy to conclude that color TV is almost three times more important than income in measuring poverty by TF measure. However, this should not be interpreted as such because the weights are the importance given to the "extremely rare poverty symptoms" (Szeles, 2004), or, as Betti, Cheli, Lemmi, and Pannuzi (2005) argue, "how representative it[a dimension] is of the community's lifestyle" rather than their importance in the "valuation function" (Brandolini & D'Alessio, 1998), considering it is computed from the frequency of *definitely poor* phenomenon in each indicators (D'Ambrosio & Silber, 2011; Betti, Cheli, Lemmi, & Verma, 2005a; Mussard & Pi Alperin, 2005). Therefore, the weights should be understood as a measure of the contribution of each indicator to the relative position of an individual between two extreme situation of 'definitely poor' and 'definitely not poor' (Miceli, 1998).

Thus, three times higher weight denotes that having color TV has three times more discriminating power than income in positioning an individual between 'definitely poor' and 'definitely nonpoor', not the former is three times more important than income. Besides, two more cautions can be suggested for table 4.12: 1) this result is sensitive to the two cut-off values in table 4.1 for continuous variable cases, and 2) for variables other than continuous, it should be noted that the distribution of levels has big impact on the weight. For the first point, figure 4.5 shows that quite many people (1,574, 24.8% of the sample) has zero membership function for income variable, while 1,250 people (19.7%) have full membership, that is, 'definitely poor'. As the change in the two values can alter the frequency of definitely poor people, the weights can be totally different. For ordinal variables in employment dimensions which are widely accepted as important indicators of poverty (Robeyns, 2000; S. Anand & Sen, 1997; Narayan et al., 2000), the extreme skewness of initial distribution to the membership function one is the main cause of the lower weights for the indicators. Since relatively higher weights for housing and durable goods dimensions are also attributable to this severely unbalanced distribution in data, the problem of weight can be eventually regarded as fundamental weakness of the membership function formula in the TF method.

Still, the weight can be a source of valuable information, examined together with subgroup categories. For instance, we can decompose each subgroup's membership function

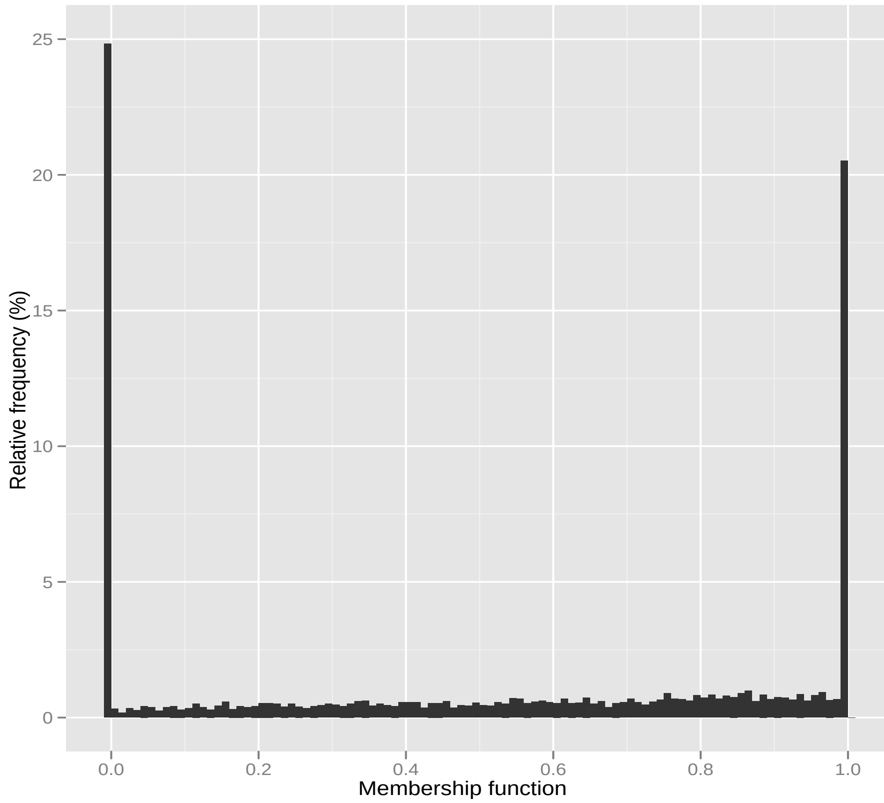

Figure 4.5: Membership function for income variable

(MF) to poverty for each attribute, applying the weights. Table 4.13 shows the poverty decomposition for attributes in six dimensions by gender. 'raw MF'[103] indicates the average membership function calculated for each indicator, so it can be interpreted as the average degree of inclusion to fuzzy subset *poor*. For example, 0.289 for male 'health status' indicator shows that the mean level of male's propensity to poverty is 0.289, ranging from zero to one. 'weighted MF', on the other hand, shows the final contribution of each indicator to aggregate TF index, because the sum of weighted M.F. constructs aggregate TF measure. Thus, weighted MF for male household income, .008, demonstrates that it contributes twice more than annual saving - 0.004 - to the overall TF index.

[103]'raw MF' is calculated by equation 4.1, and 'weighted MF' is (raw MF) $* \frac{\text{(weights in table 4.12)}}{\text{(Sum of weights in table 4.12)}}$.

Table 4.13: Membership function (MF) decomposition by gender

Dimension	Variable	raw MF		weighted MF	
		Male	Female	Male	Female
Economic resources	HH income	0.460	0.566	0.008	0.010
	Amount saved	0.702	0.739	0.004	0.004
	Amount inherited	0.988	0.985	0.0002	0.0002
	Financial situation	0.251	0.285	0.011	0.012
Health	Health status	0.289	0.318	0.012	0.013
	Satisf. with health	0.355	0.380	0.013	0.013
	Inhibits activ.	0.348	0.358	0.008	0.009
Employment	Perm./temp. job	0.406	0.498	0.004	0.005
	Security satisf.	0.534	0.590	0.005	0.005
	Overall satisf.	0.545	0.597	0.005	0.006
Housing	No heating	0.024	0.038	0.001	0.001
	Leaky roof	0.029	0.035	0.001	0.001
	Short space	0.177	0.198	0.003	0.004
	Neighb. noise	0.092	0.124	0.002	0.003
	Street noise	0.129	0.156	0.003	0.003
	Not enough light	0.038	0.054	0.001	0.002
	Condensation	0.068	0.095	0.002	0.003
	Damp walls	0.052	0.068	0.002	0.002
	Rot in floors	0.032	0.046	0.001	0.002
Social Capital	Feed visitors	0.187	0.176	0.004	0.003
	Talking to neighbors	0.228	0.208	0.010	0.009
	Meeting people	0.204	0.155	0.015	0.012
	Satisf.social life	0.342	0.377	0.014	0.016

Continued on next page

Table 4.13 – continued from previous page

Dimension	Variable	raw MF		weighted MF	
		Male	Female	Male	Female
	Meeting closest frnd.	0.332	0.226	0.011	0.008
Social Participation	Attend groups	0.841	0.821	0.003	0.003
	Voluntary work	0.877	0.871	0.002	0.002
	Union membership	0.552	0.602	0.005	0.006

Thus, the comparison of raw MFs for different indicators is not so meaningful, though it does provide some insights on the distribution of *definitely poor* phenomenon, for they do not take into account the distribution and relationship of each indicator. However, the comparison of different groups using raw MF is still valid and can give useful information on the unequal propensity to poverty in different groups. For example, while the first three dimensions indicate that males are in better position than females for every indicator except inheritance, 'social capital' dimension shows that generally females have more capabilities. For 'weighted MF' case, both comparisons are possible because the weights include information on the distribution and relationship of each indicator (see equation 4.3). For instance, from the third and fourth columns of table 4.13, it can be pointed that the difference in economic resources dimension is compensated by the difference in other dimensions, especially social capital dimension, which makes gender inequality of poverty less severe than expected (see table 4.7.).

Based on above discussion several points can be read from table 4.13. Firstly, from raw MF column, the raw MF of household income tells us that female has more propensity to poverty than male in terms of household income, or it can be known that the gap between genders pertaining to financial situation is much smaller than the gap in income. In social capital and social participation dimensions, it is noticeable that female are less likely to be poor than male for most of the indicators. Second, weighted MF column shows that 'satisfaction with social life' is the biggest concern for female poverty, while the deprivation in terms of the frequency of meeting people has the biggest impact on male poverty. Third, though the weight for 'No heating in housing' variable is more than two times bigger than household income, weighted MF column in table 4.13 shows that the variable's contribution to total poverty index is only a eighth of income for male, and a tenth of income for female. In addition, it seems meaningful in policy perspective that most of the social capital dimension indicators contribute to the overall level of poverty more than income, except 'feed visitors once a month' variable, while social participation indicators take much smaller portion of it. Still, it should not be ignored that union membership is very important factor in constructing TF measure.

Since reading table 4.13 is not easy due to the number of indicators, often it is more helpful to examine the contribution of each dimension, using weighted M.F. Table 4.14 provides fundamentally the same information to above table, though more intuitively understandable. It clearly demonstrates that social capital, durable goods, health dimensions affect the propensity to poverty index more than economic resources dimension, while employment, housing, social participation dimensions have low impact.

Overall, it can be concluded that though weights are fairly low for economic resources, health, employment, social capital and social participation dimension compared to housing dimension, still some of them construct more important portion of aggregate poverty index than housing. This means that the overall system of TF measure has its strength in this decomposition analysis - especially decomposition by both dimension and subgroup, though the direct interpretation of the weight system is not quite intuitive. Additionally, this simple analysis implies that policies need to have objectives other than just income: health issues take more portion than income, and the biggest problem in terms of capability is the lack of social capital.

4.1.6 Sensitivity analysis for the thresholds

What has been mainly argued against TF measure is that the arbitrary decision of setting two threshold lines is indispensable. Since there is no strong theoretical basis for the decision, this criticism is well-pointed. However, this does not mean that TF measure has a serious

Table 4.14: Contribution of each dimension by gender

	Male	Female
Economic resources	0.023	0.027
	(12.3)	(13.3)
Health	0.033	0.035
	(17.7)	(17.8)
Employment	0.014	0.016
	(7.4)	(7.8)
Housing	0.017	0.022
	(8.9)	(10.8)
Durable goods	0.036	0.042
	(19.2)	(21.2)
Social capital	0.055	0.048
	(29.3)	(23.9)
Social participation	0.010	0.010
	(5.3)	(5.1)
Total	0.187	0.199
	(100)	(100)

problem because the concept of 'poverty' itself makes the arbitrariness inevitable (S. Anand, 1977; Blank, 2008; Dercon, 2006; Sen, 1981; Townsend, 1979a). In fact, the introduction of fuzzy set measure itself needs to be understood as an attempt to model the inevitable arbitrariness itself rather than try to get rid of it like traditional approaches.

Therefore, one appropriate way to address the criticism can be a sensitivity analysis as comprehensive as possible. First, the robustness of TF measure for two income thresholds is examined as follows:

1. Lower threshold under which one can be determined to be *definitely poor* is changed from 30% of median income to 90% by 10%. The range is set as same interval from 60% of median income on which the analyses in previous sections are based.

2. Higher threshold over which one is *definitely nonpoor* is varied from 120% of median income to 180% by 10% (previous analyses are based on 150% line.).

3. Finally, the entire combinations of above two variations are examined.

For lower threshold change, table 4.15 shows that TF measure does not react to the change sensitively. In spite of the wide range of change in the number of *definitely poor* people in terms of income, the difference of two extreme change is only 0.005. Also, no significant changes in dispersion statistics are found. Table 4.16 also shows that there are no significant

Table 4.15: TF measure's sensitivity analysis for income lower threshold

	30%	40%	50%	60%	70%	80%	90%
Mean	0.196	0.195	0.194	0.193	0.192	0.192	0.191
Median	0.183	0.182	0.181	0.179	0.179	0.178	0.178
Minimum	0.016	0.016	0.016	0.016	0.016	0.017	0.017
1st Quartile	0.132	0.131	0.130	0.130	0.130	0.129	0.129
3rd Quartile	0.247	0.246	0.245	0.243	0.242	0.241	0.240
Maximum	0.666	0.668	0.670	0.671	0.671	0.670	0.670
S.D.	0.087	0.086	0.086	0.086	0.085	0.085	0.085
# of 'definitely poor'	197	392	760	1,250	1,779	2,322	2,754

change in descriptive statistics, though the number of *definitely nonpoor* people for income vary considerably. In sum, table 4.17 shows that TF measure is not so sensitive to the change in both income thresholds. This robustness can be attributed to the relatively low contribution of income variable to aggregate index that can be seen in table 4.13 or 4.14.

Second, for saving variable, it is not necessary to test lower threshold since it is reasonable to assume that no saving implies *definitely poor* condition when 64% of the sample (4,084

Table 4.16: TF measure sensitivity for income higher threshold

	120%	130%	140%	150%	160%	170%	180%
Mean	0.192	0.192	0.193	0.193	0.194	0.194	0.195
Median	0.178	0.179	0.179	0.179	0.180	0.181	0.181
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016
1st Qu.	0.128	0.129	0.129	0.130	0.131	0.131	0.132
3rd Qu.	0.242	0.242	0.243	0.243	0.243	0.244	0.244
Maximum	0.669	0.670	0.670	0.671	0.671	0.671	0.671
S.D.	0.086	0.086	0.086	0.086	0.085	0.085	0.085
# of 'definitely nonpoor'	2,440	2,120	1,807	1,574	1,348	1,160	999

Table 4.17: TF measure sensitivity for the combination of two income threshold variations

		Higher threshold						
		120%	130%	140%	150%	160%	170%	180%
	30%	0.192	0.194	0.195	0.196	0.197	0.198	0.199
	40%	0.192	0.193	0.194	0.195	0.196	0.197	0.198
	50%	0.192	0.193	0.194	0.194	0.195	0.195	0.196
Lower threshold	60%	0.192	0.192	0.193	0.193	0.194	0.194	0.195
	70%	0.191	0.192	0.192	0.193	0.193	0.193	0.194
	80%	0.191	0.191	0.191	0.192	0.192	0.192	0.193
	90%	0.191	0.191	0.191	0.191	0.192	0.192	0.192

out of 6,339) has no saving. Thus, the analysis is conducted only for higher threshold[104] as

[104] The case that the higher threshold is decreasing is analyzed in table A8 in Appendix A, and the result

table 4.18. Still, the first row of table 4.18 shows that TF measure is quite robust to the

Table 4.18: Sensitivity analysis for saving higher threshold, increasing case

	£1,200	£2,400	£3,600	£4,800	£6,000	£7,200	£8,400
Mean	0.193	0.194	0.194	0.194	0.194	0.194	0.194
Median	0.179	0.180	0.180	0.180	0.180	0.180	0.180
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016
1st Quantile	0.130	0.130	0.131	0.131	0.131	0.131	0.131
3rd Quantile	0.243	0.243	0.243	0.244	0.244	0.244	0.244
Maximum	0.671	0.671	0.671	0.671	0.671	0.671	0.671
S.D.	0.086	0.085	0.085	0.085	0.085	0.085	0.085
# of 'definitely nonpoor'	1,404	790	389	267	191	105	81

change in threshold level. This fundamentally results from the fact that very small numbers of people have savings. It implies that saving can be considered as an expensive item which only the small number of nonpoor people can have, such as a luxury car. Since many people do not have it, it contributes very little to the sense of relative deprivation for the whole society. Even in the sense of absolute deprivation, this also makes sense in that an item that belongs to only small proportion of population should not be considered as an appropriate indicator of *basic* standard of living, which implies the weight of the item needs to be very close to zero in multidimensional perspective.

Finally, several variations of inheritance threshold are tested. For the same reason to saving variable, only higher threshold is changed. Table 4.19 shows that the variation does not influence TF measure at all[105], which is consistent with the fact that the weight of the variable in table 4.12 is almost zero.

is still very robust.

[105] Same to the saving variable case, sensitivity analysis for decreasing threshold case in table A9 in Appendix A also shows strong robustness of TF measure.

Table 4.19: Sensitivity analysis for inheritance higher threshold, increasing case

	£6,000	£12,000	£18,000	£24,000	£30,000	£36,000	£42,000
Mean	0.193	0.193	0.193	0.193	0.193	0.193	0.193
Median	0.179	0.179	0.179	0.179	0.179	0.179	0.179
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016
1st Quantile	0.130	0.130	0.130	0.130	0.130	0.130	0.130
3rd Quantile	0.243	0.243	0.243	0.243	0.243	0.243	0.243
Maximum	0.671	0.671	0.671	0.671	0.671	0.671	0.671
S.D.	0.086	0.086	0.086	0.086	0.086	0.086	0.086
# of 'definitely nonpoor'	66	48	40	32	28	24	22

4.2 *TOTALLY FUZZY AND RELATIVE* MEASURE

Cheli and Lemmi (1995) suggest *Totally Fuzzy and Relative* method (TFR) for improving TF method. Though the authors accept that poverty as a fuzzy concept is an appropriate idea, they argue that TF measure has two nontrivial pitfalls: 1) The arbitrariness of setting two thresholds, and 2) linear functional form. In order to overcome these weaknesses, it is suggested that cumulative distribution function should be utilized as the functional form of membership function because not only setting two thresholds is more arbitrary than the traditional approach (Miceli, 1998), but also it ignores the notion of relative deprivation, which gets more important in a situation where generally higher economic well-being is already achieved (Townsend, 1985). In addition, they show a new way of constructing membership function for ordinal variables by which the problem of extreme modalities in the variables can be addressed. By introducing these changes, the suggested measure becomes "totally relative" because an individual's membership to fuzzy subset *poor* is determined entirely by its relative position in the population.

4.2.1 Formulas for membership function

Continuous variables: Assuming that a cumulative distribution function of indicator x is $F_x(\cdot)$, then the membership function of individual i for indicator j can be determined as follows:

$$\mu_j(i) = F_j(i) \quad \text{or} \quad 1 - F_j(i) \tag{4.6}$$

The choice between the two depends on the characteristic of the variable. For example, since it is reasonable to say that people with higher income could have more functionings, which can be translated to less inclusion to poverty, the latter is appropriate for income variable[106]. On the contrary, the former would be better fit, for instance, for 'air pollution level'.

[106] In addition, Cheli (1995) argues that in order to facilitate the comparison with conventional headcount ratio, an exponent α needs to be introduced for income variable. So, in modified form, the membership function for income variable is $\mu_j(i) = [1 - F_j(i)]^\alpha$ $(\alpha \geq 1)$, where α determines the relative weight of the poorer with respect to the less poor. However, as this introduction of new exponent is not so meaningful for this study, it will not be considered in this research.

Ordinal variables: If we also use cumulative distribution function as the functional form for ordinal variables like continuous variable case, then often the membership function cannot have a range from zero to one. This can be illustrated as the third column of table 4.20. Assuming that 1 indicates the highest level of functioning, the membership function shows

Table 4.20: An example of the membership function for ordinal variable

Level	Rel.freq.	Cum.freq.
1	0.6	0.6
2	0.15	0.75
3	0.1	0.85
4	0.1	0.95
5	0.005	1.00

that people in level 1 are quite closely included in the fuzzy subset *poor* (0.6), though they are actually never at risk with respect to the indicator. Therefore, the functional form needs adjustment. Still applying the same concept of cumulative distribution function, basic formula for ordinal variables by Cheli and Lemmi (1995) can be written as follows:

$$\mu_j(i) = \mu_{j^{(k)}}(i) = \begin{cases} 0 & \text{if } k = 1 \\ \mu_{j^{(k-1)}}(i) + \frac{F(j_i^{(k)}) - F(j_i^{(k-1)})}{1 - F(j_i^{(1)})} & \text{otherwise} \end{cases} \tag{4.7}$$

where k indicates the level of functioning in a ordinal indicator j for individual i, and $F(\cdot)$ is cumulative distribution function of the indicator j. The formula fundamentally differs from TF measure in that now the membership function only includes information on the number of people belongs to each level, which implies that the level of functioning for one specific people can be determined 'totally relatively'. Since $k = 1$ denotes the highest level of functioning, it needs caution to apply the formula. For example, if a membership function for 'frequency of social interaction' variable in which 1 denotes "never" and 5 "once a day" is calculated, the variable should be recoded to 1 being "once a day" and 5 "never".

Binary variables: For binary variable, the same formula to TF method is used (see equation 4.2).

Formula for weight function: For TFR measure, Cheli and Lemmi (1995) suggest a more generalized version of weighting system proposed by Cerioli and Zani (1990) as follows:

$$w_j = \frac{\ln\left(\dfrac{1}{\mu_j}\right)}{\displaystyle\sum_{k=1}^{K} \ln\left(\dfrac{1}{\mu_k}\right)} \tag{4.8}$$

where μ_j denotes the average membership function for indicator j and K equals to the total number of dimensions.

From formula 4.8, it can be easily known that the weight each indicator would have in aggregation is inversely proportional to their average membership function, not the frequency of *definitely poor* symptom like TF measure (see equation 4.3). This is 'generalized' in the sense that fundamentally the weighting system shares the same idea to TF measure's weight: relative deprivation. In other words, this weight system also assigns higher weight for attributes that most of the population have, the lack of which causes more severe sense of relative deprivation. However, since the membership functions of TFR measure are calculated from the cumulative distribution functions of each indicator, it can be said that the weights in TFR measure can reflect the idea of relative deprivation more fully.

TFR measure: TFR measure for an individual i is calculated as follows:

$$\mu(i) = \frac{\displaystyle\sum_{j=1}^{K} w_j \times \mu_j(i)}{\displaystyle\sum_{j=1}^{K} w_j} \tag{4.9}$$

where K is the number of indicators and $\mu_j(i)$ is the membership function of individual i for indicator j, and the aggregate TFR measure is the average of $\mu(i)$, just like TF measure.

$$\mu = \frac{1}{n} \sum_{t=1}^{n} \mu(t) \tag{4.10}$$

n is population size.

4.2.2 General interpretation

The average of TFR measure for the data is 0.166, which is a little lower than TF measure (0.193), and descriptive statistics in table 4.21 and the histogram 4.6 show that the distribution is wider compared to TF measure (see table 4.2 and figure 4.1 in previous section.), though no remarkable difference is found. Considering the calculation process, this variation is very likely to come from the new introduction of cumulative distribution function instead of two threshold lines, because it could provide more information about the distribution of the variables, especially about observations with extreme values. For example, in TF measure, people with income more than £37,673 are considered just same to the people with exactly that income. However, in terms of relative deprivation, this is not a reasonable idea since people would feel more relative deprivation if there are many people above that income. Therefore, it can be conjectured that TFR measure can reflect the unequal distribution of the economic resource variables better than TF measure. In addition, the new information on the distribution of the people across the levels in ordinal variables (equation 4.7) can also influence the change.

Table 4.21: Descriptive statistics for TFR measure

Statistic	Value	
Mean	0.166	
Median	0.147	
Range	0.020	0.798
Quartiles	(1st) 0.098	(3rd) 0.211
S.D.	0.091	

To confirm this conjecture, table 4.22 is constructed[107]. It shows that TFR measure is more widely distributed than TF measure. Figure 4.7 also indicates that TFR measure's membership function describes the distribution of economic resources variables more realistically, compared to figure 3.1 in chapter 4. However, this does not necessarily mean that

[107] The table includes three indicators: household annual income, saving amount a year, and inheritance.

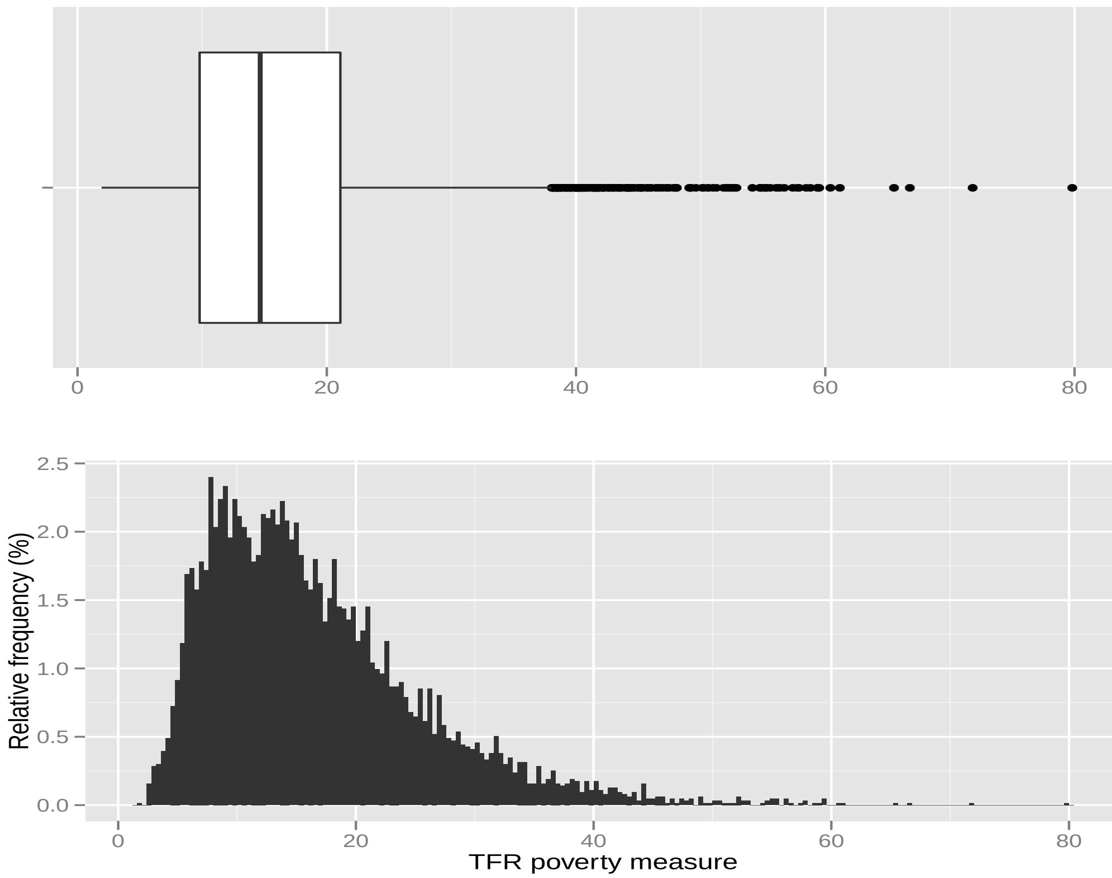

Figure 4.6: TFR measure distribution

TFR measure is better measure of poverty than TF measure because the contribution of economic resources dimension to overall measure is limited due to the small weight which reflects relatively concentrated distribution of the dimension.

Following table 4.23 shows the relationships between dimensions for TFR measure. It turns out that economic resources dimension has negative relationship with social capital which is negatively related to employment, and still all the correlations with economic resources dimension are under 0.4. However, social participation has relatively strong relationship with the dimension. For housing dimension, economic resources show relatively weak relationship with it, which is similar to the TF measure case (see table 4.3). It is also noticeable that durable goods are most strongly correlated with employment, not economic

Table 4.22: Membership function for continuous variables

TF measure			TFR measure		
Statistic	Value		Statistic	Value	
Mean	0.009		Mean	0.017	
Median	0.010		Median	0.016	
Range	0.000	0.015	Range	0.000	0.033
Quartiles	(1st) 0.005	(3rd) 0.014	Quartiles	(1st) 0.010	(3rd) 0.024
S.D.	0.005		S.D.	0.009	

resources. Still, social participation is very strongly correlated with employment dimension like TF measure, which again confirms the theory of social exclusion.

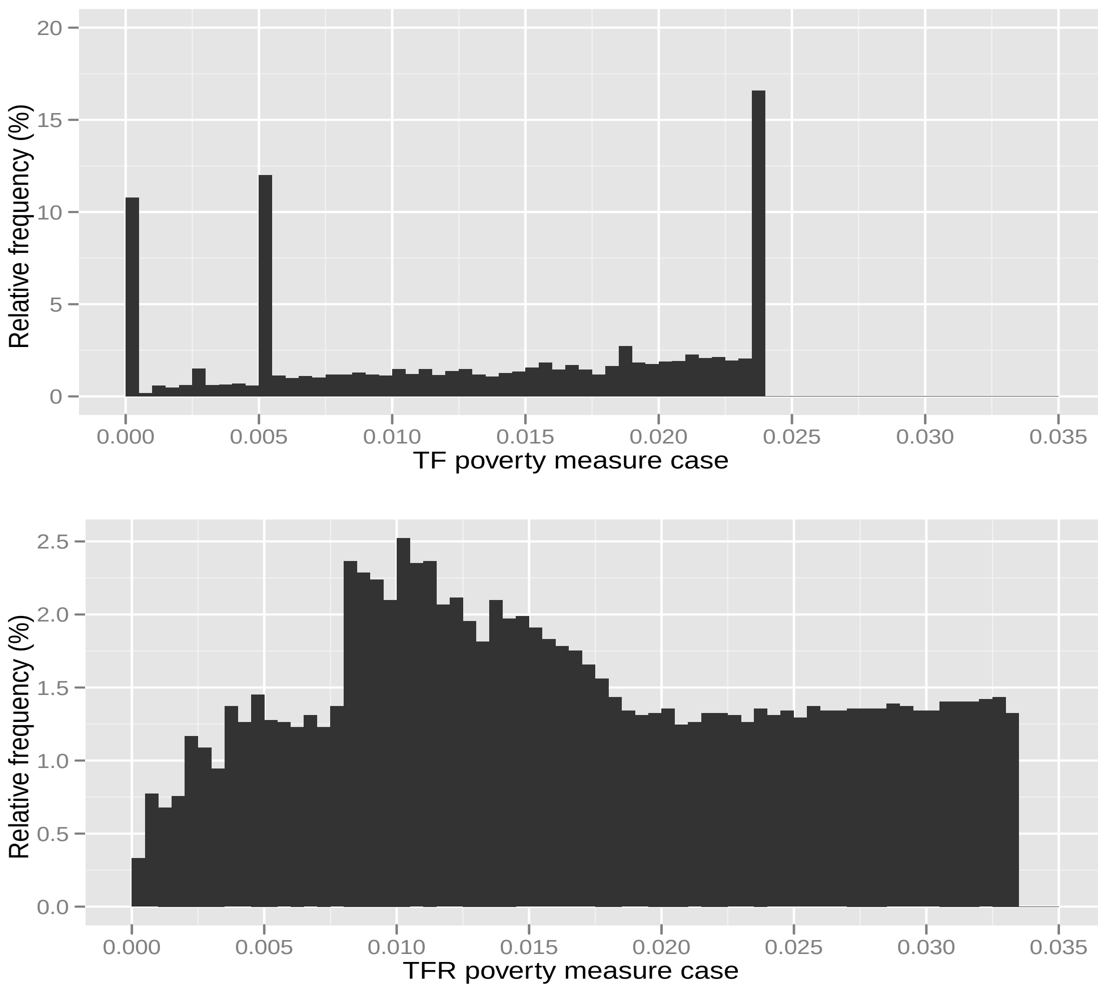

Figure 4.7: Membership function for economic resources dimension on same scale

Table 4.23: Correlation coefficients for each dimension's membership functions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) Economic resources	1.000						
	(0.000)						
(2) Health	0.255	1.000					
	(0.000)	(0.000)					
(3) Employment	0.377	0.388	1.000				
	(0.000)	(0.000)	(0.000)				
(4) Housing	0.164	0.075	0.010	1.000			
	(0.000)	(0.000)	(0.446)	(0.000)			
(5) Durable goods	0.273	0.167	0.355	0.043	1.000		
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)		
(6) Social capital	-0.026	0.109	-0.034	0.069	0.019	1.000	
	(0.040)	(0.000)	(0.006)	(0.000)	(0.136)	(0.000)	
(7) Social participation	0.386	0.327	0.795	0.030	0.335	-0.009	1.000
	(0.000)	(0.000)	(0.000)	(0.019)	(0.000)	(0.488)	(0.000)

* The numbers in parenthesis are $p - values$ from the significance test.

* All dimensions' membership functions range from zero ("definitely non-poor") to one ("definitely poor").

4.2.3 Poverty profile for subgroups

Poverty decomposition by diverse subgroups is described here. The left columns of the table 4.24 are built by computing TFR measure for four age groups. It shows that the age group 25~49, the primary working age, shows the least propensity to poverty, and people younger than 24 have the highest level, which is different from the observation in TF measure. In terms of distribution, figure 4.8 depicts almost same condition to figure A3 in Appendix A.

Table 4.24: Poverty profile by age groups and country

Age	TFR index	Country	TFR index
16~24	0.218	Britain	0.162
25~49	0.141	Wales	0.171
50~64	0.155	Scotland	0.167
65 or more	0.211	Northern Ireland	0.175

The right column of table 4.24 demonstrates that the differences in the propensity to poverty among countries are still no more than 10%[108] with Britain maintaining its position. For labor force status (table 4.25), it turns out that the general picture is concurrent to common sense in that people with disability have the highest propensity. Also, students appear to be almost the same as poor as the people with disability. In terms of marital status, widowed people show the highest level of TFR measure while married group has the lowest[109].

The decomposition by gender shows that gender inequality does exist. The results from breakdown by housing tenure is still not easy to understand intuitively as people who own housing shows higher propensity to poverty than people who own housing with mortgage in table 4.26. Since the difference between the two groups is not affected greatly by the difference in housing quality[110], further analysis is required[111]. Following analysis for economic

[108] Assuming Britain's TFR measure is one, Northern Ireland is 1.08.

[109] Although 'civil partnership' group is least inclined to be poor by the TFR measure, the sample size is too small - only five - to make the inference.

[110] In table A10 in Appendix A for housing quality dimension, it appears that the propensity for 'owned with mortgage' group is higher than 'owned' group, which means that basically people with mortgage live in inferior quality housing.

[111] As table A11 in Appendix A shows, calculation without housing dimension still does not answer why

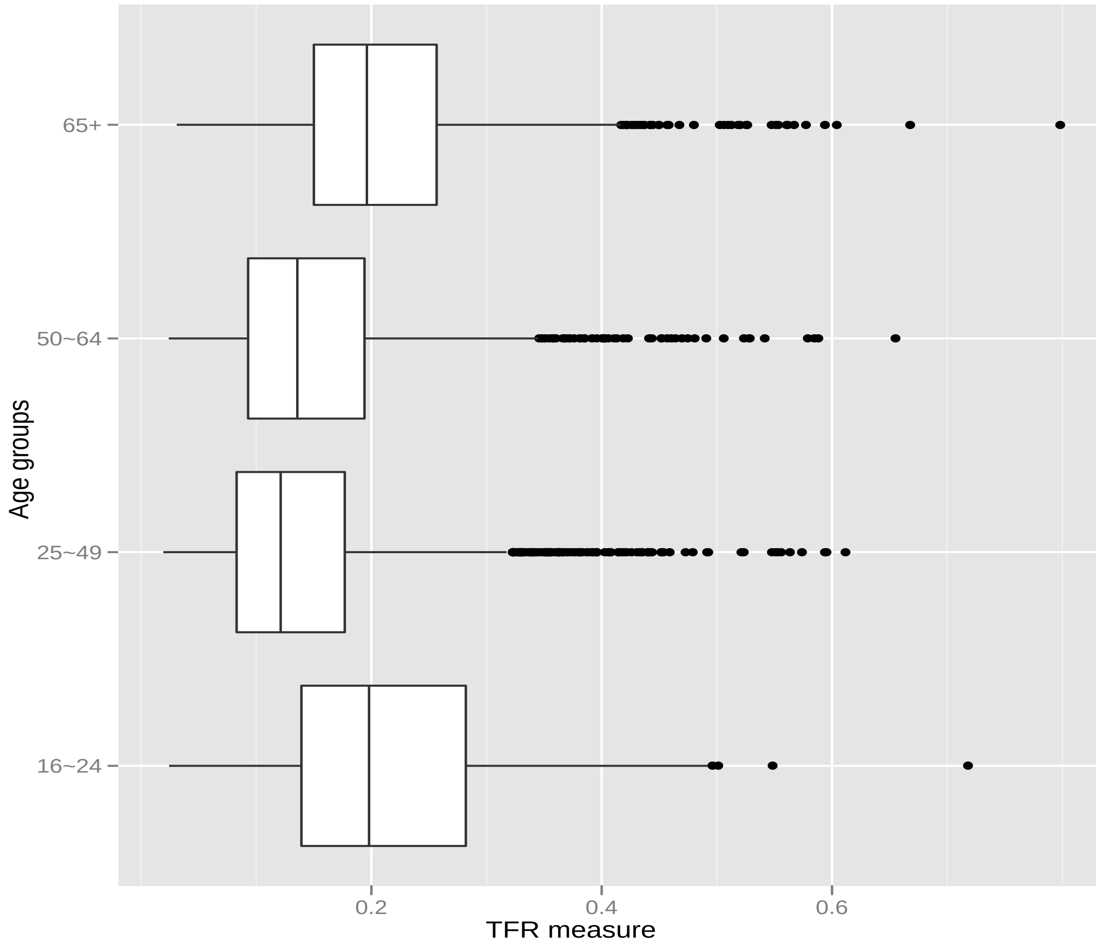

Figure 4.8: Poverty profile for age groups

resources dimension (table 4.27) shows that the difference in economic resources dimension could be one factor for the difference. In regional comparison, it is shown that the TFR measures shows that the regions are closely distributed, which is consistent with the fact that the difference between countries is not big, either. Still, all the regions in Britain except North West region have lower TFR index than the other three countries. (figure 4.9)

For different occupational classification in table 4.28, general pattern seems to be congruous with common expectation, in which managers are least likely to have a higher tendency

this happens.

Table 4.25: Poverty profile for labor force and marital status

Labor force status	TFR measure	Marital status	TFR measure
Self-employed	0.125	Married	0.143
Employed	0.127	Widowed	0.219
Unemployed	0.219	Divorced	0.185
Retired	0.206	Separated	0.168
Student	0.238	Never married	0.207
Disabled	0.239	Civil partnership	0.108

Table 4.26: Poverty profile for gender, housing tenure and regions

Housing tenure		Value	Regions	Value
Owned		0.174	London	0.160
Owned with mortgage		0.122	South East	0.153
Rented from local authority		0.230	South West	0.163
Rented from housing association		0.234	East Anglia	0.166
Rented from employer		0.192	East Midlands	0.161
Rented private housing unfurnished		0.200	West Midlands	0.159
Rented private housing furnished		0.228	North West	0.168
Gender	Male	0.143	Yorks & Humberside	0.148
	Female	0.159	North East	0.156

to poverty, and agriculture shows the highest. For different household types, it is interesting to see that single household, whether they are elderly or not, is in worse situation than lone parent with dependent children, which is contradict to previous findings in U.S. (McLanahan,

Table 4.27: Comparison of economic resources dimension for housing-owned groups

Group	Economic resources dimension
Owned	0.022
Owned with mortgage	0.019

Figure 4.9: TFR measure by Regions & Countries

1985; Snyder & McLaughlin, 2004; U.S. Census Bureau, 2009; Zinn, 1989).[112]

[112]In U.S., the academic and policy interests on the subject focus on the African-American female-headed family. A comprehensive historical review of the subject can be found in Patterson (1994).

Table 4.28: Poverty profile for occupations and household type

Occupations	Value	Household type	Value
Managers	0.109	Single non-elderly	0.192
Professionals	0.115	Single elderly	0.235
Technical Prof.	0.122	Couple: no child	0.152
Clerks	0.127	Couple: dependent child	0.133
Service workers	0.144	Couple: non-dependent child	0.135
Agriculture	0.185	Lone parent: dependent child	0.175
Craft related	0.130	Lone parent: non-dependent child	0.169
Machine operators	0.140	2+ unrelated adults	0.188
Elementary occup.	0.145	Other types	0.179

It is found again in table 4.29 that metropolitan areas have no less propensity to poverty than 'other regions.' Only London and Greater Manchester show the same level to the 'other regions' for TFR measure (see figure A5 in Appendix A). Finally, the profile for household size still shows the non-linear relationship between household size and poverty degree shown by TF measure.

Table 4.29: Poverty profile for metropolitan areas and household size

Metropolitan area	Value	Household size	Value
London	0.140	1	0.215
West Midland Conurbation	0.157	2	0.157
Greater Manchester	0.140	3	0.143
Merseyside	0.179	4	0.133
South Yorkshire	0.159	5	0.146
West Yorkshire	0.154	6	0.170
Tyne & Wear	0.170	7	0.219
Other regions	0.140	8	0.263

4.2.4 Poverty profile for dimensions

Following table 4.30 shows the weights for dimensions as percentage terms.

Table 4.30: Weights used in TFR measure calculation

Dimension	Variable	Weight(%)	Dimension	Variable	Weight(%)
Economic resources	HH income	1.1	Durable goods	Color TV	7.4
	Amount saved	1.1		VCR	4.5
	Amount inherited	1.1		Freezer	4.8
	Financial situation	1.1		Washing machine	5.0
Health	Health status	1.0		Dish washer	0.8
	Satisf. with health	1.0		Microwave	4.0
	Inhibits activ.	1.5		Home computer	1.8
Employment	Perm./temp. job	1.3		CD player	2.8

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Table 4.30 – continued from previous page

Dimension	Variable	Weight(%)	Dimension	Variable	Weight(%)
	Security satisf.	0.9		Telephone	3.8
	Overall satisf.	0.8		Cellphone	3.3
Housing	No heating	5.6		Internet	1.4
	Leaky roof	5.6		Cars	0.8
	Short space	2.7	Social capital	Feed visitors	2.7
	Neighb. noise	3.6		Talking to neighb.	1.4
Housing	Street noise	3.1		Meeting people	1.3
	Not enough light	5.0		Satis.social life	1.0
	Condensation	4.0		Meeting friends	1.4
	Damp walls	4.5	Social participation	Attend groups	0.4
	Rot in floors	5.2		Voluntary works	0.3
				Union member	1.0

Comparing to the TF measure (see table 4.12 in previous section), the most noticeable change is the weight for savings and inheritance get much bigger (from 0.5 to 1.1 and 0.02 to 1.1, respectively). The changes can be attributed to the changes in the functional form of membership function. Whereas the information that there is huge variation in people's saving or inheritance is simply ignored in TF measure, the introduction of cumulative distribution function makes it possible to integrate the information in calculation, which results in bigger relative deprivation for people. However, this change is not manifested in one direction since having savings or inheritance is still quite rare event, which means smaller relative deprivation in the variables. For health dimension, the contribution of the indicators decreases greatly, by more than a half at least, especially the weight for health status is reduced by a factor of four. This also reflects the fact that generally high level of health status is reflected in the measure more appropriately due to the new weight calculation method. That is, if people are very healthy on average, then it should diminish the contribution of health dimension in the aggregate index because of the "totally relative" characteristic of the index. The weights for housing and durable goods dimensions are also reduced, but this is simply due to the calculation process which reflects the change in overall weight (see equation 4.8). In social capital dimension, it is noticeable that all the weights except the binary "intention to feed visitors" variable are reduced considerably, by a sixth for "the frequency of meeting people" indicator. This can also be attributable to the introduction of cumulative distribution like health dimension. The weights for social participation do not differ much probably because the distribution of the indicators are extremely skewed to the membership function value 1. For instance, 82.19% of the sample answer that they never participate in voluntary work[113].

Still, as the fact that the weight of income variable is a seventh of the weight of color TV is not easy to accept, simple illustration can be helpful to understand the weights correctly. Two membership functions for employed and disabled group is illustrated in table 4.31. The case is chosen because above analysis of subgroup decomposition shows that there is a great gap in TFR measure between these two groups, which makes the contrast more intuitively acceptable. This exercise helps us to interpret the weights as discriminating power of an individual between *definitely poor* and *definitely nonpoor*.

Table 4.31: Membership function decomposition by gender

Dimension	Variable	raw MF		weighted MF	
		Employed	Disabled	Employed	Disabled
Economic resources	HH income	0.378	0.695	0.004	0.008
	Amount saved	0.397	0.677	0.004	0.008
	Amount inherited	0.398	0.675	0.004	0.008
	Financial situation	0.471	0.729	0.005	0.008
Health	Health status	0.459	0.898	0.005	0.009
	Satisf. with health	0.485	0.867	0.005	0.009
	Inhibits activ.	0.255	0.878	0.004	0.013

Continued on next page

113 74.87% says that they never attend local group activities, and only 28.71% is union members.

Table 4.31 – continued from previous page

Dimension	Variable	raw MF		weighted MF	
		Employed	Disabled	Employed	Disabled
Employment	Perm./temp. job	0.006	0.971	0.0001	0.013
	Security satisf.	0.264	0.980	0.002	0.008
	Overall satisf.	0.333	0.980	0.003	0.007
Housing	No heating	0.026	0.057	0.001	0.003
	Leaky roof	0.032	0.063	0.002	0.004
	Short space	0.207	0.244	0.006	0.007
	Neighb. noise	0.110	0.178	0.004	0.006
	Street noise	0.147	0.209	0.005	0.007
Housing	Not enough light	0.040	0.083	0.002	0.004
	Condensation	0.076	0.138	0.003	0.006
	Damp walls	0.055	0.115	0.002	0.005
	Rot in floors	0.036	0.069	0.002	0.004
Social Capital	Feed visitors	0.148	0.261	0.004	0.007
	Talking to neighbors	0.500	0.397	0.007	0.005
	Meeting people	0.476	0.390	0.006	0.005
	Satisf.social life	0.545	0.735	0.005	0.007
	Meeting closest frnd.	0.464	0.396	0.006	0.005
Social Participation	Attend groups	0.801	0.834	0.003	0.003
	Voluntary work	0.849	0.873	0.003	0.003
	Union membership	0.186	0.977	0.002	0.010

First of all, not surprisingly from raw MF, employed people are much less inclined to be poor than disabled people in all dimension. Especially the difference in employment dimension shows that disabled group is almost definitely poor in terms of permanent job, which is closely related to the high propensity in the other indicators. For economic resources dimension, employed group has almost half of the propensity of disabled group in terms of objective indicators like income or saving, which also manifested in the perception indicator of financial situation. The big difference in health indicator is even more easy to understand since one group is 'disabled'. However, membership function of the employed group on health status variable is almost 0.5, which is quite high considering the range of membership function. While there is not much difference in housing dimension, social capital dimension shows some interesting findings. On 'the frequency of talking to neighbors', 'the frequency of meeting friends or relatives', and 'the frequency of meeting the closest friend' variable, disabled group has less propensity to poverty. In a sense, this is understandable since disabled people cannot but depend more on social interaction than employed people, just for basic functionings such as cooking or transporting. In social participation dimension, two points stand out: 1) both groups are almost definitely poor in terms of social participation, and 2) union membership could reflect employment situation.

Secondly, weighted MF also provides some interesting insights on the two groups. It shows that employed people suffers from the perception of their financial situation more than their objective conditions imply in economic resources dimension, considering weighted M.F. for financial situation is the highest in the dimension. Also, it tells us that the employed group experiences the poverty as multidimensional concept mostly as the form of low social capital, for instance, they are less likely to talk to neighbors, meet friends or relatives, and meet the closest friend. On the other hand, for disabled people, it turns out that employment dimension is the biggest concern, accepting the highest weighted M.F. for "health inhibits activities" indicator is inevitable for the group. Especially, the remarkably high level of propensity for 'permanent job' indicator even seems to point out the exceptionally unequal job opportunity for the group. Thus, the weighted MF indicates that the policy priority for the disabled should be employment issue, specifically job stability[114].

[114] For the comparison between dimensions, see table A12 in Appendix A.

4.3 *INTEGRATED FUZZY AND RELATIVE* MEASURE

Although TFR measure tries to deal with concerns about the previous measure, still some problems remain to be addressed. Most of all, one important concern for TFR measure is that it is a 'totally relative' measure. For the context of economically advanced countries, this is not so problematic since the relatively higher level of economic resources in those countries can make it confident to assume that the absolute concept of poverty is much less important than the relative concept. However, as Sen (1983) argues, absolute concept still needs to take a crucial role in measuring poverty because in the dimension of capabilities, what eventually matters is whether one has capability or not, not how much capability one has compared to other people in the society. Quoting Adam Smith's example, he asserts that "a person needs leather shoes not so much to be less ashamed than others but simply not to be ashamed" (Sen, 1983). This means that the concept of poverty has different nature in different aspects - absolute in capability and relative in commodities or characteristics. That is, a commodity that is needed not to be ashamed in one society does not have to be a 'leather shoes', but by any stretch, people should not be ashamed. Thus, the 'totally relative' view on poverty needs to be modified to include an absolute notion in the space of capabilities. Besides, though the debates between relative and absolute perspective on poverty have not come up with a definitive answer (Hagenaars & Praag, 1985; Sen, 1985b; Townsend, 1979a, 1985), it cannot be denied at least that the entirely relative perspective on poverty is not realistic in developing countries' situation where a number of people are still suffering from absolute lack of resources, such as $1 a day[115].

Also, Cheli (1995) points that there is a technical problem in TFR measure - the mean of membership function for continuous variables is always 0.5, a problem that is inescapable due to the use of cumulative distribution function, which makes it difficult to compare the results of the fuzzy measures of poverty (Betti & Verma, 1998; Betti, Cheli, Lemmi, & Verma, 2005a, 2005b). In addition, they consider it debatable that the previous fuzzy set theory based measures contain both monetary indicators and non-monetary indicators in one

[115] *World Development Indicators 2011* reports that in 2005 PPP, 1.37 billion people still live on less than $1.25 a day, which is 25.2% of world population.(`http://data.worldbank.org/data-catalog/world-development-indicators`)

index, because they see that monetary variables still have a fundamental role in measuring poverty[116] (Betti et al., 2002, 2004).

To address these concerns, Betti, Cheli, Lemmi, and Verma (2005b) suggest a new measurement method based on the fuzzy set theory, "*Integrated Fuzzy and Relative* method" (IFR). Introducing the Lorenz curve which represents the share of the commodities received by all individuals less poor than the person concerned into the calculation (see figure A11 in Appendix A), this measure now can take into account both the relative position and the absolute share of each individual, which can make the measure more sensitive to the actual disparities in diverse dimensions (Betti, Cheli, Lemmi, & Verma, 2005a; Betti & Verma, 2008). Besides, the Lorenz curve is not only introduced for income indicators, but also for other indicators by grouping binary or ordinal variables.

4.3.1 Formulas for membership function

Fuzzy monetary measure: Let a Lorenz function of income indicator x is $L_x(\cdot)$ and cumulative distribution function $F_x(\cdot)$, then the membership function for individual i ("FM_i", stands for "Fuzzy Monetary") can be calculated by following formula 4.11[117], and the membership function for the population can be obtained as $FM = \frac{1}{n}\sum_{k=1}^{n} FM_i$.

$$FM_i = [1 - F_{income}(i)][1 - L_{income}(i)] \tag{4.11}$$

Since Betti and Verma (1998) argue that the equation needs to be adopted for monetary variables, the opposite case where the increase of $F_j(i)$ is considered as the decrease of well-being is not considered here.

The application here, however, is not simple because in this study monetary dimension is measured by four indicators, one of which is an ordinal variable. In order to integrate the four indicators[118], I adopt the same procedure as FS measure case below. First, the membership

[116] Boarini and d'Ercole (2006) even suggest that it is more appropriate to talk of "poverties" rather than poverty due to the incomplete match between income poverty and material deprivation.

[117] It is also argued that a parameter α needs to be induced to facilitate the comparison with traditional measure (Betti & Verma, 1998). However, as the arbitrariness of the parameter complicates the comparison between different measurement methods, this study does not follow the suggestion.

[118] The conclusions from analyses in this section do not change if only income variable is used for FM measure.

functions for each indicator are calculated by the equations used in TF method. Then the weights are computed using equation 4.12. Once the membership functions are integrated as one membership function using the weights, FM "score"[119] is obtained by subtracting the membership function from one. Finally FM measure is obtained by equation 4.11.

Fuzzy supplementary measure: For non-monetary indicators, the basic strategy is to group the indicators according to their underlying dimensions because the manner in which different indicators cluster together can be meaningful for measuring poverty (Betti & Verma, 1998; Betti, Cheli, Lemmi, & Verma, 2005b). After grouping, following procedures are applied *within each dimension.*

1. Using the formulas of TF method (see equation 4.1 and 4.2), the membership functions for each indicator are calculated.
2. Weight function for each indicator is computed by equation 4.12, where

$$w_j = w_j^a \times w_j^b \tag{4.12}$$

   a. $w_j^a = cv_j$, which is a coefficient of variation of indicator j's membership function.

   b. $w_j^b = \left(\dfrac{1}{1+\sum_{j'=1}^{K} \rho_{j,j'} | \rho_{j,j'} < \rho_H} \right) \left(\dfrac{1}{\sum_{j'=1}^{K} \rho_{j,j'} | \rho_{j,j'} \geq \rho_H} \right)$

      - $\rho_{j,j'}$ is a correlation coefficient[120] between indicator j and j''s membership func-

[119] More detailed explanation is presented in the calculation for Fuzzy supplementary measure.

[120] Different types of correlation coefficients are used in this analysis for different types of variables as follows:

Variable type	Continuous	Ordinal	Binary
Continuous	Pearson	Spearman[a]	Point-biserial[b]
Ordinal		Spearman	Polychoric[c]
Binary			Tetrachoric[d]

[a] Lehmann and D'Abrera (2006)
[b] Glass and Hopkins (1995)
[c] Bonett and Price (2005)
[d] Greer, Dunlap, and Beatty (2003)

Fundamentally, all above correlation coefficients are diverse variations of Pearson correlation coefficients (Glass & Hopkins, 1995; Lehmann & D'Abrera, 2006). So, the coefficients can be interpreted as representing the relationship of underlying unobservable continuous variables. However, Carroll (1961) points that the coefficients can be misleading unless joint distribution is considered carefully.

tions.

- ρ_H is determined by "the greatest gap" criterion (Betti & Verma, 1998), which means that the whole set of correlation coefficients is divided into two groups based on the biggest gap among the coefficients[121].

3. Using the weight and membership functions, a membership function for dimension δ can be calculated as follows:

$$\mu_\delta(i) = \frac{\displaystyle\sum_{k\in\delta} w_k \times \mu_k(i)}{\displaystyle\sum_{k\in\delta} w_k} \tag{4.13}$$

In order to integrate multiple dimensions, the same procedure for the weight in equation 4.12 is applied for each dimension. Let the weight $w_{\delta,total}$, then membership function for individual i is

$$\mu(i) = \frac{\sum w_{\delta,total} \times \mu_\delta(i)}{\sum w_{\delta,total}} \tag{4.14}$$

However, this is not a final measure of non-monetary poverty because Betti, Cheli, Lemmi, and Verma (2005b) go one step further and argue that the same procedure used in monetary variable case need to be adopted here to maintain consistency in the measurement method. Thus, they introduce a new concept of "score", which is an indicator of the lack of deprivation, as $S_i = 1 - \mu(i)$ (Betti & Verma, 1998, 2004). Using the score, the 'fuzzy supplementary (FS)' measure is computed as follows:

$$FS_i = [1 - F_{S_i}(i)][1 - L_{S_i}(i)] \tag{4.15}$$

where $F(\cdot)$ is a cumulative distribution function and $L(\cdot)$ is a Lorenz function. Similar to FM measure, the aggregate measure can be obtained by simple average over whole population.

IFR measure: For overall measure of poverty, the logic of fuzzy set theory is applied. Assume that we have clear criteria for distinguishing poor from non-poor in the two dimensions: monetary and non-monetary. Then, an individual can be classified as table 4.32. However, without those criteria, table 4.32 should be reconstructed as table 4.33 according to fuzzy set theory, where $\mu_{i,xy}$ is the membership function of individual i in $x \cap y$.

[121] Betti, Cheli, Lemmi, and Verma (2005b) assert that most of the time the second factor in equation b only contains correlation 1 (correlation with an indicator itself) by the greatest gap criterion.

Table 4.32: Situation of a hypothetic individual in traditional approach

Poverty dimension		Monetary	
		poor (0)	non-poor (1)
Non-monetary	poor (0)	0	0
	non-poor (1)	1	0

Source: Betti, Cheli, Lemmi, and Verma (2005b)

Table 4.33: Situation of an individual in fuzzy set approach

Poverty dimension		Monetary		
		poor (0)	non-poor (1)	total
Non-monetary	poor (0)	$\mu_{i,00}$	$\mu_{i,01}$	FS_i
	non-poor (1)	$\mu_{i,10}$	$\mu_{i,11}$	$1 - FS_i$
	total	FM_i	$1 - FM_i$	1

Since we already know FM_i and FS_i, an appropriate way to specify $\mu_{i,xy}$ enables us to analyze the fuzzy poverty measure. Among the four specifications for fuzzy set operations utilized frequently (Klir & Yuan, 1995), Betti and Verma (2004) argue for the combination of 'standard' and 'bounded' operation[122]. By this "composite operation", table 4.33 can be

[122]In standard operation, the intersection of fuzzy sets is defined by the minimum of membership functions as $\mu_{A \cap B}(x) = min[\mu_A(x), \mu_B(x)]$, the union by the maximum as $\mu_{A \cup B}(x) = max[\mu_A(x), \mu_B(x)]$, and the complement by the subtraction as $\mu_{Ac}(x) = 1 - \mu_A(x)$ (De Morgan triplets) (Celikyilmaz & Türksen, 2009). This is intuitively acceptable for *poor* or *non-poor* for both dimensions because it provides the largest intersection among the four operations. In traditional set theory, if someone belongs to *poor* in both dimensions, it is more certain to identify the person *poor*, which means that it is desirable for a multidimensional poverty measure to maximize intersection. However, for off-diagonal cases in table 4.33, the standard operation often does not work just because of the reason. Since it produces the largest intersection, sometimes it makes the marginal total of membership function over 1, which is in conflict with the substantive requirements of the fuzzy set theory (Betti, Cheli, & Verma, 2006). So, Betti and Verma (2004) propose that bounded operation can complement the standard operation, which defines intersection as $\mu_{A \cap B}(x) = max[0, \mu_A(x) + \mu_B(x) - 1]$.

re-written as table 4.34.

Table 4.34: Fuzzy joint distribution by composite operation

Poverty dimension		Monetary		
		poor (0)	non-poor (1)	total
Non-monetary	poor (0)	$min(FM_i, FS_i)$	$max(0, FS_i - FM_i)$	FS_i
	non-poor (1)	$max(0, FM_i - FS_i)$	$min(1 - FM_i, 1 - FS_i)$	$1 - FS_i$
	total	FM_i	$1 - FM_i$	1

Source: Betti, Cheli, Lemmi, and Verma (2005b)

From table 4.34, the intersection of monetary poor and non-monetary poor - $min(FM_i, FS_i)$ - is considered as "manifest" poverty which represents the propensity to both monetary and non-monetary poverty. As this implies a situation where two poverty phenomena happen simultaneously to one household, it can be regarded as more intense poverty. On the other hand, the complement of the 'non-poor for both dimension' - $1 - min(1 - FS_i, 1 - FM_i) = max(FS_i, FM_i)$ - can be called "latent" deprivation that indicates an individual is subject to at least one of the two aspects of poverty (Betti & Verma, 1998, 2004; Betti, Cheli, Lemmi, & Verma, 2005a).

4.3.2 General interpretation

Simple descriptive statistics for the two IFR measures are in table 4.35. The average level of propensity to monetary poverty is higher than that of non-monetary dimension. Thus, it can be said that monetary dimension presents more problem for the population than non-monetary dimension. Besides, the mean of FS measure is smaller than the median of FM measure, which suggests that the distribution of FM measure heads more toward membership function value one, *definitely poor*. Considering these two observations, it can be conjectured that FM measure needs to be the focus for policies more than FS measure. Also, from figure 4.10 and figure 4.11, it can be known that the histogram of FM measure has more thick right tail, which indicates that more people have higher propensity to poverty with respect

Table 4.35: Descriptive statistics for IFR measures

Statistic	FM measure		FS measure	
Mean	0.423		0.351	
Median	0.386		0.274	
Range	0.000	1.000	0.000	1.000
Quartiles	(1st) 0.117	(3rd) 0.708	(1st) 0.071	(3rd) 0.593
S.D.	0.320		0.304	

to monetary dimension. The correlation between the two measures is very high and positive, 0.991, which is statistically significant at $p < .000$.

As the complexity of the calculation procedure of FS measure makes the interpretation of high correlation coefficient less intuitive, the relationship needs to be analyzed to more detailed level. Table 4.36 shows the correlations between FM measure and the membership functions for each dimension.

The first interesting finding from table 4.36 may be the fact that all the correlation coefficients except one are much smaller than a half. Even the relationship between FM measure and employment dimension - in other words, income and being employed - is not particularly strong. Second, as in TFR measure, it is found again that the relationship between employment and social participation is quite strong ($r = 0.828$). Thirdly, there are three statistically insignificant correlations - between FM measure and social capital, employment and housing, and social participation and housing, which seems to be unexpected because it is repeatedly argued in the studies of social exclusion that they are closely interconnected[123], especially income and social capital (Berman & Phillips, 2000; Robila, 2006; Room, 1995). For further understanding, table 4.37 shows the correlations between FM measure and the membership functions of the indicators that construct social capital dimension (numbers in parenthesis are $p - value.$). It turns out that three out of the five indicators have negative

[123] Using seven west European countries' data, Paugam (1996) finds that the lack of job security is highly correlated with weak social ties and poor housing conditions in U.K.

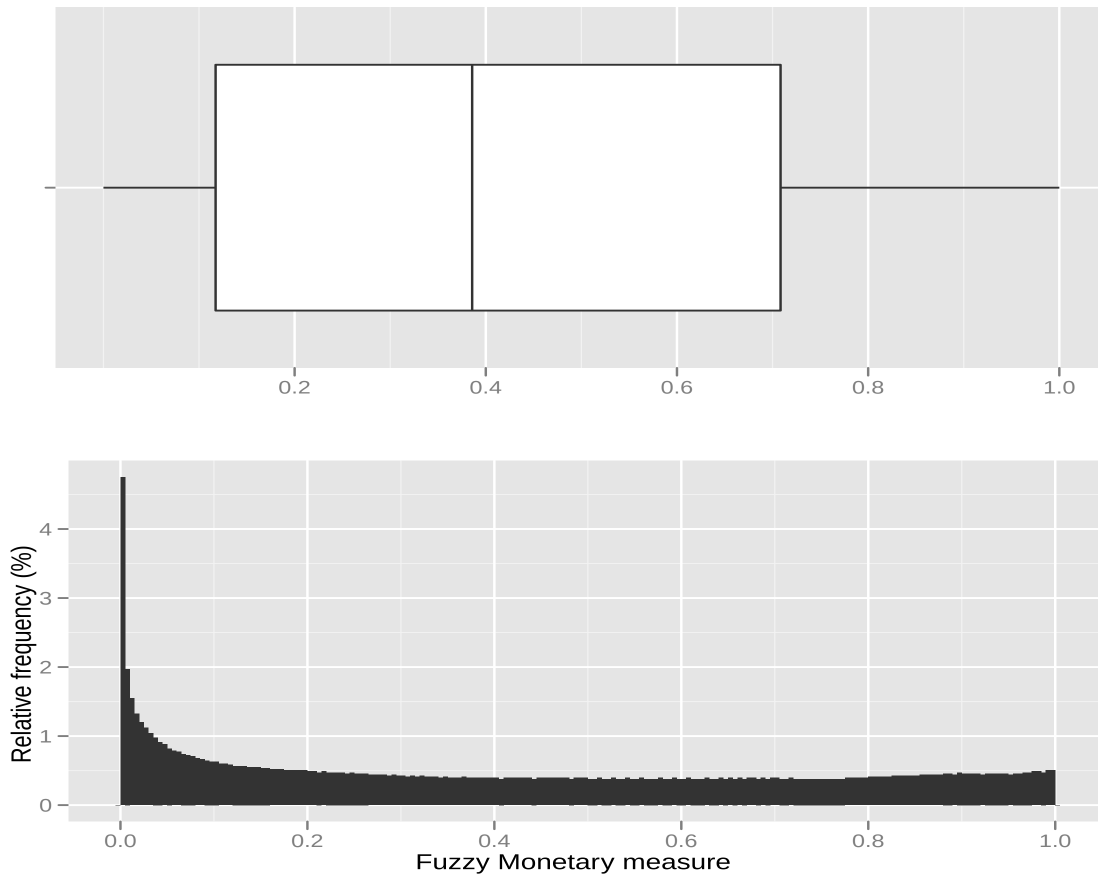


Figure 4.10: Distribution of FM measure

relationship with FM measure, with generally low strength of relationships. Though further analysis is beyond the scope of this dissertation, this finding seems to affirm the complexity of the multidimensional poverty.

In order to understand more general picture that IFR measure is describing, it is necessary to set a perspective on poverty (see table 4.34) and integrate above two measures based on it. Adopting the concept of *manifest* poverty, the population can be described as table 4.38 and figure 4.12. Since the *manifest* poverty implies the common propensity to poverty in both measures, in other words, both monetary and non-monetary aspects, this number can be interpreted as the propensity to more 'intense' poverty. This can be explained by the concept of intersection in set theory - in order to experience 'more intense' poverty

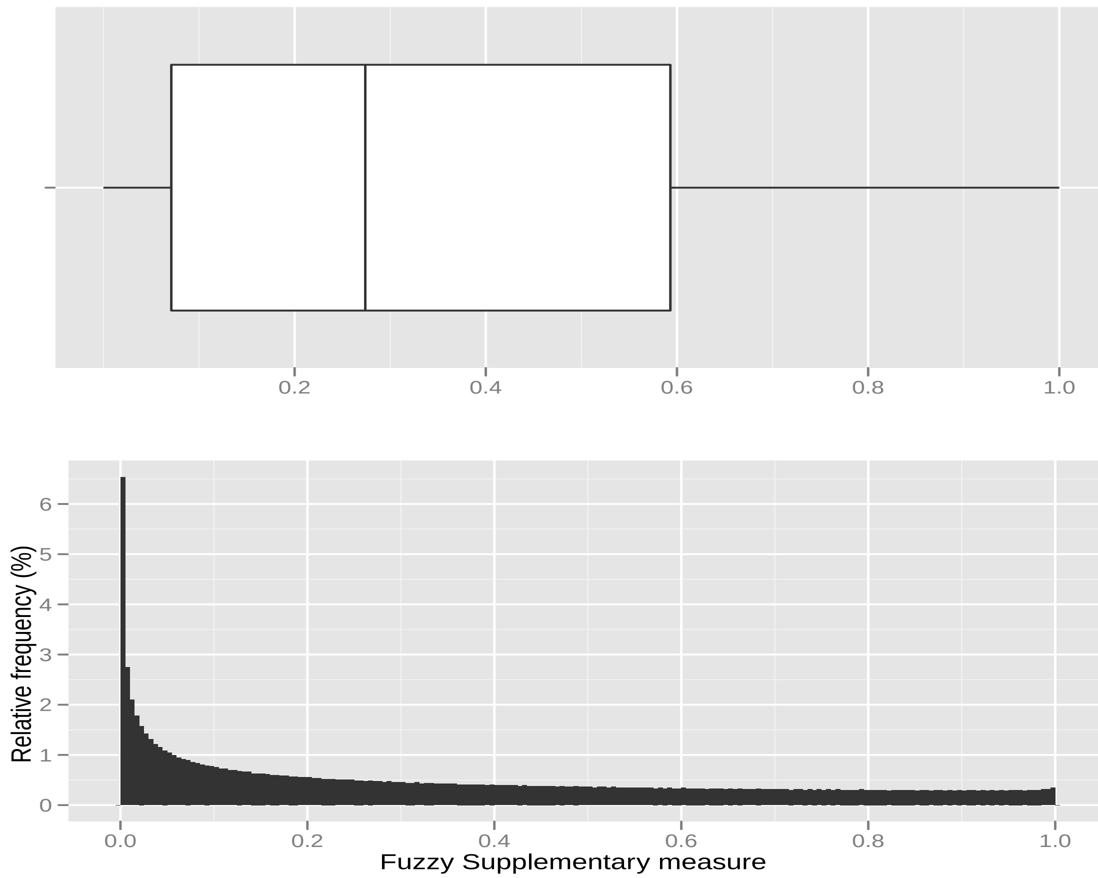


Figure 4.11: Distribution of FS measure

(*manifest* poverty), one should belong to the intersection of different kinds of poverty. Thus, the fuzzy intersection of the two measures[124] can be regarded as the propensity to more intense poverty. In other words, *manifest* poverty can be interpreted as the propensity to the overlapped poverty (Betti, Cheli, & Verma, 2006; Celikyilmaz & Türksen, 2009; Klir & Yuan, 1995).

Additionally, it would be informative to consider the concept of *latent* poverty since it can describe the maximum scope of poverty in the society. Table 4.39 and figure 4.13 show how the latent poverty is distributed. As the concept of *latent* poverty can be understood

[124]Since intersection cannot be greater than each set in set theory, the propensity to *manifest* poverty needs to be smaller than the propensity to monetary or non-monetary poverty.

Table 4.36: Correlation coefficients for each dimension's membership functions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) FM measure	1.000						
	(0.000)						
(2) Health	0.218	1.000					
	(0.000)	(0.000)					
(3) Employment	0.390	0.407	1.000				
	(0.000)	(0.000)	(0.000)				
(4) Housing	0.099	0.077	0.014	1.000			
	(0.000)	(0.000)	(0.260)	(0.000)			
(5) Durable goods	0.250	0.153	0.326	0.053	1.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		
(6) Social capital	0.009	0.131	0.033	0.086	0.075	1.000	
	(0.477)	(0.000)	(0.011)	(0.000)	(0.000)	(0.000)	
(7) Social participation	0.397	0.357	0.828	0.020	0.312	0.039	1.000
	(0.000)	(0.000)	(0.000)	(0.112)	(0.000)	(0.002)	(0.000)

* The numbers in parenthesis are $p - values$ from the significance test.
* All dimensions' membership functions range from zero ("definitely non-poor") to one ("definitely poor").

a union of two different kinds of poverty, the numbers in table 4.39 can be interpreted as the 'maximum' propensity to wider concept of poverty. For example, the mean of 0.423 implies that considering monetary and non-monetary aspects simultaneously, average 'maximum' propensity to poverty is 0.436 for the population. Put differently, the population belongs to *latent* poverty by the degree of 0.436 on average. Compared to figure 4.12, figure 4.13 looks similar to the FM measure since FM measure generally shows higher propensity to poverty.

Table 4.37: Correlations between FM measure and social capital indicators

	(1)	(2)	(3)	(4)	(5)	(6)
(1) FM measure	1.000					
	(0.000)					
(2) Feed visitors once a month	0.067	1.000				
	(0.000)	(0.000)				
(3) Frequency of talking to neighbors	-0.080	0.011	1.000			
	(0.000)	(0.370)	(0.000)			
(4) Frequency of meeting people	-0.108	0.081	0.174	1.000		
	(0.000)	(0.000)	(0.000)	(0.000)		
(5) Satisfaction with social life	0.115	0.089	0.097	0.092	1.000	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
(6) Frequency of seeing the closest friend	-0.079	0.035	0.076	0.254	0.036	1.000
	(0.000)	(0.006)	(0.000)	(0.000)	(0.004)	(0.000)

Table 4.38: Descriptive statistics for *manifest* poverty

Statistic	Manifest poverty measure	
Mean	0.351	
Median	0.274	
Range	0.000	1.000
Quartiles	(1st) 0.071	(3rd) 0.593
S.D.	0.304	

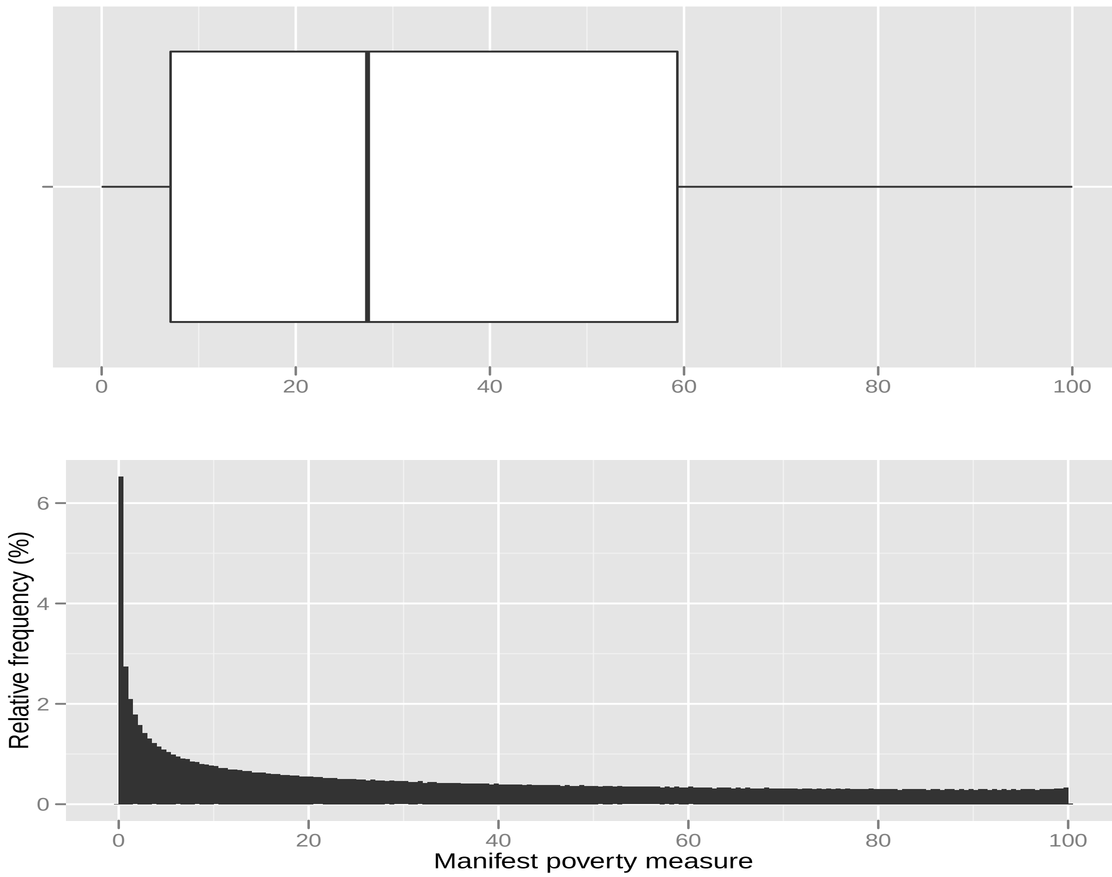


Figure 4.12: Distribution of *manifest* poverty

Table 4.39: Descriptive statistics for *latent* poverty

Statistic	Latent poverty measure	
Mean	0.423	
Median	0.386	
Range	0.000	1.000
Quartiles	(1st) 0.117	(3rd) 0.708
S.D.	0.320	

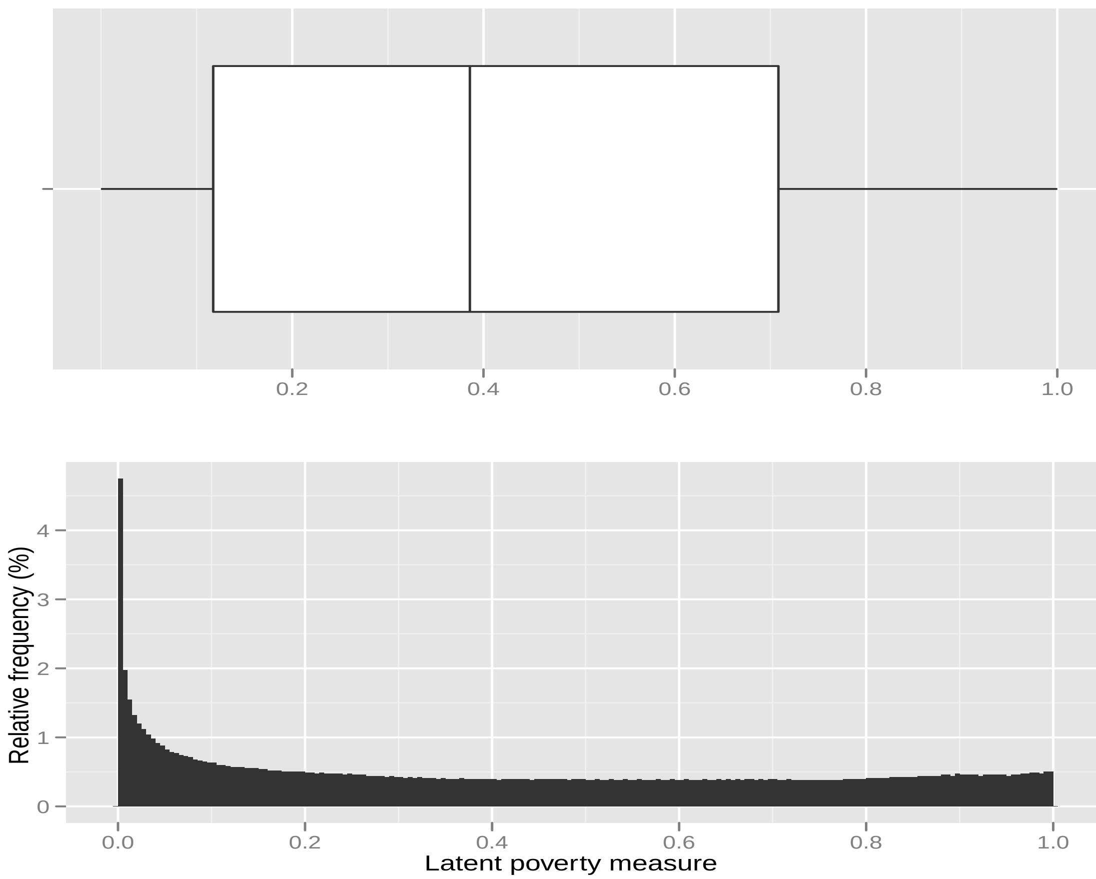


Figure 4.13: Distribution of *latent* poverty

4.3.3 Poverty profile for subgroups

Before going further into the poverty profile, it needs to be decided which concept would be used as the concept of poverty in this analysis - *manifest* or *latent*. This chapter discusses poverty profile based on the manifest poverty because it can be expected to show more intense aspects of poverty[125]. Thus, "IFR index" in following discussion indicates *manifest* poverty.

Poverty profile for age is in table 4.40. The table shows that the order of age groups is same to those of previous measures, where primary working age, 25~49, always has the smallest propensity to poverty, and people in age 50~64 mark second place. A slight difference can be found in figure 4.14, which describes that the distribution of IFR measure for the youngest age group looks much less skewed than those of the other measures (see figure A6 in Appendix A). For country comparison, the general picture is same to the conclusion by the other measures. Also, in terms of two different dimensions, still almost same order of countries can be observed in table 4.41. However, one difference can be found: Northern Ireland is in better condition in terms of monetary dimension than Wales, while the opposite is true for non-monetary dimension.

Table 4.40: Poverty profile by age groups and country

Age	IFR index	Country	IFR index
16~24	0.510	Britain	0.333
25~49	0.298	Wales	0.380
50~64	0.319	Scotland	0.353
65 or more	0.444	Northern Ireland	0.385

Poverty decomposition by labor force status and marital status in table 4.42 shows similar description of the country, compared to the results from previous measures. This analysis can be complemented by further analysis of two components of IFR measure like table 4.43, which shows that the difference between 'Employed' and 'Disabled' is still very big in both

[125] This implies relatively higher propensity to poverty would be observed.

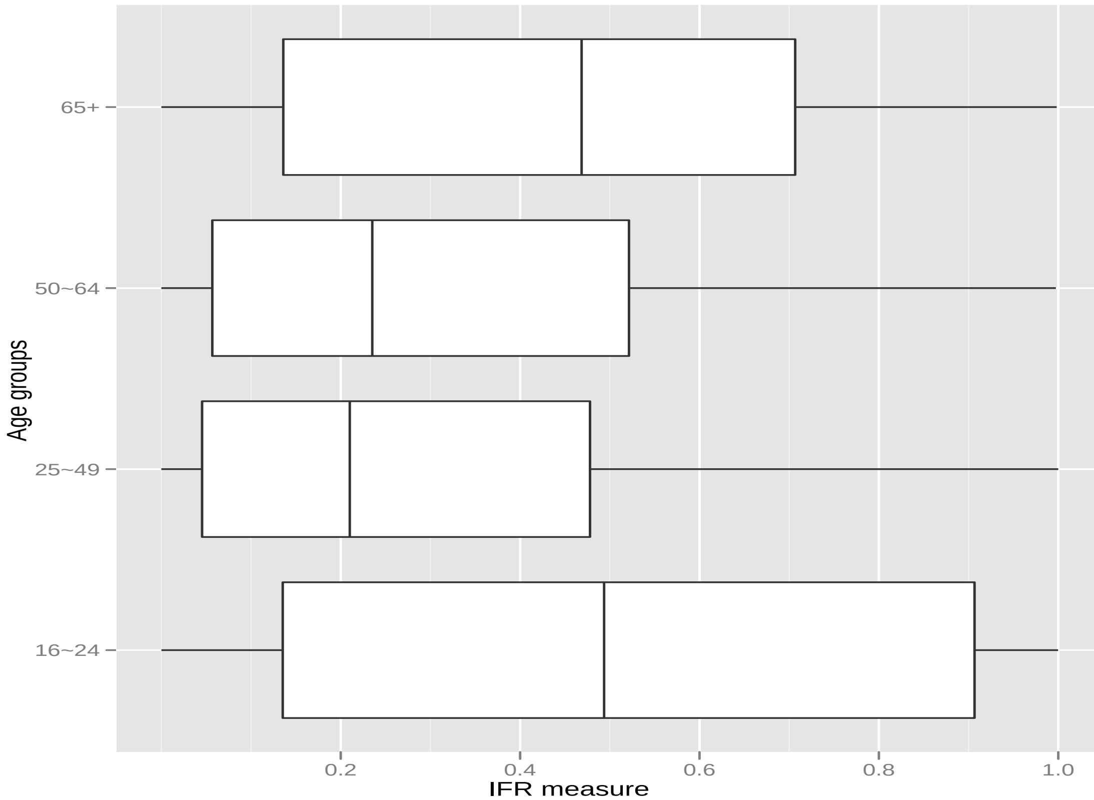


Figure 4.14: Poverty profile for age groups

Table 4.41: Country by FM and FS measure

	FM measure	FS measure
Britain	0.405	0.333
Wales	0.456	0.380
Scotland	0.426	0.353
Northern Ireland	0.454	0.385

dimensions. Also, it is interesting that 'Student' group has much higher propensity in terms of monetary dimension than 'Disabled'. For marital status groups, the order of groups is

Table 4.42: Poverty profile for labor force and marital status

Labor force status	IFR measure	Marital status	IFR measure
Self-employed	0.297	Married	0.307
Employed	0.240	Widowed	0.435
Unemployed	0.555	Divorced	0.419
Retired	0.440	Separated	0.428
Student	0.656	Never married	0.401
Disabled	0.544	Civil partnership	0.117

Table 4.43: FM and FS measure distribution by labor force status

	Self-employed	Employed	Unemployed	Retired	Student	Disabled
FM measure	0.364	0.309	0.627	0.521	0.703	0.621
FS measure	0.297	0.240	0.555	0.440	0.656	0.544

different from that of TF and TFR measure[126]. Although 'Married' group still shows the smallest propensity to poverty[127], now 'Never married' which generally shows second highest propensity takes the second least propensity. More decomposition in table 4.44 shows that the biggest difference between 'Married' and 'Widowed' group is in employment dimension where 'Widowed' people appears to be almost *definitely poor*. Besides, 'Widowed' group also turns out to experience extreme poverty of capability in social participation dimension.

Following table 4.45 shows poverty decomposition by gender, housing tenure and regions. It is not surprising to see that females have higher propensity to poverty than males. According to table 4.46, this is mainly attributable to the difference in FM measure, which

[126] For TF and TFR measure cases, see table 4.5 and 4.25, respectively.

[127] In fact, 'Civil partnership' group shows the least propensity to poverty. However, this group is excluded in this comparison because its sample size is too small, 5.

Table 4.44: Dimensions by marital status

	Married	Widowed	Divorced	Separated	Never married	Civil p.
FM measure	0.380	0.516	0.493	0.501	0.464	0.176
FS measure	0.308	0.435	0.419	0.428	0.401	0.117
Health	0.310	0.460	0.413	0.376	0.321	0.137
Employment	0.438	0.922	0.541	0.440	0.511	0.130
Housing	0.066	0.044	0.073	0.060	0.096	0.099
Durable goods	0.062	0.180	0.119	0.105	0.169	0.030
Social capital	0.220	0.225	0.224	0.218	0.225	0.163
Social participation	0.620	0.890	0.672	0.643	0.678	0.444

provides evidence for income inequality between gender. In terms of housing tenure, the general order of groups is not different from previous measures, but one change needs to be examined: the position of homeowners. While the group takes the second place in previous measures, now the group is in much worse situation than 'Rented from employer' group. Table 4.47 indicates that 'Employment' dimension is one main reason for the change. For regional comparison, though there are some gaps between regions in Briatin (figure 4.15), still the gaps between different regions are not as big as the difference between other population subgroups. For example, the gap between homeowners with mortgage and private housing renters is much bigger than the gap between London and East Midlands.

In the poverty profile by occupations and household type, the general pattern between subgroups makes sense in that 'Managers' or 'Professionals' constitute a cluster which shows the least propensity while 'Elementary occupations', 'Agriculture' and 'Service workers' construct the highest propensity group. Although this order can be conjectured as a result from income inequality, IFR measure decomposition in table 4.49 shows that this is also attributable to the non-monetary dimension, especially 'Agriculture' group case. Besides, it

Table 4.45: Poverty profile for gender, housing tenure and regions

Housing tenure		Value	Regions	Value
	Owned	0.376	London	0.286
	Owned with mortgage	0.241	South East	0.290
	Rented from local authority	0.516	South West	0.343
	Rented from housing association	0.486	East Anglia	0.365
	Rented from employer	0.258	East Midlands	0.345
	Rented private housing unfurnished	0.432	West Midlands	0.327
	Rented private housing furnished	0.506	North West	0.354
Gender	Male	0.327	Yorks & Humberside	0.390
	Female	0.374	North East	0.315

Table 4.46: FM and FS measure by gender

	FM measure	FS measure
Male	0.399	0.327
Female	0.446	0.374

is noticeable that the entire population has relatively lower propensity to poverty in terms of non-monetary dimension. Among nine types of household, it turns out that couples with non-dependent child are the closest to *definitely non-poor*, whereas Lone parent with dependent child is the closest to *definitely poor*.

Still, like in previous measures, non-metropolitan areas are observed to have less propensity to poverty than most of the metropolitan areas in table 4.50, except London, Greater Manchester and Tyne & Wear. (see figure A7 in Appendix A.). Poverty profile for household size shows interesting pattern: the propensity to *manifest* poverty is decreasing until

Table 4.47: Dimension by housing tenure groups

	Owned	Rented from employer
Health	0.396	0.283
Employment	0.753	0.277
Housing	0.045	0.144
Durable goods	0.110	0.134
Social capital	0.226	0.202
Social participation	0.784	0.595

households size reaches four, and begins to increase very rapidly afterwards. Additional decomposition in table 4.51 also shows the same pattern.

Figure 4.15: IFR measure by Regions & Countries

Table 4.48: Poverty profile for occupations and household type

Occupations	Value	Household type	Value
Managers	0.169	Single non-elderly	0.358
Professionals	0.149	Single elderly	0.455
Technical professionals	0.209	Couple: no child	0.304
Clerks	0.243	Couple: dependent child	0.321
Service workers	0.350	Couple: non-dependent child	0.269
Agriculture	0.377	Lone parent: dependent child	0.488
Craft related	0.272	Lone parent: non-dependent child	0.376
Machine operators	0.295	2+ unrelated adults	0.383
Elementary occupations	0.322	Other types	0.354

Table 4.49: FM and FS measure by occupations

	FM measure	FS measure
Managers	0.228	0.169
Professionals	0.205	0.149
Technical professionals	0.272	0.209
Clerks	0.311	0.243
Service workers	0.425	0.350
Agriculture	0.476	0.377
Craft related	0.353	0.272
Machine operators	0.375	0.295
Elementary occupations	0.401	0.322

Table 4.50: Poverty profile for metropolitan areas and household size

Metropolitan area	Value	Household size	Value
London	0.286	1	0.411
West Midlands Conurbation	0.420	2	0.320
Greater Manchester	0.285	3	0.320
Merseyside	0.369	4	0.315
South Yorkshire	0.369	5	0.398
West Yorkshire	0.335	6	0.479
Tyne & Wear	0.320	7	0.538
Other regions	0.329	8	0.660

Table 4.51: FM and FS measure by household size

	FM measure	FS measure
1	0.482	0.411
2	0.390	0.320
3	0.395	0.320
4	0.388	0.315
5	0.476	0.398
6	0.552	0.479
7	0.604	0.538
8	0.720	0.660

4.3.4 Poverty profile for dimensions

The analysis for different subgroups of population in previous section demonstrates that IFR measure shows subtle different picture of the society, compared to TF and TFR measures. As one possible reason for the difference, this section investigates the weight given for each indicator. Since IFR measure treats monetary dimension and non-monetary dimension separately, the weights need to be interpreted in the same way - 'economic resources' dimension and the others[128].

Table 4.52: Weights used in IFR measure calculation

Dimension	Variable	Weight(%)	Dimension	Variable	Weight(%)
Economic resources	HH income	27.9	Durable goods	Color TV	9.0
	Amount saved	21.0		VCR	3.1
	Amount inherited	4.6		Freezer	4.0
	Financial situation	46.6		Washing machine	3.6
Health	Health status	1.9		Dish washer	0.7
	Satisf. with health	1.7		Microwave	3.6
	Inhibits activ.	2.5		Home computer	1.2
Employment	Perm./temp. job	2.1		CD player	2.0
	Security satisf.	1.4		Telephone	3.5
	Overall satisf.	1.3		Cellphone	2.3
Housing	No heating	6.4		Internet	1.0
	Leaky roof	6.6		Cars	0.4
	Short space	2.9	Social capital	Feed visitors	2.0
	Neighb. noise	3.9		Talking to neighb.	0.8
	Street noise	3.4		Meeting people	1.0
	Not enough light	5.5		Satis.social life	0.5

Continued on next page

[128]Therefore, the sum of the weights for the indicators in economic resources dimension is 100%.

Table 4.52 – continued from previous page

Dimension	Variable	Weight(%)	Dimension	Variable	Weight(%)
	Condensation	3.6		Meeting friends	0.9
	Damp walls	4.3	Social participation	Attend groups	1.2
	Rot in floors	5.9		Voluntary works	1.1
				Union member	4.7

Within economic resources dimension in table 4.52, it is noticeable that self-evaluation of financial situation has almost twice the weight than household income. As the weights in IFR measure is a multiplication of the coefficients of variation and a measure of correlations between indicators in same dimension, a high weight can provide a basis of two conjectures: 1) the dispersion of financial situation variable is bigger than other variables in the dimension, or 2) financial situation variable has weak correlation with other variables. In either way, this implies that the variable has more discriminating power than other variables in the dimension. Within non-monetary dimensions, it is still observed that housing and durable goods dimension has higher weights than other dimensions, not because they are more important than the other indicators in nature, but because they contribute more to identifying *definitely poor*.

Though the weights in table 4.52 can provide some information on the contribution of each indicator, they are not exactly their contribution to final IFR measure because the final step of the calculation procedure - a multiplication of cumulative distribution function and Lorenz function - makes it impossible to identify them. This might be one of the weaknesses of IFR measure.

4.4 COMPARISON WITH TRADITIONAL POVERTY MEASURES

For the comparison with traditional measures, a poverty line needs to be set. I can use a real poverty criterion that is used in U.K. now, but since the purpose of this analysis is a comparison between two measurements, and the distribution of income data in dataset is different from official statistics, the real poverty line makes the analysis more complicated than it should be. Therefore, I use 60% of the median income of the dataset as the poverty line, which is generally accepted as 'relative poverty line' (Atkinson et al., 2002; Bradshaw & Finch, 2003; Wagle, 2008b). Using the poverty line of £13,919, computed from the data, following table 4.53 of poverty measures can be calculated.

Table 4.53: Poverty measures from BHPS 16th wave

Poverty measures	Value
Headcount ratio	0.197
Poverty gap ratio	0.056
Sen index	0.081
Watts index	0.082

4.4.1 Aggregation comparison

First of all, simple correlation analysis can show a brief picture of the relationship between the measures. Table 4.54 shows correlation coefficients between fuzzy measures and poverty indicator in traditional measures[129]. It turns out that TF measure and TFR measure is strongly correlated, but the correlation coefficients with IFR measure are relatively small, which means that IFR measure identifies the propensity to poverty for individuals quite differently[130]. Also, it is noticeable that IFR measure is correlated with poverty indicator

[129] A poverty indicator is a dummy variable which has value one if an individual's income is smaller or equal to the poverty line, and zero if income is greater than the poverty line.

[130] This difference between the fuzzy measures can also be investigated by the comparison of the orderings from *definitely poor* to *definitely nonpoor* by the individual propensity to poverty. As the fuzzy measures

more than the other fuzzy measures, in other words, 'poor' in terms of income. Partly, this is attributable to the fact that IFR measure weights monetary and non-monetary dimensions equally[131], while TF and TFR measure treats income as just one of indicators, which implies that the weight of income can be quite small.

Table 4.54: Correlation coefficients between fuzzy measures of poverty

	(1)	(2)	(3)	(4)
(1) TF measure	1.000			
(2) TFR measure	0.939	1.000		
(3) IFR measure[a]	0.413	0.415	1.000	
(4) Poverty indicator[b]	0.321	0.329	0.602	1.000

[a] *Manifest* poverty
[b] The numbers in this row are point-biserial correlation coefficients (Glass & Hopkins, 1995).

Second, it needs to be tested whether the prediction from welfare economic theory can be confirmed by investigating the difference between the poor identified by traditional poverty measure and the poor identified by fuzzy measures. According to the calculation of headcount ratio, it turns out that 1,250 out of 6,339 in the data are poor (henceforth, I call the people 'income-poor'). Following table 4.55 summarizes the three fuzzy measures for income-poor and nonpoor group. Since the averages of fuzzy measures of income-nonpoor are lower than those of income-poor, that is, the former has lower propensity to poverty in all three measures, the general implication from the two approaches seems to be same[132]. However, the graph 4.16 and table 4.55 seem to suggest one important question: is the difference in fuzzy measures between income poor and non-poor big enough to consider them as qualitatively

do not provide any simple distinction between the *poor* and *nonpoor* like traditional approach, this is one alternative for comparing the three measures. Table A14 in Appendix A shows the Spearman rank correlation coefficients for the orderings by the three measures, and it affirms again that IFR measure provides a different ordering from the other measures which give quite a similar ordering.

[131] The intersection operation for the *manifest* poverty implies that the the two components of IFR measure - FM measure and FS measure are not qualitatively different.

[132] Independent sample t-tests for each fuzzy measure show that the difference is statistically significant at $\alpha = .000$.

Table 4.55: Fuzzy measures for income-poor and nonpoor

Group		TF measure	TFR measure	IFR measure
Income-nonpoor	Mean	0.180	0.151	0.260
	S.D.	0.080	0.083	0.222
	Min / Max	0.030 / 0.628	0.019 / 0.798	0.000 / 0.732
Income poor	Mean	0.249	0.227	0.720
	S.D.	0.087	0.097	0.313
	Min / Max	0.049 / 0.605	0.050 / 0.668	0.000 / 1.000

different groups as in traditional approach?, because both of them indicates that some people in income-nonpoor group have even higher fuzzy indices than the income-poor. At the same time, a graph for IFR index shows that some people who are income-poor have quite low propensity to poverty.

This discrepancy between the measures can be seen in the analysis based on fuzzy measures. Accepting Cerioli and Zani (1990)'s interpretation that fuzzy measures are the proportion of individuals belonging to fuzzy subset *poor*[133] (call this 'fuzzy-poor'), and assuming that fuzzy measures and the headcount ratio provide same information, we can say that 19.7% of the population according to the descending order of fuzzy measures is *poor* in the fuzzy sense. Based on this interpretation, 1,249 people are poor in the fuzzy sense[134]. Then we can investigate the distribution of income-poor among *poor* as fuzzy term. The calculation shows that 488 income-poor are also poor by TF and TFR measure, and 1,018 income-poor are also fuzzy-poor by IFR measure as in table 4.56. Nevertheless, it turns out

[133] According to set theory, the headcount ratio can be interpreted as the cardinality of the subset *poor* divided by the size of population, $P = \frac{|\text{Poor}|}{n}$, in non-fuzzy('crisp') set (Klir & Yuan, 1995; Smithson, 2006). If the set is fuzzy, then it can be written as $P = \frac{|\text{Poor}|}{n} = \frac{1}{n}\sum_{i=1}^{n} \mu_{\text{poor}}(i)$, which is exactly same to the TF method calculation (Cerioli & Zani, 1990).

[134] However, this does not mean that we can distinguish fuzzy-poor from fuzzy-nonpoor, which is exactly against the basic idea of fuzzy set approach. This categorization is just for presentation. Betti and Verma (1998); Cheli, Ghellini, Lemmi, and Pannuzi (1994); Cheli (1995); Maniu (2009) also adopt this categorization for comparison of fuzzy set measures with traditional measures, especially the headcount ratio.

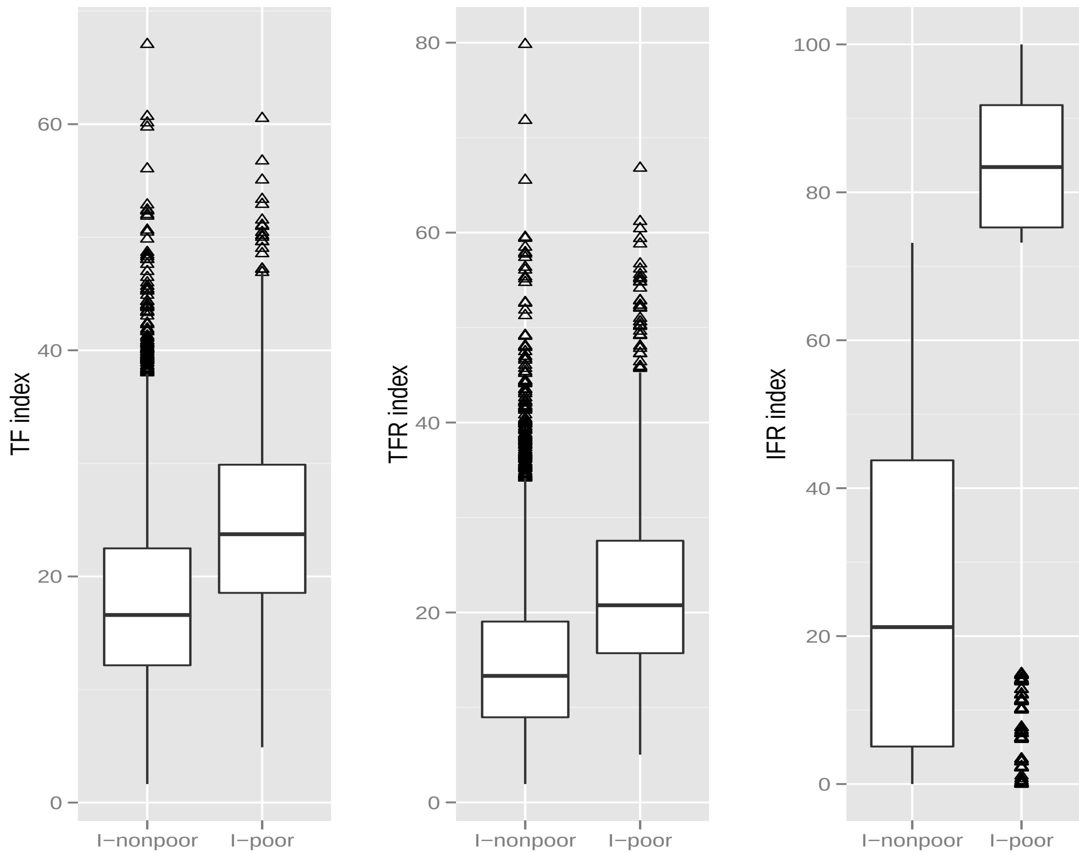


Figure 4.16: Fuzzy measure distributions for income poor and non-poor

that 761 people who have income more than the poverty line are fuzzy-poor by TF and TFR measure, and 231 people by IFR measure, in other words, significant numbers of people who are not poor in traditional sense can be regarded *poor* by the fuzzy measures, or at least have high degree of propensity to poverty. Although this does not mean whether one measure is right or wrong, it clearly shows that there is significant difference between the two approaches in terms of their representation of reality.

Since the difference can be a artifact of aggregation, further analysis based on demographic variables can make the point clearer. Considering gender, table 4.57 demonstrates that the fuzzy measures can present two unique information on the income poor and non-poor: 1) males show consistently higher propensity to poverty than females among income-

Table 4.56: Analysis of fuzzy-poor by income-poverty

	TF measure	TFR measure	IFR measure
Income non-poor	761	761	231
Income poor	488	488	1,018
Sum	1,249	1,249	1,249

poor by all three measures, while the conventional knowledge on gender inequality still applies to income-nonpoor, and 2) the fuzzy measures for income non-poor is not particularly close to zero, though they are still lower than income-poor. It seems that these observations support the argument that traditional approach to poverty is not enough to represent the well-being of people in a society adequately.

Table 4.57: Fuzzy measures for gender / traditional approach

		Income-poor		Income-nonpoor	
	Stats.	Male	Female	Male	Female
TF measure	Mean	0.328	0.326	0.160	0.161
	Min,Max	0.261,0.605	0.259,0.671	0.022,0.261	0.016,0.258
TFR measure	Mean	0.314	0.310	0.128	0.133
	Min,Max	0.231,0.718	0.231,0.798	0.024,0.231	0.019,0.231
IFR measure	Mean	0.840	0.832	0.230	0.234
	Min,Max	0.693,1.000	0.660,0.999	0.000,0.691	0.000,0.659

Applying the same logic in previous paragraph, table 4.57 can be reconstructed based on fuzzy-poor. In table 4.58, The discrepancy between fuzzy measures and the headcount ratio seems to be even bigger in the breakdown, since the number of income-nonpoor in female is greatly different from that in male. Also some discrepancy between the fuzzy measures can be observed in the table.

Table 4.58: Distribution of income-poor among fuzzy-poor by gender

	Male		Female	
Headcount ratio	0.160		0.234	
# of F-poor	502		748	
	Income-poor	Non-poor	Income-poor	Non-poor
TF measure	307	195	454	294
TFR measure	312	190	445	303
IFR measure	78	424	154	594

Judging from the tables 4.57 and 4.58, it can be hypothesized that there is significant information discrepancy between the fuzzy measures and the headcount ratio. To search for the source of the discrepancy, next section compares the difference in individual indicators in detail.

4.4.2 Individual comparison

In previous aggregate comparison, the first question that needs to be answered is why non-poor people in terms of income are poor with regard to fuzzy measures. In order to have an answer, ultimately it is required to look into individual difference in multiple indicators since the differences in the fuzzy measures are reflecting those. Here, I take a very specific subgroup of the population and examine their concrete situations: 25~49, employed in agriculture, and married household, which consists of only seven household as in table 4.59. Most remarkably, it turns out that the seventh household is 'poor' in terms of income but 'definitely non-poor' with respect to IFR measure, while TF and TFR measures show that the household is worse-off than average level[135]. On the other hand, household (1) is quite easy to understand since it is income-poor and has relatively high propensity to poverty. Thus, a contrast between the two households - (1) and (7) - may provide an insight for the

[135]The mean TF measure is 0.193, and TFR measure is 0.166.

Table 4.59: Fuzzy measures for a subgroup

	TF	TFR	IFR	Income poverty
(1)	0.234	0.168	0.934	Poor
(2)	0.175	0.168	0.549	Non-poor
(3)	0.175	0.103	0.499	Non-poor
(4)	0.219	0.220	0.465	Non-poor
(5)	0.116	0.085	0.391	Non-poor
(6)	0.217	0.238	0.367	Non-poor
(7)	0.221	0.174	0.000	Poor

reason of the different identification as *poor*. Following table 4.60 shows the selected indicators for the two households. Several points from table 4.60 are in order. First, though the income of household (7) is under poverty line (£13,919), it is hard to tell household (7) is poor even in traditional income-centered approach, considering it inherits £20,000, which is bigger than poverty line. The result in table 4.59 that household (7) is 'definitely nonpoor' in IFR measure is strongly influenced by the big inheritance because the indicator is more heavily weighted in IFR measure than in the other measures. However, we needs to be cautious to consider this point as one of the strengths of the fuzzy measures firstly because inheritance can be easily incorporated in traditional approach, and finally because TF or TFR measure in table 4.59 say that household (7) certainly has some degree of propensity to poverty even if inheritance is much less important for measuring poverty. Second, it turns out that job-related variables are not influencing factor in that the households are almost identical in terms of the indicators. Third, there is a difference in health status indicator. Considering that relatively high weight is assigned to the indicator (see table 4.12, 4.30, and 4.52.), this can be one factor for the difference between the households. Fourth, the two households are quite different with respect to the possession of durable goods. As very small portion of the sample does not have durable goods in table 4.60 (see table 3.10.), which means

Table 4.60: Selected indicators for two households

	(1)	(7)		(1)	(7)
Income	£8,271	£12,561	Gender	Male	Male
Inheritance	£0	£20,000	Permanent job	Yes	Yes
Job	Employed	Employed	Conversation	<one/month	Once a week
Job satisfaction	6[a]	6	Health status	Poor[b]	Fair
Marital status	Married	Married	Meeting people	On most days	Once a week
Occupation	Agriculture	Agriculture	Voluntary work	Never	Never
No heating	No	No	Dish washer	No	No
Rot in windows	No	No	Internet	No	Yes
Noisy neighbors	No	Yes	Phone	No	Yes
Condensation	No	No	Cellphone	Yes	Yes

[a]Measured by seven categories, 1 being "Completely not satisfied", 7 "Completely satisfied".
[b]Five points, 1 "Excellent", 5 "Very poor"

that people who could not possess the goods can feel severe relative deprivation, they could make big difference in aggregate fuzzy measure. In sum, the analysis in table 4.60 suggests that two households who are just income-poor in traditional approach do have difference in terms of diverse functionings, which are reflected in the fuzzy measures of poverty.

Besides, the discrepancy within each fuzzy method shown in figure 4.16 can be analyzed in the same way. For example, table 4.61 shows selected important indicators for the four households who show higher TF measure than most of the income-poor, though they are not poor in terms of income[136]. For comparison, I also include one household who has the highest TF index among income-poor group. First, in spite of the significant income gap between household (2) and (4), TF measure indicates that (2) is *poorer* in the fuzzy sense. Second, comparing the difference in indicators between household (3) and (4) to the difference in TF

[136]Similar analyses for TFR measure can be found in table A15 in Appendix A.

Table 4.61: TF measure: Individual comparison for extreme fuzzy-poor

	Income nonpoor			Income poor	Coding
	(1)	(2)	(3)	(4)	
TF measure	0.632	0.668	0.756	0.629	
Household income	£16,072	£20,062	£16,249	£12,907	Annual income
Health status	2	2	3	2	1(excellent)~5(very poor)
Permanent job	0	2	0	0	0(no job),1(temp.),2(perm.)
Shortage of space	2	2	2	2	1(yes),2(no)
Washing machine	0	0	0	0	0(don't have),1(have)
Satisf. with social life	4	5	7	4	1(not at all)~7(completely)
Voluntary work	2	5	5	5	1(once a week)~5(never)

measure, it seems that 'health status' is more important than 'satisfaction with social life'. Third, the combination of the highest income and permanent job cannot keep household (2) out of fuzzy-poor, and fourth, generally no big difference in indicators exists except income between income-poor and income-nonpoor. This could be interpreted that the TF measure is revealing the unobservable, complex condition called 'poverty' which cannot be figured out only by income, because the four households does not only have many characteristics in common, but also income alone cannot explain the difference in TF measures at all. This point gets more evident when we look into the weight structure in table 4.12. According to the table, income takes only 1% of TF measure variation, which explains why the income difference between household (2) and (4) is not so influential. Also the portion of 'permanent job' in TF measure (1.2%) appears to be smaller than satisfaction with social life (1.6%). On the contrary, living in housing or going without washing machine has higher weight, 2.6% and 4.7% respectively, while doing voluntary work has almost no weight(0.2%)[137]. Similar

[137] As mentioned in previous section, this does not mean having washing machine is almost five times more important than income in TF measurement. It just means that having washing machine is five times more discriminating than income in measuring poverty as a fuzzy concept.

insights can be examined by table 4.62 which examines some part of 'definitely nonpoor' people by IFR measure who appear income-poor in figure 4.16. Column (4) in the table is a household whose IFR measure is very similar to the average level of IFR measure for income-nonpoor group.

Table 4.62: IFR measure: Individual comparison for extreme fuzzy-nonpoor

	Income poor			Income nonpoor	Coding
	(1)	(2)	(3)	(4)	
IFR measure	0.000	0.000	0.000	0.213	
Household income	£13,390	£12,562	£8,384	£40,726	Annual income
Health status	3	3	2	3	1(excellent)~5(very poor)
Permanent job	0	2	2	2	0(no job),1(temp.),2(perm.)
Shortage of space	2	2	1	2	1(yes),2(no)
Washing machine	1	1	1	1	0(don't have),1(have)
Satisf. with social life	1	6	5	6	1(not at all)~7(completely)
Voluntary work	5	5	5	5	1(once a week)~5(never)

Still, the analyses in table 4.61 and 4.60 show a rather confusing representation of the phenomenon of poverty since it cannot provide a definitive criteria to identify *poor*. For example, it does not give any answer to the questions such as: should we consider household (2) in table 4.61, who has permanent job and fairly decent income *poor*, or even poorer than household (4) who has income less than 60% of median income with no job but relatively satisfied with social life?, because the answers really depend on the definition of 'being *poor*'. However, it does not mean that the information the fuzzy measures can provide is only ambiguous. Rather, it just means that the concept of poverty itself - multidimensional poverty, more specifically - does not allow any easy way to interpret the measure due to its inherent complexity and ambiguity (Brady, 2003, 2009; Chiappero-Martinetti, 2000; Sugden, 1993). Thus, the questions need to be rephrased: for instance, in table 4.61, a meaningful question should be whether we can consider household (3) is in a worse situation than household (4) due to the health status, though the former has more income. If the answer would be affirmative, then it can be argued that the fuzzy measures are showing more 'accurate' picture. Even if the answer is negative, still it cannot be denied that the fuzzy measures of poverty can be considered as a source of new possibilities of identifying *poor* according to their multidimensional approach.

Finally, for understanding the discrepancy in table 4.58, table 4.63 is built for selected indicators in case of TFR measure, where all the numbers are the average of the indicators for each group.[138].

Table 4.63: TFR measure: Comparison of fuzzy-poor and fuzzy-nonpoor for male

Indicators	Fuzzy-poor		F-nonpoor		Weights	Remark
	I-nonpoor	I-poor	I-nonpoor	I-poor		
Household income	£24,648	£9,574	£34,160	£10,113	1.1	Annual income
Health status	2.56	2.82	2.03	2.28	1.0	1(excellent)~5(very poor)
Permanent job	0.78	0.20	1.42	0.49	1.3	0(no job),1(temp.),2(perm.)
Shortage of space	1.64	1.74	1.85	1.86	2.7	1(yes),2(no)
Washing machine	0.80	0.74	0.99	0.98	5.0	0(don't have),1(have)
Satisf. with social life	4.43	4.58	5.03	5.05	1.0	1(not at all)~7(completely)
Voluntary work	4.74	4.68	4.47	4.51	0.3	1(once a week)~5(never)
# of people	312	190	2,329	312		

Table 4.64: TFR measure: Comparison of fuzzy-poor and fuzzy-nonpoor for female

Indicators	Fuzzy-poor		F-nonpoor		Weights	Remark
	I-nonpoor	I-poor	I-nonpoor	I-poor		
Household income	£23,371	£9,853	£31,558	£10,122	1.1	Annual income
Health status	2.77	2.80	2.09	2.22	1.0	1(excellent)~5(very poor)
Permanent job	0.53	0.19	1.30	0.70	1.3	0(no job),1(temp.),2(perm.)
Shortage of space	1.66	1.66	1.84	1.85	2.7	1(yes),2(no)
Washing machine	0.84	0.80	0.99	0.99	5.0	0(don't have),1(have)
Satisf. with social life	4.23	4.12	4.93	4.78	1.0	1(not at all)~7(completely)
Voluntary work	4.62	4.60	4.46	4.40	0.3	1(once a week)~5(never)
# of people	445	303	2,003	445		

The left panel of table 4.63 indicates that some people with significantly high income are still considered as poor in fuzzy sense. This is understandable in the sense that income-poor and income-nonpoor in the fuzzy-poor group do not have dramatic difference in indicators except

[138]Tables for TF (table A16) and IFR measure (table A17) can be found in Appendix A.

income, considering weights given respective indicators. Both of them are much more likely to have temporary job or no job at all, neutral satisfaction with social life, and almost no voluntary works. This point can be clarified by the contrast to the right panel of table 4.63, which reveals the difference between fuzzy-poor and fuzzy-nonpoor in male. For instance, income-poor people in fuzzy-nonpoor group are healthier and more satisfied with social life than income-nonpoor people in fuzzy-poor group in spite of the average income less than the relative poverty line of £13,919 and less permanent job. Almost same conclusion can be warranted in female case, as seen in table 4.64. This result also supports the previous conjecture that the fuzzy measures are indicating the complex and fuzzy condition of poverty that cannot be disclosed fully by income.

4.5 CONCLUSION

Since we do not have an unequivocal standard to decide who are poor, the most fundamental question of poverty is ambiguous by nature. As one attempt to model the ambiguity, the fuzzy measures of poverty suggest a promising way to see reality with different perspective. Previous analyses in this chapter show that the simple binary distinction of traditional approach to poverty is not enough to represent multidimensional, complex and vague nature of the concept. There are significant diversities within income-poor group as well as income-nonpoor group, and the fuzzy measures appear to summarize individual household's situation more acceptably, considering the multiple dimensions of capability.

It turns out that conclusions from TF measure is quite robust to the possibly arbitrary choices of two thresholds. However, the analyses of distribution provide some basis of concern for the influence of indicators' initial distribution, since the weights are calculated from the frequency of *definitely poor* phenomenon. TFR measure shows quite consistent results with TF measure, though the propensity to poverty by this measure is always lower than the previous measure. IFR measure which attempts to incorporate the absolute notion into the concept of poverty appears to be distributed more widely than above two measures, and the distinction of monetary and non-monetary dimension provides more intuitive implications. However, since the introduction of Lorenz function into the calculation makes it impossible to identify each indicator's contribution to the aggregate index, it has a limitation for application. Also, the conclusion from IFR index is not easy to compare to the conclusions from previous two measures since it represents different concepts of poverty - *manifest* or *latent*.

Overall, the comparison of the fuzzy measures with traditional measure unequivocally indicates that the pictures from two approaches are quite different. Especially, it can be clearly seen that the fuzzy measures at least provide a rich ground on which we can clarify the definition of poverty.

5.0 STATISTICAL BEHAVIORS OF THE FUZZY MEASURES

According to Sen (1976), there are two important aspects in the functions of a poverty measurement: "aggregation" and "identification" (Sen, 1979b; Foster & Shorrocks, 1988). The former refers to the "measurement of the extent of poverty" (Callan & Nolan, 1991), and the latter to identifying who are the poor (Ravallion, 1992; Sen, 1992, 1997). Both of these aspects are important because each contributes in its own way to making a more effective policy. For instance, the latter can be a crucial information for anti-poverty policies since a scarcity of resources requires policymakers to aim their policy measures at the most relevant population, while the former can demonstrate the seriousness of a social phenomenon of poverty very effectively, which can make it easier for anti-poverty policies to become a priority. Following the distinction, statistical behaviors of the fuzzy measures are examined in two ways

5.1 AGGREGATION ASPECT

I focus on the sampling distribution and small-sample behavior of the measures (Voinov & Nikulin, 1993). Each of the focus can be considered as the first step of examining two desirable statistical properties of a 'good' estimator: unbiasedness and consistency.[139] In addition, since most of the data in social sciences cannot satisfy the assumptions of a statistical model entirely (Lehmann & Casella, 1998), and one of the most important challenges for a poverty measurement is the robustness of the measurement method to both measurement

[139] According to Sage and Melsa (1971), there are four desirable properties of a 'good' estimator: unbiasedness, minimum-variance and unbiased, consistency, and efficiency.

errors in indicators and setting fixed lines (Nolan & Whelan, 1996; Ravallion, 1992, 1996), the robustness of the fuzzy measures is analyzed.

5.1.1 Sampling distribution

The sampling distribution of a statistic is simply the distribution of the statistic based on infinite numbers of random samples (Bain & Engelhardt, 1992; Devore & Peck, 2001). So, the simplest way to estimate a sampling distribution is to draw many random samples from a population and calculate the statistic. Using the bootstrapping (Chernick, 2007; Efron, 1982; Efron & Tibshirani, 1993), the difficulty in drawing a number of samples from one sample can be addressed. Following table 5.1[140] and figure 5.1[141] summarize the sampling distribution estimated by the parametric bootstrapping.

Table 5.1: Simulated mean comparison

	Original index	100 Sims.	1,000 Sims.	5,000 Sims.
TF measure	**19.33**	19.49	19.50	19.50
	S.D.	(3.640×10^{-3})	(1.333×10^{-3})	(5.817×10^{-4})
TFR measure	**16.59**	16.62	16.62	16.62
	S.D.	(5.562×10^{-4})	(1.770×10^{-4})	(7.813×10^{-5})
IFR measure	**35.08**	34.90	34.90	34.90
	S.D.	(2.009×10^{-3})	(6.275×10^{-4})	(2.927×10^{-4})

[140] Originally, the fuzzy measures range from zero to one by definition. However, for readability, all fuzzy measures in this paper are multiplied by 100.

[141] In the graphs, $\hat{\theta}$ indicates a calculated value from BHPS data because strictly speaking, it also is an estimate of an unknown population parameter, θ. $E(\hat{\theta})$ is the mean of simulated values.

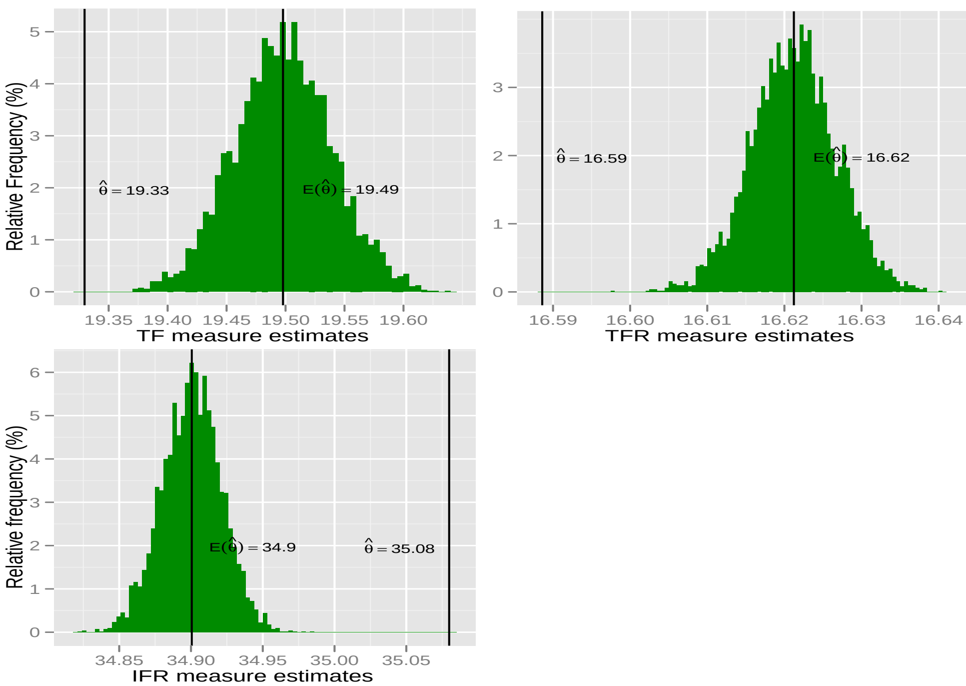


Figure 5.1: Sampling Distribution of the fuzzy measures

Table 5.2: Monte Carlo confidence intervals

# of sims.	TF measure		TFR measure		IFR measure	
	Lower	Upper	Lower	Upper	Lower	Upper
100	0.19497417	0.19497418	0.16621825	0.16621825	0.34902633	0.34902633
1,000	0.19496615	0.19496615	0.16621172	0.16621172	0.34900224	0.34900224
5,000	0.19497855	0.19497855	0.16621227	0.16621227	0.34900540	0.34900540

Although all the sampling distributions cannot reject the null hypothesis of normality test[142], figure 5.1 and table 5.1 reveals that there is some concern for bias of the measures. It turns out that even after 5,000 times iterative calculation, still there are gaps between fuzzy measures calculated from original data and simulated results. The differences can be said not

[142] *p-values* from Kolmogorov-Smirnov test for normality are 0.6068 (TF measure), 0.3625 (TFR measure), and 0.1058 (IFR measure), respectively.

to be trivial because Monte Carlo confidence intervals of the three measures do not contain the original indices within them, which can be seen in table 5.2. To determine whether the nontrivial gaps, albeit small[143], originate from the simplistic assumption of multivariate normality, I carry out a nonparametric bootstrapping which allows me to avoid the unconfirmed assumptions on the population. Figure 5.2 shows it is highly likely that the bias in figure 5.1 comes from the assumption of multivariate normality, as the sampling distribution still can be considered normally distributed.[144]

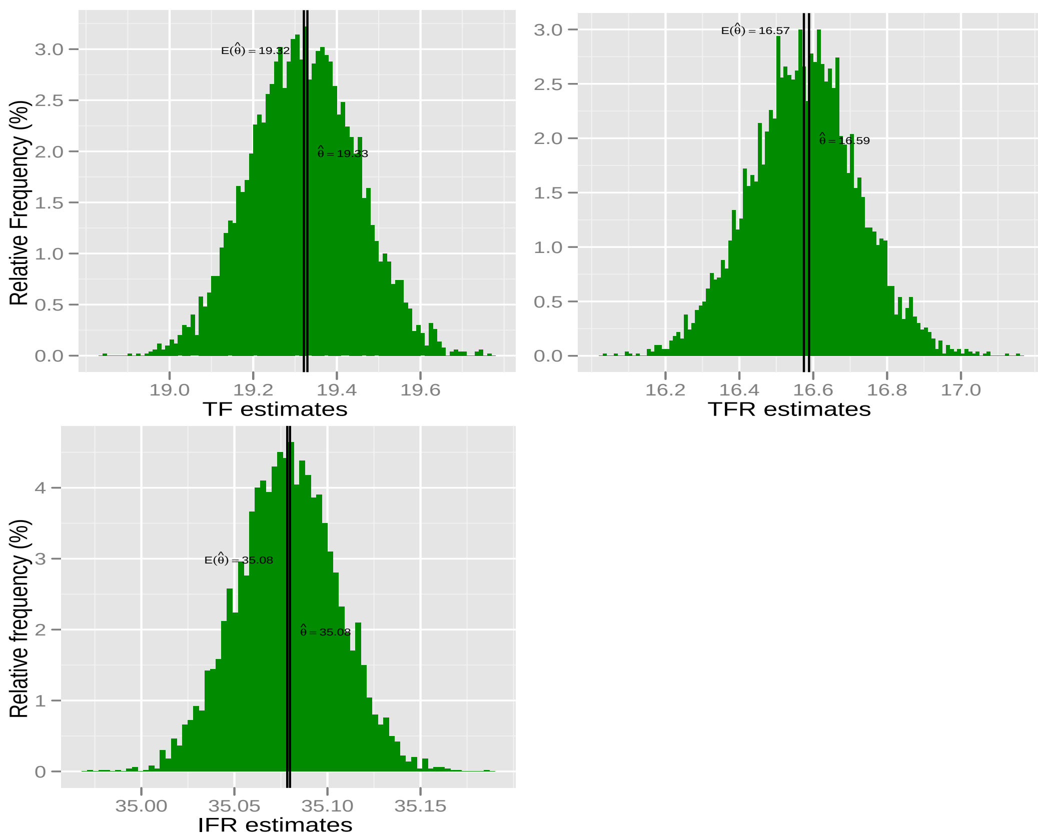


Figure 5.2: Sampling Distributions by nonparametric bootstrapping

From the two simulation results, three different confidence intervals for the fuzzy measures can be constructed: 1) by *percentile* method suggested by Efron (1982), 2) by standard

[143] I calculate the size of bias using mean square error, and the size of biases are 0.168 (TF), 0.033 (TFR), and 0.208 (IFR), all of which are less than 1% of each fuzzy measure.

[144] All the results from nonparametric bootstrapping also fail to reject the null hypothesis of normality test.

method where an estimator can be assumed to be normally distributed, at least approximately, and 3) by "Bias-corrected" percentile method (Chernick, 2007; Davison & Hinkley, 1997; Efron & Tibshirani, 1986; Efron, 1987; Efron & Tibshirani, 1993). In standard method, a confidence interval of $\hat{\theta}$ with confidence level α is defined as:

$$[\hat{\theta} - \hat{\sigma}z^{(\alpha)}, \hat{\theta} + \hat{\sigma}z^{(\alpha)}] \tag{5.1}$$

, where $z^{(\alpha)}$ indicates x which satisfies the condition $z(p < x) = \alpha$ in the standard normal distribution. In percentile method, confidence interval is $[\hat{G}^{-1}(\alpha), \hat{G}^{-1}(1-\alpha)]$, where G denotes the cumulative bootstrap sample distribution for $\hat{\theta}$. Finally, "Bias-corrected" confidence intervals can be computed by fundamentally same principle as percentile method. But the confidence level is adjusted considering possible bias in the bootstrap distribution. The bias-corrected confidence interval can be expressed as:

$$[\hat{G}^{-1}(\Phi\{2z_0 + z^{(\alpha)}\}, \hat{G}^{-1}(\Phi\{2z_0 + z^{(1-\alpha)}\})] \tag{5.2}$$

, where Φ^{-1} is the inverse of the cumulative Gaussian distribution, $z_0 = \Phi^{-1}\{\hat{G}(\hat{\theta})\}$, and $z^{(\alpha)}$ satisfies $\Phi(z^{(\alpha)}) = \alpha$.

Considering that Kolmogorov-Smirnov test for normality test cannot reject the null hypothesis[145] and Q-Q plots for the simulated measures (See Appendix A figure A8), the assumption of normal distribution of the population parameter is not far-fetched. Table 5.3 displays the 95% confidence intervals, and it can be easily seen that each confidence interval is generally overlapped to a significant extent. To check the unexpected variability in the bootstrap procedure, 100 confidence intervals for each measurement method are computed, and the results (figure 5.3) show that there is no great variability in the confidence intervals.

[145]Due to the possible sensitivity of the test, I perform three additional normality tests, and the results show consistent failure of rejection. See Appendix A table A18.

Table 5.3: Estimated 95% confidence intervals

	TF		TFR		IFR	
	Lower	Upper	Lower	Upper	Lower	Upper
Standard	19.085	19.575	15.796	17.601	35.029	35.131
Percentile	19.073	19.578	16.023	17.363	35.021	35.124
Bias-Corrected	19.085	19.599	16.074	17.370	35.038	35.140

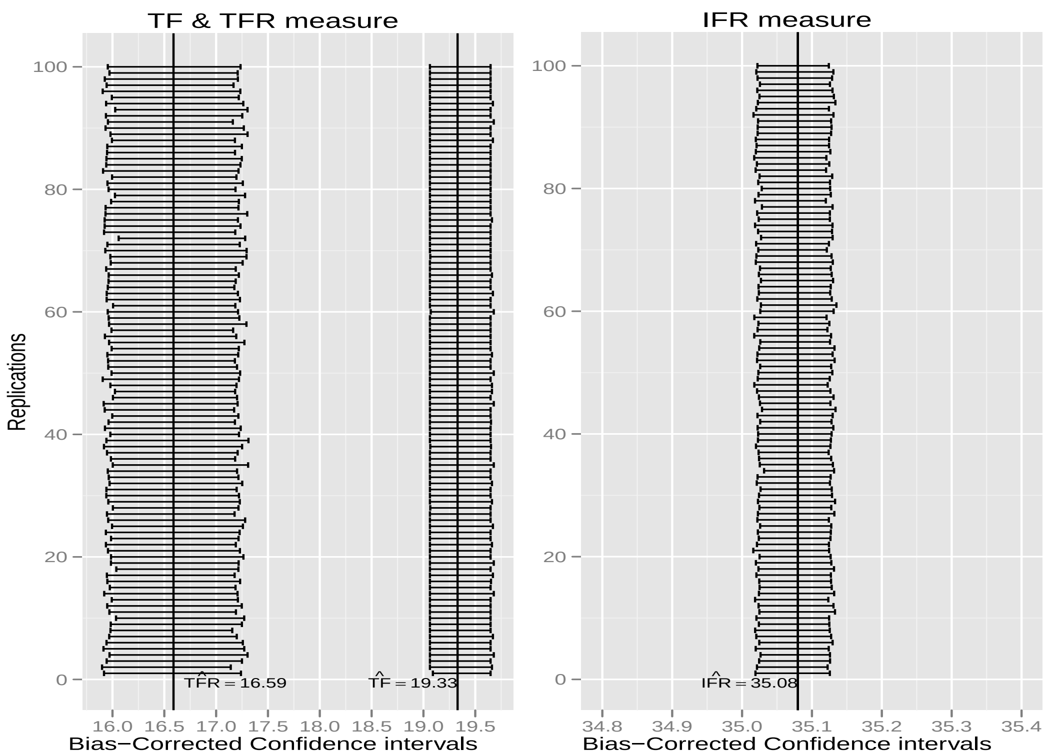


Figure 5.3: Replicated 95% confidence intervals

It turns out that the range of 95% confidence interval is quite small for TF and IFR measures, but relatively big for TFR. However, considering the relative size of the intervals, it is reasonable to argue that 95% confidence intervals for the three fuzzy measures are small enough to describe the esimtation methods precise.[146]

The persistently small confidence intervals[147] along with symmetric and smooth sampling distribution suggest at least very small variability in the fuzzy measures that is attributable to the estimation methods themselves.

5.1.2 Small-sample behavior

Due to the missing data problem, especially in social survey data, often only a small observations are available for a researcher, and since this problem gets more serious in multidimensional context. For example, assuming we only have four variables, with 10% of the observations missing independently for each indicator, then it is possible that we end up with only 65.6% of total observations that can be used for any statistical analysis. Thus, the small-sample behavior of an estimator becomes more crucial for multidimensional measurements. In a sense, small-sample behavior can be connected to the property of consistency of a statistic, which indicates an estimator becomes more accurate as the number of observations increases. Formally, it can be expressed as follows(Lehmann & Casella, 1998; Sage & Melsa, 1971; Voinov & Nikulin, 1993):

$$E[(\hat{\theta}_n - \theta)^2] \to 0 \quad \text{as} \quad n \to \infty \tag{5.3}$$

where $\hat{\theta}_n$ indicates an estimator when the number of observations is n. If an estimator turns out to have a strange small-sample behavior, then all estimations from even a relatively big sample size needs to be interpreted very cautiously. On the contrary, if an estimator has a nice small-sample behavior, then even some bias in the estimation can be considered

[146]Although there are different opinions on the definition of "precision", it is in general considered as a measure of the variations of one measurement around its nominal value, regardless of the validity of the nominal value (Dodge & Marriott, 2003; Pearson, 2011; Rice, 2007; J. K. Taylor & Cihon, 2004). van Belle (2002) argues that the concept corresponds to 'reliability' in social sciences, while accuracy to validity. However, Kaplan (1964/2003) claims that the distinction could be "the fiction of the true measure", which originates from the misconceived idea of a measurement free from error. Thus, he suggests that the concept of validity needs to be based on a convergence toward some particular value, which is quite similar to the idea of precision.

[147]Since the fuzzy measures are "nonpivotal" in the sense of Hall (1992), in other words, we cannot be sure of the exact distribution of parameters, this bootstrap confidence interval may differ from the true confidence interval. However, as the difference decreases at the rate of $\frac{1}{\sqrt{\text{sample size}}}$, it is reasonable to assume the bootstrap error in this study is negligible, considering the big sample size.

as a small problem since it is highly likely that the estimator has asymptotic unbiasedness (R. Deutsch, 1965; Kennedy, 2008). The stability requirement[148] of an estimator is also an important reason to examine small-sample behavior of an estimator (Altman et al., 2004; L. Wilkinson, 1994).

Applying different numbers of cases - 100, 500, 1,000, 2,000, and 4,000 - the following figure 5.4 of the square root of mean square error is built.[149] It needs to be noted that since the variances of the simulated indices are extremely small in table 5.1, the figure can be interpreted as a graph of $Bias^2$, because the mean square error (MSE) can be decomposed into the sum of variance of an estimator and squared bias as following equation (R. Deutsch, 1965; Kennedy, 2008; Lehmann & Casella, 1998; Sage & Melsa, 1971):

$$\text{Mean Square Error}(\hat{\theta}) = E\{(\hat{\theta} - \theta)^2\} = \text{Var}(\hat{\theta}) + \Big(\text{Bias}(\hat{\theta}, \theta)\Big)^2 \tag{5.4}$$

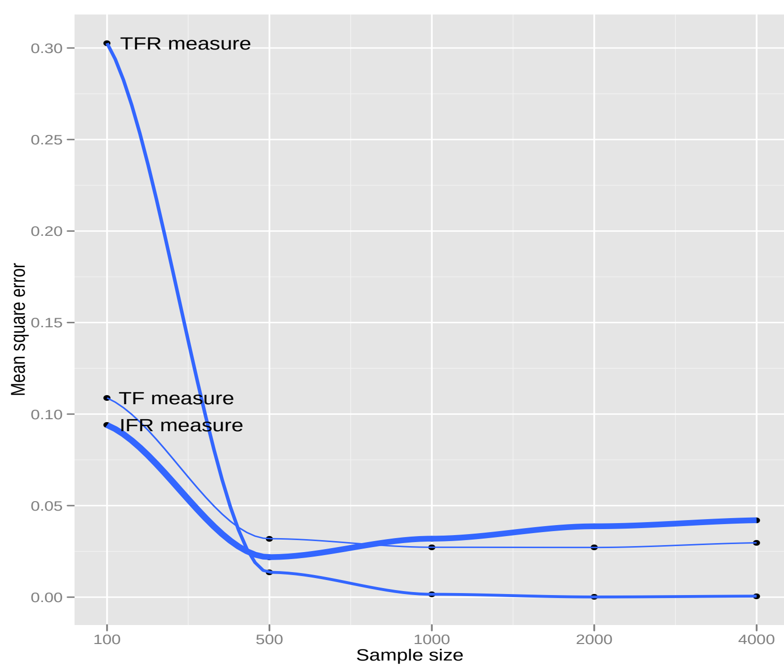


Figure 5.4: Changes in mean square error

[148] Altman, Gill, and McDonald (2004) argue that a good estimator should yield 'stable' results, such as, monotonic and smooth reduction of variance, with increasing sample size to make a correct inference.
[149] For each sample size, 5,000 simulations are run.

For the TF and the TFR measure, even though there is some difference in the mean square error when the sample size is quite small, it gets smaller as sample size increases. This can be interpreted as one evidence of consistency, which indicates the property of asymptotic unbiasedness. Especially for the TFR measure, MSE decreases drastically when sample size goes up from 100 to 500, and after sample size gets to 1,000, MSE appears almost zero (see table 5.4). Thus, it can be concluded that when sample size is extremely small, under 100, TFR measure has more concern about bias than the other two measures. As the sample size increases over 500, however, the three measures show consistently small mean square errors, almost close to zero. From a practical point of view, this has one good implication: in order to have a relatively accurate fuzzy measurement of poverty, we do not have to have a very large sample. However, for the IFR measure, it turns out that the mean square error is increasing with sample size, though the magnitude is small enough to be considered as zero. Still, for relatively small sample size, it can be pointed that the IFR measure can produce more accurate estimates than the other measures.

Table 5.4: Trends in MSE for three fuzzy measures

Sample size		TF measure	TFR measure	IFR measure
100	MSE	1.088×10^{-5}	3.026×10^{-5}	9.402×10^{-6}
	Variance	(2.449×10^{-10})	(2.478×10^{-11})	(3.725×10^{-10})
500	MSE	3.193×10^{-6}	1.355×10^{-6}	2.187×10^{-6}
	Variance	(1.859×10^{-11})	(2.101×10^{-13})	(4.484×10^{-12})
1,000	MSE	2.726×10^{-6}	1.560×10^{-7}	3.195×10^{-6}
	Variance	(9.269×10^{-12})	(1.237×10^{-14})	(3.554×10^{-12})
2,000	MSE	2.715×10^{-6}	1.061×10^{-8}	3.871×10^{-6}
	Variance	(5.328×10^{-12})	(2.497×10^{-16})	(2.101×10^{-12})
4,000	MSE	2.968×10^{-6}	5.564×10^{-8}	4.202×10^{-6}
	Variance	(3.068×10^{-12})	(1.030×10^{-15})	(1.113×10^{-12})

5.1.3 Robustness

Finally, one important property that is required for the fuzzy measures as a poverty measurement is robustness. Since the datasets used in real-world measurement studies are very prone to having various kinds of irregularities in them, the previously examined two statistical properties are, in fact, not the central interest for social scientists. Instead, whether a measurement is robust to diverse sources of irregularities has been a major concern, especially for multidimensional poverty measurement case (Alkire & Foster, 2009; Booysen, van der Berg, Burger, Maltitz, & Rand, 2008; Desai, 1991; Foster, McGillivray, & Seth, 2009; Marlier & Atkinson, 2010; Nolan & Whelan, 1996).

For the fuzzy measures of poverty, there can be two possible sources of non-robustness: 1) error from setting lines that divide the sample into different groups[150], and 2) error from indicator measurements (Zheng, 2000). Additionally, Duclos, Sahn, and Younger (2006b) point out that the relationship between indicators also matters especially for multidimensional poverty measurement. Since the calculation procedures for the TFR and the IFR measure do not involve setting lines, the investigation of robustness to setting lines is limited to the TF measure. However, for measurement error, all three measures' robustness is examined, considering the change in relationships between indicators reflected in variance-covariance matrix.

5.1.3.1 Robustness to setting minimum and maximum lines For the TF measure, the biggest criticism is that it fundamentally adopts "two poverty lines"(Miceli, 1998; Qizilbash & Clark, 2005), which can never be free from some degree of arbitrariness. To show robustness, I assume that the minimum line for income variable (j_{min} in equation 2.4) under which everyone is considered as *definitely poor* is higher than the existing poverty line (60% of median income), which is equivalent to saying that we can agree unanimously that a person is 'poor' when the person's income is lower than 60% of median income, and the maximum line (j_{max}) is lower than 200% of median income. Here, it is certain that the decision itself is not free from the criticism for arbitrariness. However, I would argue that

[150] In traditional sense, this is a problem of setting a "poverty line".

the arbitrariness is defensible in that the arbitrariness fundamentally originates from the fact that poverty is a complex and 'fuzzy' concept. For the saving variable, I consider the minimum line fixed at zero because 64.4% of the sample does not save at all, and the upper limit of the maximum line is assumed to be the median of saving among people who have saving - £1,200[151], the lower limit being the first quartile of saving (£600)[152]. In sum, the robustness of the TF measure to poverty line[153] is examined as the two lines are determined randomly[154] within the range in table 5.5.

Table 5.5: Range for the two lines

	Income	Saving
Minimum	60% of median~Median	0 (fixed)
Maximum	150% of median~200% of median	£600~£1,200

Under these assumptions, table 5.6 is obtained from 5,000 simulations. It turns out that when the two lines for income changes, the mean of simulated TF indices is about 2.6% larger than the index calculated from BHPS data. Given the 95% confidence interval of the TF index is [0.1909,0.1958], assuming normal distribution, it is hard to conclude that the difference is trivial. However, judging from the small variance of the simulated indices, it can be concluded that the influence of setting two lines to the TF measure is limited. Figure 5.5 shows that even the simulated index which has the biggest error is a little bigger than 0.20. Although the mean and variance is smaller, the same conclusions can be made from the simulation for criteria changes for the saving variable (see Appendix A figure A9). Finally, I put the two variations in the calculation of the TF measure at the same time, and the last row of table 5.6 shows that still the arbitrary decision for criteria on two continuous variables does not drastically influence the aggregate index. The difference is just about 2.7% of the

[151] By this criterion, 996 out of 6,339 in the BHPS data (15.7%) are *definitely nonpoor*.

[152] It turns out that 1,846 observations, or 29.1% of the sample in the BHPS data have saving greater than £600. In fuzzy perspective, it is not unreasonable to assume that this level of saving makes a person *definitely nonpoor* at least with regard to saving.

[153] Since quite few people have inheritance (only 125 cases, 2.0% of sample, receive money from their ancestors), the inheritance variable is not considered here.

[154] Each line is drawn from a uniform distribution.

original calculation with very small variance (for distribution, see Appendix A figure A10.).

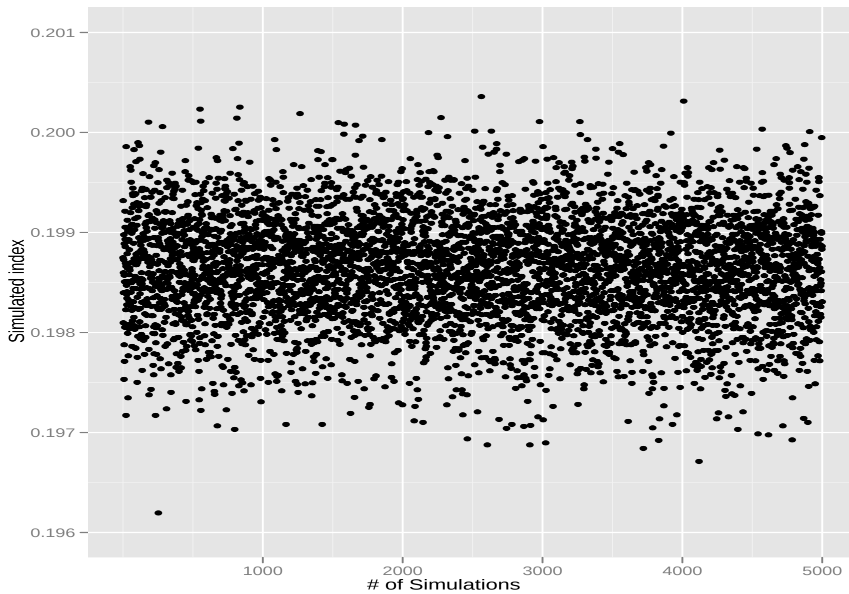


Figure 5.5: Distribution of the simulated TF index, change in income criteria

Table 5.6: Robustness to minimum and maximum lines

	Mean	Variance	S.D.
Real index	19.33		
Simulated for income criteria only	19.86	2.77×10^{-3}	5.26×10^{-2}
Simulated for saving criteria only	19.74	1.86×10^{-3}	4.31×10^{-2}
Simulated for both criteria	19.86	2.63×10^{-3}	5.12×10^{-2}

5.1.3.2 Robustness to measurement errors Since all kinds of social measurements contain measurement errors in them (Black, 1999; Deaton, 2010), it is quite important for composite indices such as the fuzzy poverty measures to be robust to measurement errors. Especially in multidimensional poverty index case where many indicators are combined to produce small numbers of indices, as small measurement errors in each indicator can be accumulated to build up a large bias in the resulting index (Voinov & Nikulin, 1993), the robustness gets more important.

To test the robustness, I introduce random errors for continuous indicators - income, saving, and inheritance - that are drawn from a normal distribution, $N(\mu, \sigma^2)$, where both μ and σ are equal to the 10% of an individual's income. Thus, in case of income, I assume that income is measured incorrectly by 10% on average, and in 84.1% of the case it is *under-reported* [155](Deaton, 1997; Federman & Garner, 1996; Haughton, 2009). Here, it is assumed that the error can have both positive and negative sign because many factors besides underreporting can affect the sign. For example, if income is measured weekly, a calculation of yearly income, a multiplication of 50, can certainly build up an error in the measurement, the sign of which is not clear. For ordinal variables, I assume that measurement errors turn out to be one greater or smaller than 'true' value. Thus, I simulate the random measurement errors as a multiplication of two random variables drawn from two Bernoulli distributions, in which one decides the sign of error from $Bern(0.5)$, and the other determine the amount of error by $Bern(0.67)$. The latter probability is not a half because I assume that the number of people who 'over-evaluate' their preference is roughly same to the number of people who 'under-evaluate' it. But I do not include random measurement error in binary indicators since the possibility of measurement error seems to be quite low for this level of measurement. As a result, I include 18 random errors in the 5,000 simulations of the three fuzzy measures of poverty[156].

Table 5.7 shows that the fuzzy measures of poverty are quite robust to the measurement errors. In terms of expected value from the simulation, it turns out that the biggest difference between the original index and the expectations, the TF measure case, is just 1.3. For the IFR measure, the difference is only 0.2, both with extremely small variances. Although all the differences are non-trivial in the sense that the Monte Carlo confidence intervals - [20.47,20.77], [16.42,16.46], and [34.84,34.92], respectively - do not contain the

[155]In a normally distributed random variable, say $Z \sim N(k, k^2)$, $Pr(Z > -k)$ =0.841.

[156]It is certainly possible to make the errors more relevant by modeling correlated part in error, instead of assuming a simple random error. For example, let a variable $X_{simulated} = X_{BHPS} + \epsilon^*$, in place of a seemingly random error ϵ^*, I can put error $\rho X_{BHPS} + \epsilon$, where ρ represents a relationship between error and observation, and ϵ truly random error. By the formulation, a tendency in error can be modeled explicitly, i.e., people with higher income are more inclined to under-report their income.

Table 5.7: Robustness to measurement errors

	TF	TFR	IFR
Original index	19.33	16.59	35.08
E(Simulation)	20.62	16.44	34.88
$S.D.$(Simulation)	0.1058	0.0178	0.0281

original indices, it is highly likely that the difference originates from the problem of random number generation, not the calculation method itself, because figure 5.1 and table 5.1 above clearly show that the parametric bootstrapping - the assumption of multivariate normality, to be exact - induces bias in the estimation, comparing to the results from nonparametric bootstrapping (see figure 5.2.).

5.2 IDENTIFICATION ASPECT

Callan and Nolan (1991) argue that recent studies on poverty measurements have usually focused on the aggregation part of them since Sen (1976)'s seminal work. However, Sen (1997) himself also emphasizes that 'identification' which is about identifying who are the poor should not be ignored, because it provides not only an answer to the fundamental question of what is poverty, but also a crucial information for more effective policy (Hagenaars, 1991; Miceli, 1998; Ravallion, 1992). Although it is against the nature of the fuzzy approach to poverty to make a binary distinction between the poor and the non-poor, still it is reasonable to examine how the fuzzy measures for each individual are distributed, and to see how each individual is positioned in the distribution.

As a first step, the distribution of the fuzzy measures needs to be examined since even simple visualization of the distribution can provide interesting insights about the identification performace of the multidimensional poverty measures. In figure 5.6, the first point that can be observed is that IFR measure shows a more even distribution than the other two measures. It turns out that the maximums of TF and TFR measure are not very close to one, which implies no one is "definitely poor". This interpretation seems reasonable judging from the fact that the data comes from U.K., one of the most advanced industrial countries. However, this does not mean that the information from IFR measure is misleading because it can be interpreted as emphasizing the relative conception of poverty. The number of people whose fuzzy poverty measure value is over 0.5, on the other hand, shows that only very small fraction of population is closer to *definitely poor* than *definitely non-poor.* For instance, only 0.36% of the sample is closer to *definitely poor* by TF measure, which, in reverse, implies 99.64% is closer to *definitely non-poor.* In spite of the context of U.K., this information is not so convincing, especially considering that the fuzzy measures are generally based on the relative poverty concept.

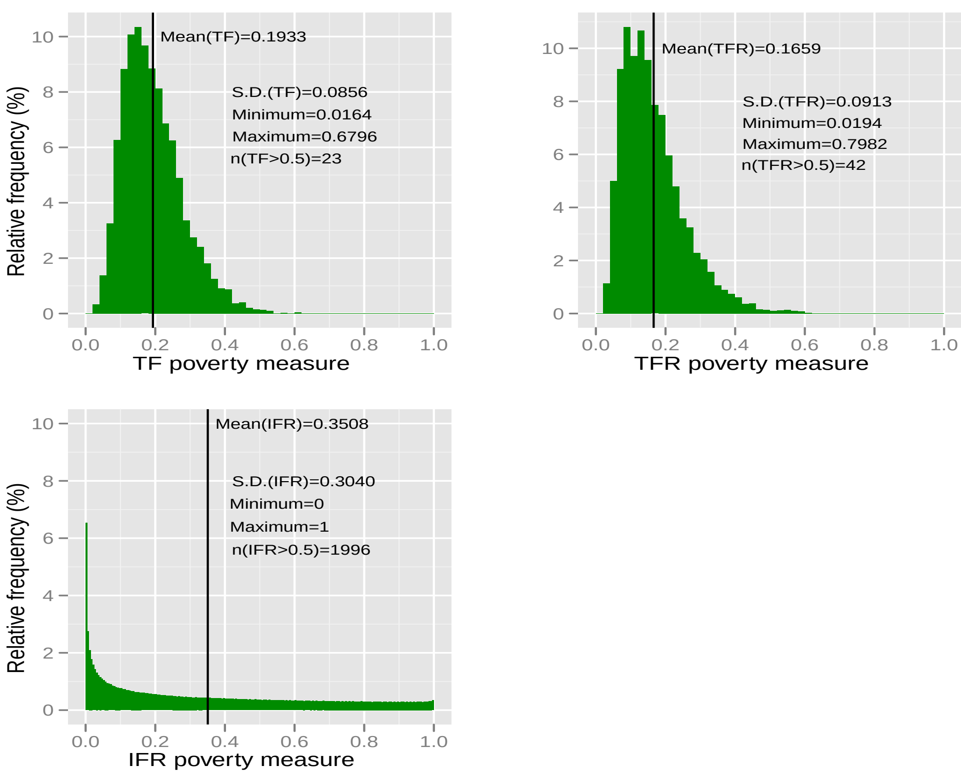


Figure 5.6: Distribution of the fuzzy measures of poverty

Considering that the three fuzzy measures are within $[0, 1]$, I calculate the number of people who have a higher value of the fuzzy measures than some values, such as, mean or 0.5. This helps us to understand the identification performance of the fuzzy measures clearly because after all, the number of people who are closer[157] to the phenomenon of 'poverty' is the most important and fundamental information for a policy-making process. Following table 5.8 shows the expectation of the number of people whose fuzzy measures are bigger than a series of values, 0.3, 0.4, 0.5, and the average.

Table 5.8: Differences in Identification[a]

	TF index		TFR index		IFR index	
	BHPS	Simulated	BHPS	Simulated	BHPS	Simulated
$\mu_i(x) > E(\mu_i(x))$[b]	2,769	2,985.90	2,620	2,839.03	2,734	2,728.06
		(24.14)[c]		(23.19)		(0.76)
$\mu_i(x) > 0.3$	729	503.85	549	298.95	3,016	3,000.17
		(17.70)		(13.68)		(1.80)
$\mu_i(x) > 0.4$	50	38.04	147	31.88	2,477	2,460.01
		(5.89)		(5.34)		(1.81)
$\mu_i(x) > 0.5$	23	1.91	42	2.57	1,996	1,978.65
		(1.02)		(1.39)		(1.78)

[a] Each measure is simulated 3,000 times

[b] $\mu_i(x)$ indicates the fuzzy measures for each individual i.

[c] The numbers in parenthesis are standard deviations of 3,000 calculations.

It turns out that both TF and TFR measure significantly underestimate the number of people whose fuzzy measure value is greater than certain thresholds, especially when the numbers are closer to one. On the contrary, IFR index appears to identify the number of people quite consistently[158]. From the observation, it can be inferred that TF and TFR

[157] If non-fuzzy perspective is adopted, then this sentence can be replaced by "the number of people who are *in* the phenomenon of poverty".

[158] For graphical presentation, see figures in Appendix A.1

measure tend to underestimate the number of people who are relatively closer to *definitely poor*, while IFR measure provides quite accurate numbers. This finding provides a crucial implication for utilizing the fuzzy measures because it means that TF and TFR measures are more likely to underestimate the number of people who are closer to *definitely poor* than IFR measure, which is a great drawback for a measurement method to inform public policy.

5.3 CONCLUSION

In this chapter, I examine the statistical behaviors of the fuzzy measures of poverty, which are recently suggested to embrace the capability approach by Sen (1979a, 1981, 1985a) but are never clearly shown to be reliable statistics. Since there are not many reliable multidimensional data that can be a basis of the calculation, I utilize the Monte Carlo method to test the properties. To be more precise, I generate a number of datasets from a multivariate normal distribution with the parameters computed from the 16th wave of BHPS data, and calculate each measure. As desirable behaviors for an aggregate statistic, I focus on the sampling distribution, small-sample behavior, and robustness, and for a function of identification, I delve into the change in the number of people who are closer to *definitely poor* than a series of values.

In terms of aggregation, it turns out that the three measures have very small confidence intervals with a symmetric and nicely-behaving sampling distribution, which implies that all three measures are quite accurate estimate of a population parameter. However, though the investigation for small-sample cases through mean square error shows that for two measures - the TF and the TFR measure - it can be concluded that they are reliable in small sample case, the IFR measure does not appear to be in spite of the smaller mean square error than the other two measures with a relatively small sample size. Finally, as a more important property for a social measurement, I test the robustness of the measures by introducing random errors in the data. The investigation shows that all three measures are quite robust to the random errors in measurement. Although the errors from simulation made by the measurement errors are statistically significant, it is more reasonable to conclude that the differences come from the unverified assumption of multivariate normality, rather than to say that they appear due to the sensitivity to measurement errors. More importantly, the absolute changes in the fuzzy measures due to measurement errors are extremely small.

I also examine the measures with respect to the performance in identification. I find that TF and TFR measure considerably underestimate the number of people who are relatively closer to *definitely poor*, while the estimation from IFR measure is very consistent.

6.0 CONCLUSIONS

6.1 SUMMARY OF THE RESEARCH

Truly, defining and measuring poverty is not a clear-cut research topic that can be done thoroughly by one study because the definition eventually rests on the social contexts of a society (Chiappero-Martinetti & Moroni, 2007; Orshansky, 1965; Øyen, 1996; Sen, 1992; Townsend, 1985). For instance, a *poor* household identified in the U.S. is in a totally different situation from a *poor* household by World Bank's criteria - one or two dollars per day. This implies that attempts to have a universal definition of poverty might really turn out to be futile, not to mention useful insights. Besides, an idea of multidimensional poverty seems to make the obstacle more difficult in that now it brings up a new question of deciding proper dimensions for poverty (Alkire, 2002; Clark, 2003; Clark & Hulme, 2005; Kakwani & Silber, 2008; Robeyns, 2005b).

In spite of these difficulties, it is an imperative to find an appropriate definition as well as measurement of poverty since the social phenomenon is related to almost every social problem in a society. Especially the context of more and more globalizing world makes the necessity more urgent because there has been a concern about the extreme polarizing effect of globalization - so-called "race-to-the-bottom" (Agénor, 2004; Bello, 2001; Gough, 2001; Rudra, 2008).

As a theoretical breakthrough for the challenges of defining poverty as a multidimensional concept, I focus on the capability approach, which emphasizes actual functionings of people in society because it provides a proper theoretical justification for the multidimensional perspective on poverty. However, since the approach is "just a general and flexible framework of thought" (Chiappero-Martinetti & Moroni, 2007), it is still not enough to get a satisfactory list of human functionings and capabilities that can work as the components of multidimensional poverty. Thus, I look into empirical studies that delve into the multiple dimensions of human well-being in order to compile a more agreeable list of dimensions, and identify seven dimensions of poverty as well as empirical indicators that measure the concept (for detailed list, see table 2.5 in Chapter 2).

For a method to aggregate the information in multiple indicators of multiple dimensions, I choose a method that is grounded on the idea of fuzzy-set because one of the most frequent critiques for traditional poverty measurement methods is that all those methods are dividing a population into two status, the criterion of which is arbitrary in nature. I argue that the very idea of fuzzy-set, which fundamentally expresses a characteristic as a degree of inclusion to a characteristic set, could be a good way to address the critique of arbitrary division. So, I apply three previously-suggested fuzzy measures of poverty - *Totally Fuzzy (TF), Totally Fuzzy and Relative (TFR),* and *Integrated Fuzzy and Relative (IFR)* - to the 16th wave of *British Household Panel Survey (BHPS)* data, and demonstrate the unique information that the fuzzy measures can show. This exercise makes it clear that the simple binary distinction of traditional approach to poverty is not enough to represent multidimensional, complex and vague nature of the concept because it turns out that there are significant diversities within income-poor group as well as income-nonpoor group, and the fuzzy measures appear to summarize individual household's situation more acceptably, considering the multiple dimensions of capability. In addition, the comparison between traditional measures and the fuzzy measures reveals unequivocally that the pictures from two approaches are quite different. Especially, it can be clearly seen that the fuzzy measures at least provide a rich ground on which we can clarify the definition of poverty.

Finally, I examine the statistical behaviors of the fuzzy measures of poverty, which have never been clearly shown to be reliable statistics. Since there are not many reliable multidimensional data that can be a basis of the calculation, I utilize the Monte Carlo method to test the properties. As desirable behaviors for an aggregate statistic, I focus on the sampling distribution, small-sample behavior, and robustness, and for a function of identification, I delve into the change in the number of people who are closer to *definitely poor* than a series of values. Based on Sen (1976, 1979b)'s distinction of the aspect of aggregation and identification, I separately run the simulation. For aggregation aspect, it turns out that the three measures have very small confidence intervals with a symmetric and nicely-behaving sampling distribution, which implies that all three measures are quite accurate estimate of a population parameter. However, though the investigation for small-sample cases through mean square error shows that for two measures - the TF and the TFR measure - it can be concluded that they are reliable in small sample case, the IFR measure does not appear to be in spite of the smaller mean square error than the other two measures with a relatively small sample size. In terms of robustness, I find that all three measures are quite robust to the random errors in measurement. Although the errors from simulation made by the measurement errors are statistically significant, it is more reasonable to conclude that the differences come from the unverified assumption of multivariate normality, rather than to say that they appear due to the sensitivity to measurement errors, because non-parametric bootstrapping, a method which does not need the restrictive assumption, shows that the errors are negligible. I also examine the measures with respect to the performance in identification. I find that TF and TFR measure considerably underestimate the number of people who are relatively closer to *definitely poor*, while the estimation from IFR measure is very consistent.

6.2 FURTHER STUDIES

Although this study shows that the fuzzy measures of poverty can be diverse sources of crucial information on poverty, still there are several questions that must be addressed properly to make the measures more usable for public policy:

1. **Exploring the dimensions of poverty:** The inevitable arbitrariness in the choice of dimensions for a multidimensional poverty measurement does not keep us from exploring more agreeable dimensions of poverty, which is an indispensable process for formulating *better* public policies. However, it is hard to deny that the choice of the dimensions of poverty still lacks some levels of transparency and objectivity. Thus, what can be included as the dimensions of poverty needs to be studied thoroughly by both quantitative

and qualitative approach. In addition, the comparison of the conception of poverty among different social strata is required in order to get closer to political consent on the multidimensional measurement of poverty.

2. **Demonstrating new insights from multidimensional poverty:** In spite of the theoretical necessity, multidimensional poverty measurement in general is not a popular topic of research because it is difficult to compare and contrast the results with extant poverty measurements, which are used widely to inform policymakers. Also, what kinds of information it could supply for policymakers is not so clearly demonstrated so far. Thus, the new insights from multidimensional perspective needs to be presented by the comparison and/or contrast with the policy implications from traditional measurements.
3. **Investigating the dynamics of multidimensional poverty:** Although there have been numerous attempts to delve into the dynamics of poverty, all those studies only adopt unidimensional perspective on poverty. Since the knowledge of the dynamic change in multidimensional poverty can provide a more comprehensive understanding of the subject, the dynamics needs to be examined. Also, this topic is important because it can give us a crucial insight on the different poverty experiences of individuals, which is one essential aspect of multidimensional poverty.

Appendix A

Figures and Tables

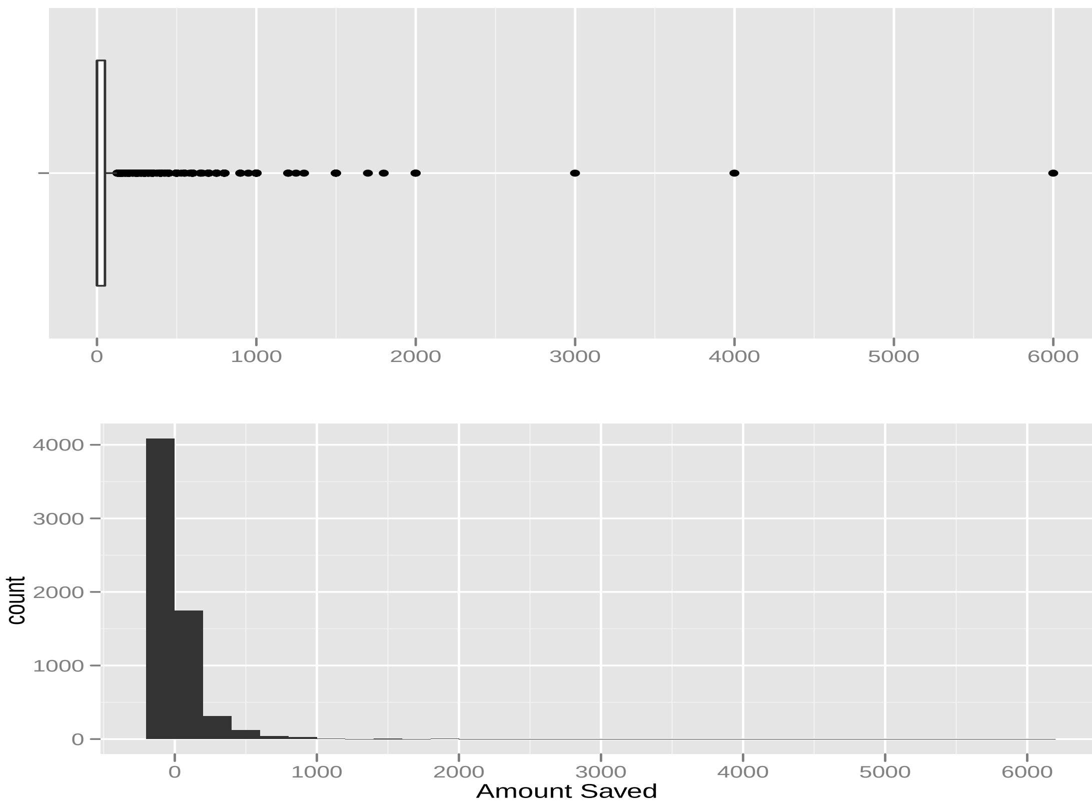


Figure A1: Individual saving distribution

Table A1: Health inhibits activities

	Frequency	Percent	Cumulative perc.
Often	773	12.19	12.19
Sometimes	1,449	22.86	35.05
Not often	1,499	23.65	58.70
Never	2,618	41.30	100
Sum	6,339	100	

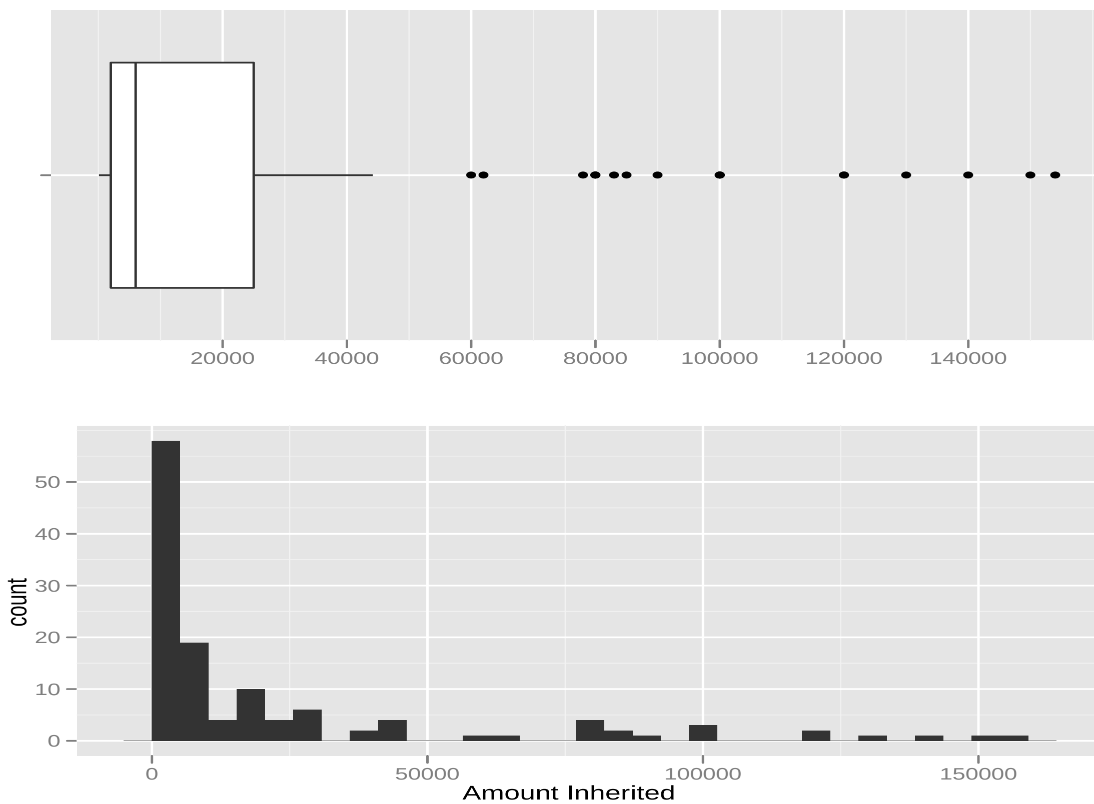


Figure A2: Individual inheritance distribution

Table A2: Satisfaction with job security

	Frequency	Percent	Cumulative perc.
No job	2,798	44.14	44.14
Completetly dissatisfied (1)	76	1.20	45.34
2	80	1.26	46.60
3	225	3.55	50.15
Neither satisfied nor dissatisfied (4)	252	3.98	54.13
5	712	11.23	65.36
6	1,411	22.26	87.62
Completely satisfied (7)	785	12.38	100
Sum	6,339	100	

Table A3: The number of cars owned

	Frequency	Percent	Cumulative perc.
None	1,429	22.54	22.54
1	2,780	43.86	66.40
2	1,777	28.03	94.43
3+	353	5.57	100
Sum	6,339	100	

Table A4: Frequency of interactions

	Talking to neighbors			Meeting people		
	Frequency	Percent	Cum.perc.	Frequency	Percent	Cum.perc.
On most days	2,700	42.59	42.59	2,841	44.82	44.82
Once or twice a week	2,329	36.74	79.33	2,635	41.57	86.39
Once or twice a month	860	13.57	92.90	687	10.84	97.22
Less often than above	329	5.19	98.09	168	2.65	99.87
Never	121	1.91	100	8	0.13	100
Sum	6,339	100		6,339	100	

Table A5: Contact with the closest friend

	Frequency	Percent	Cumulative perc.
Most days	2,558	40.35	40.35
At least once week	2,576	40.64	80.99
At least once a month	897	14.15	95.14
less often	308	4.86	100
Sum	6,339	100	

Table A6: Correlation coefficients for each dimension's membership functions after logistic transformation, TF measure

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) Economic resources	1.000						
	(0.000)						
(2) Health	0.275	1.000					
	(0.000)	(0.000)					
(3) Employment	0.331	0.367	1.000				
	(0.000)	(0.000)	(0.000)				
(4) Housing	0.212	0.093	0.009	1.000			
	(0.000)	(0.000)	(0.474)	(0.000)			
(5) Durable goods	0.302	0.194	0.405	0.059	1.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		
(6) Social capital	0.000	0.148	-0.055	0.097	-0.066	1.000	
	(0.981)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
(7) Social participation	0.331	0.303	0.777	0.017	0.382	-0.053	1.000
	(0.000)	(0.000)	(0.000)	(0.164)	(0.000)	(0.000)	(0.000)

* The numbers in parenthesis are $p - values$ from the significance test.

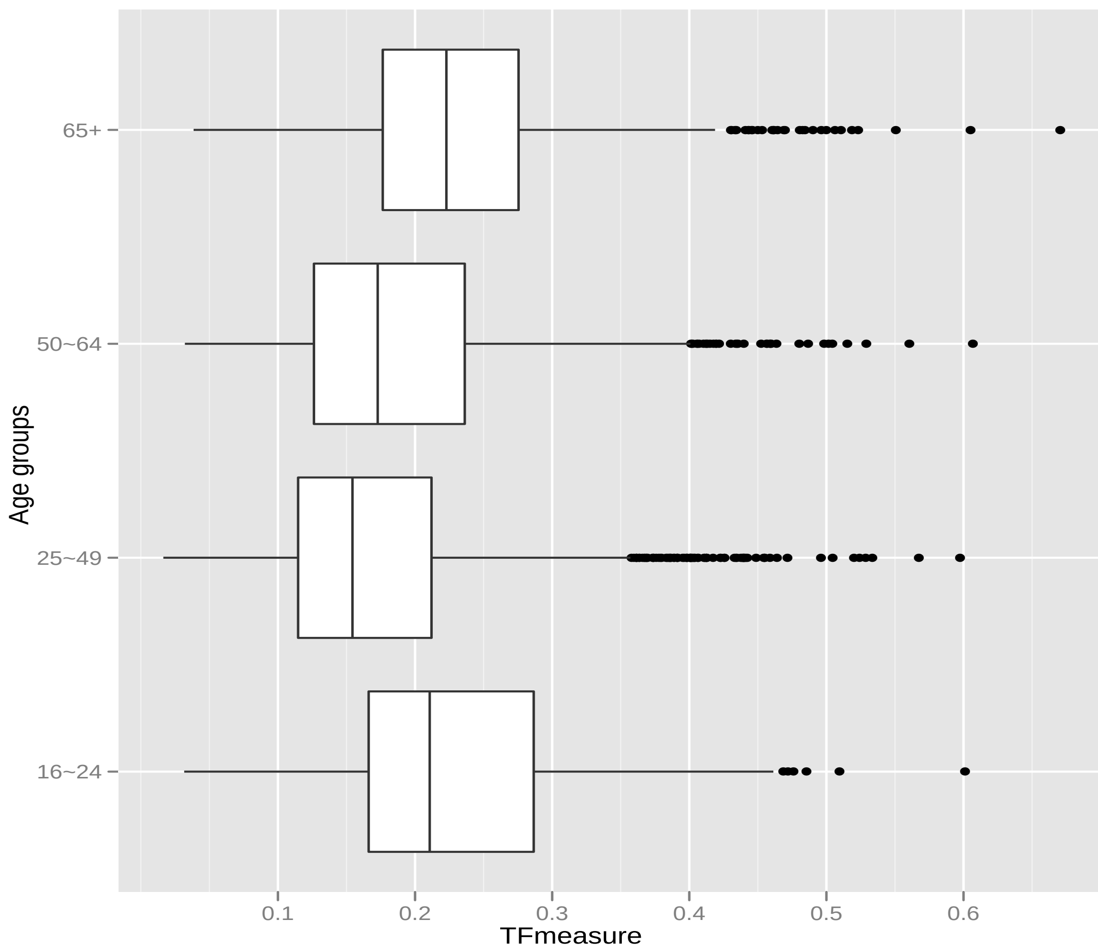


Figure A3: TF distribution for age groups

Table A7: Subgroup decomposition by age and household type, TF measure

	16~24	25~49	50~64	65+
Single non-elderly	0.252	0.194	0.239	
Single elderly			0.213[a]	0.256
Couple: no child	0.207	0.158	0.171	0.204
Couple: dependent child	0.227	0.159	0.184	0.238
Couple: non-depedent child	0.175	0.175	0.163	0.191
Lone parent: dependent child	0.254	0.200	0.192	0.286
Lone parent: non-dependent child	0.192	0.189	0.192	0.233
2+ unrelated adults	0.217	0.179	0.242	0.301
Other types	0.229	0.205	0.191	0.236

* Blanks indicates no observation for the cell.

[a]110 cases report that their houshold type is 'single elderly', though the age of householder is under 65.

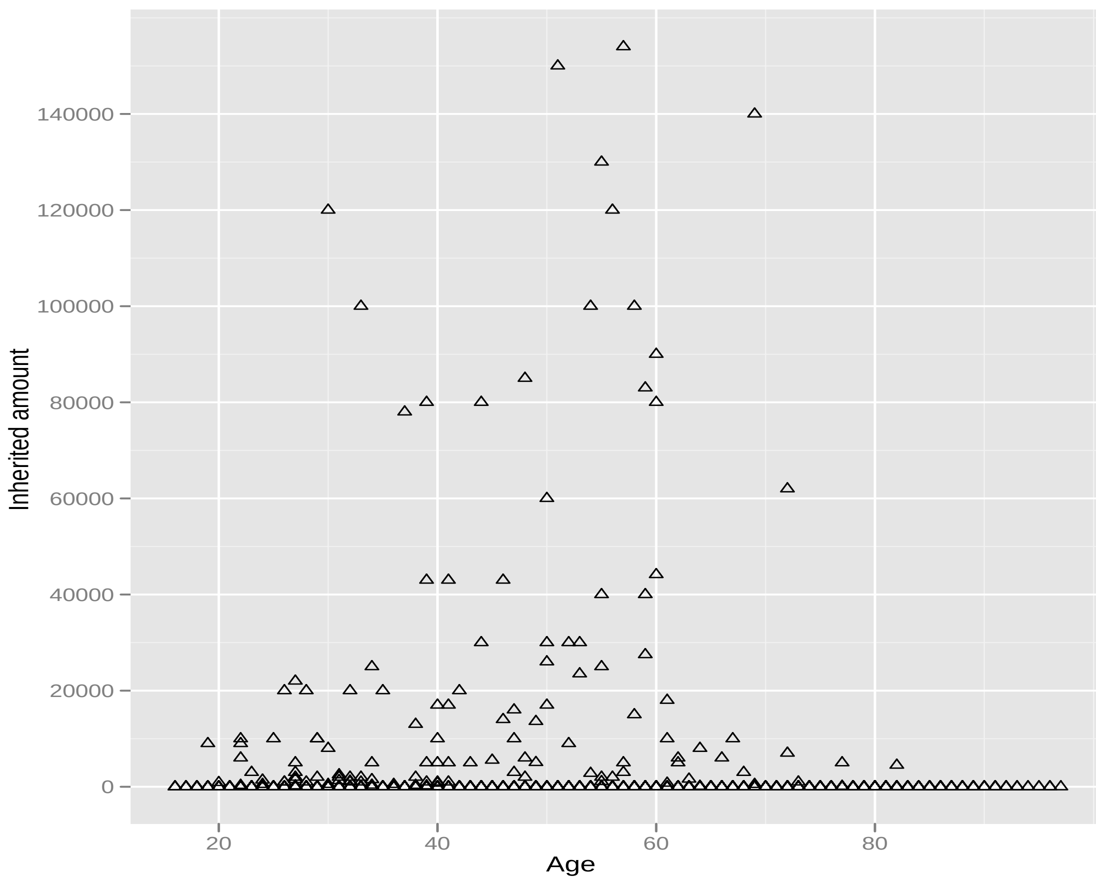


Figure A4: Inheritance distribution across age

Table A8: Sensitivity analysis for saving higher threshold, decreasing case

	£1,100	£1,000	£900	£800	£700	£600	£500
Mean	0.193	0.193	0.193	0.193	0.193	0.193	0.193
Median	0.179	0.179	0.179	0.179	0.179	0.179	0.179
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016
1st Quantile	0.130	0.130	0.130	0.130	0.130	0.130	0.130
3rd Quantile	0.243	0.243	0.243	0.243	0.243	0.243	0.243
Maximum	0.671	0.671	0.671	0.671	0.671	0.671	0.671
S.D.	0.086	0.086	0.086	0.086	0.086	0.086	0.086
# of 'completely non-poor'	1,404	1,414	1,504	1,525	1,570	1,846	1,851

Table A9: Sensitivity analysis for inheritance higher threshold, decreasing case

	£5,300	£4,600	£3,900	£3,200	£2,500	£1,800	£1,100
Mean	0.193	0.193	0.193	0.193	0.193	0.193	0.193
Median	0.179	0.179	0.179	0.179	0.179	0.179	0.179
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016
1st Quantile	0.130	0.130	0.130	0.130	0.130	0.130	0.130
3rd Quantile	0.243	0.243	0.243	0.243	0.243	0.243	0.243
Maximum	0.671	0.671	0.671	0.671	0.671	0.671	0.671
S.D.	0.086	0.086	0.086	0.086	0.086	0.086	0.086
# of 'completely non-poor'	67	77	78	78	85	94	101

Table A10: Comparison of housing dimension for housing-owned groups

Group	Housing dimension
Owned	0.018
Owned with mortgage	0.022

Table A11: Group comparison according to housing tenure without housing dimension

Housing tenure	TFR measure without housing dimension
Owned	0.257
Owned with mortgage	0.164
Rented from local authority	0.308
Rented from housing associaton	0.316
Rented from employer	0.223
Rented private housing unfurnished	0.252
Rented private housing furnished	0.293

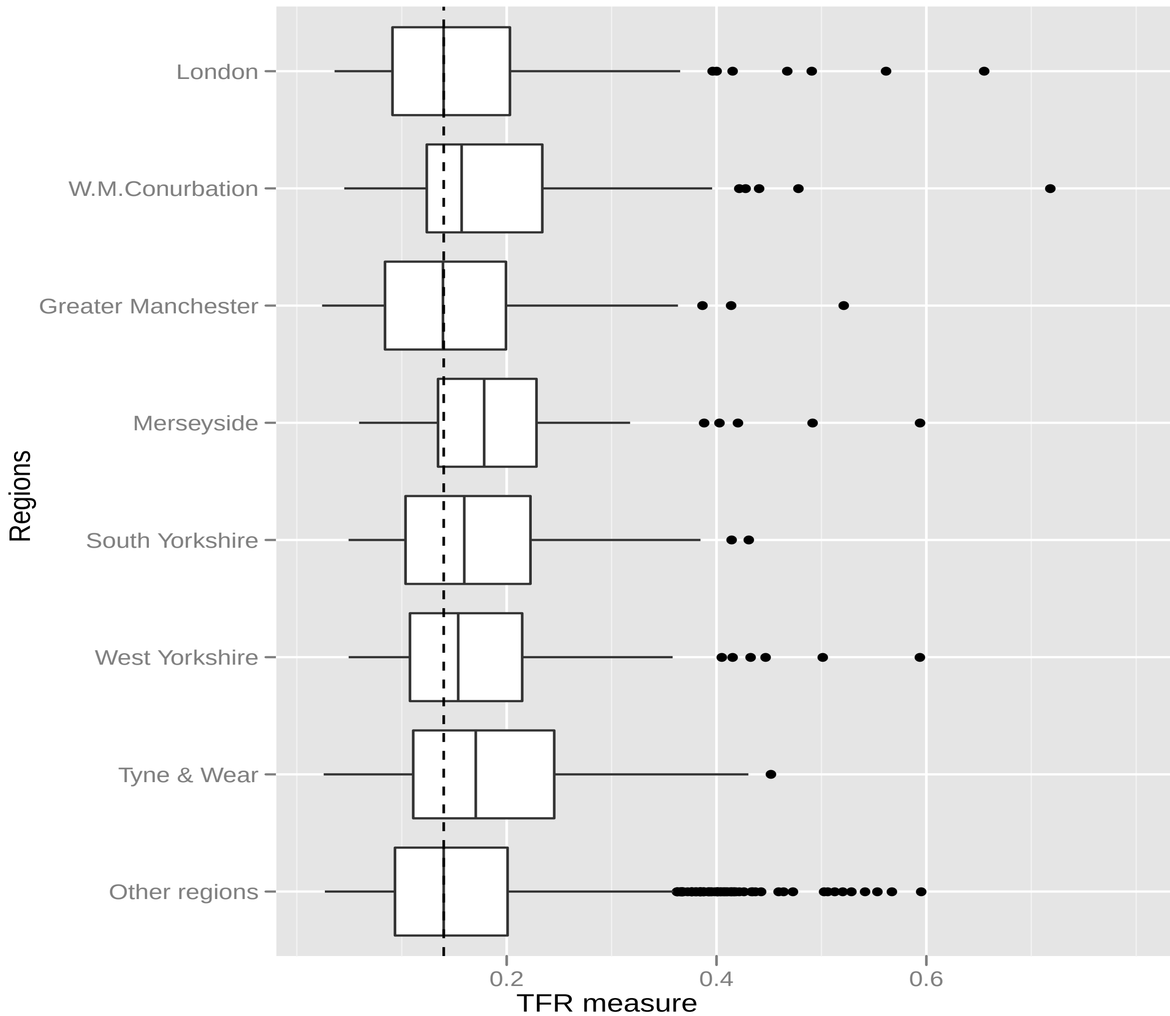


Figure A5: TFR measure distribution for metropolitan areas

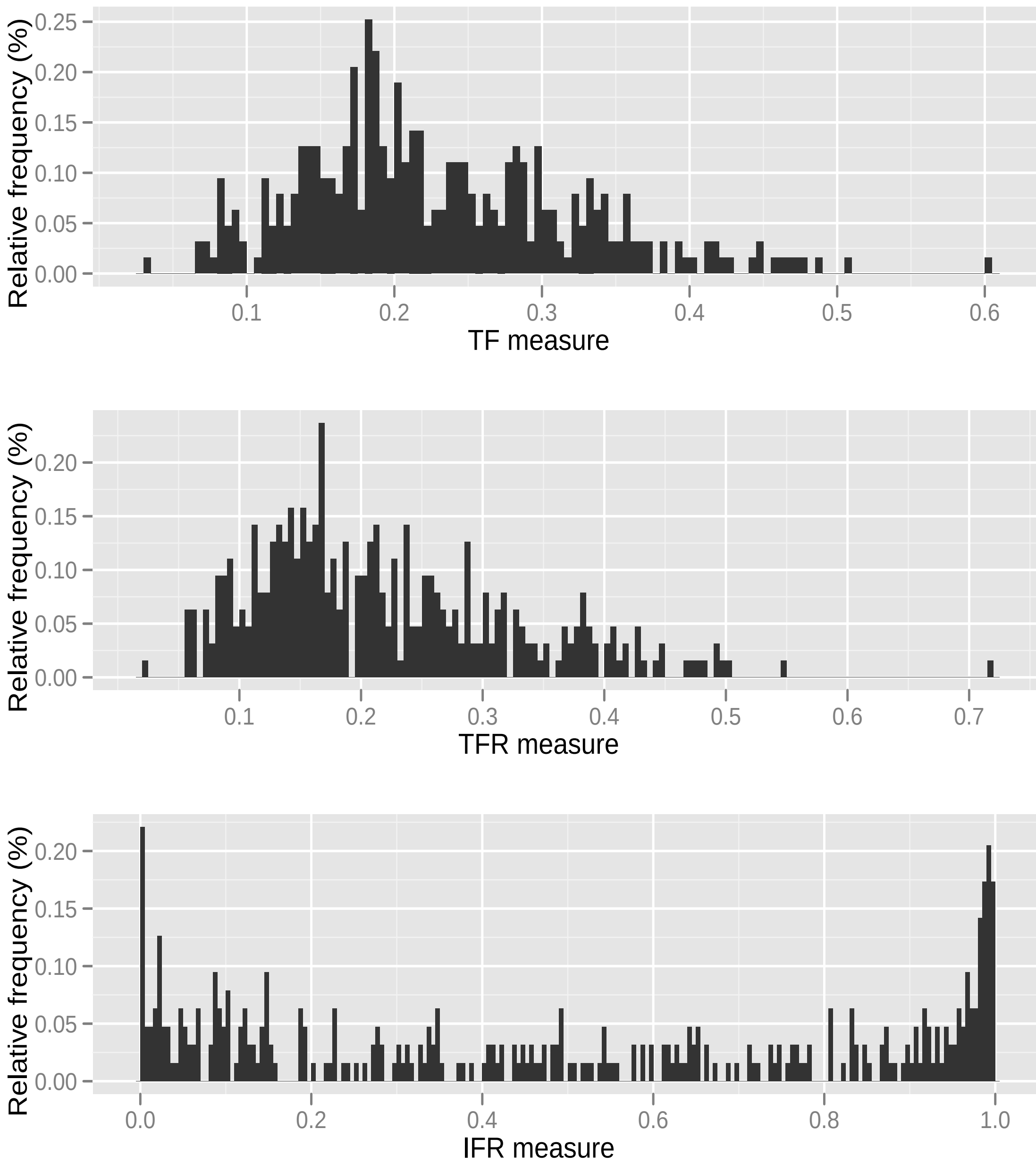


Figure A6: Poverty measure distributions for age group 16~24

Table A12: Contribution of each dimension by labor force status

	Employed	Disabled
Economic resources	0.019	0.031
	(14.6)	(13.1)
Health	0.013	0.030
	(10.4)	(12.8)
Employment	0.005	0.029
	(3.8)	(12.0)
Housing	0.027	0.045
	(21.1)	(18.7)
Durable goods	0.027	0.058
	(21.6)	(24.4)
Social capital	0.029	0.030
	(22.5)	(12.6)
Social participation	0.008	0.016
	(6.1)	(6.5)
Total	0.127	0.239
	(100)	(100)

Table A13: Dimensions by labor force status

	Self-employed	Employed	Unemployed	Retired	Student	Disabled
Health	0.297	0.249	0.352	0.445	0.245	0.750
Employment	0.400	0.139	0.979	0.996	0.771	0.975
Housing	0.047	0.067	0.110	0.043	0.150	0.112
Durable goods	0.042	0.061	0.130	0.151	0.177	0.127
Social capital	0.235	0.219	0.198	0.230	0.190	0.257
Social participation	0.646	0.450	0.939	0.932	0.821	0.948

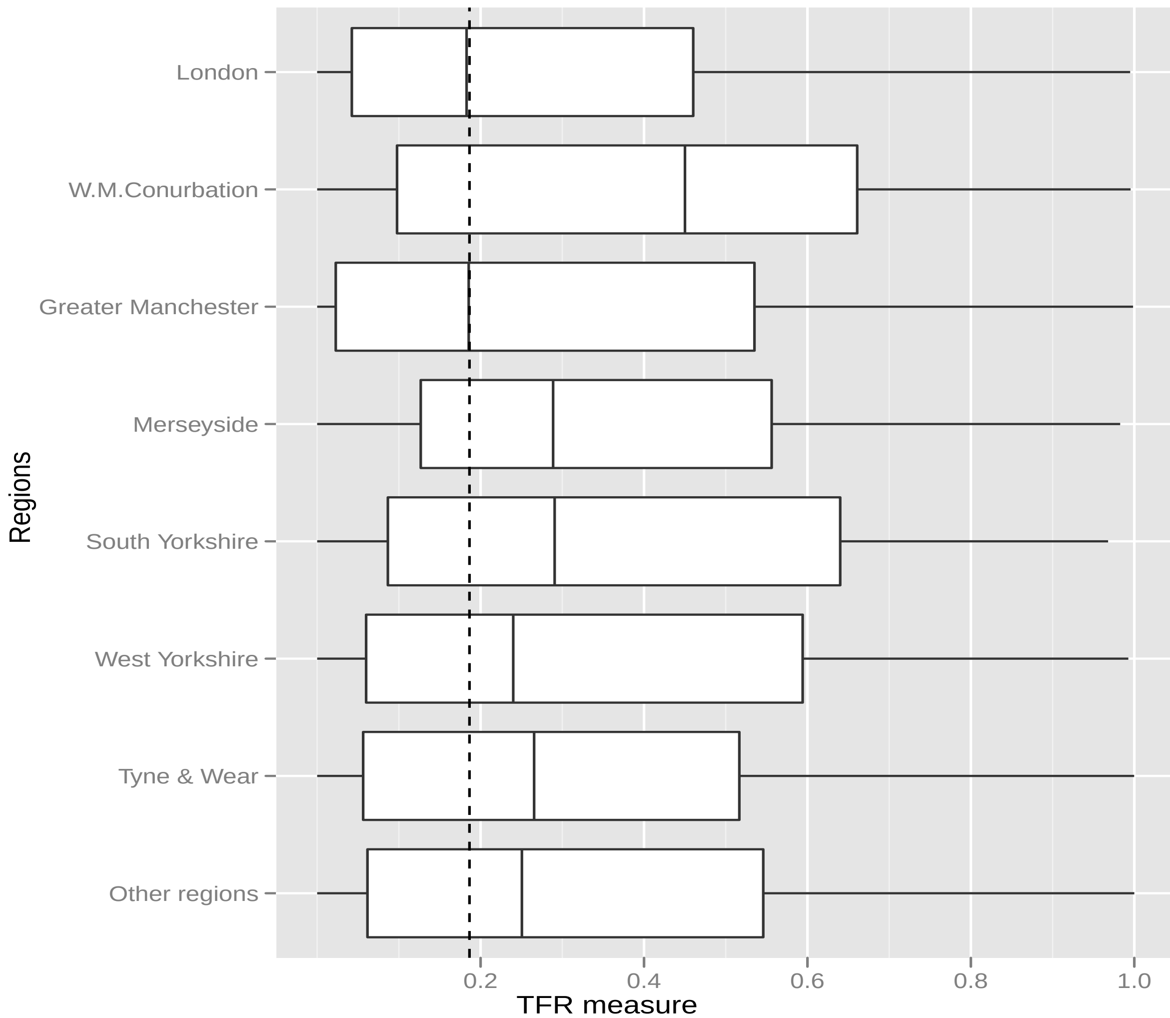


Figure A7: IFR measure distribution for metropolitan areas

Table A14: Rank order correlations between fuzzy measures

	TF	TFR	IFR
TF	1.000		
TFR	0.936	1.000	
IFR	0.419	0.446	1.000

Table A15: TFR measure: Individual comparison for extreme fuzzy-poor

	Income nonpoor		Income poor	Coding
	(1)	(2)	(3)	
TFR measure	0.798	0.718	0.668	
Household income	£16,250	£20,062	£12,908	Annual income
Health status	3	2	2	1(excellent)~5(very poor)
Permanent job	0	2	0	0(no job),1(temp.),2(perm.)
Shortage of space	2	2	2	1(yes),2(no)
Washing machine	0	0	0	0(don't have),1(have)
Satisf. with social life	7	5	4	1(not at all)~7(completely)
Voluntary work	5	5	5	1(once a week)~5(never)

Table A16: TF measure: Comparison of fuzzy-poor and fuzzy-nonpoor for male

Indicators	Fuzzy-poor		F-nonpoor		Weights	Remark
	I-nonpoor	I-poor	I-nonpoor	I-poor		
Household income	£24,601	£9,695	£34,146	£10,045	1.8	Annual income
Health status	2.91	2.98	1.99	2.17	4.2	1(excellent)~5(very poor)
Permanent job	0.67	0.15	1.43	0.53	0.9	0(no job),1(temp.),2(perm.)
Shortage of space	1.69	1.73	1.84	1.87	1.9	1(yes),2(no)
Washing machine	0.82	0.76	0.99	0.97	3.5	0(don't have),1(have)
Satisf. with social life	3.97	4.28	5.09	5.26	4.1	1(not at all)~7(completely)
Voluntary work	4.69	4.67	4.47	4.50	0.2	1(once a week)~5(never)
# of people	307	196	2,334	307		

Table A17: IFR measure: Comparison of fuzzy-poor and fuzzy-nonpoor for male

Indicators	Fuzzy-poor		F-nonpoor		Weights	Remark
	I-nonpoor	I-poor	I-nonpoor	I-poor		
Household income	£14,400	£9,818	£33,604	£10,403	27.9	Annual income
Health status	2.74	2.48	2.07	2.50	1.9	1(excellent)~5(very poor)
Permanent job	0.45	0.36	1.37	0.50	2.1	0(no job),1(temp.),2(perm.)
Shortage of space	1.85	1.81	1.82	1.82	2.9	1(yes),2(no)
Washing machine	0.96	0.88	0.97	0.92	3.6	0(don't have),1(have)
Satisf. with social life	4.50	4.87	4.97	4.87	0.5	1(not at all)~7(completely)
Voluntary work	4.56	4.56	4.50	4.62	1.1	1(once a week)~5(never)
# of people	78	424	2,563	78		

Table A18: Various normality tests for simulations

Normality	TF measure		TFR measure		IFR measure	
test	statistic	*p-value*	statistic	*p-value*	statistic	*p-value*
Shapiro-Wilks	0.9995	0.1744	0.9995	0.1189	0.9995	0.2437
Anderson-Darling	0.2881	0.6182	0.5017	0.2067	0.6190	0.1071
Cramer von Mises	0.0332	0.8010	0.0659	0.3160	0.1122	0.0766
Kolmogorov-Smirnov	0.0080	0.6068	0.0093	0.3625	0.0116	0.1058

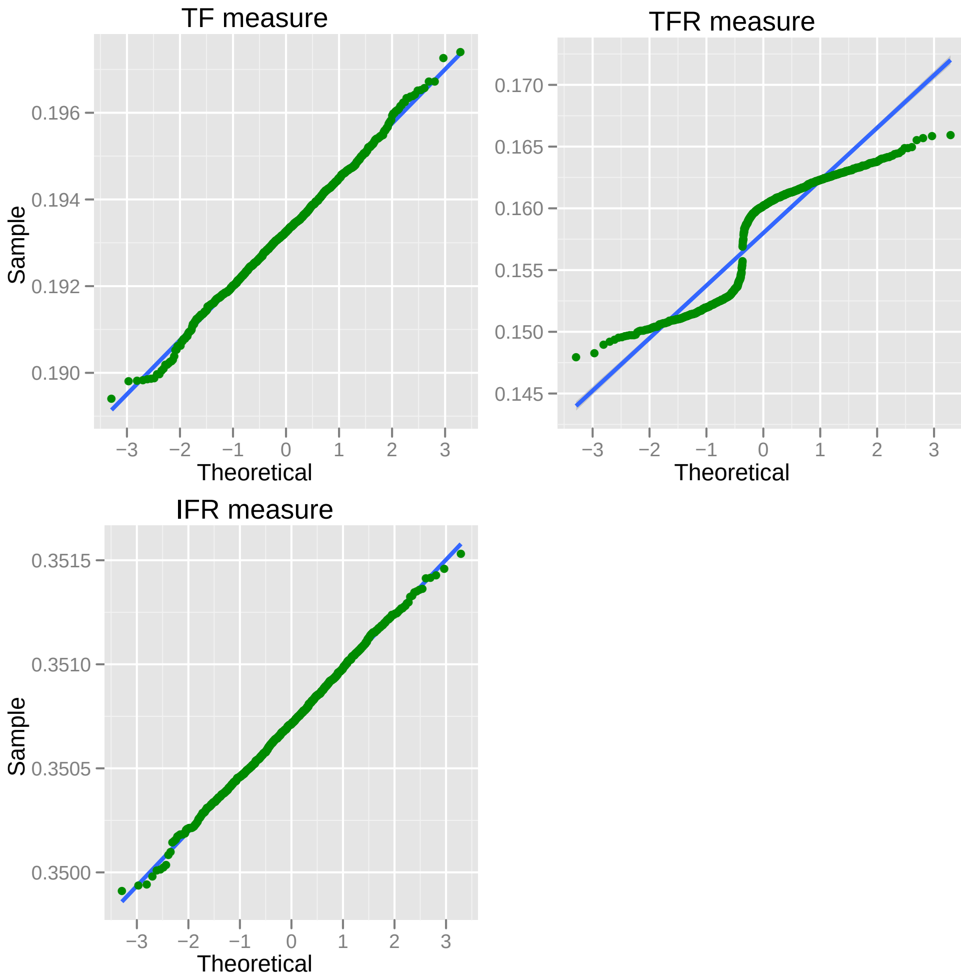


Figure A8: Q-Q plots for simulated fuzzy measures

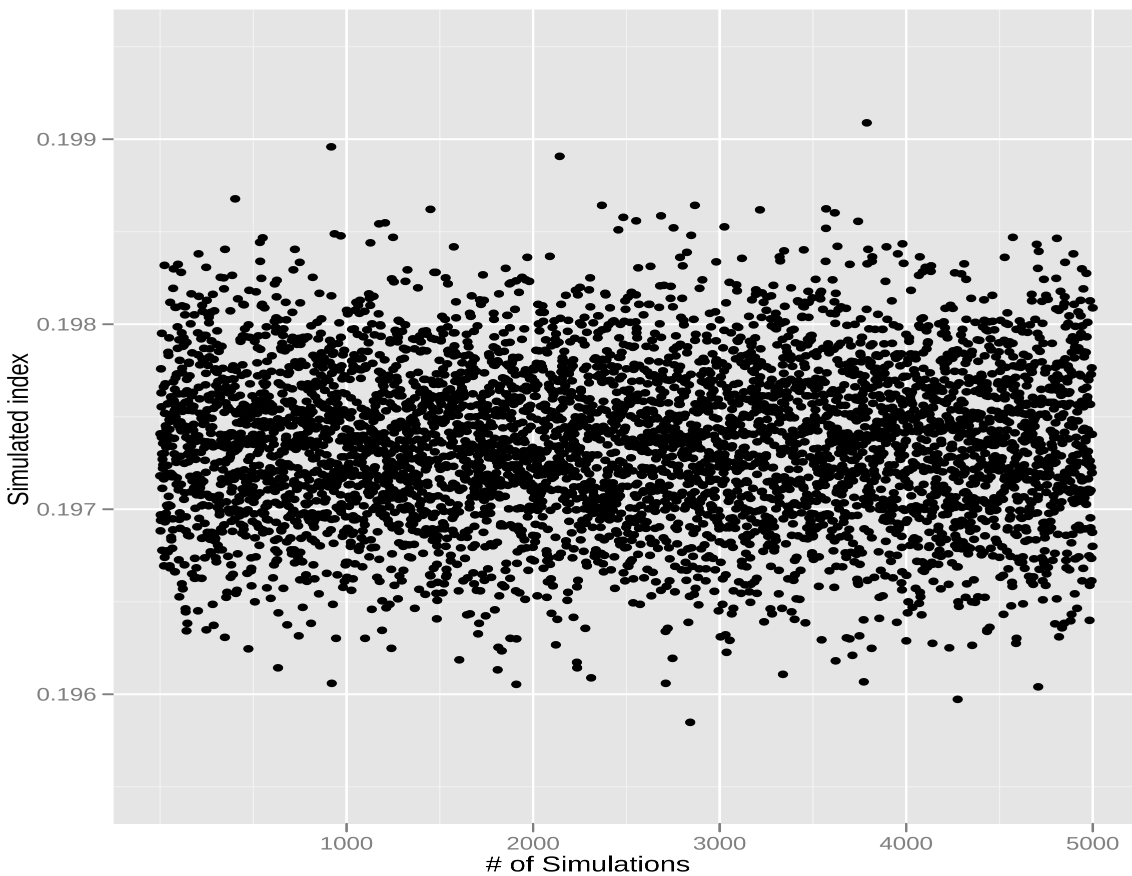


Figure A9: Distribution of the simulated TF index, change in saving criteria

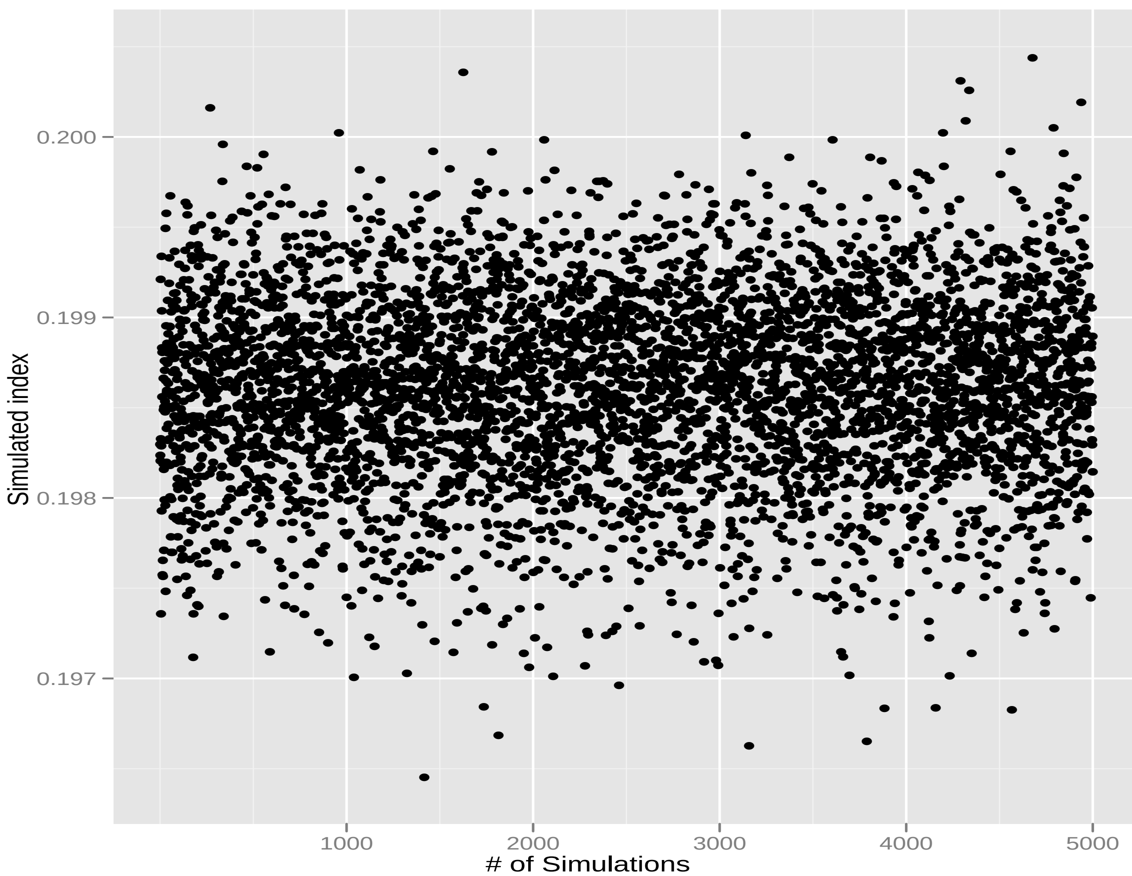
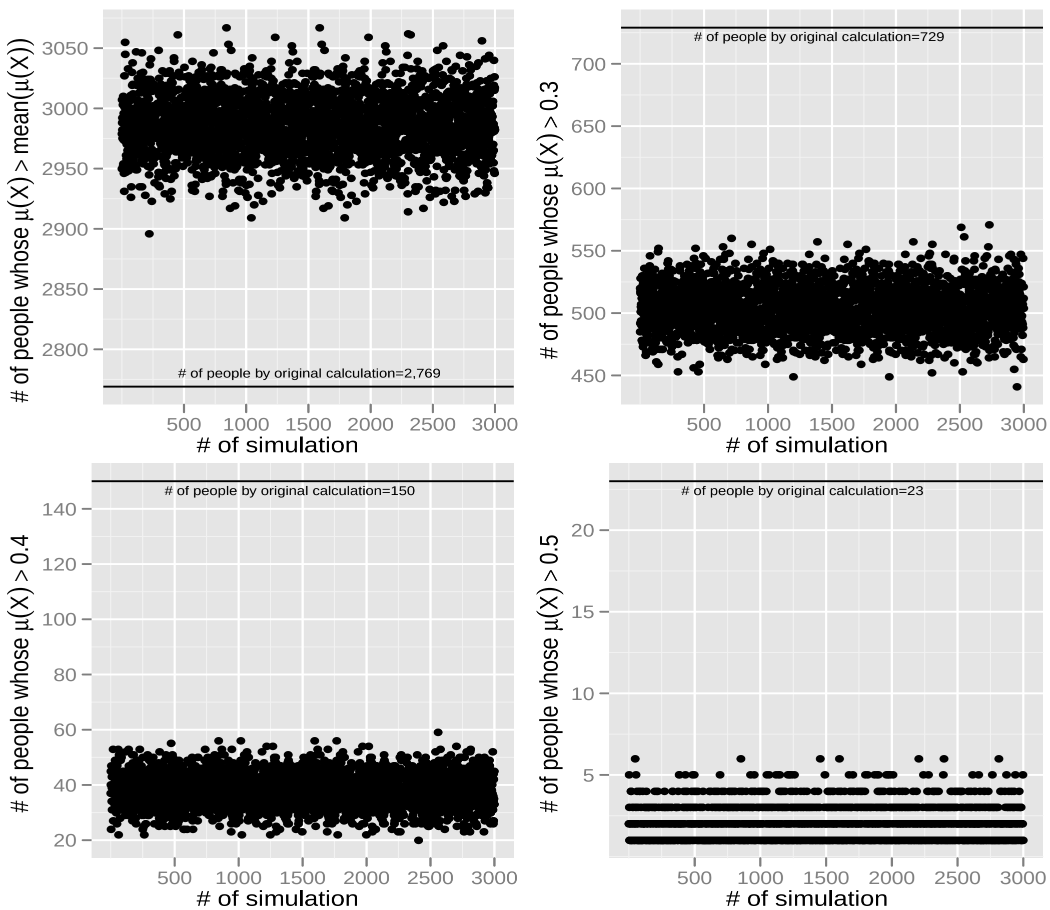


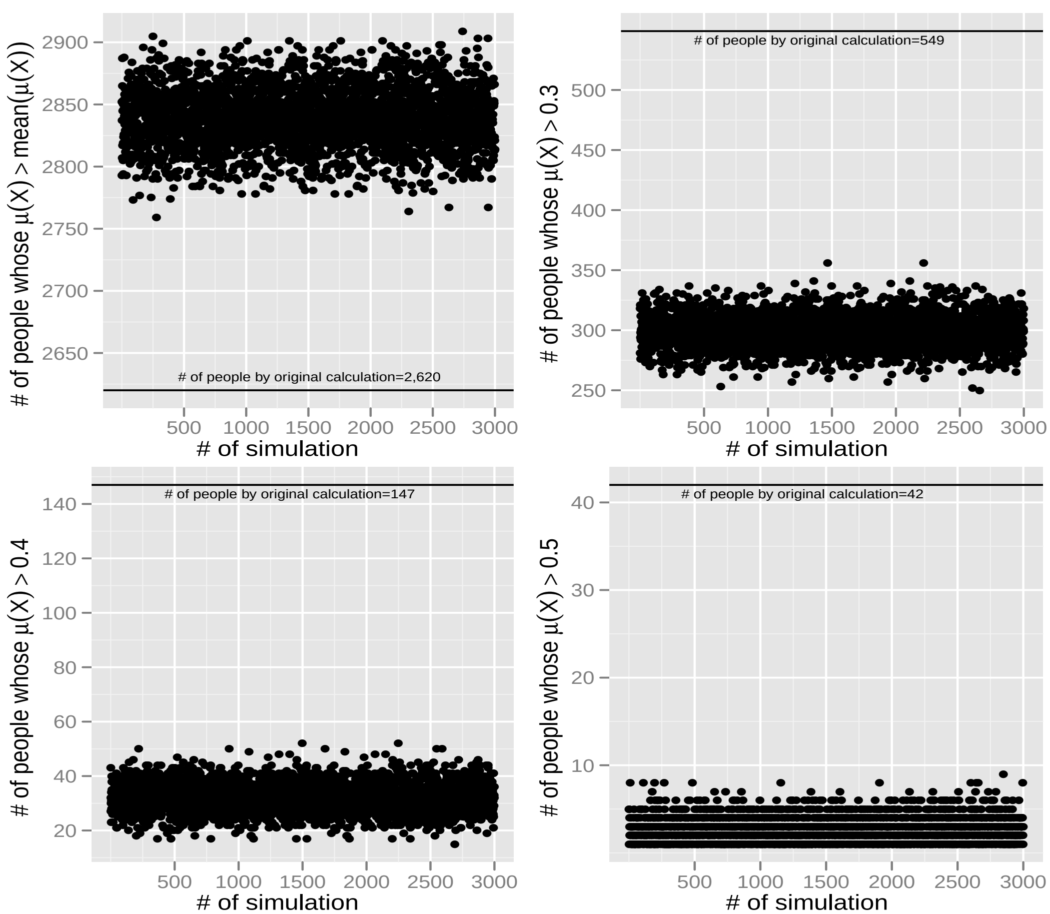
Figure A10: Distribution of the simulated TF index, change in both variables

A.1 SIMULATION RESULTS FOR IDENTIFICATION PERFORMANCE

TF measure

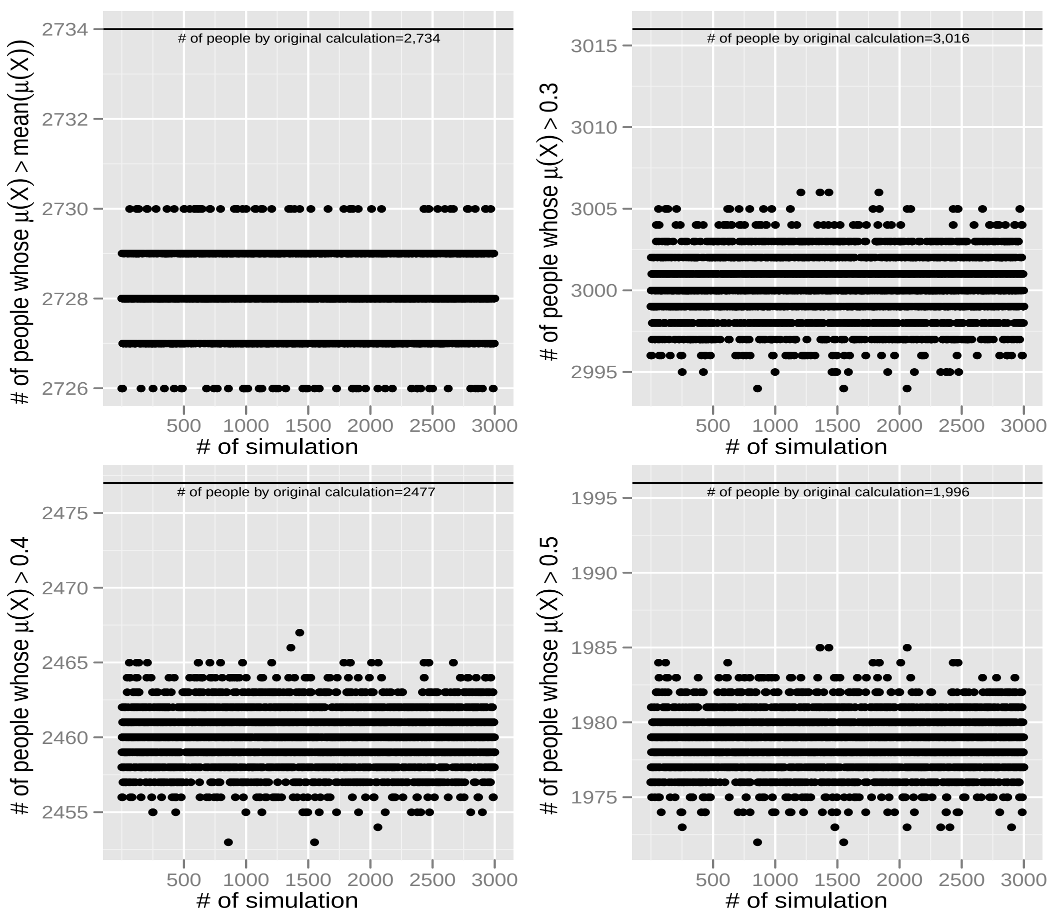


TFR measure



# of people whose µ(X) > mean(µ(X))
# of people by original calculation=2,620
# of simulation
# of people whose µ(X) > 0.3
# of people by original calculation=549
# of simulation
# of people whose µ(X) > 0.4
# of people by original calculation=147
# of simulation
# of people whose µ(X) > 0.5
# of people by original calculation=42
# of simulation

IFR measure



# of people by original calculation=2,734
# of people whose µ(X) > mean(µ(X))
# of simulation
# of people by original calculation=3,016
# of people whose µ(X) > 0.3
# of simulation
# of people by original calculation=2477
# of people whose µ(X) > 0.4
# of simulation
# of people by original calculation=1,996
# of people whose µ(X) > 0.5
# of simulation

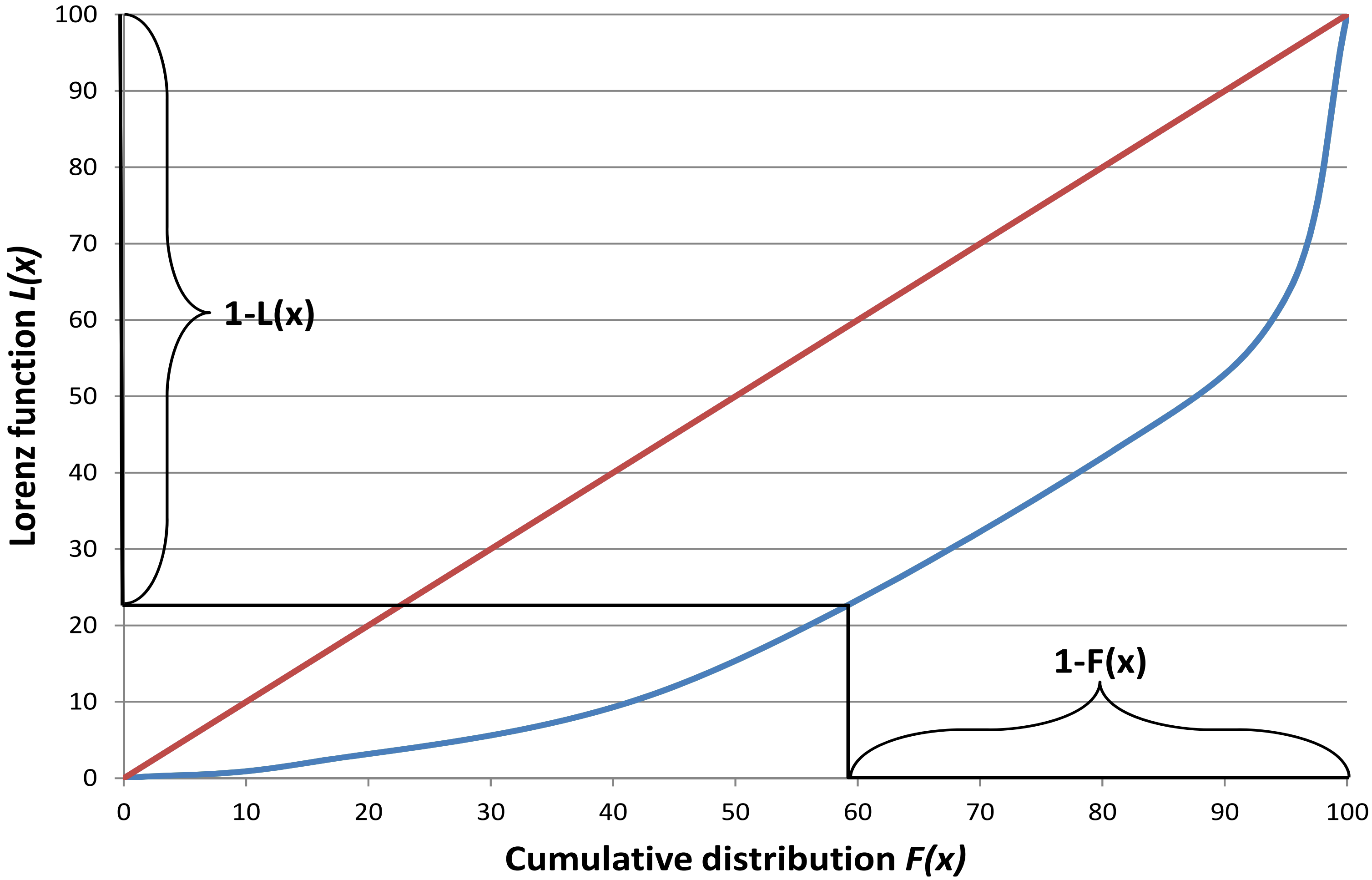


Figure A11: The membership function in IFR method (source: Betti, Cheli, Lemmi, & Verma (2006))

Appendix B

Bourguignon & Charkravarty (2003)'s axiomatic multidimensional index

As a non-additive functional form for multidimensional extension of FGT measure, Bourguignon & Chakravarty (2003) suggest following index for two dimensions case:

$$P_\alpha^\theta(X;z) = \frac{1}{n}\sum_{i=1}^{n}\left[a_1\left[\text{Max}\left(1-\frac{x_{i1}}{z_1},0\right)\right]^\theta + a_2\left[\text{Max}\left(1-\frac{x_{i2}}{z_2},0\right)\right]^\theta\right]^{\frac{\alpha}{\theta}} \tag{B.1}$$

where n equals to population size, a respective weights, z poverty thresholds, α a non-negative parameter for the aversion to poverty, and $\theta > 1$ a parameter for the elasticity of substitution between the dimensions.

If parameter θ is bigger than α - let's say θ is infinity and α is one, which is the extreme case that the contribution of each dimension to aggregate index is respectively infinite, the index goes to simple sum of poverty gap in two dimensions as follows:

$$\begin{aligned} P_\alpha^\theta(X;z) &= \frac{1}{n}\sum_{i=1}^{n}\left[\text{Max}\left[\text{Max}\left(1-\frac{x_{i1}}{z_1},0\right)\right] + \left[\text{Max}\left(1-\frac{x_{i2}}{z_2},0\right)\right]\right] \\ &= \frac{1}{n}\sum_{i=1}^{2}\sum_{i\in I_j}\left(1-\frac{x_{ij}}{z_j}\right) \end{aligned} \tag{B.2}$$

where I_j is an indicator function for poor people in dimension j. The final result B.2 implies that poverty gaps in both dimensions matter in the same way for aggregate index. Therefore, given distribution of each dimension, the only way to reduce poverty is to increase correlation between two dimensions.

On the contrary, if θ is smaller than α, and very close to 1, which means that one dimension can perfectly compensate the lack of the other dimension, the equation B.1 becomes:

$$P_\alpha^\theta(X;z) = \frac{1}{n}\sum_{i=1}^{n}\left[a_1\text{Max}\left(1-\frac{x_{i1}}{z_1},0\right) + a_2\text{Max}\left(1-\frac{x_{i2}}{z_2},0\right)\right]^\alpha \tag{B.3}$$

The equation B.3 shows that the contribution of one dimension to the whole index can be compensated by that of the other dimension, depending on the weights. If this is the case, increasing correlation between two dimensions cannot reduce poverty.

Appendix C

Choice of random number generation method

In following tables, *M.N.* indicates numbers truncated after obtained from an untruncated multivariate normal distribution, *T.N.* means numbers from a truncated multivariate normal distribution, and *Real* is the numbers calculated from the 16th wave of BHPS data. Covariance is a covariance vector with income variable (V1 in the table). For detailed information on the variables, see chapter 3.2

Table C1: Moments comparison - TF measure case

	Mean			Covariance		
	M.N.	T.N.	Real	M.N.	T.N.	Real
V1	0.4970	0.5037	0.5130	0.1282	0.1209	0.1580
V2	0.7096	0.7078	0.7206	0.0386	0.0130	0.0556
V3	0.9872	0.9870	0.9863	0.0006	0.0001	0.0023
V4	0.2728	0.3073	0.2684	0.0258	0.0135	0.0320
V5	0.3101	0.3448	0.3038	0.0181	0.0031	0.0223
V6	0.3795	0.4424	0.3680	0.0123	0.0004	0.0152
V7	0.3523	0.3677	0.3532	0.0236	0.0015	0.0307
V8	0.4523	0.4523	0.4520	0.0629	0.0191	0.0896
V9	0.6066	0.6505	0.5619	0.0514	0.0115	0.0714
V10	0.6158	0.6763	0.5712	0.0509	0.0107	0.0698
V11	0.0308	0.0308	0.0308	0.0020	0.0005	0.0058
V12	0.0317	0.0317	0.0317	0.0007	0.0001	0.0021
V13	0.1873	0.1873	0.1873	0.0055	0.0027	0.0093
V14	0.1079	0.1079	0.1079	0.0040	0.0010	0.0079
V15	0.1429	0.1429	0.1429	0.0038	0.0004	0.0069
V16	0.0459	0.0459	0.0459	0.0020	0.0006	0.0053
V17	0.0819	0.0819	0.0819	0.0048	0.0015	0.0103
V18	0.0601	0.0601	0.0601	0.0026	0.0005	0.0062
V19	0.0390	0.0390	0.0391	0.0018	0.0006	0.0048
V20	0.0103	0.0103	0.0103	0.0003	-0.0000	0.0014
V21	0.0598	0.0598	0.0598	0.0051	0.0001	0.0120
V22	0.0499	0.0499	0.0499	0.0018	-0.0004	0.0043
V23	0.0461	0.0461	0.0461	0.0039	0.0007	0.0099
V24	0.6124	0.6124	0.6124	0.0371	0.0080	0.0549
V25	0.0831	0.0831	0.0831	0.0023	-0.0006	0.0050
V26	0.3204	0.3204	0.3204	0.0385	0.0050	0.0586
V27	0.1731	0.1731	0.1731	0.0186	0.0022	0.0322
V28	0.0953	0.0953	0.0953	0.0086	0.0022	0.0174
V29	0.1249	0.1249	0.1249	0.0147	0.0020	0.0279
V30	0.4136	0.4136	0.4136	0.0458	0.0081	0.0671
V31	0.6125	0.6596	0.6112	0.0388	0.0119	0.0462
V32	0.1813	0.1813	0.1813	0.0082	0.0011	0.0140
V33	0.2229	0.2635	0.2177	-0.0085	-0.0027	-0.0112
V34	0.1754	0.2111	0.1792	-0.0067	-0.0030	-0.0090
V35	0.3662	0.4255	0.3598	0.0079	0.0026	0.0096
V36	0.2817	0.3130	0.2784	-0.0098	-0.0039	-0.0123
V37	0.8420	0.8279	0.8314	0.0031	0.0021	0.0051
V38	0.8780	0.8704	0.8742	0.0015	0.0003	0.0026
V39	0.5755	0.5842	0.5771	0.0582	0.0171	0.0754

Table C2: Moments comparison - TFR measure case

	Mean			Covariance		
	M.N.	T.N.	Real	M.N.	T.N.	Real
V1	0.4999	0.5346	0.4999	0.0715	0.0440	0.0833
V2	0.4999	0.5278	0.4999	0.0491	0.0227	0.0578
V3	0.4999	0.5232	0.4999	0.0481	0.0219	0.0567
V4	0.4510	0.4510	0.4899	0.0301	0.0122	0.0359
V5	0.5042	0.4921	0.5430	0.0183	0.0026	0.0210
V6	0.5243	0.5278	0.5372	0.0091	-0.0005	0.0107
V7	0.3728	0.3506	0.3983	0.0178	0.0005	0.0228
V8	0.3692	0.3195	0.4424	0.0439	0.0111	0.0654
V9	0.5648	0.5729	0.5865	0.0365	0.0073	0.0489
V10	0.6045	0.6263	0.6254	0.0330	0.0054	0.0434
V11	0.0308	0.0308	0.0308	0.0016	0.0004	0.0043
V12	0.0317	0.0317	0.0317	0.0005	0.0001	0.0014
V13	0.1873	0.1873	0.1873	0.0044	0.0018	0.0070
V14	0.1079	0.1079	0.1079	0.0034	0.0008	0.0063
V15	0.1429	0.1429	0.1429	0.0030	0.0001	0.0051
V16	0.0459	0.0459	0.0459	0.0015	0.0003	0.0036
V17	0.0819	0.0819	0.0819	0.0038	0.0011	0.0075
V18	0.0601	0.0601	0.0601	0.0020	0.0003	0.0044
V19	0.0390	0.0390	0.0391	0.0014	0.0004	0.0036
V20	0.0103	0.0103	0.0103	0.0003	0.0000	0.0012
V21	0.0598	0.0598	0.0598	0.0042	0.0002	0.0092
V22	0.0499	0.0499	0.0499	0.0017	-0.0002	0.0038
V23	0.0461	0.0461	0.0461	0.0033	0.0006	0.0078
V24	0.6124	0.6124	0.6124	0.0293	0.0057	0.0404
V25	0.0831	0.0831	0.0831	0.0021	-0.0002	0.0042
V26	0.3204	0.3204	0.3204	0.0295	0.0032	0.0417
V27	0.1731	0.1731	0.1731	0.0148	0.0018	0.0236
V28	0.0953	0.0953	0.0953	0.0074	0.0018	0.0141
V29	0.1249	0.1249	0.1249	0.0114	0.0015	0.0199
V30	0.4136	0.4136	0.4136	0.0350	0.0054	0.0479
V31	0.6231	0.6468	0.6425	0.0293	0.0074	0.0360
V32	0.1813	0.1813	0.1813	0.0063	0.0008	0.0101
V33	0.3809	0.3758	0.4233	-0.0144	-0.0032	-0.0179
V34	0.3894	0.3770	0.4438	-0.0119	-0.0028	-0.0153
V35	0.5249	0.5209	0.5384	0.0053	0.0011	0.0063
V36	0.4073	0.3925	0.4554	-0.0111	-0.0029	-0.0137
V37	0.7042	0.5838	0.7736	0.0045	0.0014	0.0069
V38	0.7440	0.6282	0.8300	0.0031	0.0003	0.0046
V39	0.5027	0.4824	0.5448	0.0429	0.0114	0.0572

Table C3: Moments comparison - IFR measure case

	Mean			Covariance		
	M.N.	T.N.	Real	M.N.	T.N.	Real
V1	0.5103	0.5280	0.5130	0.1053	0.0088	0.1580
V2	0.6651	0.6131	0.7206	0.0313	0.0070	0.0556
V3	0.9490	0.9065	0.9863	0.0010	0.0004	0.0023
V4	0.2728	0.3073	0.2684	0.0239	0.0078	0.0320
V5	0.3101	0.3447	0.3038	0.0168	0.0116	0.0223
V6	0.3796	0.4424	0.3680	0.0114	0.0113	0.0152
V7	0.3524	0.3677	0.3532	0.0220	0.0184	0.0307
V8	0.4523	0.4523	0.4520	0.0582	0.0381	0.0896
V9	0.6065	0.6504	0.5619	0.0477	0.0267	0.0714
V10	0.6157	0.6763	0.5712	0.0472	0.0241	0.0698
V11	0.0308	0.0308	0.0308	0.0018	0.0025	0.0058
V12	0.0317	0.0317	0.0317	0.0007	0.0025	0.0021
V13	0.1873	0.1873	0.1873	0.0050	0.0121	0.0093
V14	0.1079	0.1079	0.1079	0.0037	0.0007	0.0079
V15	0.1429	0.1429	0.1429	0.0035	0.0008	0.0069
V16	0.0459	0.0459	0.0459	0.0019	0.0004	0.0053
V17	0.0819	0.0819	0.0819	0.0045	0.0009	0.0103
V18	0.0601	0.0601	0.0601	0.0025	0.0007	0.0062
V19	0.0390	0.0390	0.0391	0.0016	0.0003	0.0048
V20	0.0103	0.0103	0.0103	0.0003	-0.0000	0.0014
V21	0.0598	0.0598	0.0598	0.0048	0.0003	0.0120
V22	0.0499	0.0499	0.0499	0.0016	-0.0001	0.0043
V23	0.0461	0.0461	0.0461	0.0036	0.0001	0.0099
V24	0.6124	0.6124	0.6124	0.0345	0.0015	0.0549
V25	0.0831	0.0831	0.0831	0.0022	-0.0001	0.0050
V26	0.3204	0.3204	0.3204	0.0357	0.0019	0.0586
V27	0.1731	0.1731	0.1731	0.0172	0.0014	0.0322
V28	0.0953	0.0953	0.0953	0.0079	-0.0000	0.0174
V29	0.1249	0.1249	0.1249	0.0138	0.0010	0.0279
V30	0.4136	0.4136	0.4136	0.0424	0.0027	0.0671
V31	0.6124	0.6595	0.6112	0.0360	0.0032	0.0462
V32	0.1813	0.1813	0.1813	0.0075	0.0009	0.0140
V33	0.2230	0.2635	0.2177	-0.0079	-0.0011	-0.0112
V34	0.1754	0.2111	0.1792	-0.0061	-0.0001	-0.0090
V35	0.3662	0.4254	0.3598	0.0073	0.0044	0.0096
V36	0.2817	0.3129	0.2784	-0.0090	0.0001	-0.0123
V37	0.8420	0.8278	0.8314	0.0028	-0.0003	0.0051
V38	0.8780	0.8704	0.8742	0.0014	-0.0002	0.0026
V39	0.5755	0.5843	0.5771	0.0540	0.0208	0.0754

Appendix D

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